Searching for a Metric for Financial Stability
By O. Aspachs, C. Goodhart, M. Segoviano, D. Tsomocos and L. Zicchino

1. The Problem

In the ECB Financial Stability Review (December, 2005, p. 131), it is stated bluntly that “there is no obvious framework for summarising developments in financial stability in a single quantitative manner.” This is, to say the least, a considerable disadvantage when attempting to analyse financial stability issues. As the same ECB Special Feature on ‘Measurement Challenges in Assessing Financial Stability’, (ibid)¹ put it, “Financial stability assessment as currently practiced by central banks and international organisations probably compares with the way monetary policy assessment was practised by central banks three or four decades ago – before there was a widely accepted, rigorous framework.”

But how can you have a rigorous framework if one cannot even measure financial stability? Even five, or six, decades ago there was a comparative plenitude of data relating to monetary policy, e.g. monetary aggregates, interest rates, inflation. By contrast, there is still no good way to provide quantitative comparisons of financial stability across countries, or over time in a single country.

Economics is a quantitative social science. Without an ability to make numerical comparisons, it becomes hard to undertake rigorous analysis. This is one of the

¹ Also see Fell and Schinasi (2005).
reasons why the growing multitude of Financial Stability Reviews are mostly
descriptive and backwards-looking in character, whereas the Inflation Reports are
more analytical and forwards-looking. While Central Banks have two main
responsibilities, to provide price stability and also systemic financial stability,
achieving the first is technically far easier than for the second objective. As the Table
below shows, when the pursuit of price stability is contrasted with that of systemic
financial stability, the former is generally far easier to undertake.

Table 1: Contrasts between Price and Financial Stability

<table>
<thead>
<tr>
<th></th>
<th>Price Stability</th>
<th>Financial Stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>Measurement and Definition</td>
<td>Yes, subject to technical queries</td>
</tr>
<tr>
<td>b)</td>
<td>Instrument for control</td>
<td>Yes, subject to lags</td>
</tr>
<tr>
<td>c)</td>
<td>Accountable</td>
<td>Yes</td>
</tr>
<tr>
<td>d)</td>
<td>Forecasting Structure</td>
<td>Central tendency of distribution</td>
</tr>
<tr>
<td>e)</td>
<td>Forecasting Procedure</td>
<td>Standard Forecasts</td>
</tr>
<tr>
<td>f)</td>
<td>Administrative Procedure</td>
<td>Simple</td>
</tr>
</tbody>
</table>

Our colleague, Prof. E.P. Davis, has run an internet chat-room on financial regulation.
In 2004, he ran a competition to find the best definition of financial stability. While
there were many entries, the one that seemed to garner most support was, as noted in
the top right hand cell of Table 1, ‘the absence of financial instability’.
Indeed a major bank crisis is all too obvious to those involved. So much of the analytical work on financial stability has, at least until recently, been based on the identification of bank crises, and has then analysed common antecedent factors, in the attempt to examine what factors may cause instability. A number of examples of this approach are Kaminsky and Reinhart (1996, 1999), Logan (2000), Berg (1999), Disyatat (2001), Dermirguc-Kunt and Detragiache (1998) and Vila (2000). Illing and Liu (2003) provide a critique and a literature review of related papers, especially those aiming to discover common ‘Early Warning Indicators’ of assessed crisis events, see their Appendix A, pp 49-50.

While this approach, studying the event of banking crisis, has achieved considerable success, it has several limitations. The dividing line between a crisis event, and non-crisis, is bound to be fuzzy; the dating both of its onset and, even more so, of its ending will be uncertain; the intensity of crises varies. Any reduced form relationship between a banking crisis and prior conditions is likely to be subject to the Lucas critique, in that the identification of prior regularities will change the behaviour of actors in the system, such as regulators, depositors, etc. Finally, most of the time financial systems are not in crisis mode, so focussing only on crisis events is tantamount to throwing most of the observations away.

So, one of our main objectives, in a wider programme of work on financial stability, has been to refute the opening quote, and to find a metric for measuring financial stability. Two papers that have followed this same route are Hanschel and Monnin (2004), and Illing and Liu (2003). In both cases the variables in their financial stress index are not derived from any structural model, and their exercises are limited to
single countries (Switzerland and Canada respectively). We obtain our key variables from a general equilibrium model, and apply our results to seven countries.

2. Modelling Financial Stability

If everyone always fully paid their debts, with certainty, there would be no credit risk, probably no money (since everyone’s IOUs could be used in trade) and no need for financial intermediaries, (Goodhart, 2005). So the existence of credit risk, i.e. that a borrower may not repay in full, is central to the analysis of money, financial intermediation and financial (in)stability, (Goodhart, Tsomocos, Zicchino and Aspachs, 2006).

“Indeed, the probability of default (PD) is a key concept in any analysis of financial fragility. It is, of course, central to the Basel II exercise. At the more formal level, modelling of default, (following on from the approach pioneered by Martin Shubik and his co-authors), is the crucial element for the analysis of financial fragility that we have been developing, see Tsomocos (2003a and b), Goodhart, Tsomocos and Sunirand (2004, 2005, 2006 a, 2006 b), Tsomocos and Zicchino (2005), and Goodhart and Zicchino (2005).”

