Bank Size, Leverage, and Financial Downturns

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Abstract

I construct a macroeconomic model with a heterogeneous banking sector and an interbank lending market. Banks differ in their ability to transform deposits from households into loans to firms. Bank size differences emerge endogenously in the model, and in steady state, the induced bank size distribution matches two stylized facts in the data: bigger banks borrow more on the interbank lending market than smaller banks, and bigger banks are more leveraged than smaller banks. I use the model to evaluate the impact of increasing concentration in US banking on the severity of potential downturns. I find that if the banking sector in 2007 was only as concentrated as it was in 1992, GDP during the Great Recession would have declined by 40% less it did, and would have recovered twice as fast.

JEL Classifications: E02, E44, E61, G01, G21

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1 Introduction

The financial crisis of 2008 began in the housing sector but eventually affected employment and output in many other sectors of the economy. The banking sector played a crucial role in this story - banks held mortgages and their derivatives, and when the value of these assets declined, financial institutions engaged in a deleveraging process which reduced their investments dramatically, driving a large and persistent downturn in the real economy.

This project aims to further our understanding of one possible mechanism for the transmission of financial crises. A strand of previous work[1] considers the transmission of crises through the liquidity banks provide to each other through interbank loans and other forms of short-term debt. Banks rely on other banks for funds to efficiently meet day-to-day investment demands. If these funds are hard to come by in times of weakness in the economy, otherwise unaffected banks may not be able to function as efficiently as in good times. This leads to a decrease in the investments they make and amplifies the real effects of the initial shock.

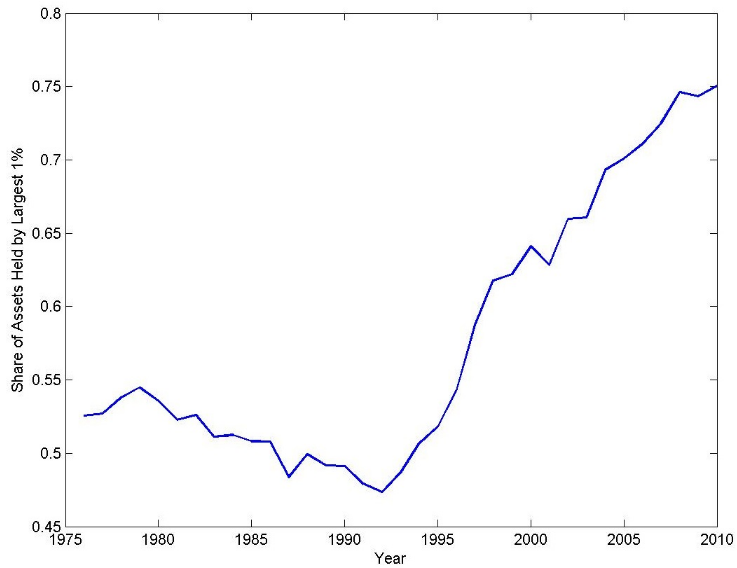
In this paper, I go a step beyond this existing literature and investigate the following questions: does bank size, and the distribution of bank sizes in the industry, affect the transmission of crises in this way? If so, how?

Why think about size? First, the financial crisis showed us that there may be good reasons to think that, on their own, big banks de-stabilize the economy in a crisis - in particular, the knock-on effects from the failures of large banks and the potential moral hazard problems generated by large bank bailouts have been pointed out as inherently destabilizing to the system. Second, there are good reasons to think that big banks operate in qualitatively different ways than small banks. Big banks are more leveraged, lose more in downturns, and rely more on interbank lending to finance their day-to-day operations. Last, a key trend in US financial markets in recent decades has been a remarkable rise in the concentration of the banking system, as shown in figure 1. Higher concentration means that big banks make up a larger share of the banking sector than in the past, and therefore the actions taken by big banks, and the interactions between big and small banks, matter more for outcomes in the entire banking sector.

I build a model in which households, banks, and firms are divided across a continuum of islands. Banks take deposits from households and lend them to firms on the same island, and also have access to a national interbank market where they can lend to or borrow from

[1] Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) consider this mechanism through interbank lending specifically. He and Krishnamurthy (2012) and Boissay et al. (2013) consider similar effects, but liquidity problems arise between households and banks instead of between banks.

Figure 1: Concentration in the US Banking Sector



Note: Share of assets held by largest 1% of banks calculated for all commercial banks from 1976 to 2010. Data from call reports (forms FFIEC 031 and 041), retrieved from Federal Reserve Bank of Chicago (2013).

other banks. Banks will face a moral hazard problem when taking funds from other banks - they can divert a fraction of their funds (deposits and interbank loans) for their private purposes. Crucially, banks will differ in the productivity, or intermediation ability, with which they transform deposits into investments in firms.

In equilibrium, the moral hazard problem will limit the amount each bank can borrow from other banks; as a result, rather than efficiently allocating all funds to the most able bank, the interbank market will misallocate funds, directing some to less able banks. More able banks will borrow more on the interbank lending market and invest more in the firms on their island. The model generates an endogenous asset size distribution of banks which matches two features of the data: (1) large banks will tend to be net borrowers on the interbank market, while small banks will tend to be net lenders, and (2) larger banks will be more leveraged than smaller banks.

I focus on financial downturns in the model by inducing a shock that destroys the value of net worth of all banks. The net effect on aggregate credit and investment will depend on two factors: the average ability of borrowing banks, or "extensive margin", and size of the investments/borrowing of each borrowing bank, or "intensive margin". In a downturn, the average ability of borrowing banks falls, worsening the extensive margin, and with it, the misallocation of funds across banks. At the same time, borrowing constraints tighten as bank net worth falls, reducing size on the intensive margin. In the typical case, both effects combine to prolong and deepen downturns. Moreover, the richness of the model enables us to examine the difference in the impact of downturns on individual banks. In particular, downturns in the model mirror downturns in the data: the collapse in interbank lending disproportionately hurts large banks, large banks de-lever and lose more in downturns, and firm investment financed by large banks falls more than investment financed by small banks.

The dispersion of bank sizes will end up mattering for the depth of downturns after financial shocks. The more dispersed the bank sizes, the more banks rely on interbank lending for efficiently allocating funds. Downturns interrupt interbank lending, and they therefore have a bigger impact in an economy with a more dispersed banking sector.

I quantify these effects in two ways. First, I compare the effects of financial shocks in the model with heterogeneous banks to a model with a homogenous banking sector. I calibrate both models to the US economy in 2007, just before the financial crisis. Introducing realistic heterogeneity in the banking sector yields significant amplification and propagation of financial shocks. Recessions in the heterogeneous model lead to a 15% deeper drop in output.

Next, I use the model to answer another question: if the US banking sector had not experienced the long-term increase in concentration that it did, would potential downturns have been any less severe? If so, by how much?

Over the last three decades, increases in concentration have been accompanied by increases in the dispersion of bank sizes and the skewness of the size distribution, or relative frequency of big banks to small banks. I calibrate the model to roughly match the changes in concentration, dispersion, and skewness of the bank size distribution, from their lowest values in 1992 to the pre-crisis economy of 2007. I then consider the effect of a financial shock which produces a drop in output similar to that seen in 2007/08 financial crisis in both economies. If the banking sector in 2007 had maintained the lower concentration, dispersion, and skewness of the banking sector in 1992, the same financial shock would have produced a 40% smaller drop in output, and would have recovered twice as fast.

This paper follows a strand of literature in macroeconomics that considers the impact of information frictions in the financial sector on financial crises. The model in this paper builds on the model of Gertler and Kiyotaki (2010), who incorporate an interbank lending market in a macroeconomic model with a banking sector composed of identical banks. The agency problem in that model keeps aggregate investment below its efficient level in normal times, and amplifies the adverse effects of financial shocks in downturns. Boissay et al. (2013) consider the effects of this agency problem with a heterogeneous banking sector, and are able to create an economy in which banking crises arise endogenously.[2]

My paper differs from this related work in two ways. First, my focus is different: I am more interested in the connection between banking industry characteristics and downturns, and as such, I restrict my attention to the analysis of industry trends. Second, my model is different: leverage, or the assets banks hold per dollar net worth, will differ with size. This margin will generate a connection between the depth of downturns and the variance of the size distribution - the larger the highest bank is relative to the smallest, the more output declines in response to a given decrease in liquidity, the deeper the recession.

Concentration in the banking sector has been steadily increasing over the last two decades, and a great deal of research has been done to consider its effects on the likelihood and severity of banking sector crises. One view is that concentration makes the economy less prone to crises because large banks are also more diversified, and are therefore better able to maintain the credit they provide to firms in even of a downturn. Beck et al. (2007) perform a reduced-form, cross-country study using a global dataset of banks, finding that concentration is associated with fewer crises (though they do not find evidence supporting the diversification theory). On the other hand, another strand of literature maintains that large banks are less disciplined by competition than smaller banks, make poorer lending choices, and lose more in downturns. The model of Boyd and De Nicolo (2005) predicts that banks in less competitive environments charge higher interest rates to firms, which induces firms to take on greater risk and default more often. De Nicolo et al. (2004) performs a cross-country study using a dataset of large banks, and finds that higher concentration is associated with a higher fragility of the largest five banks. In some ways, this division in the research still persists. Corbae and D'Erasmo (2010) builds a two-tiered model of the banking sector, where a few dominant banks interact with a competitive fringe. They find evidence that an increase in banking concentration (implemented as an increase in entry costs) has offsetting effects: bank exit decreases, increasing stability, while interest rates increase, decreasing stability as in Boyd and De Nicolo (2005).

[2]This paper is also related to a broader literature considering the quantitative effects of financial frictions in macroeconomic models. Carlstrom and Fuerst (1997) implement a friction generated by costly monitoring of borrowers, and focus on the effects of a shock to net worth, as in this paper. Jermann and Quadrini (2012) consider the effects of a friction generated by the possibility of default by firms, similar to the friction considered here, and find that the addition of this friction is important in explaining dynamics during the crisis. Christiano et al. (2010) find a more general result, weighing the quantitative effects of several frictions in the literature, finding their addition to generally improve the output of this class of models. For a good review of this literature, see Christiano et al. (2010) and Christiano and Ikeda (2011).

The paper proceeds as follows. I begin by presenting some stylized facts about the banking sector. The next section presents the model framework, describing its solution and discussing general properties. Then, I lay out the two quantitative exercises described above and briefly analyze them. A third section concludes.

2 Stylized Facts

All banks are not the same, and size appears to be a key determinant of differences in bank behaviors. In this section, I document three properties of financial institutions which vary systematically with size and play a central role in my model. I then revisit concentration in the US banking sector, along with two related trends that will be important for quantitative analysis. Because the data exhibits a strong time trend, I will also display values for 1992 and 2007, the two reference years I consider in the quantitative analysis.

