A General Martingale Approach to Measuring and Valuing the Risk to the FDIC Deposit Insurance Funds

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November 24, 2003

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Abstract

This paper presents a general methodology for measuring and valuing the risk of the FDIC deposit insurance funds using the martingale valuation approach. The FDIC insurance funds capitalize a portfolio of insurance policies, each issued against the deposits of an individual commercial bank. To evaluate this portfolio, our methodology evaluates each individual bank's insurance policy and aggregates to obtain the risk of the entire portfolio. Our methodology includes the four relevant risks: interest rate, credit, deposit growth, and loss. To adequately model these four correlated risks, a multi-dimensional system is formulated. The risk measurement and valuation results are based on Monte Carlo simulation. The resulting methodology is flexible and easily modified to incorporate extensions and generalizations.

1 Introduction

A rigorous approach to risk measurement and valuation of the risks of the FDIC insurance guarantees is essential for effective risk management. The FDIC deposit insurance funds can be viewed as capital that is held against the portfolio of insurance guarantees that the FDIC provides. This paper develops a rigorous approach and provides a valuable tool for evaluating the risks posed to the deposit insurance funds from potential bank failures. The approach utilized herein is based on the martingale valuation methodology explored in a previous paper by Duffie, Jarrow, Purnanandam, and Yang [6]. The martingale valuation methodology characterizes financial risks in an arbitrage free and complete market setting.

To evaluate the potential losses on the FDIC deposit insurance fund, our methodology evaluates each individual bank's potential failure and aggregates to obtain the risk of the entire fund. The four relevant risks of these insurance guarantees are included: interest rate, credit, deposit growth, and loss. To adequately model these four correlated risks, a multi-dimensional system is formulated. The risk measurement and valuation results obtained are based on a Monte Carlo simulation. The resulting risk management tool is constructed to be flexible and easily modified to incorporate extensions and generalizations.

Of the four risks, we model interest rate risk using a four-factor Heath-Jarrow-Morton [7], or HJM, model. Credit risk is modeled using the reduced form methodology introduced by Jarrow and Turnbull [9],[10]. Following the recent insights of Duffie and Lando [5] and Cetin, Jarrow, Protter, Yildirim [4] in this regard, an intensity process is used because regulators and the market have less information than do bank management. Less information can generate a totally inaccessible default time for the regulators and the market, whereas it may be a predictable stopping time for bank management.[1] Merton's [14] structural approach to credit risk is more appropriate when valuing these claims from the bank management's perspective. Deposit growth is modeled using various bank specific, local- and macro-economic variables in a time series regression. The loss (or equivalent, recovery rate) process depends on the asset and liability structure of the bank.

The results of the Monte Carlo simulation provide a complete characterization of the risks faced by the FDIC's deposit insurance fund. Over a one, three, five, and ten year horizon we compute various quantiles and summary statistics for the number of bank failures, the total assets in the failed banks, the total deposits in the failed banks, and the current values of the potential losses to the FDIC. From these computations, one obtains various risk measures and market valuations. For example, the value at risk measure (VaR) over a one year horizon using a 99 percent probability is the 99th percentile loss over a one year horizon, or $1.4 billion. Analogous VaRs for the three, five, and ten year horizons are $1.5 billion, 2.0$ billion, and $3.0 billion respectively. The market value of these losses to the FDIC insurance fund over the various horizons, valued as if traded on public capital markets, are given by the mean of the loss distribution. For a one year horizon, the market value of the FDIC's losses are computed to be $191 million. Analogous market values over the three, five, and ten year horizons are $238 million, $387 million, and $678 million respectively.

The remainder of this paper is organized as follows. Section 2 presents the martingale valuation methodology. This section characterizes the four risks present in FDIC insurance guarantees and it presents both the risk measure construction and valuation technology. The simulation model is the content of section 3. Section 4 presents the parameter estimation used in the simulation, whose results are discussed in section 5. Section 6 concludes the paper.

[1] The definitions of totally inaccessible, accessible, and predictable stopping times can be found in Protter [17].

2 The Martingale Valuation Methodology

This section introduces the notation and economic logic underlying the valuation model for the FDIC deposit insurance funds. The valuation methodology is based on Duffie, Jarrow, Purnanandam, and Yang [6].

2.1 Model Structure

We are given a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\in[0,\mathcal{T})}, \mathcal{P})$ satisfying the usual conditions[2] with $\mathcal{P}$ the statistical probability measure. The trading interval is $[0, \mathcal{T}]$. Default free bonds of all maturities $T \in [0, \mathcal{T}]$ are traded with time t prices denoted $p(t, T)$, and various stock price indices introduced below. The spot rate of interest at time t is denoted r_t. We assume that markets are complete and arbitrage free so that there exists a unique equivalent martingale probability measure $\mathcal{Q}$ under which discounted prices are martingales.[3]

Let $i = 1, ..., I$ represent the banks insured by the FDIC. Let $Y_j^i(t)$ for $j = 1, ..., N_y$ be a collection of characteristics of bank i at time t adapted to the given filtration, for example, the loan to deposit ratio of bank i at time t. These variables are known to the banks and the regulators, but perhaps not all of them, such as examination ratings, are available to the market.

2.2 Term Structure Model

Since we will be using a simulation to evaluate the future losses to the FDIC deposit insurance fund, we employ the most general form of a term structure evolution available. This is a multi-factor HJM [7] model.

2.2.1 Forward Rate Process

We specify the evolution of the term structure using forward rates under the martingale measure $\mathcal{Q}$. Let $f(t, T)$ be the instantaneous (continuously compounded) forward rate at time t for the future date T. We use a K factor model.

$$df(t,T) = \alpha(t,T)dt + \sum_{j=1}^{K} \sigma_j(t,T)dW_j(t) \tag{1}$$

where K is a positive integer, $\alpha(t,T) = \sum_{j=1}^{K} \sigma_j(t,T) \int_t^T \sigma_j(t,u)du$, $\sigma_j(t,T) \equiv \min[\sigma_{rj}(T)f(t,T), M]$ for M a large positive constant, $\sigma_{rj}(T)$ are deterministic functions of T for $j = 1, ..., K$, and $W_j(t)$ for $j = 1, ..., K$ are uncorrelated Brownian motions initialized at zero. Forward rates are approximately lognormally distributed under expression (1).

As shown in HJM [5], under the statistical measure $\mathcal{P}$, forward rates follow the process

$$df(t,T) = \alpha(t,T)dt + \sum_{j=1}^{K} \theta_j(t)\sigma_j(t,T)dt + \sum_{j=1}^{K} \sigma_j(t,T)d\widetilde{W}_j(t)$$

where $\theta_j(t)$ is the market price of risk (a stochastic process) associated with the jth factor, and $\widetilde{W}_j(t)$ for $j = 1, ..., K$ are independent, standard Brownian motions under the statistical measure $\mathcal{P}$.

[2] See Protter [17] for the definition of the usual conditions.

[3] The discount factor at time t is $e^{\int_0^t r_s ds}$.

2.2.2 Spot Rate Process

The spot rate process can be deduced from the forward rate evolution. Let $r_t \equiv f(t,t)$, then

$$dr_t = [\partial f(t,t)/\partial T]dt + \alpha(t,t)dt + \sum_{j=1}^{K} \sigma_j(t,t)dW_j(t). \tag{2}$$

But $\alpha(t,t) = \sum_{j=1}^{K} \sigma_j(t,t) \int_t^t \sigma_j(t,t)du = 0$, so

$$dr_t = [\partial f(t,t)/\partial T]dt + \sum_{j=1}^{K} \min[\sigma_{rj}(t)r(t), M]dW_j(t) \tag{3}$$

under the martingale measure $\mathcal{Q}$. Under the statistical measure $\mathcal{P}$, its evolution is

$$dr_t = [\partial f(t,t)/\partial T]dt + \sum_{j=1}^{K} \min[\sigma_{rj}(t)r(t), M](\theta_j(t)dt + d\widetilde{W}_j(t)).$$

2.3 State Variable Processes

We have two sets of state variables. Let $V_j(t)$ for $j = 1, ..., N_v$ be a collection of macro-variables, adapted to the filtration, that are independent of a particular bank. These state variables are intended to capture the health of the economy at time t. Second, let $X_j(t)$ for $j = 1, ..., N_x$ represent another collection of state variables, adapted to the filtration, also characterizing the state of the economy at time t. The difference between these two sets of state variables is that X_t represents the prices of traded assets, while $V_j(t)$ need not. We assume that these state variables give equivalent characterizations of the state of the economy.