We began our current program of work with ‘A model to analyse financial fragility’, (Goodhart, Sunirand and Tsomocos, 2006), based on earlier work by Tsomocos (2003a and b). This is a micro-founded general equilibrium model with endogenous default and heterogeneous agents.²

² While the representative agent approach has many uses and advantages, applying it to the banking system inevitably obscures many of the economic and behavioural relationships, notably between banks, in which a regulatory authority is closely interested. For example, with a single ‘representative’ bank, there can be no interbank market. Again, either the whole banking system, as represented by the one agent, fails, or the whole banking system survives in the face of some assumed shock. Typically in reality individual banks have differing portfolios, often reflecting differing risk/return preferences. So, typically, failures occur with the greatest probability amongst the riskiest banks. Such failures in turn generate interactions in the system more widely that may threaten the survival of other banks, a process of contagion. This may have several channels, both in interbank relationships more directly, and via changes in asset market flows and prices that may involve other sectors, e.g. persons and companies. Such interactions can hardly be studied in a model with a single representative bank, since many of these interactions, e.g. the interbank market, are ruled out by definition. (Ibid, p. 108)
The model incorporates heterogeneous banks and capital requirements in a general equilibrium model with incomplete markets, money and default. It extends over two periods and all uncertainty is resolved in the second period. Trade takes place in both periods in the goods and equity markets. In the first period agents also borrow from, or deposit money with banks, mainly to achieve a preferred time path for consumption. Banks also trade amongst themselves, to smooth out their individual portfolio positions. The Central Bank intervenes in the interbank market to change the money supply and thereby set the interest rate. Capital adequacy requirements (CARs) on banks are set by a regulator, who may, or may not, also be the Central Bank. Penalties on violations of CARs, and on the default of any borrower, are in force in both periods. In order to achieve formal completeness for the model, banks are liquidated at the end of the second period and their profits and assets distributed to shareholders. The chart below makes the time line of the model explicit.
In the first period trades by all agents take place against a background of uncertainty about the economic conditions (the state of nature) that will prevail in the second period. Agents are, however, assumed to have rational expectations, and to know the likelihood (the probability distribution) of good or bad states occurring when they make their choices in period one. In period two the actual economic conjuncture (the state of nature) is revealed and all uncertainty is resolved.

The model incorporates a number of distinct, i.e. heterogeneous, commercial banks, each characterised by a unique risk/return preference and different initial capital. Since each bank is, and is perceived as being, different, it follows that there is not a single market for either bank loans or bank deposits. In addition, we introduce limited access to consumer credit markets, with each household assigned (by history and custom) to borrow from a predetermined bank. This feature allows for different interest rates across the commercial banking sector. In sum, multiple credit and deposit markets lead to different loan rates among various banks and to endogenous credit spreads between loan and deposit rates.

Individual non-bank agents are also assumed to differ in their risk attitudes and hence in their preferences for default. We model the incentive for avoiding default by penalising agents and banks proportionately to the size of default. Banks that violate their capital adequacy constraint are also penalised in proportion to the shortfall of capital. Both banks and households are allowed to default on their financial obligations, but not on commodity deliveries.
As the Abstract to this paper (Goodhart, Sunirand and Tsomocos, 2006a) noted:-

“This paper sets out a tractable model which illuminates problems relating to individual bank behaviour, to possible contagious inter-relationships between banks, and to the appropriate design of prudential requirements and incentives to limit ‘excessive’ risk-taking. Our model is rich enough to include heterogeneous agents, endogenous default, and multiple commodity, and credit and deposit markets. Yet, it is simple enough to be effectively computable and can therefore be used as a practical framework to analyse financial fragility. Financial fragility in our model emerges naturally as an equilibrium phenomenon. Among other results, a non-trivial quantity theory of money is derived, liquidity and default premia co-determine interest rates, and both regulatory and monetary policies have non-neutral effects. The model also indicates how monetary policy may affect financial fragility, thus highlighting the trade-off between financial stability and economic efficiency.”

As noted above, one of the purposes of the exercise was to design a framework which could be used in practice for policy purposes, in any country, to assess and to estimate whether the banking system would be robust in the face of adverse shocks. So we followed up our theoretical modelling first with a numerical simulation, (Goodhart, Sunirand and Tsomocos, 2004), and subsequently by calibrating the model to values for the UK banking system, (Goodhart, Sunirand and Tsomocos, 2004 and 2006b). Subsequently independent applications of this same exercise have been done for Colombia, Greece and Norway [References still to come].

The calibration/simulation procedure was set out in Goodhart, Sunirand and Tsomocos (2005 and 2006b).3 Having chosen the initial values of the relevant

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3 As noted in GST (2006b, p. 14):-

“Our calibration procedure follows directly that presented in Goodhart, Sunirand and Tsomocos (2004b). In each period $t$, excluding the Lagrange multipliers, [the four] conditions in the previous section imply that we have a system of 56 equations in 143 unknown variables, 87 of which are exogenous variables/parameters in the model. This implies that there are 87 variables whose values have to be chosen in order to obtain a numerical solution to the model. Thus, they represent the degrees of freedom in the system and can either be set appropriately or calibrated against the real data. In particular, we choose the values of these variables such
variables, by calibration or exogenously, we can then reach an initial equilibrium value for all the endogenous variables, including interest rates, bank profitability, monetary aggregates and repayment rates, the latter being directly inversely related to the probability of default. The next stage is to shock the initial equilibrium, for example to simulate a recession and then see how the banking system responds. An example is given in Table 2, taken from GTZA, when the central bank cuts the stock of base money by 10%.