First, bigger banks intermediate a lot more funds than small banks. The first row of table 1 shows the differences in the volume of loans performed by banks of different sizes calculated from Federal Reserve Bank of St. Louis (2013), where large is defined as the largest 25 banks by asset size. With respect to commercial and industrial loans in isolation or all loans and leases in general, the largest banks lend twice as much as the smallest.[3] On the deposit side, banks also perform more intermediation activity than small banks; again, large banks receive twice as many deposits as small banks.

Second, bigger banks tend to be more leveraged than smaller ones. Table 1 shows the inverse of the tier 1 leverage ratio, calculated for all bank holding companies by the Federal Reserve Bank of New York (2009). The measure in the graph is the ratio of the bank's risk-weighted assets, where assets are weighted by their regulator-determined credit risk, to the bank's tier 1 capital, which consists of a bank's equity and its retained earnings. Expressed in this way, larger values of this ratio imply that less of a bank's asset holdings are funded through the bank's equity; in this sense, leverage is a measure of the bank's vulnerability to downturns. Both before and after the onset of the financial crisis, large banks were more leveraged than smaller banks. In the run up to the financial crisis, leverage increased for all banks, and in the midst of the crisis, leverage decreased substantially. This partly

[3] Literature since the financial crisis focusing on systemic risk, and more recently Bremus et al. (2013), point out the degree to which this is true. A key component of these argument is that the largest banks are so large that they can individually influence macroeconomic outcomes. The difference in systemic importance is another key difference between banks of different sizes - for example, that because they are systemically important, big banks will be able to rely on implicit guarantees of government bailouts in the event of a crisis. (See Admati and Hellwig (2013) for a good discussion of this.) I do not consider this point here, but plan to address it in future work.

Table 1: Large and Small Bank Differences: Intermediation Volume and Leverage

Volume of Intermediation

Measure	Large Banks	Small Banks	Ratio Large to Small
All Loans	2314	1210	1.91
Commercial and Industrial Loans	497	244	2
Total Deposits	2516	1318	1.9

Leverage

Measure	Large Banks	Small Banks
Assets/Equity		
Average	13.1	11
1992	17	13
2007	10	9.5
Assets/Tier 1 Capital		
Average	15.4	12.7
1996	14.8	11.9
2007	17	14.3

Note: Top panel: loans, deposits, and interbank loans in units of billions of US dollars for large and small commercial banks, where large is defined as the largest 25 banks by asset size. Averages calculated from monthly data over the period April 1988 Q1 to 2013 Q3. Recessions are defined by NBER recessions. Data are also displayed in 1992 and 2007, the reference years considered in the quantitative analysis, to show the effects of industry trends.
Bottom panel: ratio of total assets to equity for large and small commercial banks, where large is defined as banks with asset size larger than \$20 bn, for quarterly data over the period April 1985 to September 2013. Next, ratio of risk-weighted assets to tier 1 capital for large and small banks, where large is defined as all banks with asset size larger than \$500 bn, for quarterly data from 1996 Q1 to 2013 Q2.
Sources: *Label - Series Name*
Top Panel: All Loans - Loans and Leases from Bank Credit, Large/Small Domestically Chartered Banks; C+I Loans - Commercial and Industrial Loans, Large/Small Domestically Chartered Banks; Total Deposits - Deposits, Large/Small Domestically Chartered Banks
Bottom Panel: Assets/Equity Large - inverse of Total Equity/Total Assets, Banks with Total Assets over \$ 20B [EQTA5]; Assets/Equity Large - Total Equity/Total Assets, Banks with Total Assets less than \$ 20B [EQTA1-4]
Data from Board of Governors of the Federal Reserve System, retrieved from Federal Reserve Bank of St. Louis (2013).
Assets/Tier 1 Capital - inverse of leverage ratio, from Federal Reserve Bank of New York (2009).

reflects the tightening of borrowing constraints (margins) banks faced when the value of many assets was deemed uncertain.[4]

Banks rely on interbank lending and short-term debt to fund their day-to-day operations. With respect to interbank lending, there are several studies that indicate that when interbank liquidity markets stop functioning, the real economy suffers. Ivashina and Scharfstein (2010) finds that banks that relied less on interbank liquidity reduced their lending to nonbank borrowers in the wake of the financial crisis. Puri et al. (2011) find that banks that were affected worse by liquidity shortages rejected more potential borrowers than banks that weren't.

Big banks tend to rely more on short-term liquidity markets than small banks do. With respect to interbank lending, Furfine (1999) finds that net borrowers of Fed funds tend to be larger in asset size than net lenders of funds. Cocco et al. (2009) finds a similar result in the Portuguese interbank lending market, finding that larger banks borrow more often and borrow more when they do. With respect to repo, a form of securitized lending banks use in a similar way as interbank loans, Fecht et al. (2011) find that, in auctions for repo, banks that bid for repo were larger than non-bidders.

Finally, I document a few more facts about the increasing concentration observed in the banking sector over time. A primary objective of this paper is to understand whether concentration has made downturns worse or better. The answer to this question will depend crucially on how concentration manifests itself.

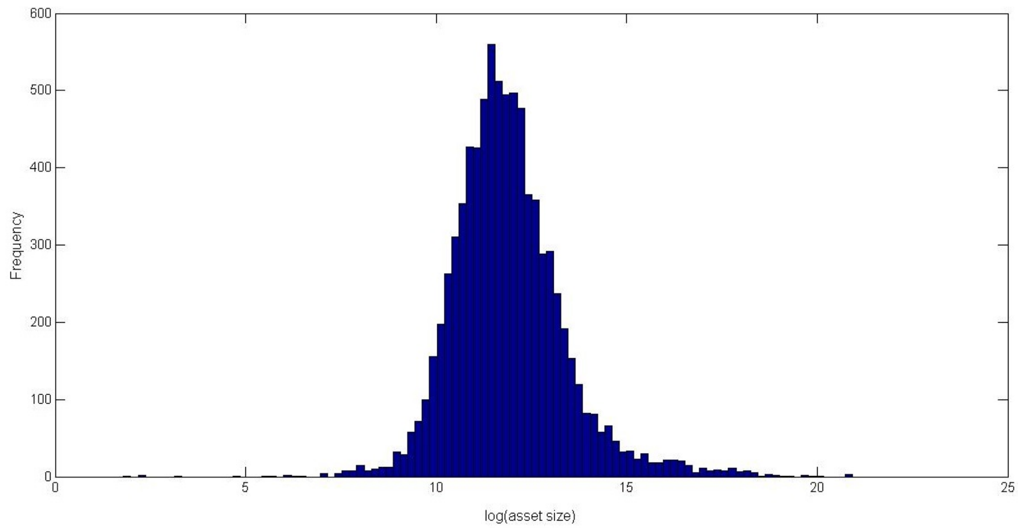
Increasing concentration is typically characterized by an increase in the share of assets held by the largest banks. However, for nearly three decades in the US, this trend has been accompanied in the data by two other relevant trends. First, banks have become more dispersed by asset size, that is, the size of the biggest and smallest banks have become more extreme relative to the mean. Second, there are relatively more big banks in the banking sector today, that is, the bank size distribution has become more skewed to the right.

Though it has changed over time, the distribution has maintained three typical characteristics: it has a large mass of small banks, so the the left side of the distribution is heavy, and it has a long right tail, representing a few very large banks. Figure 2 is a histogram of banks in 2007 by their log asset size. Janicki and Prescott (2006) considers similar bank size distributions over time, and concludes that the distribution is best captured by a lognormal distribution with a Pareto tail. [5]

[4] See Brunnermeier (2009) for a detailed explanation of this mechanism and its consequences.

[5] For computational convenience, I will model the bank size distribution with a truncated Pareto distribution in the quantitative section.

Figure 2: Typical Bank Size Distribution



Note: frequency plot of banks by their log(asset size) in 2007, where asset size is measured in thousands of US dollars. Calculated from call reports FFIEC 031 and 041, retrieved from Federal Reserve Bank of Chicago (2013).

In answering this question, I choose to examine the banking sector in two reference years, one to represent a low concentration sector, another to represent a high concentration sector. I choose 1992 as the reference low concentration year for several reasons.[6] I choose 2007 as a reference high concentration year for comparison to the financial crisis.

Table 2 shows the relevant facts about the size distribution in each of the reference years. This increase corresponds to an increase in the share of total assets held by the top banks. The overall upward trend is robust to changes in the definition of concentration; as we see in the table, whether measured with the top 1% share, top 10% share, or the GINI coefficient, concentration has increased.

As the banking sector has become more concentrated, it has also become more dispersed and heavier in the right tail. Dispersion is quantified in two ways in the table: first, in the

[6] In figure 1, we saw that the share of assets held by the largest 1% of banks slightly decreases over the 1976-1992 period, and then increases until 2010. This apparent trend break in concentration is accompanied by trend breaks in dispersion, or the standard deviation relative to the mean, and skewness/kurtosis, which for the bank size distribution are synonymous with an increase in the fatness of the right tail. In both cases, these roughly move with concentration over the period - between 1976 and 1992, these measures decrease, while between 1992 and 2010, these measures increase. Historically, this roughly coincides with the end of the savings and loan crisis, which took place during the mid-1980s and early 1990s. During this period, many smaller commercial banks faced competitive pressure from savings and loans, leading to a reduction in the number of small banks.

Table 2: Increasing Concentration and Related Trends

Measure	1992	2007
Concentration		
Top 1% Share of Assets	0.47	0.72
Top 10% Share of Assets	0.81	0.91
GINI Coefficient	0.845	0.92
Dispersion		
Coeff. of Variation	7.65	15.6
Interquartile Range	97063	180933
Right Tail		
Skewness	33.7	39.49

Note: measures of concentration, dispersion, and skewness calculated from the distribution of total assets across banks in 1992 and 2007. Assets and interquartile range measured in thousands of US dollars. (Interquartile range in 2007 adjusted to 1992 dollars using series GDP Implicit Price Deflator in the US, from OECD, retrieved from Federal Reserve Bank of St. Louis (2013).) Data from call reports FFIEC 031 and 041, retrieved from Federal Reserve Bank of Chicago (2013).

coefficient of variation, or the ratio of the standard deviation to the mean, and second, in the interquartile range, or the difference in size between the banks at the 25th and 75th percentiles in size.[7] Both measures have increased, indicating that banks have become more dispersed over time. The number of large banks has also increased relative to the number of smaller banks. This is indicated by an increase in the sample skewness over the period. The bank size distribution typically has a large mass on the left side and a long right tail. Therefore, the increase in the skewness indicates an increase in the mass of the right tail of the size distribution.