For the subsequent analysis, we do not need to specify the evolution of $V_j(t)$, but we do need to do so for the traded assets. We assume that the traded state variables follow a diffusion process under the martingale measure $\mathcal{Q}$.

$$dX_j(t) = r_t X_j(t)dt + \sigma_{xj} X_j(t)dZ_j(t) \tag{4}$$

where σ_{xj} is a constant, $Z_j(t)$ are correlated Brownian motions with $dZ_i(t)dZ_j(t) = \rho_{ij}dt$, and with respect to the term structure of interest rates, $dW_i(t)dZ_j(t) = \eta_{ij}dt$. Because the state variables represent traded prices, the drift of this process is the spot rate of interest r_t.

By Girsanov's theorem, under the statistical measure $\mathcal{P}$, the evolution of these state variables is

$$\frac{dX_j(t)}{X_j(t)} = \mu_j(t)dt + \sigma_{xj}d\widetilde{Z}_j(t) \quad \text{where} \tag{5}$$

$$d\widetilde{Z}_j(t) \equiv \left(\frac{r(t) - \mu_j(t)}{\sigma_{xj}}\right) dt + dZ_j(t)$$

is a standard Brownian motion under the statistical measure $\mathcal{P}$, and $\mu_j(t)$ is an adapted process. Note that $\left(\frac{r(t)-\mu_j(t)}{\sigma_{xj}}\right)$ is known as the market price of risk for the jth state variable. For the subsequent analysis, we define the *detrended state variables* $x_j(t)$ as

$$\frac{dx_j(t)}{x_j(t)} \equiv \frac{dX_j(t)}{X_j(t)} - \mu_j(t)dt = \sigma_{xj}d\widetilde{Z}_j(t). \tag{6}$$

Under the martingale measure $\mathcal{Q}$, it evolves as

$$\frac{dx_j(t)}{x_j(t)} = [r(t) - \mu_j(t)]dt + \sigma_{xj} dZ_j(t). \tag{7}$$

2.4 Deposit Growth Model

Consider a particular bank with index i. The FDIC insurance guarantee covers the insured bank deposits.[4] If the insured bank defaults, the FDIC pays the insured depositors and stands in their place as a claimant in the receivership.[5] Although the FDIC covers insured deposits we chose to model the evolution of total deposits primarily due to data limitations. Insured deposits were not reported quarterly for much of our sample period and the numbers that are reported are estimates of insured deposits. We let the total deposits of bank i follow the stochastic process[6]

$$D_t^i \equiv D^i(t, Y_j^i(t), V_k(t) \text{ for all } \ j,k) \tag{8}$$

$$with \quad D^i(0, Y_j^i(0), V_k(0) \text{ for all } \ j,k) = D_0^i$$

where D_0^i are the observed balances at time 0. The deposit balance evolution depends on the variables $Y_j^i(t), V_k(t)$ for all j,k. These variables are known to the banks and the regulators, but perhaps not the market.

2.5 Bank Default Model

Consider a particular bank with index i. Let τ_i be the random default time on this bank and denote its point process by $N_i(t) \equiv 1_{\{\tau_i \leq t\}}$.

2.5.1 The Default Intensity Process.

Following Lando [12], we assume that the default point process follows a Cox process with an intensity $\lambda_t^i = \lambda^i(t, Y_j^i(t), V_k(t)$ for all $j,k)$ under the statistical measure $\mathcal{P}$.[7] The Cox process assumption implies that conditional upon the information set generated by $(Y_j^i(t), V_k(t)$ for all $j,k)_{t\in[0,\mathcal{T}]}$ up to time $\mathcal{T}$, $N_i(t)$ behaves like a Poisson process. We assume that these conditional Poisson processes are independent across banks.

In general, this intensity process is different under the martingale measure $\mathcal{Q}$. Under an equivalent change of measure, this default intensity can be written as

$$\kappa_t \cdot \lambda^i(t, Y_j^i(t), V_k(t) \text{ for all } \ j,k)$$

[4] Bank deposits are insured up to \$100,000 per ownership category. For example, suppose a bank customer has an account in their name of \$105,000 and a joint account with a spouse with a balance of \$280,000. The individual account is insured up to \$100,000 and the individual's portion of the joint account is insured up to \$100,000. Uninsured deposits for this individual would be \$5,000 of the individual account and \$40,000 from the joint account. For more information on deposit insurance coverage please visit www.fdic.gov.

[5] Here we have provided the most conceptually simple example, a payout, to describe the resolution of a bank. The FDIC resolves banks in the least costly manner which typically involves selling some of the assets and liabilities to an acquirer, otherwise known as a purchase and assumption transaction.

[6] The deterministic function $D^i : R \times R^{N_y} \times R^{N_v} \to R$ is Borel measurable, so that the process given in expression (8) is adapted to the filtration. A similar qualification applies to the loss rate process δ^i defined below.

[7] This intensity process and the other intensity processes introduced below are assumed to satisfy the necessary measurability and integrability conditions required to guarantee that the related expressions in expression (14) are well-defined and exist, see Lando [12] for details.

where κ_t is a suitably bounded and integrable stochastic process, adapted to the filtration generated by $Y_j^i(t), V_k(t)$ for all j, k.[8] This is the intensity process used for valuation. If, after conditioning upon the state variables $Y_j^i(t), V_k(t)$ for all j, k, default risk is idiosyncratic, then Jarrow, Lando, Yu [11] show that $\kappa_t = 1$.

2.5.2 The Loss Rate Process

If default occurs, the loss to the insurance fund as a percent of the banks deposits is assumed to be equal to $\delta_t^i \equiv \delta^i(t, Y_j^i(t), V_k(t)$ for all $j, k)$ at the time of default. Note that this loss rate process depends on the same set of state variables as the default intensity process and the deposit growth model.

2.6 Risk Measures and Valuation

This section discusses risk measures and valuation of the FDIC insurance guarantees.

For bank i, the loss faced by the FDIC at some future date T is given by

$$\delta_{\tau_i}^i D_{\tau_i}^i e^{+\int_{\tau_i}^T r_s ds} 1_{\{\tau_i \leq T\}}. \tag{9}$$

If default occurs before time T, then the FDIC incurs the losses $\delta_{\tau_i}^i D_{\tau_i}^i$, future valued to time T using the spot rate of interest. If default does not occur, then this is zero (due to the indicator variable $1_{\{\tau_i \leq T\}} = 1$ if $\tau_i \leq T$, 0 otherwise). The entire FDIC insurance funds losses are the aggregate losses across all banks:

$$L_T = \sum_{i=1}^{I} \delta_{\tau_i}^i D_{\tau_i}^i e^{+\int_{\tau_i}^T r_s ds} 1_{\{\tau_i \leq T\}}. \tag{10}$$

Given the stochastic processes for the forward rate process and state variables, the distribution for the losses L_T is completely determined by expression (10). Due to the dimension of the problem, a Monte Carlo simulation will be used to compute various risk measures and values.

2.6.1 Loss Risk Measures

Given the losses as quantified in expression (10), we can compute the loss distribution for the FDIC fund at any time T, i.e.

$$\mathcal{P}(L_T \leq k) \text{ for any } k \geq 0. \tag{11}$$

One might also be interested in the $\alpha-$quantile of this distribution, defined as

$$k_\alpha = \inf\{k : \ \mathcal{P}(L_T \leq k) \geq \alpha\}. \tag{12}$$

2.6.2 Present Value of Losses

The present value of this loss due to bank i is

$$E_t^{\mathcal{Q}}\{\delta_{\tau_i}^i D_{\tau_i}^i e^{-\int_t^{\tau_i} r_s ds} 1_{\{\tau_i \leq T\}}\} \tag{13}$$

[8] See Jarrow, Lando, Yu [11].

where $E_t^{\mathcal{Q}}(\cdot)$ corresponds to conditional expectation under the martingale measure $\mathcal{Q}$ using the filtration $(\mathcal{F}_t)_{t\in[0,\mathcal{T})}$. This is the cost of the FDIC insurance guarantee for bank i. Analogously, one can compute the present value of the insurance proceeds to determine whether or not FDIC insurance is properly priced, see Duffie, Jarrow, Purnanandam, and Yang [6].