Table 2: % change in key variables given a negative 10% shock to \(M\) at \(t=1\)

<table>
<thead>
<tr>
<th>Bank</th>
<th>(r^b)</th>
<th>(r^b)</th>
<th>(\rho)</th>
<th>(\pi^b)</th>
<th>(\pi^b)</th>
<th>(e^b)</th>
<th>(e^b)</th>
<th>(k^b)</th>
<th>(k^b)</th>
<th>(\nu^b)</th>
<th>(\nu^b)</th>
<th>(GDP^i)</th>
<th>(GDP^{ii})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank</td>
<td>(\delta)</td>
<td>2.8</td>
<td>3</td>
<td>2.4</td>
<td>0.06</td>
<td>0.1</td>
<td>0.005</td>
<td>0.003</td>
<td>0.35</td>
<td>0.38</td>
<td>-0.02</td>
<td>-0.04</td>
<td>-0.18</td>
</tr>
<tr>
<td>Bank</td>
<td>(\gamma)</td>
<td>2.4</td>
<td>3.3</td>
<td>2.4</td>
<td>0.12</td>
<td>0.3</td>
<td>0.03</td>
<td>0.08</td>
<td>0.5</td>
<td>0.6</td>
<td>0.005</td>
<td>-0.04</td>
<td>-0.18</td>
</tr>
<tr>
<td>Bank</td>
<td>(\tau)</td>
<td>2.4</td>
<td>3.3</td>
<td>2.4</td>
<td>0.02</td>
<td>0.03</td>
<td>0.005</td>
<td>0.005</td>
<td>0.47</td>
<td>0.5</td>
<td>0.003</td>
<td>-0.01</td>
<td>-0.18</td>
</tr>
</tbody>
</table>

Legend:
\(r^b\) = lending rate offered by bank \(b \in B = \{\delta, \gamma, \tau\}\),
\(r^b\) = deposit rate offered by bank \(b\),
\(\rho\) = interbank rate,
\(\pi^b\) = profits of bank \(b\) in state of the world \(s = \{i, ii\}\),
\(e^b\) = capital held by bank \(b\) in state \(s\),
\(k^b_s\) = repayment rate of bank \(b\) to all its creditors in state \(s\),
\(\nu^b_s\) = repayment rate of bank \(b\) to all its creditors in state \(s\),
\(GDP_s\) = GDP in state \(s\).

that they capture realistic features of the UK banking sector in 1997. It is important to note that these variables, which are exogenous when solving the system of equations, do not necessarily have to be those which are exogenous in the model. We report the values of exogenous parameters/variables in the model and the resulting initial equilibrium in table I. The table also summarises whether the value of each variable reported is (1) calibrated against real data, (2) arbitrarily selected, or (3) endogenously solved.”
The point to note here is not so much the details of the table, though a brief explanation of what is happening is given in the footnote below, but that the crucial aspects of the impact of shocks on the banking system are contained in two variables, bank profitability and bank repayment rate, which in turn is equivalent to its probability of default (PD), see Tsomocos and Zicchino (2005).

In normal economic cycles bank repayment rates (PDs) will be positively (negatively) correlated with profitability. When economic conditions improve, bank profitability rises and PDs fall. However there can be shifts in preferences, or in conditions, that change the risk aversion of banks. In such cases the previous normal relationship can be reversed. One such example, taken from GTZA, pp 24-25, involves an increase in the penalties imposed on banks should they default. The outcome is shown in Table 3 below.

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4 As can be seen from the table, the interbank rate increases by 2.4 percent (from 4.41% to 4.52%). Given a higher rate of return on interbank loans, other things equal, bank δ invests more in this market. To do so, it seeks more funds from the deposit markets and it cuts on lending to its customer, β (banks' loans are not shown in Table 2). This portfolio adjustment causes a 2.8% increase in δ's deposit rate and a 3% increase in its lending rate (from 3.88% to 3.99% and from 7.45% to 7.68%, respectively). Banks γ and τ, who are net borrowers in the interbank market, respond to a higher ρ by reducing their interbank borrowing, by increasing their demand for deposits, and by reducing loan supply to their customers, α and θ. This, in turn, causes the deposit and lending rates of these to banks to increase.