3 Model

I consider an infinite-horizon economy comprised of a continuum of islands $a \in [0, 1]$. On each island, there are many households that supply labor and save funds, many firms which produce consumption goods from capital goods and labor, and many banks which intermediate funds from households and lend them to the firms. Households and firms will be identical across islands, but banks will not.

[7]Because banks are generally larger today than yesterday, the variance has increased over time, the interquartile range (measured in units of dollars of assets) has increased much more than the coefficient of variation (a unitless quantity).

Following Gertler and Kiyotaki (2010) and Gertler and Karadi (2011), households and bankers on each island will belong to the same family, or economy-wide household. This induces two desirable features in the model: households will own banks, and receive dividends that bankers pay out. Second, households will insure each other against consumption risk across islands. This setup is maintained to ensure that we can represent the household side of the model with a single, economy-wide representative household. All the heterogeneity in the model will be isolated in the banking sector.

Banks will differ in a single dimension, the quantity of assets they can invest in with a given dollar of deposit; I call this intermediation ability. Ability differences will generate differences in investment demands across islands, which will be satisfied through the reallocation of funds through an interbank lending market. Reallocation will be imperfect, however, and this imperfection will generate size differences between banks. In order to abstract away any distributional consequences other than what I am interested in, I will specify the model in a way that the size distribution does not endogenously evolve over time.

I will denote a single island by a, and an ability type by κ. I will refer to the quantity x on island a with $x(a)$. The mass of any set of islands A will be given by the measure $\mu(A)$.

3.1 Banks

There are many potentially infinitely lived, risk-neutral banks on each island. These banks raise deposits from households on the same island, raise interbank loans from banks on all other islands, and invest in the firms on the same island. All banks on the same island are identical, but banks on different islands are not.

Banks intermediate between households and firms, and differences between banks arise purely through differences in intermediation ability. After they receive deposits but before they borrow interbank loans, all the banks on an island a receive a random ability draw $\kappa(a)$. Draws are assumed to be distributed with cumulative distribution function $\kappa(a) \sim F(\cdot)$. I make several assumptions on the distribution of draws:

Assumption 1. *The random ability draws for each island a in each period t, $\kappa(a)$, are independent and identically distributed over time, where $\kappa(a) \sim F(\cdot)$ has the following properties:*

- $E(\kappa(a)) = 1$
- *Draws have bounded support, so that $\kappa(a) \in [\underline{\kappa}, \overline{\kappa}]$.*

- *The distribution of draws F admits a density function, denoted $p(\cdot)$.*

Banks with higher ability get better returns per dollar invested in firms[8]. I introduce ability into the model as a productivity term in a simple bank production function, so that if banks are producing assets S valued at price Q from liabilities and net worth L:

$$QS = \kappa L \tag{1}$$

Banks in this model are intermediaries, taking deposits from households and investing them as loans to firms. I implement size heterogeneity in this model through heterogeneity in productivity differences in this intermediation. [9]

In this model, big banks are big because they are better. As we will see later, it will turn out that they are also more leveraged because they are better. This represents a decidedly neutral, benign stance regarding big banks. For example, I could have instead assumed that managers at big banks prefer risk more than managers at small banks, as has been suggested since 2008.[10] I take this stance for two reasons. First, it is easier to implement this stance in the model. Second, any negative consequences of concentration that my model generates will represent a lower bound for more general cases where large banks prefer risk.

Banks raise deposits in an economy-wide deposit market. At the time they visit the deposit market, banks are identical, so that all island representative banks offer the same deposit rate R_t to raise deposits D_t from all households. Every bank will thus demand the same quantity of deposits, which means the banks on each island will hold $d_t \equiv d_t(a) = \mu(a)D_t$.

Ability increases bank demands for investment. Because deposits are made before ability is realized, deposits will not be allocated according to ability, and therefore a reallocation of funds after ability is realized can improve the profits of all banks. This reallocation is the primary function of the interbank lending market.

[8] Ability differences, and their correlation with size, can reflect a number of different forces: bigger banks provide more services (e.g. consulting services, business contacts) that firms value, but don't manifest as differences in loan interest rates. These services reduce costs for firms, and the firms realize higher returns on projects as a result. Bigger banks may also have more productive investment hunters, so that per hour of bank employee labor, bigger banks find a higher number of investment opportunities.

[9] There is some evidence to suggest that such differences exist in reality. Rezitis (2006) considers the production of assets from deposits, labor, and capital in the Greek banking sector, and finds that large banks enjoy higher productivity than small banks. Altunbas et al. (2001) views bank productivity as the quantity of assets and deposits generated for a given level of labor hours and capital, and finds that banks enjoy economies of scale in this type of production.

[10] Perhaps the most widely discussed explanation for this is that too big to fail policies, or the implicit guarantee of a government bailout in the event of a crisis, influences the behavior of bank managers and executives. See Admati and Hellwig (2013) for a good explanation of this.

Banks on any island are able to borrow or lend to banks on any other island. Since there are many banks on each islands, instead of characterizing loans between individual banks, I will be interested in the net loans made between all banks on one island with all banks on another. Ability and amount borrowed are observable, so we can characterize all interbank loans with a contract specifying the interest rate and the borrowing quantity, $(R_{bt}, b_t(a))$, where a is the island where the borrower banks live.[11]

Banks lend a mixture of capital and consumption goods to the firms on the same island. Firms can use these loans to buy new capital goods in period t, and then use the old and any new capital to produce consumption goods in period $t+1$. The firm then gives the bank its output (less wages) along with any undepreciated capital goods, also at time $t+1$.

Banks are potentially infinitely-lived, but face an incentive constraint that I will present below. In order to prevent the bank from saving its way out of this constraint, a constant proportion σ of banks on each island exit every period. Upon exit, a bank transfers its earnings to the household on its island. Second, new banks enter to replace the old banks; in order to ensure that these banks have something to invest with, these banks receive a "start-up transfer" equal to a constant fraction ξ of the total assets held by all banks on the island.

Banks carry wealth from period to period in the form of net worth, defined as the payoff from assets less deposits and interbank loans. The net worth of the island a representative bank at time t is given by:

$$n_t(a) = [Z_t + (1-\delta)Q_t(a)]\psi_t(\sigma+\xi)s_{t-1}(a) - R_{t-1}d_{t-1}(a) - R_{bt-1}b_{t-1}(a) \qquad (2)$$

The first term gives the bank's returns on investments in the firm: Z_t is the economy-wide representative firm's gross profits from investments (per unit invested), $Q_t(a)$ is the price of capital on island a, and s_{t-1} is the units of capital held by the firm in period $t-1$. The second term gives the bank's repayments for deposits and interbank loans: b_{t-1} is the funds borrowed on the interbank market last period, and d_{t-1} is the deposits made by households last period.

ψ_t is a shock to the quality of capital, and the key source of uncertainty in this model; it typically takes value 1. This kind of shock is crucial to the model, because in order to

[11] More precisely, an interbank loan contract is bilateral, that is, we should specify $(R_{bt}(a,a'), b_t(a,a'))$, where a, a' are the island where the borrower and lender banks live, respectively. However, since there are a continuum of potential lenders, lenders compete with each other for borrowers across all islands. All potential lenders then offer the same interest rate, since otherwise a borrower would go to another lender. The borrowed amount will be borrower-dependent because of the financial friction I describe below. Then the debt contract will have the property that $(R_{bt}(a,a'), b_t(a,a')) = (R_{bt}, b_t(a))$.

precipitate a "financial crisis", we need a way to exogenously affect the value of net worth. In addition, because the price of capital is endogenous in this model, this shock serves as an exogenous trigger for the type of asset price dynamics that were an important feature of the recent downturn.

Without intermediation ability, banks would balance the dollars invested in assets with the dollars taken as liabilities and equity, i.e. deposits, interbank loans, and net worth. With intermediation ability, banks balance assets against the intermediated value of liabilities and equity. We can summarize the balance sheet of the bank with a flow of funds constraint:

$$Q_t(a)s_t(a) = \kappa_t(a)\left[n_t(a) + d_t(a) + b_t(a)\right] \tag{3}$$

In every period, banks are supposed to repay depositors and banks they borrowed from in the previous period. Instead of repayment, however, banks can choose to default on their loans, in which case they take a fraction $\theta \in [0,1]$ of the total funds $Q_t(a)s_t(a)$ and exit forever. If a bank chooses to default, it does so at the very end of a period, and sells its shares to another bank on the island. Depositors and potential lender banks know this, and though default will not occur in equilibrium, it will impose a limit on the amount of interbank loans any bank can borrow in a period.[12] On the other hand, there will be no friction between banks and firms on the same island: banks will be able to enforce full repayment of loans made to firms.

We can now state the bank's maximization problem. Bank managers choose a quantity of deposits, loans to firms, and interbank loans today to maximize the expected present value of future dividends. Households are bank owners, and the banker internalizes this when calculating the expected value of dividends, discounting it by $\Lambda_{t,t+i}$, the stochastic discount factor of the economy-wide representative household.[13] At time t, the bank formulates a plan for the path of deposits, assets, and interbank loans. The plan is state-contingent, so that it chooses a different level of deposits d_t for each value of aggregate shock it faces in that period, ψ_t, and a different level of $s_t(a)$ and $b_t(a)$ for each level of aggregate shock and ability shock $\kappa_t(a)$ it receives in that period.

[12] I interpret this "running away" as capturing a bankruptcy cost: if a borrower bank decides to declare bankruptcy, creditors can capture some, but probably not all, of the repayments they are owed. Despite the perhaps unrealistic form of the constraint, it generates the more realistic property that better banks can borrow more.

[13] As mentioned in the setup, households insure each other against differences in consumption across islands.

$$V_t(d_{t-1}, s_{t-1}(a), b_{t-1}(a)) = \underset{\{d_{t+i},(s_{t+i}(a_i))_a,(b_{t+i}(a))_a\}_{i=0}^{\infty}}{max} E_t \sum_{i=1}^{\infty} (1-\sigma)\sigma^{i-1}\Lambda_{t,t+i} n_{t+i}(a_{t+i}) \quad (4)$$

$$s.t.\ V_t \geq \theta Q_t(a) s_t(a) \quad (5)$$

Note that $n_t(a)$ is a state variable here. At the time the bank makes its deposit demand decision, it only knows the expected value of its net worth, but at the time it makes its asset and interbank loan decisions, it knows its net worth precisely.

The constraint in the above problem is an incentive constraint imposed by the bank's ability to default. In equilibrium, the continuation value of staying on the equilibrium path has to exceed the dollar value of assets the bank can run away with.