The present value (PV) of the losses to the entire FDIC insurance fund over the time interval $[0, T]$ is given by

$$PV(L_T) = \sum_{i=1}^{I} E_t^{\mathcal{Q}}\{\delta_{\tau_i}^i D_{\tau_i}^i e^{-\int_t^{\tau_i} r_s ds} 1_{\{\tau_i \le T\}}\}. \tag{14}$$

To evaluate expression (14) we use Monte Carlo simulation.

3 The Simulation Model

This section presents the simulation model used to evaluate losses to the FDIC insurance funds. To compute the risk measures and present values of the FDIC insurance losses using Monte Carlo simulation, we must first simulate the time series processes for $Y_j^i(t), V_k(t)$ for all j, k, i. Unfortunately, this direct simulation has two problems. One, the complexity of the default, loss, and deposit growth models (see the subsequent sections) makes the direct simulation of these models problematic. Second, even if this were not the case, the dimension of a direct simulation would be too large given that the number of banks (I) is approximately 9,000 (the dimension of the problem is $(N_y \times N_v \times I)$). To make the simulation tractable, we need to reduce the dimension of this problem. In addition, to avoid the need to estimate the market prices of risk associated with the state variable processes $V_k(t)$, we will use the traded state variables $X_k(t)$ instead of the local- and macro-variable indices $V_k(t)$.

3.1 Projection to a Smaller Dimensional Problem

For simulation, we use the traded asset prices $X_j(t)$, and only a static subset of the bank's characteristics, denoted by $\overline{Y}_j^i$ for $j = 1, ..., \overline{N}_y$. This subset includes characteristics like the bank's geographical location.

The simulated insured deposit growth process is given by its conditional expectation, given the reduced information set, i.e.

$$\widehat{D}_t^i \equiv E^{\mathcal{P}}\{D^i(t, Y_j^i(t), V_k(t) \text{ for all } j, k) \mid \overline{Y}_j^i,\ X_j(t) \text{ for all } j\} \tag{15}$$

with $\widehat{D}_0^i = D_0^i$. Using the strong Markov property of a diffusion process, we can write this as

$$\widehat{D}_t^i \equiv \widehat{D}^i(t, \overline{Y}_j^i, X_k(t) \text{ for all } j, k) \tag{16}$$

where $X_j(t)$ follows the process in (5) under the statistical measure $\mathcal{P}$, and where $X_j(t)$ follows the process in (4) under the martingale measure $\mathcal{Q}$. We compute risk measures under $\mathcal{P}$, and valuation using expression (16) under the martingale measure $\mathcal{Q}$.

Analogous to the deposit growth model, we use the following intensity process in the simulation:

$$\widehat{\lambda}_t^i \equiv E^{\mathcal{P}}\{\lambda^i(t, Y_j^i(t), V_k(t) \text{ for all } j, k) \mid \overline{Y}_j^i,\ X_j(t) \text{ for all } j\} \tag{17}$$

$$= \widehat{\lambda}^i(t, \overline{Y}_j^i, X_k(t) \text{ for all } j, k).$$

This is the intensity process used for valuation. Under the martingale measure $\mathcal{Q}$, as previously discussed, this default intensity can be written as

$$\kappa_t \cdot \widehat{\lambda}^i(t, \overline{Y}^i_j, X_k(t) \text{ for all } j, k) \tag{18}$$

where κ_t is a suitably bounded and integrable stochastic process, adapted to the filtration generated by $X_k(t)$ for all k. This is the intensity process used for valuation. In the simulation, we set $\kappa_t = \kappa$, a constant. Furthermore, we assume that default risk is conditionally diversifiable as in Jarrow, Lando, Yu [11] and set $\kappa = 1$.[9].

Following a similar line of reasoning, the loss rate process is

$$\widehat{\delta}^i_t \equiv E^{\mathcal{P}}\{\delta^i(t, Y^i_j(t), V_k(t) \text{ for all } j, k) \mid \overline{Y}^i_j,\ X_j(t) \text{ for all } j\} \tag{19}$$

$$= \widehat{\delta}^i(t, \overline{Y}^i_j, X_k(t) \text{ for all } j, k) \quad with \quad \widehat{\delta}^i_0 = \delta^i_0$$

where δ^i_0 is the observed loss rate on deposits defaulting at time 0, $X_j(t)$ follows the process in (5) under the statistical measure $\mathcal{P}$, and $X_j(t)$ follows the process (4) under the martingale measure $\mathcal{Q}$.

Although the lower dimensional projection described above is used to faciliate computation, this projection has an economic interpretation. This formulation is consistent with only bank management and regulators observing the bank specific characteristics $Y^i_j(t)$ for all i, j, perhaps because this is proprietory information. In contrast, the market sees only a static subset of these bank characteristics represented by the variables $\overline{Y}^i_j$ for $j = 1, ..., \overline{N}_y$. Consequently, given the market's reduced information set, the deposit growth and bankruptcy processes are given by expressions (16), (17), and (19). Under this interpretation, these processes are the correct ones to use for market valuation of the FDIC insurance guarantees (see Duffie and Lando [5] and Cetin, Jarrow, Protter, Yildirim [4]).

3.2 Algorithm

To compute the loss distribution to the FDIC insurance fund (11) or its present value (14), we need to be able to simulate the forward rate process, the traded state variables, and the bankruptcy processes. The simulation algorithm is now described under the martingale measure $\mathcal{Q}$. The analogous simulation can take place under the statistical measure, with the appropriate change in drift.

- Step 1. Discretize the time interval $[0, T]$ as $t = 0, 1, 2, ..., T$. Generate a sample path for $W_1(t), ..., W_K(t), Z_1(t), ..., Z_{N_x}(t)$ over this discretization, called a scenario. Note that these variables are normally distributed with covariance matrix given by

$$dW_1(t) \quad ... \quad ... \quad dW_K(t) \quad dZ_1(t) \quad ... \quad ... \quad dZ_{N_x}(t)$$

[9] In subsequent research, we will explore the impact of utilizing market determined estimates for $\kappa \neq 1$.

$$
\begin{matrix} dW_1(t) \\ \vdots \\ \vdots \\ dW_K(t) \\ dZ_1(t) \\ \vdots \\ \vdots \\ dZ_{N_x}(t) \end{matrix}
\begin{bmatrix}
dt & 0 & ... & 0 & \eta_{11}dt & \eta_{12}dt & ... & \eta_{1N_x}dt \\
0 & dt & \ddots & \vdots & \eta_{21}dt & \ddots & & \vdots \\
\vdots & \ddots & \ddots & 0 & \vdots & & \ddots & \vdots \\
0 & ... & 0 & dt & \eta_{K1}dt & ... & ... & \eta_{KN_x}dt \\
\eta_{11}dt & \eta_{12}dt & ... & \eta_{1N_x}dt & dt & \rho_{12}dt & ... & \rho_{1N_x}dt \\
\eta_{21}dt & \ddots & & \vdots & \rho_{21}dt & dt & & \\
\vdots & & \ddots & \vdots & \vdots & & \ddots & \\
\eta_{K1}dt & ... & ... & \eta_{KN_x}dt & \rho_{1N_x}dt & ... & ... & dt
\end{bmatrix}
$$

- Step 2. Given a sample path for $W_1(t), ..., W_K(t), Z_1(t), ..., Z_{N_x}(t)$, use expressions (1) and (4) to obtain a sample path for the forward rates $f(t,T)$ and traded state variables $X_j(t)$.[10]

- Step 3. Use the sample paths for the forward rates and state variables, to obtain realizations of the deposits $\widehat{D}^i(t, \overline{Y}^i_j, X_k(t)$ for all $j,k)$, the intensity process $\kappa \cdot \widehat{\lambda}^i(t, \overline{Y}^i_j, X_k(t)$ for all $j,k)$, and the loss rate $\widehat{\delta}^i(t, \overline{Y}^i_j, X_k(t)$ for all $j,k)$.