All banks anticipate that a lower credit availability will cause a higher rate of households' default as the decrease in liquidity affects next-period income negatively (GDP decreases in both states of the world). Households' repayment rates decrease by approximately 0.1% in the good state of the world and by 0.14 percent in the bad state (not shown in Table 2). Thus, the expected rate of return on loans decreases for all banks and their willingness to supply credit decreases even further. The lower rate of return on household loans and the higher cost of funds have a negative effect on the profits of banks γ and τ while bank δ benefits from the higher return on interbank market investments. However, since banks are subject to a capital requirement and therefore need to increase their profits to accumulate capital, they choose to increase their default rates (the default rates increase when the repayment rates \(v^b\) decrease). Put in another way, banks adopt riskier strategies to counteract the negative effect on profits of the liquidity contraction.
This table reports the consequences of the regulator increasing the penalties on banks who default on their debt (to depositors and other banks). We assume a 2 percent increase in both states of the world. (from 0.9 to approximately 0.92 in the state 1 and from 1.1 to approximately 1.12 in state 2). Since defaulting is now more costly, banks increase their repayment rates (the percentage changes of $\nu_{b_1}$ and $\nu_{b_2}$ are positive for all banks, as shown in Table 3). Banks' more prudent investment choices induce a decline in profits (and, therefore, in capital and capital to risk-weighted asset ratios). Because banks $\gamma$ and $\tau$ increase their repayment rates to all creditors considerably, bank $\delta$ is willing to invest more in the interbank market. As a result, the interbank rate $\rho$ decreases. Since $\gamma$ and $\tau$ are able to borrow more and at a lower cost from the interbank market, their demand of deposits decreases and so do their deposit rates. The overall level of aggregate credit to households decreases as a result of the negative households' wealth effect of lower bank equity values.

The point of all this, and the conclusion of this sub-section, is that the effects of shocks on the stability of the overall banking system can reasonably be represented by

<table>
<thead>
<tr>
<th>Bank</th>
<th>$\delta$</th>
<th>$\gamma$</th>
<th>$\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_{b_1}$</td>
<td>-9.2</td>
<td>-18</td>
<td>-18</td>
</tr>
<tr>
<td>$r_{b_2}$</td>
<td>-3.3</td>
<td>-10</td>
<td>-10</td>
</tr>
<tr>
<td>$\pi_{b_1}$</td>
<td>-14.8</td>
<td>-13.2</td>
<td>-16.9</td>
</tr>
<tr>
<td>$\pi_{b_2}$</td>
<td>-27.6</td>
<td>-34.4</td>
<td>-23.3</td>
</tr>
<tr>
<td>$e_{b_1}$</td>
<td>-1.4</td>
<td>-2.8</td>
<td>-3.5</td>
</tr>
<tr>
<td>$e_{b_2}$</td>
<td>-0.9</td>
<td>-1.8</td>
<td>-3.5</td>
</tr>
<tr>
<td>$k_{b_1}$</td>
<td>-0.38</td>
<td>-8</td>
<td>-2.6</td>
</tr>
<tr>
<td>$k_{b_2}$</td>
<td>0.2</td>
<td>-8</td>
<td>-2.6</td>
</tr>
<tr>
<td>$\nu_{b_1}$</td>
<td>0.24</td>
<td>0.24</td>
<td>0.44</td>
</tr>
<tr>
<td>$\nu_{b_2}$</td>
<td>0.14</td>
<td>0.58</td>
<td>0.29</td>
</tr>
<tr>
<td>$GDP_i$</td>
<td>-0.9</td>
<td>-1.5</td>
<td></td>
</tr>
<tr>
<td>$GDP_{ii}$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 3: % change in key variables given a 2% increase in default penalties imposed on banks on both states of the world
a two factor model, where the two factors are profitability and PD.\textsuperscript{5} It would have been nice, in searching for a metric, to employ a one factor model, since that makes quantification easier. With a two factor model, quantification requires giving weights to each factor to arrive at a single number, or metric, and the choice of weights will be conditioned by the results of the particular empirical model employed.

Thus our search for a metric, in the shape of a weighted two factor model, is derived from our prior (micro-founded, general equilibrium) theoretical modelling. This is not ‘measurement without theory’, but measurement based on theory.

\textsuperscript{5} If the normally expected inverse correlation between profitability and PD had been very high (close to -1.00) in practice, there could have been a case for concentrating only on PD. However the simple correlation between our measures of banking profitability and PDs, described in the next Section, was only about -0.32 on a quarterly, and -0.61 on an annual, basis, being the average of the simple correlations in our seven countries. This is far too low to allow one to ignore the separate effect of bank profitability.
3. The Data

So, we sought variables that would give a good measure of banking profitability and default probabilities. Initially we tried variables taken from bank accounts, of profitability and of non-performing loans and/or write-offs, with the latter being treated as a possible measure of banks’ own PD. But these accounting data did not work well in our empirical exercises. We think that this comparative failure may have been because of:-

(1) shifts in accounting practices both over time and between countries;
(2) some continuing ability of bank management (and their auditors) to manipulate and smooth the published accounts;
(3) the relatively long delays between the current effect of events on banks and their appearance in the accounts; thus a rise in n.p.l.s and write-offs tends to follow bank crises by many quarters.