We can rewrite the above problem in terms of a Bellman equation:

$$V_t = \underset{d_t}{max}\, E_t \underset{s_t(a),b_t(a)}{max} E_{t,a}\Lambda_{t,t+1}\left[(1-\sigma)n_{t+1}(a) + \sigma \underset{d_{t+1}}{max}\left(\underset{s_{t+1}(a),b_{t+1}(a)}{max} V_{t+1}\right)\right] \quad (6)$$

$$s.t.\ V_t \geq \theta Q_t(a) s_t(a) \quad (7)$$

3.1.1 Additional Assumptions

I make two additional assumptions to ensure a tractable solution to the model. First, I make the ability distribution in any period independent of the ability distribution in previous periods. Even if ability draws are independent across periods, the interbank loans from the previous period will make the expected returns from assets, or the ratio of net worth to capital, unequal. In this case, the history of previous draws will matter for bank returns today. To prevent this, I follow Gertler and Kiyotaki (2010) and allow individual banks to arbitrage these return differences away, before their new ability type is realized:

Assumption 2. *At the beginning of a period, banks can move between islands to equalize expected returns on assets.*

To see how this works, consider two islands, a_L and a_H. a_L has low expected returns when it enters the period, that is, the representative bank has high interbank debt obligations, and a_H has high expected returns. When given the opportunity, an individual bank on a_L decides to move to a_H. It currently holds assets (investments) in firms on a_L, interbank debt to banks on a_H, and deposits (economy-wide). Before it moves, it trades its assets with another bank on a_L for more interbank debt to a_H. It then takes its net worth and deposits and moves; this reduces the interbank debt of a_L to a_H but maintains the total asset held in a_L firms. In equilibrium, this process will continue until returns are equalized.

Another assumption has to do with the capital goods that carry over from previous periods. Intermediation ability also applies to the undepreciated capital that banks carry between periods - banks re-allocate, or re-intermediate, the undepreciated portion of the capital stock among firms on the same island. In equilibrium, banks on low ability islands will only invest enough to maintain the undepreciated capital stock. Re-intermediation of capital will leave some islands with higher production capacity than others. In order to ensure that we can maintain a simple law of motion for capital, I assume that these changes to production capacity can be realized as gains in the output of firms in the same period:

Assumption 3. *When an island's existing capital $k_t(a)$ is intermediated, any gains (losses) in the production possibilities of that capital are realized as an increase (decrease) in output of all island firms $y_t(a)$ in the same period.*

If we think of re-intermediation as bankers giving management advice to firm owners, this assumption essentially requires that any extra productivity realized by the firm gets paid to the banker as a consulting fee in the same period that the advice was given.

3.2 Households

Households on each island a are infinitely-lived, supply up to one unit of labor per period, save in the form of deposits, and consume consumption goods. The household makes its deposit and labor supply decisions before the banks on its island realize their ability. Because households across islands are members of the same family, we can represent the decisions of each household with an economy-wide representative household.

Households on each island save by making riskless one-period deposits D_t in the economy-wide deposit market[14] at the interest rate R_t. Deposits made last period are repaid at the beginning of this period.

I make another simplifying assumption on labor:

Assumption 4. *Workers can supply labor on any island.*

With the above, wages will be identical across islands - call this economy-wide wage W_t. The household also owns the bank, and receives dividends from exiting banks every period Π_t. Then the household's budget constraint is

[14] We could alternatively restrict the household to only making deposits in the banks on their island; with the timing assumptions below, nothing about the model changes. I maintain this formulation for ease of explanation.

$$C_t = W_t L_t + \Pi_t + R_{t-1} D_{t-1} - D_t \tag{8}$$

The household's maximization problem becomes

$$\underset{(C_t, L_t, D_t)_{t=0}^{\infty}}{max} E_t \sum_{i=0}^{\infty} \beta^i \left[ln(C_{t+i} - \gamma C_{t+i-1}) - \frac{\chi}{1+\varphi} (L_{t+i})^{1+\varphi} \right] \tag{9}$$

$$s.t. \; C_t = W_t L_t + \Pi_t + R_{t-1} D_{t-1} - D_t \text{ (for each } t) \tag{10}$$

where $\beta \in (0,1)$ is the discount factor and $\gamma \in [0,1)$ is a habit formation parameter[15]. φ is the inverse Frisch elasticity of labor supply.

Taking first order conditions, we first work out a condition for aggregate deposits:

$$R_t (E_t \Lambda_{t,t+1}) = 1 \tag{11}$$

and a condition for aggregate labor:

$$W_t E_t (u_{Ct}) = \chi (L_t)^{\varphi} \tag{12}$$

where $u_{Ct} \equiv (C_t - \gamma C_{t-1})^{-1} - \beta \gamma (C_{t+1} - \gamma C_t)^{-1}$ is the marginal utility of consumption and $\Lambda_{t,t+1} \equiv \beta \frac{u_{Ct+1}}{u_{Ct}}$ is defined as the household's stochastic discount factor.

3.3 Firms

There are many identical, competitive firms on each island. Immediately after the bank's ability is realized, the firm uses the capital it held from last period to produce output. It makes its loan repayment to the bank, and the bank then makes its loan to the firm for next period production.

Coming into the period, there is capital $k_t(a)$ on island a. The firm's problem then reduces to one of choosing how much labor to input. The representative firm on island a chooses $l_t(a)$ to maximize:

$$y_t(a) = A_t k_t(a)^{\alpha} l_t(a)^{1-\alpha} \tag{13}$$

[15] These preferences exhibit habit formation when $\gamma \in (0,1)$, a feature that is included for comparison to other models in the literature. The model is not fundamentally different when we turn off habit formation, i.e. set $\gamma = 0$.

where A_t is the (economy-wide) total factor productivity of the firm.[16] Since labor is mobile, firms on every island face the same wage W_t. Firms will choose labor to equate the economy-wide wage with the marginal product of labor:

$$W_t = (1-\alpha)\left(\frac{y_t(a)}{l_t(a)}\right) = (1-\alpha)\frac{Y_t}{L_t} \tag{14}$$

where the second inequality also follows from the fact that the ratio of capital to labor is constant across islands. Thus, to find the optimal capital/labor ratio on each island, we need only solve the economy-wide representative firm's problem:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{15}$$

Even though the ratio of net worth to capital is identical across islands by assumption 2, the capital on each island is not known. As a result, the size distribution of firms across islands in equilibrium will be indeterminate.

After production happens, capital depreciates, leaving $(1-\delta)k_t(a)$ on the island. As part of the repayment to banks, firms transfer ownership of this capital to banks. Banks then decide on the loan package $s_t(a)$, which consists of this undepreciated capital and (possibly) of cash (basic goods) for new investment, which I denote by $i_t(a)$. This implies that, for the asset markets on each island to clear, it should be the case that

$$s_t(a) = (1-\delta)k_t(a) + i_t(a) \tag{16}$$

3.4 Capital Goods Producers

When firms decide to expand their existing capital stock, they travel to a central (economy-wide) market for new capital. The market is perfectly competitive, but all producers face adjustment costs. Capital goods producers sell new capital to firms for the price Q_t^i. These producers then choose I_t to maximize

$$E_t \sum_{\tau=t}^{\infty} Q_\tau^i I_\tau - \left(1 + f(\frac{I_\tau}{I_{\tau-1}})\right) I_\tau \tag{17}$$

From the FOC for this profit maximization problem, the new capital price should satisfy

[16] This will be constant and equal to 1 throughout the paper, though TFP shocks can be induced in the model through this channel.

$$Q_t^i = 1 + f\left(\frac{I_t}{I_{t-1}}\right) + \left(\frac{I_t}{I_{t-1}}\right) f'\left(\frac{I_t}{I_{t-1}}\right) - E_t \Lambda_{t,t+1} \left(\frac{I_{t+1}}{I_t}\right)^2 f'\left(\frac{I_{t+1}}{I_t}\right) \tag{18}$$

3.5 Other Conditions

I'll add another assumption to ensure that the law of motion for capital has a simple form. (In equilibrium, this will also lead to more able lenders lending more than less able lenders.)

Assumption 5. *Once installed, existing capital cannot be used on any other island.*

With this assumption, banks will always find it optimal to reinvest their capital stock. This also implies that capital (asset) prices on each island are not necessarily the same. Denote the price of capital on island a by $Q_t(a)$.

When we combine the household budget constraint with the equation for firm output and new capital, we can write an economy-wide resource constraint for basic goods:

$$Y_t = C_t + \left(1 - f\left(\frac{I_t}{I_{t-1}}\right)\right) I_t \tag{19}$$

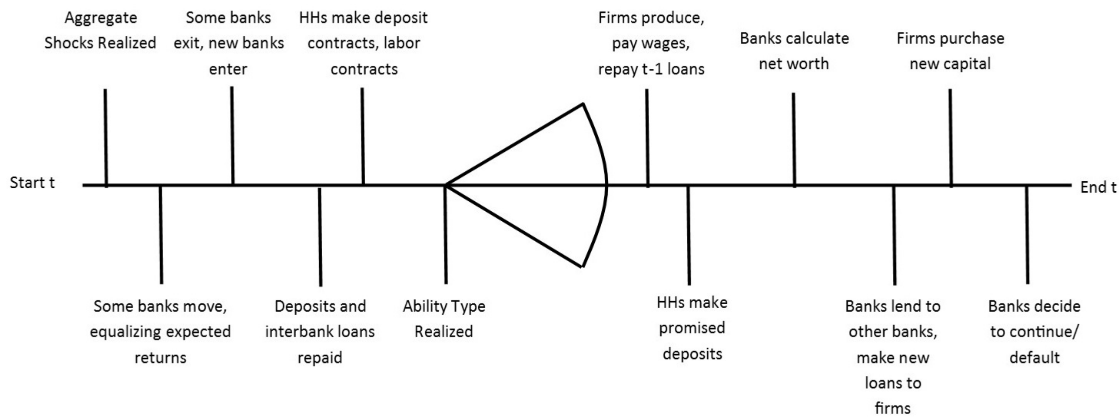
With the assumptions above, we can write a simple law of motion for capital:

$$K_{t+1} = \psi_{t+1} \left(I_t + (1-\delta) K_t\right) \tag{20}$$

A period is divided into two parts: before and after intermediation ability is realized. Deposits are made in the first part, while firm loans, interbank loans, and investment purchases are made in the second. Figure 3 gives the timing of the model. First, the aggregate shocks ψ_t and A_t are realized. Banks then move to equalize expected returns, and some banks exit exogenously. Banks then pay dividends to households, and new banks enter in their place. Deposits and interbank loans are then repaid, and households make their deposit and labor supply decisions.