- Step 4. Generate I independent unit exponentially distributed random variables E^i for $i = 1, ..., I$. Compute

$$\tau_i \equiv \inf\{s \in [0,T] : \int_0^s \kappa \cdot \widehat{\lambda}^i(t, \overline{Y}^i_k, X_j(t) \text{ for all } k,j)dt \geq E^i\} \tag{20}$$

  and compute

$$N_i(t) \equiv \{1 \text{ if } t \geq \tau_i, 0 \ \ otherwise\}. \tag{21}$$

  This point process determines the time of failure for all the banks under the given scenario.[11] Given a failure, the loss rate process $\widehat{\delta}^i(t, \overline{Y}^i_k, X_j(t)$ for all $k,j)$ then applies to the deposits $\widehat{D}^i(t, \overline{Y}^i_j, X_k(t)$ for all $j,k)$ to determine the loss to the insurance funds.

- Step 5. For this scenario, compute L_T in expression (10).

- Step 6. Repeat steps 1 - 5 m times. Let $\omega \in \{1, ..., m\}$ represent an arbitrary scenario. From this collection of scenarios, the risk measures and values can be computed. For example, expression (14) is approximated by

$$PV(L_T) \simeq \sum_{\omega=1}^{m} \frac{1}{m}[L_T(\omega)e^{-\int_0^T r_s(\omega)ds}]. \tag{22}$$

4 The Parameter Estimation

This section presents the parameter estimation procedures and results for the underlying stochastic processes, including the forward rates, the stock price indices, and the deposit growth and loss rate models.

[10] In the subsequent simulation, we actually use the detrended trade state variables $x_j(t)$ and add the spot rate of interest r_t.

[11] For efficiency, we assume that the intensity process is constant between quarters. This simplification allows us to approximate the default process in expression (20) using a Bernoulli representation, see the appendix.

4.1 Term Structure Model

To estimate the forward rate process given in expression (1), we employ a principal component analysis as discussed in Jarrow [8]. Given a time series of discretized forward rate curves $\{f(t,T_1), f(t,T_2), ..., f(t,T_{N_r})\}_{t=1}^m$ where N_r is the number of discrete forward rates observed, the interval between sequential time observations is Δ and m is the number of observations. Then, percentage changes are computed $\{\frac{f(t+\Delta,T_1)-f(t,T_1)}{f(t,T_1)}, ..., \frac{f(t+\Delta,T_N)-f(t,T_{N_r})}{f(t,T_{N_r})}\}_{t=1}^m$. From the percentage changes, the $N_r \times N_r$ covariance matrix (from the different maturity forward rates) is computed, and its eigenvalue/eigenvector decomposition calculated. The normalized eigenvectors give the discretized volatility vectors $\{\sigma_{rj}(T_1)\sqrt{\Delta}, ..., \sigma_{rj}(T_{N_r})\sqrt{\Delta}\}$ for $j = 1, ..., N_r$.

The term structure data was provided by Kamakura Corporation.[12] It consists of quarterly forward rates with maturities $T_1 = 1/4$, $T_2 = 1/2$, $T_3 = 3/4$, $T_4 = 1$, $T_5 = 2$, $T_6 = 3$, $T_7 = 5$, $T_8 = 7$, $T_9 = 10$, $T_{10} = 15$, $T_{11} = 20$ measured in years. The data set starts on January 4, 1982 and goes to August 2, 2003. Table 1 provides the volatility coefficients for the .25, .5, .75, 1, 2, 3, 5, 7, 10, 15 and 20 year forward rates and the percentage variance explained by the factors based on quarterly observation intervals ($\Delta = 1/4$). As indicated, the first four factors explain 93.16 percent of the variation in quarterly forward rate movements. For the subsequent analysis, we set $K = 4$ in expression (1).

4.2 State Variable Processes

To compute the parameters of expression (4), we use the quadratic variation, which is invariant under a change of equivalent probability measures. Given is a time series of $\{X_i(t)\}_{t=1}^m$ where the interval between sequential time observations is Δ, a quarter, and m is the number of observations. Define $\Delta X_i(t) \equiv [X_i(t+\Delta) - X_i(t)]$.[13] We compute

$$\sum_{t=1}^{m} \left(\frac{\Delta X_i(t)}{X_i(t)}\right)^2 \frac{1}{m} \quad \text{giving an estimate of} \quad \sigma_{xi}^2 \Delta. \tag{23}$$

Next we calculate

$$\sum_{t=1}^{m} \left(\frac{\Delta X_j(t)}{X_j(t)} \frac{\Delta X_i(t)}{X_i(t)}\right) \frac{1}{m} \text{ giving an estimate of } \sigma_{xj}\sigma_{xi}\rho_{ji}\Delta. \tag{24}$$

To obtain the correlation between the forward rates, the house price index, and the bank stock price index index η_{ji} for $j = 1, ..., K$ we compute

$$\sum_{t=1}^{m} \left(\frac{\Delta f(t,T_k)}{f(t,T_k)} \frac{\Delta X_i(t)}{X_i(t)}\right) \frac{1}{m} \text{ giving an estimate of } \sum_{j=1}^{K} \sigma_{rj}(T_k)\sigma_{xi}\eta_{ji}\Delta. \tag{25}$$

This is computed for $k = 1, ..., K$ for distinct $T_1, ..., T_K$ yielding K equations in K unknowns $\{\eta_{1i}, ..., \eta_{Ki}\}$. The estimates of $\sigma_{rj}(T_k)$ come from the forward rate principal components analysis discussed in the previous section. Solving this system gives the estimates. This is done for all i. For this estimation, we set $K = 4$ and we use the four forward rate maturities $T_1 = 1/2$, $T_2 = 1$, $T_3 = 3$, $T_4 = 5$.

[12] See www.kamakuraco.com.

[13] This could be done using log differences instead of returns or using the detrended variables $x_j(t)$ instead of $X_j(t)$.

Empirically, $\{X_i(t)\}_{t=1}^m$ includes a series of house price indices and a series of bank price indices. House prices are measured by the OFHEO indices for the nine census regions.[14]

Comparable bank price indices were compiled from CRSP data.[15] The indices include only those banks with at least $1 billion dollars (1996 dollars), a total turnover of 1 million shares per quarter, and five years of data. A total of 267 stocks was used. The banks were divided into ten groups: a set of money center banks and one set for each of the census regions. An equal weighted index was created for each group of banks

Table 2 summarizes the correlation matrix of the data. By construction, the principal components of the interest rate process are uncorrelated. These components are also weakly correlated with changes in the house and bank stock price indices. The mean correlation between interest rate components and changes in the house price index is 0.025 and the mean with the stock price index -0.063. Both the median and standard deviation of the correlations show that they are clustered around zero. Changes in the house price indices are correlated with changes in other house price indices; the mean correlation is 0.281. Changes in bank stock prices indices are even more correlated with changes in other bank stock prices; the mean correlation is 0.677. However, changes in house and bank stock prices are weakly correlated with a mean correlation of 0.051.

4.3 Bank Default Model

For the empirical default intensity, we estimated a standard bank failure model. The variables as well as the sample means and medians are found in Table 3. Most of the variables are conventional in the bank failure literature, although there are several differences. Income excludes taxes, and both it and the chargeoff rate are twelve-month totals adjusted for mergers. The CAMEL composite rating was entered as a series of dummies; in estimation, the dummy for 2-rated banks was excluded.[16] In addition, the model included the difference between the individual bank's CAMEL rating and the weighted average rating assigned to all the banks owned by the same bank holding company. The natural logarithm of assets was included as well as the square and the cube of the logarithm. The model also included the total assets of problem banks (with CAMEL ratings of 4 or 5) was entered as a percentage of total assets in all banks as was the square of the same variable.[17]

A pooled time series, cross sectional model with a logistic specification was used. The dependent variable was failure within the second quarter after the Call Report was filed. Consequently, the model used December 1990 data to estimate the probability of failure between April and June 1991. Banks receiving open bank assistance were considered failures. The sample included all banks and thrifts with the necessary data between December 1984 and December 2002.

Table 4 displays the results from the estimation of the model. The log-likelihood test ratio is 11191 and indicates that the model is statistically significant at any reasonable significance level.

[14] The Office of Federal Housing Enterprise Oversight (OFHEO) house price index is available quarterly since 1975.

[15] CRSP is the Center for Research in Security Prices, an affiliate of the University of Chicago Business School. Among other things, CRSP compiles daily data on stock trades.