Anyhow, this led us to switch from accounting data to market data, which is not so affected by the above disadvantages. In particular we looked at equity market data. We took the % change in equity values of the banking sector as our index of the market’s perception of the change in the present value of returns to banks. Meanwhile the IMF has been calculating Distance to Default indicators for banking sectors for a number of countries going back to 1990. With the assistance of Miguel Segoviano we used their time series to obtain a time series for country PDs. The procedure for obtaining country banking sector PD series is taken from Goodhart, Hofmann and Segoviano (2005, pp 14/15) and reproduced as Appendix 1.
Estimates of banking sector PD are based on both equity values and their volatility, so this number would appear to incorporate in a single number most of the key elements for which we were searching. So why is PD not a sufficient statistic by itself? There are a number of reasons, mainly empirical, why we continued with our two factor approach. First, as already noted, the correlation between movements in equity valuations and in PDs is less strong than we had originally expected. Second, in our empirical findings movements in equity prices did have a significant separate effect on real output. Third, the volatility of bank equity is composed of the volatility of the overall market plus the volatility of the banking sector relative to the market. Our examination of the time series for bank PD leads us to worry that market volatility may have too high a weight (and relative bank volatility too low a weight) in current estimates of PD. But a re-estimate of PD based on a volatility decomposition remains for future work.

Initial exercises, relating PD and the % change in banking sector values (over the previous year), $\text{Eq}$ to % changes in real output revealed, however, that these two variables were both threshold variables. When PD and $\text{Eq}$ were sufficiently bad, (high PD, big fall in $\text{Eq}$), they brought about a decline in real growth; but once the values of these variables were good enough, further improvement had no effect on GDP. We estimated the threshold empirically by a grid search. This is, in effect, data mining. We believe that treating PD and $\text{Eq}$ as threshold variables with a non-linear effect on GDP makes sense, but, given the few countries and short data period in our sample, our particular form of non-linearity may well need to be revised when more observations become available.
Our data set included: Finland, Germany, Japan, Korea, Norway, Sweden and the UK over the period 1990 Q4 till 2004 Q4. Whereas values of PD were available throughout, bank equity values were only available from Norway and Finland from 1996 onwards, and for Sweden from 2000 onwards. In order to maximise available data, we assumed that equity values were better than the threshold (i.e. constant) up to the first available data point; this may bias the coefficient on PD (making it absolutely somewhat larger) for these three countries.

The Scandinavian countries, Japan and Korea all had spells of severe financial fragility during this period; there was a brief period of financial weakness in the UK in 1992/93, and in Germany in 2002. So the data set should provide a good template for this exercise. Charts of PD and Eq for each country, also indicating the threshold, are shown in Figures 1-14. Note the general increase in PD, (and in some cases decline in Eq), in 2002. This partly reflects the decrease in general equity values, and concomitant rise in equity market volatility, at the trough of the dot.com bust. In our view this somewhat exaggerates the fragility of banking sectors at this juncture (apart from Germany), which explains why we shall also want to look at decompositions of the PD variable.

Our tests sought to examine whether financial fragility, measured as described by threshold values for PD and Eq, would have an impact on economic welfare. We treated real output, GDP, as our index of social welfare. Given that GDP and financial fragility have a, possibly complex, simultaneous relationship, we reckoned that Vector Auto Regressions would be an appropriate technique. Other variables
included in (some of) the VARs were inflation, defined as the % change in the CPI index, property prices and short term interest rates. There is often a close correlation between sharp increases in PD and collapses in property prices, see Goodhart and Hofmann (forthcoming). Omitting property prices might give an upwards bias to the estimated effect of PD. The macroeconomic variables were obtained from the IMF’s International Financial Statistics and the OECD. Residential property prices were obtained from the BIS. For each country an index of banking sector equity was obtained from Bloomberg.

We used both individual country VARs and a panel data VAR methodology for our empirical investigation. This latter technique combines the traditional VAR approach, which treats all the variables in the system as endogenous, with the panel data approach, which allows for unobserved heterogeneity. We specify our model of order $s$ as follows:

$$z_{i,t} = \Gamma_0 + \Gamma_1 z_{i,t-1} + \Gamma_2 z_{i,t-2} + \ldots + \Gamma_s z_{i,t-s} + f_i + e_t \quad (1)$$

In our main model $z_{i,t}$ represents a four-variable vector $\{pod, Gdp, eq, inf\}$, where $pod$, a transformation of the distance to default, is our measure of the banking sector's default risk, $gdp$ is the growth rate of GDP, $eq$ is the annual growth rate of the bank equity index, and $inf$ is the inflation rate. In all models, the variable $pod$ is further transformed so that it has a value greater than a constant only in those quarters in which it is above a given threshold, otherwise it is set equal to that constant. Earlier testing had shown that $pod$ had a non-linear relationship with GDP. Below a certain

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$^6$ The analysis has been done using the programme developed by Love (2001).
threshold, whose value was estimated by empirical examination, fluctuations in pod had no effect on GDP. Similarly, fluctuations in bank equity values also appear to have a threshold effect on GDP, with the threshold also empirically estimated. It is only when bank equity declines, fairly sharply, that GDP is adversely affected.