The second half of the period begins when intermediation ability types are realized. Firms use household labor and capital to produce output, then pay profits Z_t to banks and wages W_t to workers. Firms borrow from banks, and those that want to expand their capital stock use the loans to purchase capital from the capital goods producers on the central island. Simultaneously, banks borrow from other banks. Banks then make their plan for the future, deciding in the process whether to default between periods.

Figure 3: Timing



3.6 Bank Solution

As mentioned above, the size distribution of firms across islands will be indeterminate in equilibrium. To get around this, I will define and solve a version of the model aggregated to the level of ability type. To make things easier, I will first characterize the solution to the bank's maximization problem.

To solve the bank's problem, I'll follow a method similar to that presented in Gertler and Kiyotaki (2010). First, solve the bank's problem for the difference between the returns from investment and the deposit interest rate; there will be at least one such spread which clears the interbank lending market. (I will focus on situations where only one such spread exists.) Once this spread is known, solve the household and firm problems; only one combination of spread and deposit interest rate will solve the problems of these agents as well. We'll then solve for the remaining quantities by aggregating up to the ability type level.

The bank on island a maximizes net worth, which itself is a linear function of assets, deposits, and loans. Because of this property, the bank's value function will also turn out to be linear. The problem then boils down to solving for the coefficients of this value function.

First, guess that in every period a bank guided by a some linear function of shares, deposits, and interbank loans will maximize its expected net worth; we'll verify this guess later. Specifically:

$$V_t = \nu_{st} s_t(a) - \nu_{bt} b_t(a) - \nu_t d_t \tag{21}$$

In what follows, I restrict attention to equilibria with interior solutions to the above guess, so that banks can only hold nonzero quantities of all choice variables - only these equilibria will have positive interbank lending.

Next, use the guess to set up a Lagrangian for the bank's problem. First, substitute the flow of funds constraint into the guess to reduce the number of choice variables to two:

$$
\begin{aligned}
V_t &= \nu_{st} s_t(a) - \nu_t d_t - \nu_{bt} b_t(a) \\
&= \frac{\kappa(a)\nu_{st}}{Q_t(a)}(n_t(a) + d_t + b_t(a)) - \nu_t d_t - \nu_{bt} b_t(a) \\
&= \frac{\kappa(a)\nu_{st}}{Q_t(a)} n_t(a) + (\frac{\kappa(a)\nu_{st}}{Q_t(a)} - \nu_t) d_t + (\frac{\kappa(a)\nu_{st}}{Q_t(a)} - \nu_{bt}) b_t(a)
\end{aligned} \tag{22}
$$

Note again that $n_t(a)$, the bank's net worth, is a state variable. Next, substitute the guess into the incentive constraint, and use it to construct a Lagrangian:

$$
\begin{aligned}
L(d_t, b_t(a)) = &\left((1+\lambda_t(a))\frac{\kappa(a)\nu_{st}}{Q_t(a)} - \lambda_t(a)\kappa(a)\theta\right) n_t(a) \\
&+ \left((1+\lambda_t(a))(\frac{\kappa(a)\nu_{st}}{Q_t(a)} - \nu_t) - \lambda_t(a)\kappa(a)\theta)\right) d_t \\
&+ \left((1+\lambda_t(a))(\frac{\kappa(a)\nu_{st}}{Q_t(a)} - \nu_{bt}) - \lambda_t(a)\kappa(a)\theta\right) b_t(a)
\end{aligned}
$$

where $\lambda_t(a)$ is the multiplier on the incentive constraint in period t for the representative bank from island a.

We obtain the following first order conditions for d_t and $b_t(a)$:

$$\nu_t = \nu_{bt} \tag{23}$$

$$\kappa(a)\theta \frac{\lambda_t(a)}{1+\lambda_t(a)} = \kappa(a)\frac{\nu_{st}}{Q_t(a)} - \nu_{bt} \tag{24}$$

Using the above, we can also rearrange the borrowing constraint into a form that will prove useful later. If $\bar{b}_t(a)$ is the maximum a bank will borrow, then

$$b_t(a) \leq \bar{b}_t(a) = \frac{\frac{\kappa(a)\nu_{st}}{Q_t(a)} - \kappa(a)\theta}{\nu_{bt} - \frac{\kappa(a)\nu_{st}}{Q_t(a)} + \kappa(a)\theta} n_t(a) - d_t(a) \equiv \phi_t(a) n_t(a) - d_t(a) \tag{25}$$

$\phi_t(a)$ is an important quantity. It is related to the leverage ratio (denoted $L_t(a)$), or the ratio of shares held to net worth; $L_t(a) = \kappa(a)(1 + \phi_t(a))$. This means $L_t(a)$ is a convex, increasing function of κ. This property arises because intermediation ability affects banks in two ways: first, intermediation ability decreases the cost of shares directly through its effect on the flow of funds constraint; second, ability increases the continuation value relative to the "running away" value, loosening the borrowing constraint.

If the guess is correct, the coefficients ν_{st} and ν_{bt} will be equal to the marginal value of shares and interbank loans. Shares produce profits from firms tomorrow, which increases the value of tomorrow's net worth. Deposits and interbank loans today reduce tomorrow's net worth through repayment. Because net worth is a linear function of each of these quantities, this marginal value will be constant. Calculating the value of these coefficients amounts to calculating the marginal effect of increasing these quantities on net worth.

Note that the above Lagrangian is also equal to the bank's maximized value written in terms of net worth. Substituting the FOCs into the Lagrangian, we get

$$V_t = (1 + \lambda_t(a))\nu_{bt} n_t(a) \tag{26}$$

If we iterate this one period forward and then plug this into the Bellman equation, we obtain:

$$\nu_{st} s_t(a) - \nu_t d_t - \nu_{bt} b_t(a) = E_{t,a'} \Lambda_{t,t+1} (1 - \sigma) n_{t+1}(a') + \sigma(1 + \lambda_{t+1}(a'))\nu_{bt+1} n_{t+1}(a') \tag{27}$$

We can obtain the value of each coefficient by taking the partial derivatives of both sides of the above equation with respect to each of the variables $b_t(a)$, d_t, $s_t(a)$:

$$\nu_{bt} = R_{bt} E_{t,a'} \Lambda_{t,t+1} \Omega^{a'}_{t+1} \tag{28}$$

$$\nu_t = R_t E_{t,a'} \Lambda_{t,t+1} \Omega^{a'}_{t+1} \tag{29}$$

$$\nu_{st} = E_{t,a'} \Lambda_{t,t+1} \psi_{t+1} (\Omega^{a'}_{t+1} (Z_{t+1} + (1 - \delta) Q^{a'}_{t+1}) \tag{30}$$

where

$$\Omega^{a'}_{t+1} = 1 - \sigma + \sigma(1 + \lambda_{t+1}(a'))\nu_{bt+1} \tag{31}$$

As long as ability draws next period are independent of the draw this period, these coefficients do not depend on ability type or level of either of the choice variables. They also maximize the value of the original objective function (4):

$$V_t = E_{t,a'} \sum_{i=1}^{\infty} (1 - \sigma)\sigma^{i-1} \Lambda_{t,t+i} n_{t+i}(a') \tag{32}$$

To see this, consider the effect of d_t on the original bank objective function. d_t only affects the bank objective when the bank exits. A fraction $(1-\sigma)$ of banks exit from each island every period. An increase in d_t affects net worth in period $t+1$ directly, by increasing the repayment in that period. Thus, banks that exit in period $t+1$ will have lower net worth. The decrease in $t+1$ net worth decreases s_{t+1} through the flow of funds constraint, which turn reduces the net worth in period $t+2$; if banks exit then, they will also have lower net worth. The full effect of the change in deposits can then be written as

$$\frac{dV_t}{dd_t} = (1-\sigma)\frac{\partial n_{t+1}}{\partial d_t} + \sigma(1-\sigma)\frac{\partial n_{t+1}}{\partial d_t}\frac{\partial s_{t+1}}{\partial n_{t+1}}\frac{\partial n_{t+2}}{\partial s_{t+1}} + \sigma^2(1-\sigma)\frac{\partial n_{t+1}}{\partial d_t}\frac{\partial s_{t+1}}{\partial n_{t+1}}\frac{\partial n_{t+2}}{\partial s_{t+1}}\frac{\partial s_{t+2}}{\partial n_{t+2}}\frac{\partial n_{t+3}}{\partial s_{t+2}} + ... \quad (33)$$

The partial effects $\frac{\partial s_{t+i}}{\partial n_{t+i}}$ are captured by the quantity $(1+\lambda_{t+i}(a'))\nu_{bt+i}$. This means that the coefficient ν_t completely summarizes the direct effect of d_t on the island objective function. The same argument holds for $b_t(a)$. Because of this, we can conclude:

Proposition 1. *A set of choices $(d^*_{t+i}, b^*_{t+i}(a))_{i=0}^{\infty}$ that maximizes the linear value function guess (21) for each i will also maximize the bank objective (32).*

3.7 Solution Properties

The linear solution has several useful properties. I will first use it to characterize the capital prices on each island a, then use that to characterize bank behavior in the interbank lending market. I will also mention two restrictions on parameters that ensure equilibrium existence.

Recall that, after production happens, the bank receives ownership of the existing capital stock, which it can then re-lend to the firms as part of a loan package $s_t(a)$. Islands that are not borrowing constrained have $\lambda_t(a) = 0$, which implies that they are indifferent between lending on the interbank market and re-lending. I will assume that, given indifference between these choices, the bank will always reinvest the existing capital stock on its island.

Assumption 6. $s_t(a) \geq (1-\delta)k_t(a)\ \forall a$

Further, from the FOC for $b_t(a)$ in the previous section, we can see that if the banks on island a are not borrowing constrained, it must be that the price of capital on its island satisfies

$$\frac{Q_t(a)}{\kappa(a)} = \frac{\nu_{st}}{\nu_{bt}} \equiv Q_t^n \quad (34)$$

Assumption 5 also implies that the price of capital on each island is free to adjust to a different value on each island. Make another guess which we can verify later:

$$Q_t(a) = \kappa(a)\frac{\nu_{st}}{\nu_{bt}} = \kappa(a)Q_t^n \;\; \forall a \in L \tag{35}$$

where L is the set of ability types for which banks are not borrowing constrained. This price essentially represents the shadow cost of re-lending existing capital - if a bank on island a were given an extra unit of basic good, it could either lend it on the interbank market, earning the bank value ν_{bt}, or it could turn it into $\frac{1}{Q_t(a)}$ units of capital, earning it $\frac{\nu_{st}}{Q_t(a)}$ value. The previous equation tells us that this shadow cost increases in intermediation ability, that is, re-investing the existing capital stock is more costly for more able banks. To put it another way, banks can demand higher repayments per unit capital lent to the firms on its island.