[16] CAMEL composite ratings are a summary of individual *C*apital, *A*sset quality, *M*anagement, *E*arnings, and *L*iquidity ratings that are assigned after bank examinations. After 1997, a market *S*ensitivity rating was added, and the ratings became CAMELS. The ratings are integers between 1 and 5 with 1 being the best rating. A rating of 2 is an appropriate benchmark because that rating is assigned to basically sound banks that have some relatively minor weakness and because most banks receive that rating.

[17] The variables were selected from a larger set of potential variables. For example, CAMEL components were considered, but they did not improve the overall fit of the model substantially. For an explanation of the details of this specification, see Nuxoll [15].

Individual variables are generally significant at the 0.1% level and generally have the expected sign. The three size variables are individually insignificant, but as a group they are significant at the 0.1% level.[18]

The model described above was used to generate estimates of the probability of failure for December 2002. The desciptive statistics for the estimated probabilities of failure are given in Table 5 along with some results from a stress scenario which is explained later. Obviously, this model indicates that most banks are very safe. The vast majority of institutions have failure probabilities of less than 0.01%. The expected number of failures in the second quarter of 2003 is 1.1, and in fact, there was one failure during this period.[19]

As explained earlier, the failure probabilities were projected into a smaller space to make the simulation tractable. The coefficients from this projection are summarized in Table 6. The mean coefficients on the bank stock price index and the house price index are positive which suggests that an increase in house prices and an increase in the bank stock index would increase the probability of failure. This result is counter-intuitive. The signs on the interest rate variables are more plausible given the conventional wisdom that banks borrow short and lend long. It should be noted that the standard deviations reported in this table are very large, indicating that there is a good deal of variability among banks.

The last column of Table 5 reports how a stress scenario and the method of projection affect estimated failure probabilities. In the stress scenario, bank stock prices decline by approximately 30% while house prices are constant.[20] The term structure is assumed flat with both interest rates at 2% The estimated probability of failure of most banks is not affected substantially by these events. The median probability actually decreases slightly. However, for a notable minority of banks, the probability of failure increases, so the mean probability increases from 0.0125% to 0.0199%. This amounts to an increase from expected failures of about one a quarter to about 1.7 a quarter.

Three further sets of refinements are contemplated. First, more bank holding company data will be tested in the basic failure model. Second, the possibility of using cross-equation restrictions will be explored in the projection methodology. Currently, each bank has its own set of coefficients. The projection methodology could be modified so that similar groups of banks share some, if not all, the same coefficients. Finally, data is not available for banks that have been in existence less than about three and a half years. The projection methodology demands ten quarters of data, and banks must have a full year's worth of data to have estimated failure probabilities. An analysis of young banks could provide the missing data.

4.4 Deposit Growth Model

Deposit growth was measured on a year-over-year basis to eliminate seasonal effects. The deposit growth model used lagged deposit growth, the equity to asset ratio, the loan to asset ratio, CAMEL ratings, and age as explanatory variables. The CAMEL variable was entered as series of dummies. Age was measured in quarters, and dummies were used for age until a bank had attained an age of ten. Ten year old banks were considered mature. The combination of year-over-year growth and

[18] For further discussion of the coefficients and especially of the non-linearities in the model, see Nuxoll [15].

[19] It should be noted that expected assets and expected deposits in failed banks are very large ($600 million and $414 million respectively). This occurs because of the larger institutions. A single bank with $100 billion of deposits and a failure rate of 0.0001 would add $10 million to expected deposits in failed banks.

[20] The house price indices have increased 4% on average. Consequently, when house prices are constant, the detrended state variable $x_k(t)$ decreases by 4%. Thus there is a negative shock in the housing market.

the presence of lagged deposit growth in the model meant that banks had to be at least two years old to be included in the model.

Data was taken from the period March 1986 to March 2003. Sample means and medians are reported in Table 7.[21]

The coefficients for the deposit growth model are shown in Table 8. The coefficients for the age dummies are averages of the dummies for the various quarters, so the coefficient for two years old is an average of the coefficients for the dummies for nine to eleven quarters old. Although standard errors and significance labels on those variables are not reported in the table, all the standard errors range from 0.40 to 0.50, making the coefficients significant at the 0.01% level.

The desciptive statistics for forecasted deposit growth based on the December 2002 data are in Table 9. Also included in the table are the forecasts for a stress scenario which will be explained later. This model forecasts that most banks will experience modest deposit growth, averaging 7.43% over the next year This growth is actually slightly lower than the 10.4% experienced by the average bank during 2002.[22]

As explained earlier, deposit growth was projected into a smaller space. The coefficients from this projection are summarized in Table 10. The average coefficients are close to zero, although the R^2 are generally respectable. Except for the house price index there is not much variation among banks.

The stress scenario shown in the last column of Table 9 is the same as that used to evaluate the bank failure projection model. In this scenario, the average deposit growth rate increases slightly. The most noticeable aspect of the stress scenario is that the standard deviation of the distribution increases with larger deposit losses among some banks and more rapid growth for other banks.

Three general sets of refinements are contemplated. First, the effects of size and age on deposit growth will be explored more fully and possibly some additional structure imposed. Second,the possibility of using cross-equation restrictions will be explored in the projection methodology. Finally, a complete set of data is not available for the banks that have not been in existence for almost five years The projection methodology demands ten quarters of data, and banks that have not filed nine call reports are excluded from the model. An analysis of young banks could provide the missing data.

4.5 Loss Rate Model

The estimate of loss for individual institutions is calculated using a model similar to that used to estimate losses for the Least Cost Test.[23] This model uses a loss rate calculated from historical data and applies it to asset types from the Call Report for the institution. The resulting estimated market value of assets is then distributed across claimants on the receivership according to their priority: secured creditors, insured depositors, uninsured depositors, general trade creditors and subordinated debt holders. The FDIC pays insured depositors in full and then stands in the place of the insured depositors in the receivership. Therefore, the estimated loss to insured depositors is the estimated loss to the FDIC. Table 11 provides an example of the loss calculation.

The loss to the FDIC in this example is $876,000. This loss would then be divided by the total

[21] For more details on the deposit growth model, see Nuxoll [16].

[22] This deceleration would be in line with the trend over the last year. The average bank experienced 12.7% deposit growth during 2001.

[23] This model described here is used only when the FDIC does not have enough time to enter the bank and value the assets on site.

deposits of the institution to arrive at the loss rate that is used in the simulation.[24]

The model uses loss rates for six types of assets: consumer loans, commercial loans, securities, mortgages, owned real estate (ORE) and other assets.[25] As noted above, losses for each asset type are defined as the the amount of assets less the present discounted value of collections and expenses.[26] Losses are then divided by the the amount of assets for that asset type to arrive at a loss rate.

The data are available from 1986 to 2002. The loss rates are calculated using a sample of 369 failures from the 1990 to 2002 period.[27] Table 12 provides the decriptive statistics for the loss rates. Also note that rather than weighting each failure equally, the loss rate is weighted by the amount of assets of that type, henceforth this is referred to as the weighted average loss rate.

Descriptive statistics for the estimated losses in the event of failure generated from this model for December 2002 are shown in Table 13. In addition, the table includes statistics on estimated losses for a stress scenario that will be explained below.

Approximately 10% of the sample has an expected loss of 0.01% of total deposits. In fact, for these institutions, the model estimated that the loss on assets were small enough to allow the FDIC to be reimbursed in full for the amount it would have paid to insured depositors. However, the loss rate was set equal to one basis point of total deposits to accomodate the functional form of the regression. The maximum loss exceeds 100% of total deposits because the FDIC pays accrued interest on deposits.

To project the estimated losses into a smaller space, an estimate of losses was calculated for each institution in each quarter from 1986 to the present using the weighted average loss rates shown in Table 12 and the method described above. This information was then used to run a regression that projected into a smaller space. The regression coefficients are summarized in the following table.

This method used to calculate losses assumes that loss rates are constant, so that the state of the credit cycle has no effect on the loss rates for the assets. However, banks' balance sheets change over the cycle, so the coefficients are identifying the effect of the shift between loans, for example, and securities or between secured borrowing, insured deposits, uninsured deposits and other borrowed funds. The coefficients indicate that decreases in the price of bank stocks, increases in house prices, higher short term rates, and lower long term rates are associated with higher estimated loss rates.