Since the actual variance-covariance matrix of the errors is unlikely to be diagonal, to isolate shocks to one of the variables in the system, it is necessary to decompose the residuals in such a way that they become orthogonal. We do this by applying a Choleski decomposition of the variance-covariance matrix of the residuals (this is equivalent to transforming the system in a recursive VAR). The identifying assumption is that the variables that come earlier in the ordering affect the following variables contemporaneously, as well as with lags, while the variables that come later affect the previous variables only with lags. In our specifications we assume that the probability of default affects all other variables in the system contemporaneously and with lags, while macroeconomic variables such as GDP and inflation affect the default risk of the banking sector only with a lag. We experimented with different ordering of the variables (and therefore different identification assumptions) and obtained results that were qualitatively similar to the ones presented here. In applying the VAR procedure to panel data, we need to impose the restriction that the underlying structure is the same for each cross-sectional unit. Since the constraint is likely to be violated in practice, one way to overcome the restriction on the parameters is to allow for individual heterogeneity in the levels of the variables by introducing fixed effects, denoted by \( f_i \) in equation (1). Since the fixed effects are correlated with the regressors due to lags of the dependent variables, the mean-differencing procedure commonly used to eliminate fixed effects would create biased
coefficients. To avoid this problem we use forward mean-differencing, also referred to as the 'Helmert procedure' (See Arellano and Bond, 1995). This procedure removes only the forward mean, i.e. the mean of all the future observations available for each country-quarter. This transformation preserves the orthogonality between transformed variables and lagged regressors, so we can use lagged regressors as instruments and estimate the coefficients by system GMM.

4. Results

Our hypothesis, based on simulations and calibrations of our ‘Model to Analyse Financial Fragility’, is that whenever banks' default rates increase and banks' profitability decrease (above the threshold), i.e. when the economy is more financially fragile, GDP (our proxy of welfare) falls.7 Our aim here is to investigate whether data give any support to our model, namely that our two measures of banking sector's distress do have the predicted impact on output. We thus proceed by analysing the impulse response functions of the VAR model. Estimate of these and their confidence intervals are shown in Figures 15-17.8

Figure 15 reports the impulse-responses for a 3 lag VAR including pod, gdp, eq, and inf. The second row in the figure shows the response of gdp to a one standard deviation shock to the other variables of the model. The response of GDP growth to

7 In the general version of the model, an increase in default and a decrease in profitability is always associated with a reduction in agents' welfare (see Goodhart, Sunirand and Tsomocos (2004).

8 We calculate standard errors of the impulse-response functions and generate confidence intervals with Monte Carlo simulations. In practice, we randomly generate a draw of the coefficients Γ in model (1) using the estimated coefficients and their variance-covariance matrix and recalculate the impulse-responses. We repeat this procedure 1000 times (we experimented with a larger number of repetitions and obtained similar results). We generate the 5th and 95th percentiles of this distribution, which we use as a confidence interval for the impulse-responses.
*pod* is negative and significant (i.e. an increase of the default probability of the banking sector induces a decrease in the growth rate of GDP). Also, the response of GDP growth to a shock to the banking sector equity index is positive and significant. Put in a different way, maintaining all other variables constant, a positive shock to the banks' probability of default (once above the threshold) has a negative impact on output while a positive shock to the banks' equity value (again above the threshold) has a positive impact on output. These results are in line with the predictions of our model. The rest of the impulse response estimates are quite standard and intuitive: the bank equity index responds negatively to a positive shock to the bank probability of default while the impact of GDP growth on the same index is positive but marginally significant. Finally, a positive innovation in output growth induces a negative and significant decrease in inflation. This would be consistent with a positive supply shock. However, we do not estimate a structural model, so we are not able to identify supply and demand factors.

In order to check the robustness of the results, we run additional regressions adding a few variables that are usually included in small macroeconomic models for the analysis of monetary transmission and monetary policy (see for example Goodhart and Hofmann, 2005). Figure 16 reports the impulse-responses of a 3 lag VAR where a property price index, *propprice*, is added to the variables of the previous specification. The first row shows the responses of *pod*: its response to a positive shock to property prices is negative and significant. A higher property price index translates in a higher bank asset values, which in turn decreases banks' probability of default. As the second row in Figure 16 shows, the impact of a shock to *pod* of *gdp* is less significant that in the previous specification, as expected, but it still goes in the right direction.
Moreover, $gdp$ responds positively to a shock to the property price index. The response of the bank equity index to a shock to bank default risk is less marked than before but still negative while the response to a positive output shock is more significant and more persistent.

Finally, we analyse a model that includes the short term interest rate, $ir$. Figure 17 shows the impulse-responses. Their behaviour is very similar to the previous model. The impact of the added variable, the short term interest rate, is quite intuitive. A positive shock to $ir$ induces a positive response of the banking sector's probability of default, a negative response of GDP growth, a negative response of property prices. The response of inflation is however positive. This result is common to most of the VAR estimations of small macroeconomic models and can often be removed by adding another variable measuring import or commodity prices.

The variance decomposition of the panel VARs confirm the main results. Although, as expected, the variation in GDP growth 10 and 20 quarters ahead is mainly explained by GDP growth itself, bank probability of default and equity index explain a significant part of its change in the basic model specification (see Table 4).
When we include a property price index in the regression, this variable explains variation in gdp more than bank equity values, as shown in Table 5.

Finally, adding the short term interest rate variable does not seem to alter the result of the previous specification: however the bank equity index no longer has explanatory power of the variation in gdp (see Table 6).