What about the value of existing capital on islands with banks that are borrowing constrained? The FOC for $b_t(a)$ also tells us that constrained banks view assets as more valuable than interbank lending, implying that these banks also desire higher investment. If these banks were indifferent between investing in assets and lending on the interbank market, as in the previous case, the price $Q_t(a)$ would get very large, as banks would demand ever higher returns per unit capital.

I maintain that firms will never allow the price $Q_t(a)$ to exceed the price for new capital, Q_t^i. This amounts to preventing banks from extracting too much from firms. Imagine a bank that wants to lend a unit of existing capital to a firm, but demands repayment $Q_t(a) > Q_t^i$ for a unit of this capital. Rather than agree to these terms, the firm instead threatens to go to another bank on the island which will give it a cash loan instead, which it could then use to buy new capital at the lower price Q_t^i. Since the firm is small, the threat is credible, and the bank should drop the price of the existing capital to Q_t^i as well. Thus, the price of capital on borrowing constrained islands gets pinned down:

$$Q_t(a) = Q_t^i \;\; \forall a \in B \tag{36}$$

where B is the set of islands on which banks are borrowing constrained.

Some sets of parameters will not admit equilibria with positive lending. I make two restrictions on parameters to prevent these cases. I will choose parameters so that these restrictions hold in steady state, and in the quantitative analysis below, I will choose the level of the shock to be small enough to ensure that the restrictions hold in the impulse responses.

First, we need to restrict $\underline{\kappa}$, the lowest possible intermediation ability, to ensure that banks on all islands can fund reinvestment of their entire capital stock. Renegotiation of the remainder of the capital stock means that each share of this portion costs $\frac{Q_t(\kappa)}{\kappa}$. For high productivity islands, this is a boon; the cost of these shares is smaller than that of the fixed capital. For low productivity islands, this is a burden; their poor managerial skill causes them to lose some of their net worth through this process. Thus, if we require that the entire existing capital stock will be reinvested, we must ensure that the ability of the worst bank is not so low that it cannot cover the cost. This is accomplished if we ensure that $\underline{\kappa}$ satisfies

$$(1-\underline{\kappa})Q_t^n(1-\delta)K_t \leq Z_tK_t - R_{t-1}D_{t-1} + D_t \tag{37}$$

In steady state, this equation becomes

$$(1-\underline{\kappa})Q^n(1-\delta) \leq Z + (1-\frac{1}{\beta})\frac{D}{K} \tag{38}$$

Second, the borrowing constraint equation only acts as an upper bound on borrowing if the denominator of $\phi_t(a)$ is positive, that is, $\nu_{bt} > \frac{\kappa\nu_{st}}{Q_t^i} - \kappa\theta$. Though equilibria exist if the inequality is reversed, all banks in these equilibria are unconstrained, which would make the incentive constraint useless. I avoid this case by choosing $\overline{\kappa}$ and θ so that

$$\theta > \frac{\nu_{st}}{Q_t^i} - \frac{\nu_{bt}}{\overline{\kappa}} \tag{39}$$

In steady state, this equation becomes

$$\theta > \nu_s - \frac{\nu_b}{\overline{\kappa}} \tag{40}$$

Given the linear solution, it will turn out that high ability banks will borrow, and be borrowing constrained, while low ability banks will lend. To see this, first note that the flow of funds constraint reduces the number of bank choice variables to two, and since deposits are chosen before ability type is realized, the only choice the bank makes after ability is realized is the level of interbank loans.

Proposition 2. *Consider an equilibrium with strictly positive lending. In each period t, there exists a κ_t^* such that:*

- *for all banks with $\kappa < \kappa_t^*$, $b_t(a) = -(n_t(a) + d_t - Q_t^n k_t(a))$, that is, the bank will lend* *its net worth and deposits less the cost of refinancing the entire existing capital stock on its island.*

- *for all banks with $\kappa > \kappa_t^*$, $b_t(a) = \overline{b_t}(a)$, that is, the bank will borrow up to its borrowing constraint and use the funds to purchase assets.*

Proof. Consider the term for borrowing in equation (22), and let us first assume

$$(\frac{\kappa\nu_{st}}{Q_t(a)} - \nu_{bt}) > 0$$

In this case, the bank gets positive value for every dollar it borrows, and it will choose to borrow as much as it can, i.e. until $b_t(a) = \overline{b_t}(a)$. Because all constrained banks face capital prices $Q_t(a) = Q_t^i$, this implies that $\kappa > \frac{\nu_{bt}}{\nu_{st}} Q_t^i$.

If $(\frac{\kappa\nu_{st}}{Q_t(a)} - \nu_{bt}) \leq 0$, the bank gets positive value for every dollar it lends (negative borrowing). By assumption 6, the bank will only lend funds left over after re-lending its existing capital stock, i.e. $b_t(a) = -(n_t(a) + d_t - Q_t^n k_t(a))$. We know that capital prices on these islands are smaller than the new capital market price, so $\kappa \leq \frac{\nu_{bt}}{\nu_{st}} Q_t(a) \leq \frac{\nu_{bt}}{\nu_{st}} Q_t^i$.

Call $\kappa^* \equiv \frac{\nu_{bt}}{\nu_{st}} Q_t^i$. Then, for any bank with ability $\kappa > \kappa^*$, borrowing and investing is strictly more profitable than lending on the interbank market. For any bank with ability $\kappa \leq \kappa^*$, lending is weakly more profitable than borrowing. $\square$

The ability differences between banks in this model generate differences in borrowing and investment behavior. The above proposition tells us that these differences are easily summarized with one equilibrium object, the ability cutoff.

It will also turn out that lower ability interbank lenders will lend less than higher ability interbank lenders.

Proposition 3. *Consider two banks on different islands with types $\kappa(a)$ and $\kappa(a')$ and $b_t(a), b_t(a') < 0$. If $\kappa(a) < \kappa(a')$, then $b_t(a) < b_t(a')$.*

Proof. The cost of refinancing the entire existing capital stock $k_t(a)$, net of the benefit derived in the form of net worth, is given by

$$\frac{Q_t(a)}{\kappa(a)} k_t(a) - Q_t(a) k_t(a)$$

Because of assumption 3, any differences in refinancing costs are reflected in output in the same period. This allows banks to lend the consumption good equivalent of the differences.

For unconstrained (lending) islands, this simplifies to

$$Q_t^n k_t(a)(1-\kappa(a))$$

This is a decreasing function of κ. Thus, for interbank lenders the cost of refinancing the capital stock is decreasing in κ. By the flow of funds constraint, the amount left over for lending to other islands is then increasing in κ. □

Since banks on islands with the same ability draw $\kappa(a)$ will make the same choices for d_t and $b_t(a)$, this result extends to all banks with the same ability κ.

3.8 Aggregating to Ability Types

The size distribution of firms across islands is indeterminate. Rather than characterizing and solving an equilibrium at the island level, I will characterize the model at the ability type level. Aggregating the model in this way will lend itself to a tractable solution concept. In this section, the quantity x on all islands with the same ability type (a sum across islands) is called $x(\kappa)$, that is, $x(\kappa) = \int_{A_\kappa} x(a)da$, where $A_\kappa = \{a : \kappa(a) = \kappa\}$.

For any island receiving ability draw κ, there will be a positive measure of islands with the same ability draw.[17] Since the distribution is assumed to be iid across periods, any positive measure of islands will be representative of the entire economy in the previous period; the net interbank loan repayment in this measure will be 0, and the total capital installed last period will just be a fraction of aggregate capital: $b_{t-1}(\kappa) = 0$, $s_{t-1}(\kappa) = p(\kappa)K_t$. Then the aggregate net worth for all islands with the same ability type will be

$$n_t(\kappa) = [Z_t + (1-\delta)Q_t(\kappa)]\psi_t(\sigma+\xi)p(\kappa)K_t - p(\kappa)\sigma R_{t-1}D_{t-1} \tag{41}$$

The first term gives the representative bank's returns on investments in the firm: Z_t is the economy-wide representative firm's gross profits from investments (per unit invested), $Q_t(\kappa)$ is the price of capital on all islands with ability κ, and K_t is the capital installed by all firms in the economy in period $t-1$. The second term gives the bank's repayments for deposits and interbank loans: b_{t-1} is the funds borrowed on the interbank market last period, and d_{t-1} is the deposits made by households last period.

[17] To be more precise, we can justify this with the following setup: specify the set of islands over the space $[0,1]\times[0,1]$, where a specific island is referred to by two coordinates, $a=(a_1,a_2)$. Now assume that in even periods, all islands with same first coordinate receive the same draw, so that $\kappa(a_1,a_2)=\kappa(a_1,\widetilde{a}_2)$, and in odd periods, all islands with the same second coordinate receive the same draw. Then for any island with a particular κ, there will be a positive measure of islands with the same κ. Since $[0,1]\times[0,1]$ has the same cardinality as $[0,1]$, I leave this setup out of the main text for ease of exposition.

On each island, banks choose deposits d_t identically, before uncertainty is realized, and choose $s_t(a)$ with the solution (21). Across islands, this solution only changes when ability type changes, that is, banks on all islands with the same ability type will find the same values for the coefficients ν_{st}, ν_{bt}, and ν_t. Banks on different islands could still have different net worth, however; rewriting the flow of funds constraint for the bank with the solution, we see that

$$s_t(a) = \frac{\kappa(a)}{Q_t(a)}(1 + \phi_t(a))n_t(a) \tag{42}$$

But the quantities $Q_t(a)$, $\phi_t(a)$, and $\kappa(a)$ are identical for any islands with the same ability type. This implies that we can represent the ability type aggregate flow of funds constraint with the same form as the individual island, so that if $Q_t(\kappa)$ and $\phi_t(\kappa)$ are type common prices,

$$s_t(\kappa) = \frac{\kappa}{Q_t(\kappa)}(1 + \phi_t(\kappa))n_t(\kappa) \tag{43}$$