The last column of Table 13 reports the results of using the projection for estimated loss rates. The stress scenario is the same as that used to evaluate the bank failure projection model This scenario has little effect on estimated loss rates for most banks. However, estimated loss rates increase notably for the fraction of the industry that already has high rates. The net result is the average estimated loss rate almost doubles.

The FDIC is currently undertaking an evaluation and redevelopment of the model used to esti-

[24] Bennett [1] provides more detail on the methodology.

[25] The model assumes the following about losses on asset categories. First, the loss rates on cash and federal funds are zero. Second, there is a category of assets including intangible assets that experience 100 percent losses. Third, fixed assets such as bank premises will have the same loss rate as that experienced on ORE.

[26] The discount rate used is the average of the 2-year Treasury-bill rate over the 1986 to 2002 period, or 5.4 percent.

[27] The concern was to use loss experience from more recent failures. A Chow test revealed that 1990 was a valid structural break in the loss data. The sample was constructed from the 1,235 BIF failures, excluding institutions where the FDIC provided assistance, from 1986 to 2002. The following types of transactions were excluded from the sample: (1) transactions where the acquirer purchased all of the assets (2) transactions with loss sharing agreements (3) transactions with bridge banks (4) transactions with cross guarantees, and (5) failures where fraud was the primary cause of failure. This resulted in a sample of 876 failures for the 1986 to 2002 period and 369 from 1990 to 2002.

mate losses from Call Report data for use in the Least Cost Test and reserving process. This development effort includes investigating different modeling strategies and testing their out-of-sample forecast accuracy. Possible refinements include more sophisticated modeling of the coefficients. Since the current method employs weighted averages for the coefficients, it is possible that more sophisticated modeling such as using regression techniques or nonparametric estimation will increase out-of-sample forecast accuracy. As refinements are made to the methodology to estimate losses they will be incorporated into the simulation model.

5 The Simulated Results

Tables 15 to 18 contain the simulation results for the one-, three-, five- and ten-year horizons. These results are based on 10,000 replications of the model with starting values given by December 2002 data. The tables show the distribution of the number of bank failures, the total assets in the failed banks, the total deposits in the failed banks, and the current value of the losses to the FDIC.[28] Total losses are discounted as in expression (22) from the estimated failure time to December 2002. Discounted losses are used so they are comparable to the dollar value of the FDIC capitalization on December 2002.

The results for the one-year horizon indicate that the FDIC should expect approximately five failures between January and December of 2003, and the expected total deposits for the failed banks should be on the order of $1.7 billion. The expected loss to the insurance funds is $87 million, and the median loss is approximately $44 million. Between January and August of 2003 two banks have failed resulting in $109 million in estimated losses to the FDIC insurance funds. In 2002, 11 banks failed resulting in a total estimated cost of approximately $621 million. Looking at the 5th and 95th percentiles, the model estimates that there is a 90% chance that between 2 and 9 banks will fail between January and December of 2003 and that losses will amount to between $4.4 million and $215 million.

The distribution presented in Tables 15 to 18 can be used to construct various risk measures. For example, the value-at-risk measure (VaR) over a one-year horizon using a 99 percent probability is the 99th percentile loss, or $1.2 billion. The market value of losses to the FDIC insurance fund over the one-year horizon as given in expression (22), valued as if traded on public capital markets, is given by the mean of the loss distribution, or $87 million.

The loss distributions presented in Tables 16, 17 and 18 can be used to construct risk measures over longer horizons. For example, the 99 percent VaR is $2.6 billion over three years, $2.0 billion over five years, and $3.1 billion over ten years. The market value of the losses amount to $307 million over three years, to $387 million over five years, and to $678 million.over ten years.

6 Conclusion

This paper provides a general methodology for measuring and valuing the risk of the FDIC deposit insurance funds. This methodology is implemented using Monte Carlo simulation. Illustrative results are provided.

[28] Note that although total deposits are as of the time of failure, total assets are as of December 2002 because asset growth is not part of the simulation.

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7 Appendix: Simulation Method

The nonhomogeneous (or nonstationary) Poisson process can be simulated in two main ways [13]

1. Inverse Transform Method — In general, this method requires integration of the rate function $\lambda(t)$, and then performing an inversion.

2. Acceptance-Rejection Method[29] — This method requires simulation of a homogeneous (stationary) Poisson process of sufficiently large rate $\lambda^* \geq \lambda(t)\ \forall t$. Event epochs epochs on a sample path $\tau_1, \tau_2, \ldots$ are "accepted" (kept) or "rejected" (deleted) according to a Bernoulli coin flip with probability $\lambda(\tau_i)/\lambda^*$.

The advantages of the Acceptance-Rejection Method are its simplicity and easy application for complicated rate functions, because it avoids the integration and inversion operations. Aside from being potentially non-trivial, these operations may also require additional storage. The chief disadvantage of the Acceptance-Rejection Method is that in general it will require a larger number of random numbers to be generated.

A special case of the Acceptance-Rejection Method is a discrete-time Bernoulli (coin flipping) where the discrete time increments are sufficiently small compared to the expected number of Poisson events in the increment (i.e., the expected number should be well under 1). In other words, it exploits the defining properties that the nonhomogeneous Poisson process still retains:

(i) Poisson distribution;

(ii) independent increments.

This special case of the Acceptance-Rejection Method is extremely simple, but a disadvantage of is that one random number (for each failure process simulated) must be generated every period.

Thus, assume that over periods of length Δ, denoted by I_i, that $\lambda(t)$ is constant with rate λ_i, and that $P(N(I_i) > 1) \approx 0\ \forall i$, where $N(I)$ is the number of Poisson events over the interval I. The Poisson assumption then implies $P(N(I_i) = 1) \approx \lambda_i \Delta = P_i$. This P_i is the conditional failure probability in a quarter that is provided by the empirical estimation of λ_t^i[30]. Thus, the failure process can be simulated as a discrete-time Bernoulli process where failure occurs in the period with probability λ_t^i. The actual failure time can be generated randomly (i.e. from a uniform distribution) over the period.

Note that the assumption of $P(N(I > 1) \approx 0$ is necessary only to use the Bernoulli random variable. More generally, one could use a Poisson random variable with parameter $\lambda\Delta$ (or $\gamma(t) = \int_0^\Delta \lambda(t)dt$ for the non-constant case). Under the assumption that the rate is constant over the period, the Poisson event (unordered) epochs would then be randomly distributed over the period using the property of a homogeneous Poisson process that given $N(I) = n$, the distribution of unordered epochs are independently and identically distributed $U(0, \Delta)$ (replace by cumulative density function. $\gamma(\cdot)/\gamma(\Delta)$ for the non-constant case). This case would not apply in the context of modeling bank failures since we assume that a bank can only fail once over the horizon (or at least in a quarter).

However, it turns out that for the constant-over-an-interval case, the Inverse Transform Method also simplifies considerably, since integrals are just rectangle areas. Again, let P_i denote the conditional failure probability in period i as above. Let $X \sim exp(1)$. Then,

$$Q = \text{failure period} = \min\{q : \sum_{i=1}^{q} P_i \geq X\}, \quad \tau = \text{failure time} = (Q-1)\Delta + \left(X - \sum_{i=1}^{Q-1} P_i\right) \Delta / P_Q.$$

[29] This method is also known as "thinning".

[30] The estimation of λ_t^i is described above in section 4.0.3.

The corresponding algorithm is as follows:

- Set $Sum := 0$ and $Q := 1$.
- Generate a random variable $X \sim exp(1)$.
- Loop until $Q >$ # quarters to simulate:
  If $Sum + P_Q \geq X$, then return failure time $\tau = (Q - 1 + (X - Sum)/P_Q)\Delta$;
  else $Sum := Sum + P_Q; Q := Q + 1$.

This method requires only a single random number per failure process simulated, and there is no complicated integration or inversion required, nor extra storage of sample path quantities, just one simple counter sum (corresponding to the cumulative integral).

In numerical test cases with 8,532 banks, 19 state variables, a 4-factor HJM interest rate model and a 10-year horizon, we found that the difference in using the two different failure processes is indistinguishable for a single replication with Δt of 1 week (1/52), because the forward rate process simulation dominates the failure rate process in terms of computational burden. However, for 200 replications with Δt of a quarter (1/4), we found a reduction in computation time of nearly 50%, as in this case, the forward rate process no longer dominates the simulation nearly as much.