Table 4. Variance decompositions: percent of variation in the row variable explained by the column variable

<table>
<thead>
<tr>
<th>Model (1)</th>
<th>Quarters ahead</th>
<th>pod</th>
<th>gdp</th>
<th>equity</th>
<th>inf</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>pod</td>
<td>10</td>
<td>97.32</td>
<td>0.87</td>
<td>1.45</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>97.18</td>
<td>0.96</td>
<td>1.48</td>
<td>0.36</td>
</tr>
<tr>
<td>gdp</td>
<td>10</td>
<td>11.12</td>
<td>86.38</td>
<td>2.09</td>
<td>0.38</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>11.28</td>
<td>86.13</td>
<td>0.27</td>
<td>0.4</td>
</tr>
<tr>
<td>equity</td>
<td>10</td>
<td>41.19</td>
<td>6.09</td>
<td>50.77</td>
<td>1.63</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>41.27</td>
<td>6.39</td>
<td>50.63</td>
<td>1.69</td>
</tr>
<tr>
<td>inf</td>
<td>10</td>
<td>16.19</td>
<td>10.71</td>
<td>10.11</td>
<td>62.97</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>16.42</td>
<td>10.87</td>
<td>11.02</td>
<td>61.68</td>
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Table 5. Variance decompositions: percent of variation in the row variable explained by the column variable

<table>
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<tr>
<th>Model (2)</th>
<th>Quarters ahead</th>
<th>pod</th>
<th>gdp</th>
<th>equity</th>
<th>inf</th>
<th>proprice</th>
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<td></td>
</tr>
<tr>
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<td>63.4</td>
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<td>3.30</td>
<td>23.35</td>
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<tr>
<td></td>
<td>20</td>
<td>62.69</td>
<td>8.57</td>
<td>1.42</td>
<td>3.85</td>
<td>23.19</td>
</tr>
<tr>
<td>gdp</td>
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<td>7.36</td>
<td>85.44</td>
<td>0.48</td>
<td>1.78</td>
<td>4.92</td>
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<tr>
<td></td>
<td>20</td>
<td>7.56</td>
<td>84.12</td>
<td>0.57</td>
<td>2.35</td>
<td>5.38</td>
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<td>equity</td>
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<td>16.16</td>
<td>16</td>
<td>58.94</td>
<td>5.17</td>
<td>3.7</td>
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<td></td>
<td>20</td>
<td>16.12</td>
<td>15.79</td>
<td>58.67</td>
<td>5.3</td>
<td>4.1</td>
</tr>
<tr>
<td>inf</td>
<td>10</td>
<td>6.6</td>
<td>3.9</td>
<td>1.2</td>
<td>87.96</td>
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<tr>
<td></td>
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<td>6.81</td>
<td>4.31</td>
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<td>86.67</td>
<td>0.71</td>
</tr>
<tr>
<td>proprice</td>
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<td>10.73</td>
<td>15.99</td>
<td>20.46</td>
<td>41.84</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>12.04</td>
<td>10.77</td>
<td>19.41</td>
<td>21.12</td>
<td>36.63</td>
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</table>
To test the robustness of the panel results, we also conducted a country level analysis, using again a VAR approach. The model specification includes four variables (pod, gdp, eq, inf). We include, in addition to the contemporary values of the variables, the first and fourth lag. We use a Choleski decomposition to identify the shocks and obtain estimates of the impulse-response functions. All the graphs are presented in Appendix II. The country-level results are in line with the evidence provided by the panel data analysis. As it is the case for the panel VAR, GDP responds negatively and significantly to a positive shock to the banking sector's probability of default for Korea, Sweden and Finland. The response is negative but not significant for the UK and Germany. This is not surprising since there were hardly any observations of pod in these two countries above the threshold level. In contrast to the panel VAR, the response of GDP is positive (but not very significant) for Norway and positive (but not significant) for Japan. The response of GDP to bank equity index is positive for Norway, Japan, Sweden and the UK (but not very significant for the last two countries) while it is not significant for Korea, Finland and Germany. On balance, the country-level analysis gives us some confidence in the robustness of the panel VAR.
results.

What we have done so far is to test the hypothesis that our two measures of banking sector fragility, PD and $Eq$, have a significant effect on welfare, as measured by GDP. For this purpose we have largely reproduced the results already obtained and presented in Goodhart, Tsomocos, Zicchino and Aspachs (2006), ‘Towards a Measure of Financial Fragility’. But now we want to go further. We want to weight these two factors in such a way as will enable us to provide a single quantitative metric, an index, for financial fragility.

We seek to do so by looking at the variance decompositions shown in Tables 4 to 6. Apart from one case (the 10 quarter ahead estimate for the effect of equity on GDP), the effect of PD on GPP is always more than 10x as important in explaining GDP than $Eq$. So what we have done is to make the weighting on $Eq$ one tenth that of PD. We did this as follows. First, we rescaled $Eq$ so that its mean absolute value was the same as PD, and its standard deviation the same as that of PD. Then we multiplied the resultant $Eq$ value by 0.1 and added it to PD, to achieve an overall index. The resulting time series, giving welfare indices for each country, are shown in Figures 18 to 24. Note that a high value is bad, and a low value good, indicating that Japan and Korea have had serious problems; that conditions in Finland have been improving, whereas they have been worsening in Germany; and that the UK has had few problems with financial instability in this historical period.
5. **Conclusions**

We began this paper with a quotation asserting that there was no obvious framework for measuring financial fragility. It has been our purpose here to demonstrate that such a framework can be obtained. This is clearly a first shot at what has been a difficult problem. We hope and expect successors to refine and to improve our methodology, but we contend that it can be done.