Thus, we can solve for the ability type aggregates for banks with the maximization problem

$$V_t = \max_{\{d_{t+i},(s_{t+i}(\kappa_{t+i}))_\kappa,(b_{t+i}(\kappa_{t+i}))_\kappa\}_{i=0}^{\infty}} E_t \sum_{i=1}^{\infty}(1-\sigma)\sigma^{i-1}\Lambda_{t,t+i}n_{t+i}(\kappa_{t+i}) \tag{44}$$

$$s.t.\ V_t \geq \theta Q_t(\kappa)s_t(\kappa) \tag{45}$$

On the firm side, since net worth is pinned down for each ability type, and the ratio $\frac{n_t(a)}{k_t(a)}$ is equal for each island by assumption 2, we can pin down the quantities $k_t(\kappa)$ and $i_t(\kappa)$ for each ability type. These will take the form

$$k_t(\kappa) = p(\kappa)K_t \tag{46}$$

$$i_t(\kappa) = s_t(\kappa) - (1-\delta)k_t(\kappa) \tag{47}$$

Second, even though firms on each island may have different sizes, when we aggregate them to the ability type level, all firms on islands with the same ability type will choose ability type aggregate labor $l_t(\kappa)$ to maximize:

$$y_t(\kappa) = A_t k_t(\kappa)^{\alpha} l_t(\kappa)^{1-\alpha} \tag{48}$$

With these in hand, we can describe the size distribution of firms across ability types. Since the household was already represented by an economy-wide single representative agent, we can define an aggregated equilibrium concept, equilibrium in ability types:

Definition 1. *A recursive competitive equilibrium in ability types consists of a sequence of economy-wide prices* $\mathbb{P}_t \equiv (R_{t+i}, R_{bt+i}, W_{t+i}, Z_{t+i}, Q^i_{t+i})_{i=0}^{\infty}$ *, a sequence of type-specific prices* $\mathbb{P}_{\kappa t} \equiv (Q_{t+i}(\kappa))_{i=0}^{\infty}$*,*

a sequence of economy-wide quantities $\mathbb{Q}_t \equiv (K_{t+i}, C_{t+i}, I_{t+i}, Y_{t+i}, L_{t+i}, L_{t+i}, D_{t+i})_{i=0}^{\infty}$*, a sequence of type-specific quantities* $\mathbb{Q}_{\kappa t} \equiv (b_{t+i}(\kappa), s_{t+i}(\kappa), d_{t+i}(\kappa), i_{t+i}(\kappa))_{i=0}^{\infty}$[18] *such that:*

(Individual Optimization) for each t,

- $(d_t, b_t(\kappa), s_t(\kappa)$ *maximizes the representative bank's expected value (44) subject to their flow of funds constraint (3) for each ability type* κ
- $(k_t(\kappa), l_t(\kappa))$ *maximizes the representative firm's profits (48) for each ability type* κ
- (C_t, L_t, D_t) *maximizes the economy-wide representative household's expected utility (9)*
- I_t *maximizes capital goods producer profits (17)*

(Market Clearing) and for each t, these markets clear

- *deposits:* $D_t = \int_\kappa d_t(\kappa)p(\kappa)d\kappa$
- *labor:* $L_t = \int_\kappa l_t(\kappa)p(\kappa)d\kappa$
- *interbank loans:* $\int_\kappa b_t(\kappa)p(\kappa)d\kappa = 0$
- *new capital, so that* $I_t = \int_\kappa i_t(\kappa)p(\kappa)d\kappa$
- *assets (for each ability type):* $s_t(\kappa) = (1-\delta)k_t(\kappa) + i_t(\kappa)$

This system can be solved for any ability type distribution satisfying assumption 1. The full set of equilibrium equations is given in appendix A.

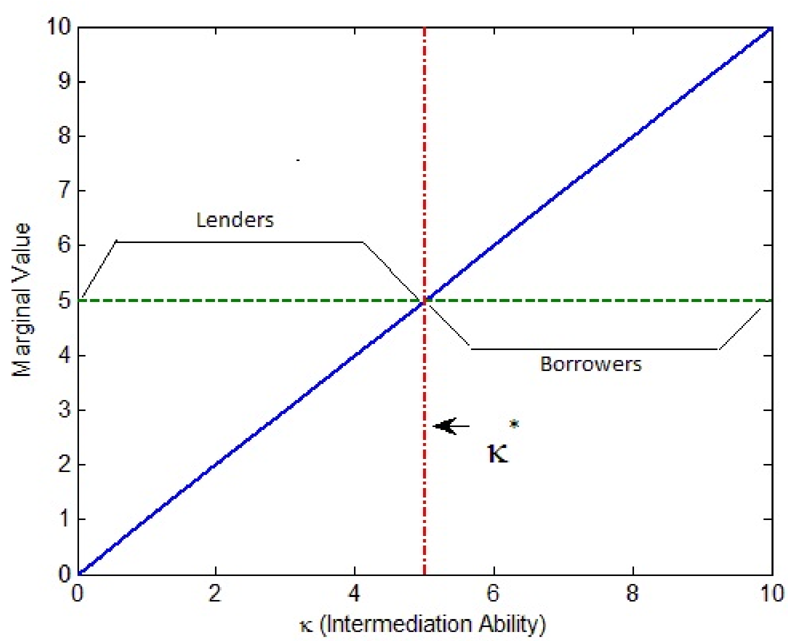
[18] Note that type-specific quantities are functions $x_t(\kappa) : [0,1] \to \mathbb{R}$

3.9 Steady State Properties

To illustrate the basic mechanism of the model, consider the case where ability is distributed according to a uniform distribution, and the economy is in steady state.

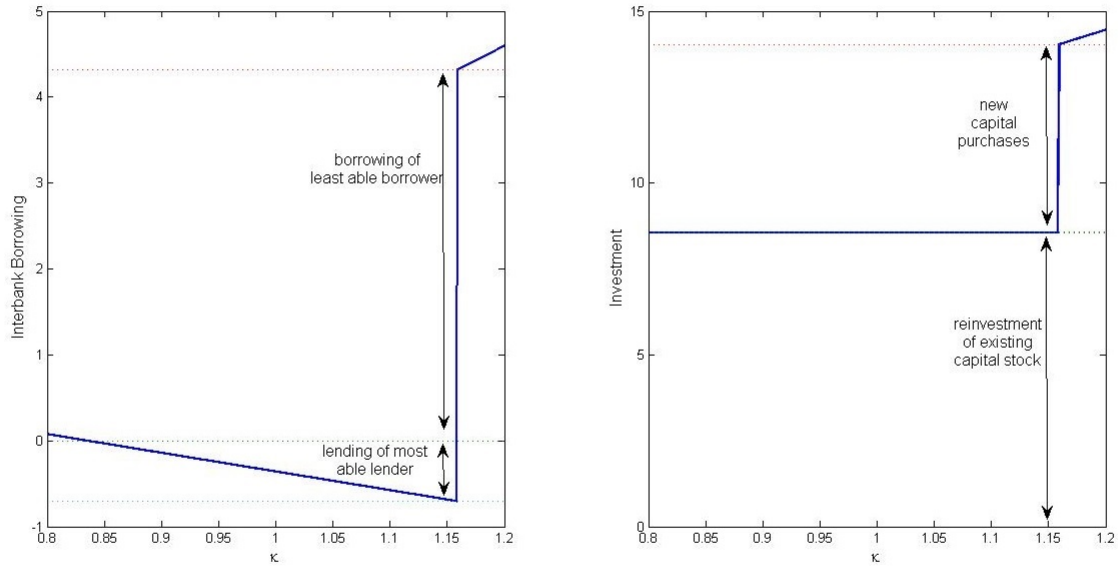
Figure 4 shows the marginal value from lending on the interbank market (dash line) and the marginal value from investing in firms. This is essentially a visualization of proposition 2. The value of lending is the same for all banks, since all banks receive the same interest rate for loans of any size. The value from investing/lending to firms, on the other hand, increases with bank intermediation ability. In any equilibrium with positive lending, the two lines have to cross - if the interbank lending line was always under the investment line, no bank would be willing to lend, and if the interbank lending line was always above the investment line, no bank would be willing to borrow. The ability level at which these two lines cross is the ability cutoff κ^* - for any bank with ability above the cutoff, the marginal value from investing is higher than the value from lending, so it becomes profitable for the bank to borrow and invest.

Figure 4: Ability Cutoff



Note: Marginal value from interbank borrowing (dash) and marginal value from holding shares (solid) versus intermediation ability. The intersection of the two lines marks the ability cutoff κ^*. Above the cutoff, banks act as net borrowers on the interbank lending market, while below the cutoff, banks act as net lenders.

Figure 5: Interbank Borrowing and Investment



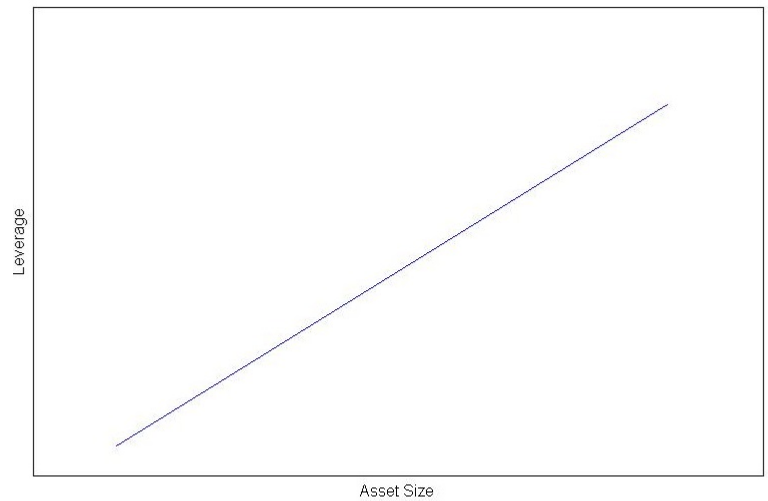
Note: Interbank borrowing (left) and investment (right) versus intermediation ability.

Ability increases the intensity with which banks borrow or lend in the interbank market. Figure 5 shows interbank borrowing versus ability in a typical case of the model. All banks with ability smaller than the ability cutoff κ^* lend on the interbank market, and because higher ability banks are more efficiently able to refinance their existing capital stock, they have more dollars left over for lending, and therefore lend more on the interbank lending market than lower ability banks. (Since the vertical axis represents borrowing, lending is represented with a negative y-coordinate.)

Banks with ability higher than the ability cutoff decide to borrow, buy new capital, and expand the production capacity of the firms on their island. The difference in borrowing between banks just above and just below the cutoff can be divided into two components. The first component represents a reduction in lending - banks just below the cutoff make these loans. The second component represents borrowing by banks up to the point that their borrowing constraint binds.

Moreover, the cap on borrowing increases in κ at an increasing rate. The function is convex, implying that the average leverage in the economy is higher than the leverage of the average bank. This convex relationship between leverage and intermediation ability will ultimately generate a linear relationship between leverage and size, as shown in figure 6. This is one of the properties we wanted to replicate.

Figure 6: Leverage and Size



Note: Leverage, or the ratio of bank assets to bank net worth, for banks of different asset sizes in a typical steady state. As ability increases, leverage increases at an increasing rate, but as size increases, leverage increases linearly.