Table 1
Forward Rate Volatility Functions
Estimated from January 4, 1982 to August 2, 2003, Quarterly Observations

	Factors										
Maturities	1	2	3	4	5	6	7	8	9	10	11
0.25	-0.211	-0.094	0.017	0.049	0.079	0.018	0.012	0.005	-0.012	0.013	0.001
0.5	-0.256	-0.066	0.022	0.039	0.032	-0.015	-0.004	0.006	0.013	-0.019	-0.004
0.75	-0.262	-0.025	0.028	0.035	-0.047	-0.032	0.008	-0.009	0.000	-0.003	0.008
1	-0.260	-0.010	0.007	0.007	-0.066	-0.002	-0.007	-0.024	-0.008	-0.008	-0.006
2	-0.240	-0.006	-0.027	-0.059	-0.004	0.057	-0.034	0.004	0.010	0.002	0.003
3	-0.200	0.038	-0.007	-0.041	-0.039	-0.017	0.011	0.043	-0.006	0.003	-0.001
5	-0.137	0.060	-0.009	-0.071	0.023	0.025	0.019	-0.012	-0.019	-0.016	0.000
7	-0.116	0.072	0.035	-0.053	0.023	0.002	0.036	-0.011	0.021	0.009	-0.001
10	-0.117	0.044	-0.148	-0.026	0.048	-0.051	-0.014	-0.007	-0.000	0.004	0.000
15	-0.082	0.152	-0.044	0.130	-0.003	0.029	0.003	0.004	0.001	-0.001	-0.000
20	-0.53	0.107	0.142	-0.011	0.041	-0.027	-0.029	-0.000	-0.005	0.002	0.000
Percent Explained.	68.44	10.39	8.15	6.18	3.49	1.70	0.73	0.50	0.23	0.17	0.02
Cumulative Percent Explained	68.44	78.83	86.98	93.16	96.65	98.35	99.08	99.58	99.81	99.98	100.00

Table 2
Descriptive Statistics
Correlation Matrix of Data Used in Projection

	Statistics	House Price Indices	Bank Stock Price Indices
Interest Rate Factors	Mean	0.025	-0.063
	Median	0.036	-0.061
	Std. Dev.	0.106	0.101
House Price Indices	Mean	0.281	0.051
	Median	0.330	0.027
	Std. Dev.	0.294	0.133
Bank Stock Price Indices	Mean		0.667
	Median		0.678
	Std Dev.		0.100

Table 3
Descriptive Statistics
Failure Model Variables

Variable	Mean	Median
Equity / Total Assets	9.64%	8.69%
Pre-tax Income / Total Assets*	3.11%	1.34%
Loans Past Due 30 - 89 Days / Total Assets	1.10%	0.80%
(Loans Past Due 90 or More Days + Nonaccual Loans + Other Real Estate Owned) / Total Assets	1.46%	0.77%
Loss Reserves / Total Assets	0.87%	0.75%
Chargeoffs / Total Assets*	0.46%	0.18%
CAMEL Composite Rating of 1 (Dummy)	30.3%	
CAMEL Composite Rating of 2 (Dummy)	53.9%	
CAMEL Composite Rating of 3 (Dummy)	10.4%	
CAMEL Composite Rating of 4 (Dummy)	4.3%	
CAMEL Composite Rating of 5 (Dummy)	1.1%	
CAMEL Composite - BHC CAMEL Composite Rating	-0.008	0
Examination Interval (Years)	0.829	0.644
Ln(Assets)	11.21	11.05
$(\text{Ln(Assets)})^2$	127.45	122.21
$(\text{Ln(Assets)})^3$	1470.03	1351.18
Percentage of Assets in Problem Banks	6.93%	7.41%
$(\text{Percentage of Assets in Problem Banks})^2$	85.49	54.91
Number of Observations	891,171	

Source: Author's calculations.

*The data are twelve-month totals and are merger-adjusted.

Table 4
Model Estimates

Variable	Coefficient	Standard Error
Intercept	-0.4562	5.8714
Equity / Total Assets	0.1504***	0.0086
Pre-tax Income / Total Assets*	0	0
Loans Past Due 30 - 89 Days / Total Assets	-0.1274%***	0.0107
(Loans Past Due 90 or More Days + Nonaccual Loans + Other Real Estate Owned) / Total Assets	-0.0872***	0.0045
Loss Reserves / Total Assets	-0.0005	0.0156
Chargeoffs / Total Assets*	-0.0148***	0.0031
CAMEL Composite Rating of 1	2.8378***	0.2999
CAMEL Composite Rating of 2		
CAMEL Composite Rating of 3	-2.1960***	0.1542
CAMEL Composite Rating of 4	-3.6029***	0.1466
CAMEL Composite Rating of 5	-5.2500***	0.1536
CAMEL Composite - BHC CAMEL Composite Rating	2.0462***	0.0915
Examination Interval	-0.6886***	0.0362
Ln(Assets)	2.1272	1.4844
$(\text{Ln(Assets)})^2$	-0.1480	0.1236
$(\text{Ln(Assets)})^3$	0.0033	0.9657
Percentage of Assets in Problem Banks	-0.1355***	0.0256
$(\text{Percentage of Assets in Problem Banks})^2$	0.00564***	0.0011
Log Likelihood Ratio	11191	
Somer's D	0.948	

Source: Authors' calculations.
*** Indicates that the coefficient is signficant at the 0.1% level.

Table 5
Descriptive Statistics
Forecasted Probability of Failure
December 31, 2002

	Baseline	Stress
Mean	0.0125%	0.0199%
Standard Deviation	0.1271%	0.3041%
Minimum	0.0000%	0.0000%
1st Percentile	0.0001%	0.0000%
10th Percentile	0.0001%	0.0001%
25th Percentile	0.0002%	0.0002%
Median	0.0024%	0.0021%
75th Percentile	0.0047%	0.0049%
90th Percentile	0.00827%	0.0104%
99th Percentile	0.1936%	0.2393%
Maximum	6.69%	18.3142%

Source: Author's calculations.

Table 6
Projection Coefficients for Changes in Probability of Failure

	Bank Stock Price Index	House Price Index	Interest Rate 3 Month	Interest Rate 3 Year	R^2
Mean	0.0661	0.7272	0.0294	-0.0211	0.0561
Standard Deviation	0.6546	8.0512	0.1318	0.1474	0.0623
Median	0.0521	0.7045	0.0283	-0.0215	0.0398

Table 7
Descriptive Statistics
Deposit Growth Model Variables

Variable	Mean	Median
Deposit Growth	7.07%	4.84%
Lagged Deposit Growth	8.53%	5.14%
Equity / Total Assets	9.32%	8.70%
Loan / Total Assets	57.22%	58.86%
Ln (Assets)	11.26	11.10
CAMEL Composite Rating of 1	30.56%	
CAMEL Composite Rating of 2	53.51%	
CAMEL Composite Rating of 3	10.27%	
CAMEL Composite Rating of 4	4.34%	
CAMEL Composite Rating of 5	1.31%	
Two Years Old	0.99%	
Three Years Old	1.34%	
Four Years Old	1.25%	
Five Years Old	1.16%	
Six Years Old	1.08%	
Seven Years Old	1.01%	
Eight Years Old	0.95%	
Nine Years Old	0.92%	
Number of Observations	824,082	

Source: Authors' calculations

Table 8
Deposit Growth Model Estimates

Variable	Coefficient	Standard Error
Intercept	-2.91	0.23
Lagged Deposit Growth	0.022***	0.0011
Equity / Total Assets	-0.77***	0.0054
Loan / Total Assets	0.114***	0.0015
Ln(Assets)	1.46***	0.019
CAMEL Composite Rating of 1	0.72***	.055
CAMEL Composite Rating of 2		
CAMEL Composite Rating of 3	-5.03***	0.080
CAMEL Composite Rating of 4	-12.12***	0.12
CAMEL Composite Rating of 5	-22.08***	0.21
Two Years Old (Average)	29.97	
Three Years Old (Average)	18.10	
Four Years Old (Average)	11.43	
Five Years Old (Average)	8.35	
Six Years Old (Average)	6.64	
Seven Years Old (Average)	5.19	
Eight Years Old (Average)	5.43	
Nine Years Old (Average)	4.93	
R^2	0.087	

Source: Authors' calculations.
*** Indicates that the coefficient is signficant at the 0.1% level.