Moreover, having a metric for financial fragility should help to improve analytical studies on how to predict it, and to generate pressure for devising policy measures to limit such fragility before it passes through the threshold at which it begins to affect welfare (GDP) adversely. As noted, the main component of our index of financial fragility is PD. It is possible to predict fluctuations in PD. Goodhart, Hofmann and Segoviano (2005) do just that. Furthermore, as the determinants of financial fragility, especially asset price bubbles associated with excessively fast bank lending, indicate a future worsening of financial fragility, prudential controls should be tightened. Ultimately the hope/intention is to get an evidenced-based, analytically rigorous, counter-cyclical structure for prudential regulation, in place of the present pro-cyclical system.
Appendix 1

The Derivation of Bank PD Time Series

The dependent variable that is used in this study is a transformation of the distance to
default (DD) indicator, which is prepared by the Monetary and Financial Systems
Department, Financial Surveillance Policy Division in the IMF.

The DD indicator is used in the FST to gauge banking sector soundness. The variables
to calculate the DD indicator are obtained from information contained in bank equity
prices and balance sheets of some of the largest financial institutions for each country
under analysis.

In a standard valuation model, the distance-to-default DD is determined by: (a) the
market value of a firm’s assets, $V_A$; (b) the uncertainty or volatility of the asset value
(risk), $\sigma_A$; and (c) the degree of leverage or the extent of the firm’s contractual
liabilities, measured as the book value of liabilities at time $t$, $D_t$ (with maturity $T$).

The DD indicator is computed as the sum of the ratio of the estimated current value
of assets to debt and the return on the market value of assets, divided by the volatility
of assets. The formula is given by:

$$DD_t = \ln(V_A_t / D_t) + \mu - \frac{(1/2)\sigma_A^2 T}{\sigma_A \sqrt{T}},$$

(III.1)

Where $\mu$ measures the mean growth of $V_A$.

Using market data of equity and annual accounting data, the market value $V_A$ and the
volatility of assets $\sigma_A$ are typically estimated using Black and Scholes (1973) and

Once the DD is computed, the theoretical probability of default (PoD) is obtained as:

$$PoD_t = N(-DD_t),$$

(III.2)

Where $N$ is the cumulative probability distribution function (cdf) for a variable that is
normally distributed with a mean zero and a standard deviation of 1. (Vassalou and
Xing, 2002).

The theoretical probabilities of default (PoD) at each period $t$, are grouped in the T-
dimensional vector PoD. Since each observation in the vector PoD is restricted to lie
between 0 and 1, we make the following transformation:

$$Y = N^{-1}(PoD) + 5$$

(III.3)
where $N^{-1}$ is the inverse standard normal cdf. We are interested in modelling the PoD as a function of identifiable macroeconomic and financial developments $X$. We formalize the relationship as:

$$Y = XB + e$$

(III.4)

An alternative way to look at this issue is to assume that defaults reflect an underlying, continuous credit change indicator ("normal equivalent deviate" in the language of probit analysis) that has a standard normal distribution. Thus, we can state the relationship as: $PoD = N(XB + e)$, where the inverse normal cdf transformation converts this equation to a linear problem $Y = XB + e$. 

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Bibliography


Probability of Default (Pod) series and graphs

Figure 1: Pod Finland

Figure 2: Pod Germany

Figure 3: Pod Japan
Figure 4: Pod Korea

Figure 5: Pod Norway
Equity Growth series and graphs

Figure 8: Bank Equity Index annual growth. Finland

Figure 9: Bank Equity Index annual growth. Norway

Figure 10: Bank Equity Index annual growth. Sweden
Impulse-responses for 3 lag VAR of pod gdp equity inf

Errors are 5% on each side generated by Monte-Carlo with 1000 reps
Figure 16

Impulse-responses for 3 lag VAR of pod gdp equity inf propprice

Errors are 5% on each side generated by Monte-Carlo with 1000 reps
Figure 17

Impulse-responses for 3 lag VAR of pod gdp equity inf ir proprice

Errors are 5% on each side generated by Monte-Carlo with 1000 reps
Metric series

The metric is a weighted average of the pod and the equity of each country. The weights are obtained in the VAR and must reflect that the welfare changes due to financial instability are produced mainly through changes of the POD (90% of the variation). Therefore the metric is constructed as:

\[
\text{Metric}(i) = 0.9 \times \text{POD}(i) - 0.1 \times \left[ \frac{\text{SDpodi}}{\text{SDeqi}} \times \text{Eq}(i) + \text{av}(\text{pod}) - \text{av}(\text{eqi'}) \right]
\]

Where:
- \(\text{av}(\text{eqi'})\): is the average of the transformed equity series of country \(i\)
- \(\text{av}(\text{pod})\): is the average of the pod of country \(i\)
- \(\text{SDpodi}\): is the standard deviation of pod of country \(i\)
- \(\text{SDeqi}\): is the standard deviation of eq of country \(i\)