Table 9
Descriptive Statistics
Forecasted Deposit Growth
December 2002

	Baseline	Stress
Mean	7.43%	8.13%
Standard Deviation	5.86	8.90
Minimum	-54.16%	-52.18%
1st Percentile	-8.72%	-11.51%
10th Percentile	1.61%	0.34%
25th Percentile	4.80%	4.08%
Median	7.80%	7.67%
75th Percentile	10.31%	10.88%
90th Percentile	13.12%	14.78%
99th Percentile	21.20%	39.79%
Maximum	39.13%	167.34%

Source: Author's calculations.

Table 10

Projection Coefficients for Deposit Growth

	Bank Stock Price Index	House Price Index	Interest Rate 3 Month	Interest Rate 3 Year	R^2
Mean	0.0013	-0.0353	0.0001	0.0000	0.4607
Standard Deviation	0.0192	0.2733	0.0030	0.0042	0.2030
Median	-0.0001	0.0020	0.0001	-0.0002	0.4521

Table 11

Example of Least Cost Test Model

	Book Value (000 omitted)	Loss (000 omitted)
Cash and Due Froms	600	0
Fed Funds	1,300	0
Securities	1,300	30
Consumer Loans	4,100	251
Mortgages	3,900	1,209
Commercial Loans	4,700	2,336
Fixed Assets	200	136
Other Assets	1,300	355
Loss Assets	400	400
Trading Assets	0	0
Customer Liabilities	0	0
ORE	50	34
Total Gross Assets/Loss	17,250	4,751
Less: Loss on Assets	4,751	
Net value of assets available for distribution	12,499	
Less: Claims on Receivership	16,009	
Total Loss to Creditors in Receivership		3,510
Distribution of Claims and Losses	Claim	Loss
Secured and Preferred Creditors	900	0
FDIC	9,138	876
Uninsured Depositors	3,691	354
General Creditors	2,136	2,136
Subordinated Creditors	144	144
Total	16,009	3,510

Source: Author's calculations.

Table 12
Descriptive Statistics
Loss as a Percent of Assets
1990-2002 Sample

Asset Type	Consumer Loans	Commercial Loans	Securities	Mortgages	ORE	Other Assets
Weighted Average	18.4%	40.0%	1.1%	22.0%	62.2%	25.9%
Number	343	345	341	337	338	349
Mean	26.6%	40.0%	1.7%	28.3%	65.8%	25.2%
Standard Deviation	19.68	17.6	6.8	17.8	21.6	19.5
Minimum	0.4	0.3	-34.9	-48.4	0.35	-30.7
25th Percentile	12.6	27.7	0.0	17.8	53.5	10.7
Median	22.0	37.6	0.0	25.6	65.8	22.2
75th Percentile	35.31	51.1	1.3	34.9	77.8	37.4
Maximum	135.4	152.5	86.7	145.5	154.2	100.2

Source: Author's calculations. Excludes observations where initial assets are less than $10,000, pre-expenses losses are over 100 percent or where total direct expenses are more than 110 percent of assets. Negative values for direct expenses are set to zero.

Table 13
Descriptive Statistics
Estimated Losses as a Percent of Total Deposits
December 2002

	December 2002	Stress Scenario
Mean	6.94%	13.31%
Standard Deviation	5.23%	23.25%
Minimum	0.01%	0.01%
1st Percentile	0.01%	0.01%
10th Percentile	0.01%	0.01%
25th Percentile	2.63%	1.49%
Median	7.03%	7.02%
75th Percentile	10.51%	12.62%
90th Percentile	13.27%	24.72%
99th Percentile	18.33%	110.00%
Maximum	101.13%	110.00%

Source: Author's calculations. Note that losses can be larger than total deposits since the FDIC pays accrued interest on deposits.

Table 14
Projection Coefficients for Estimated Loss Rates

	Bank Stock Price Index	House Price Index	Interest Rate 3 Month	Interest Rate 3 Year	R^2
Mean	-0.3654	3.0063	0.0791	-0.0694	0.4309
Standard Deviation	2.1434	27.4325	0.4153	0.6105	0.1902
Median	-0.0890	0.6789	0.0401	-0.0364	0.4310

Table 15
Simulation Results
December 2002

One-Year Horizon				
	Number of Failures	Total Assets in Failed Banks (000 omitted)	Total Deposits in Failed Banks (000 omitted)	Total Losses to the FDIC (000 omitted)
Mean	4.96	2,535,383	1,710,570	86,653
Standard Deviation	2.14	18,253,448	11,349,264	294,951
Minimum	0	0	0	0
1st Percentile	1	16,196	13,317	61
5th Percentile	2	104,369	90,528	4,425
10th Percentile	2	172,505	152,167	9,369
25th Percentile	3	319,143	275,940	22,552
Median	5	569,501	483,805	44,103
75th Percentile	6	1,106,227	923,366	79,816
90th Percentile	8	2,624,251	2,067,607	140,265
95th Percentile	9	12,985,079	7,888,961	215,245
99th Percentile	10	20,023,384	12,865,840	1,166,497
Maximum	15	633,194,383	413,867,996	19,808,656

Table 16
Simulation Results
December 2002

Three-Year Horizon				
	Number of Failures	Total Assets in Failed Banks	Total Deposits in Failed Banks	Total Losses to the FDIC
		(000 omitted)		
Mean	14.46	7,885,398	5,208,262	307,306
Standard Deviation	3.52	31,889,148	19,307,108	808,190
Minimum	3	131,207	101,976	5,107
1st Percentile	7	580,431	492,516	33,708
5th Percentile	9	828,049	706,451	56,225
10th Percentile	10	1,024,487	867,190	73,700
25th Percentile	12	1,478,213	1,236,868	111,446
Median	14	2,521,024	2,050,133	169,715
75th Percentile	17	4,837,810	3,640,753	258,419
90th Percentile	19	17,321,182	11,565,855	444,574
95th Percentile	21	21,081,382	13,443,997	794,415
99th Percentile	23	53,740,926	33,328,379	2,606,415
Maximum	29	639,188,968	413,500,219	25,535,710

Table 17
Simulation Results
December 2002
Five-Year Horizon

	Number of Failures	Total Assets in Failed Banks	Total Deposits in Failed Banks	Total Losses to the FDIC
		(000 omitted)		
Mean	22.96	12,819,552	8,601,416	386,574
Standard Deviation	4.32	43,016,590	25,999,019	790,760
Minimum	7	523,754	481,276	43,157
1st Percentile	14	1,296,296	1,108,938	87,708
5th Percentile	16	1,734,299	1,474,439	122,650
10th Percentile	18	2,063,842	1,746,618	143,462
25th Percentile	20	2,883,950	2,404,249	187,149
Median	23	4,449,118	3,639,744	255,141
75th Percentile	26	14,841,599	8,688,974	370,045
90th Percentile	29	20,718,702	14,301,658	636,334
95th Percentile	30	30,001,850	19,045,706	1,194,580
99th Percentile	34	96,030,875	62,951,947	2,040,721
Maximum	47	652,024,795	414,060,254	41,137,936

Table 18
Simulation Results
December 2002
Ten-Year Horizon

	Number of Failures	Total Assets in Failed Banks	Total Deposits in Failed Banks	Total Losses to the FDIC
		(000 omitted)		
Mean	41.54	24,797,297	16,623,473	678,283
Standard Deviation	5.63	60,234,903	37,339,418	1,134,199
Minimum	17	2,079,341	1,763,453	108,835
1st Percentile	29	3,380,380	2,840,576	206,501
5th Percentile	33	4,312,643	3,615,688	258,206
10th Percentile	34	5,024,319	4,183,269	291,264
25th Percentile	38	6,704,073	5,482,686	359,622
Median	41	13,985,348	8,788,646	467,896
75th Percentile	45	23,119,224	16,339,595	676,392
90th Percentile	49	37,390,491	24,471,547	1,280,989
95th Percentile	51	54,008,330	34,714,308	1,669,792
99th Percentile	55	340,601,035	222,384,122	3,072,332
Maximum	65	684,093,273	431,320,735	58,733,274