Interbank lenders lend different quantities based on their intermediation ability. This is a result of both assumptions 5 and 6. The cost of refinancing the capital stock is smaller for more able banks, and since all lending banks refinance their capital stock, more able banks have more funds left over to lend out to other banks.

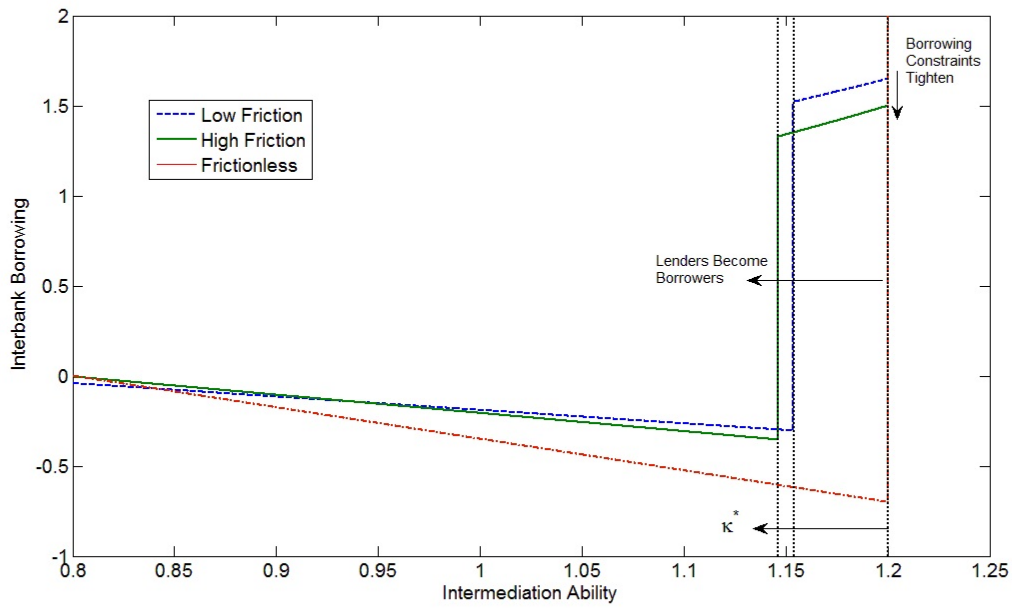
The right panel of figure 5 shows the value of investments (measured in units of consumption goods) by banks of different abilities. All banks reinvest their existing capital stock - capital prices on lending islands adjust to ensure that this is optimal. Once ability increases above the cutoff, we see a jump in the quantity of investments as banks suddenly switch from a "lend and refinance" strategy to a "borrow and buy new capital" strategy. As ability increases, borrowing limits increase, and because the price of new capital is pinned down by the new capital market, this translates into higher investment.

If there were no financial friction in the model, no bank would be constrained in its borrowing. Depositors would make deposits in all banks as before, but once ability is realized, the interbank lending market would funnel all deposits to the most able bank. This bank would transform these loans into assets, which firms would use to purchase the highest amount of capital possible. This borrowing curve would look something like the steepest, dashed curve in figure 7.

Introducing the friction limits the maximum the most able bank can borrow through the interbank lending market. By limiting the maximum the bank can borrow, funds that would go to the most able bank go to less able banks. Economy-wide investment and average returns banks obtain on investments decrease.

To visualize this, compare the steady state from the previous case with the steady state that obtains after an increase in the level of financial friction, θ. In figure 7, the steady state borrowing curve from the model above are plotted along with steady state borrowing from $\theta = 0.4$ (solid green), as in the previous section, to 0.5 (dashed blue).

Figure 7: Borrowing and Financial Friction



Note: Steady state interbank borrowing in model with low friction (dash), high friction (solid), and frictionless (dash-dot) by banks of different abilities. Ability cutoff κ^* is largest for the frictionless case, then the low friction, then the high friction case. In all three cases, ability is distributed uniformly in $[0.8, 1.2]$.

Among interbank borrowers, a change in θ has both an "intensive" and an "extensive" effect on steady state behavior. The intensive effect is a change in leverage for all banks: the value of the leverage ratio $\phi_t(a)$ decreases as θ increases, so any borrower that continues to be a borrower will face a tighter constraint on the level of their borrowing, and thus will not be able to invest as much. Moreover, the leverage curve gets less steep, so an increase in ability results in less additional borrowing for higher θ.

On the extensive side, as the friction increases and demand for interbank loans drops, the price of borrowing should decrease, at least relative to the value from lending to firms. This decrease in price causes some banks that were interbank lenders to become borrowers and invest. The increase in demand for borrowing that results from this extensive movement will offset some of the initial fall.

Both of these effects result in a decrease in the volume of interbank loans that are made in the economy. This is what we should expect - both demand and supply of interbank loans falls.

The magnitude of a given change in the level of the friction decreases as the initial friction increases. For parameter values for which steady state equilibria exist, an increase in the level of the friction always results in a decrease in total interbank borrowing. But the size of this decrease itself decreases as the initial level of the friction increases. Since the leverage ratio increases at an increasing rate in the friction, a higher initial friction level essentially means that we are starting higher up on the leverage ratio curve, where the increases are larger.

Among interbank lenders, we see a decrease in the slope of the line that determines how much lenders lend; more able lenders still lend more than less able ones, but the size of the difference between the two decreases as the level of the friction increases. This is because the steady state level of capital decreases as the friction increases, and because the cost of refinancing is a linear function of the steady state level of capital, the slope decreases.

4 Homogeneous and Heterogeneous Banking Sector Comparison

In this section, I compare the response of this model to that of Gertler and Kiyotaki (2010), a model with a homogeneous banking sector. I calibrate both models to match characteristics of the 2007 economy, and show that downturns in the heterogeneous banks model are deeper.

4.1 Calibration

The model demands the choice of 9 parameters, the adjustment costs function, and the distribution of ability types. Five of the parameters control the standard preference and technology shocks from from the literature: the discount rate β, the habit parameter γ, the utility weight of labor χ, the share of capital in production α, and the depreciation rate δ. These parameters are drawn from Gertler and Kiyotaki (2010) and Christiano et al. (2005) and are given in Table 3 below.

The parameter φ, the inverse elasticity of labor supply, is chosen so that the Frisch elasticity is ten. This choice is made to induce realistic labor responses in a model with no other labor market frictions. The parameter ξ, the start-up transfer to new bankers, governs the average spread between the interbank lending rate and the average return on assets. The parameter σ, the exogenous probability of exit by a bank, is chosen so that the average bank survives for approximately 15 years.

Adjustment costs take the quadratic form:

$$f\left(\frac{I_t}{I_{t-1}}\right) = \frac{c_I}{2}(\frac{I_t}{I_{t-1}} - 1)^2 \tag{49}$$

The parameter θ and the distribution of ability types are the main quantities of interest. Together, they govern the average leverage held by all banks and the bank size distribution that is generated in the model. The equilibrium existence restriction (39) on θ limits how small this parameter can be chosen.

The distribution of ability types will always take the form of a bounded Pareto distribution, for two reasons: first, the bank size distribution over the period takes this approximate shape[19], and second, using the Pareto distribution is computationally convenient. This distribution has a pdf, given by

$$p(x) = \frac{\rho \underline{\kappa}^{\rho} \overline{\kappa}^{\rho}}{\overline{\kappa}^{\rho} - \underline{\kappa}^{\rho}} x^{-\rho-1} \tag{50}$$

where L and H are the lower and upper bounds of the distribution. In accordance with assumption 1, the distribution will always be chosen to have mean 1.

In what follows, the level of θ, the bounds of the Pareto distribution L and H, and the shape parameter ρ will be adjusted differently in each case.

In Gertler and Kiyotaki (2010), all banks are identical in ability. Demand for interbank loans is created by exogenously specifying that a constant fraction π^i of islands are the only ones allowed to purchase new capital every period. In my model, the banks with firms that purchase new capital are those with ability above the cutoff; therefore, the fraction of all islands that purchase new capital is an endogenous object. Therefore, I choose π^i in the simulation of the model so that in steady state, the fraction of banks that purchase new capital in my model (i.e. those with ability higher than the cutoff) matches the fraction of banks that purchase new capital in theirs.

[19] Janicki and Prescott (2006) suggests that the tail of the bank size distribution is best modeled with a Pareto distribution, while the remainder is best modeled with a lognormal distribution. For this exercise, I exclusively use the Pareto distribution for computational convenience.

Table 3: Parameters for Heterogeneous and Homogenous Banks Comparison

Parameter		Value	Target
Inverse Elasticity of Labor Supply	φ	0.33	Gertler/Kiyotaki (2011)
Start-up Transfer	ξ	0.002	Gertler/Kiyotaki (2011)
Probability of Bank Exit	σ	0.982	Average Bank Age
Discount Factor	β	0.99	Gertler/Kiyotaki (2011)
Habit Parameter	γ	0.8	Christiano, Eichenbaum, Evans (2005)
Depreciation Rate	δ	0.025	Christiano, Eichenbaum, Evans (2005)
Effective Capital Share	α	0.36	Christiano, Eichenbaum, Evans (2005)
Utility Weight of Labor	χ	5.584	Gertler/Kiyotaki (2011)
Adjustment Cost Parameter	c_I	1.5	Gertler/Kiyotaki (2011)
Heterogeneous Banks Model			
Parameter		Value	Target
Friction	θ	0.3	Leverage Ratio - 2007
Min Ability	$\underline{\kappa}$	0.9	High Investing Bank Fraction
Max Ability	$\overline{\kappa}$	1.18	Mean 1 Assumption
Shape	ρ	5	Skewness - 2007
Homogeneous Banks Model			
Parameter		Value	Target
Fraction of Banks that Invest	π^i	0.1017	Heterogeneous Banks Investing Fraction

I choose a calibration that matches the baseline calibration of their paper closely, with two exceptions. Their choice of the investing fraction is 25%. My model tends to generate smaller investing fractions for a Pareto ability distribution, so though I choose parameters for the ability distribution so that the induced investing fraction is as close to their baseline choice as possible, I am only able to generate a fraction in my model around 10%.

All impulse responses are generated using the full set of equilibrium equations in Appendix A with Dynare.

4.2 Crisis Response

The 2008 financial crisis, and banking crises more broadly, have their roots in more fundamental weaknesses in the wider economy. In this paper, I do not take a stand on the sources of these weaknesses - given some initial weakness in the economy, I am interested in any additional negative effects imperfect financial intermediation might create.

As such, I initiate a downturn in this economy with an exogenous shock to the value of capital in the economy. Since banks in the model derive their net worth (equity) partly from the capital investments they've already made, this shock to capital translates to a shock to the net worth of all banks.

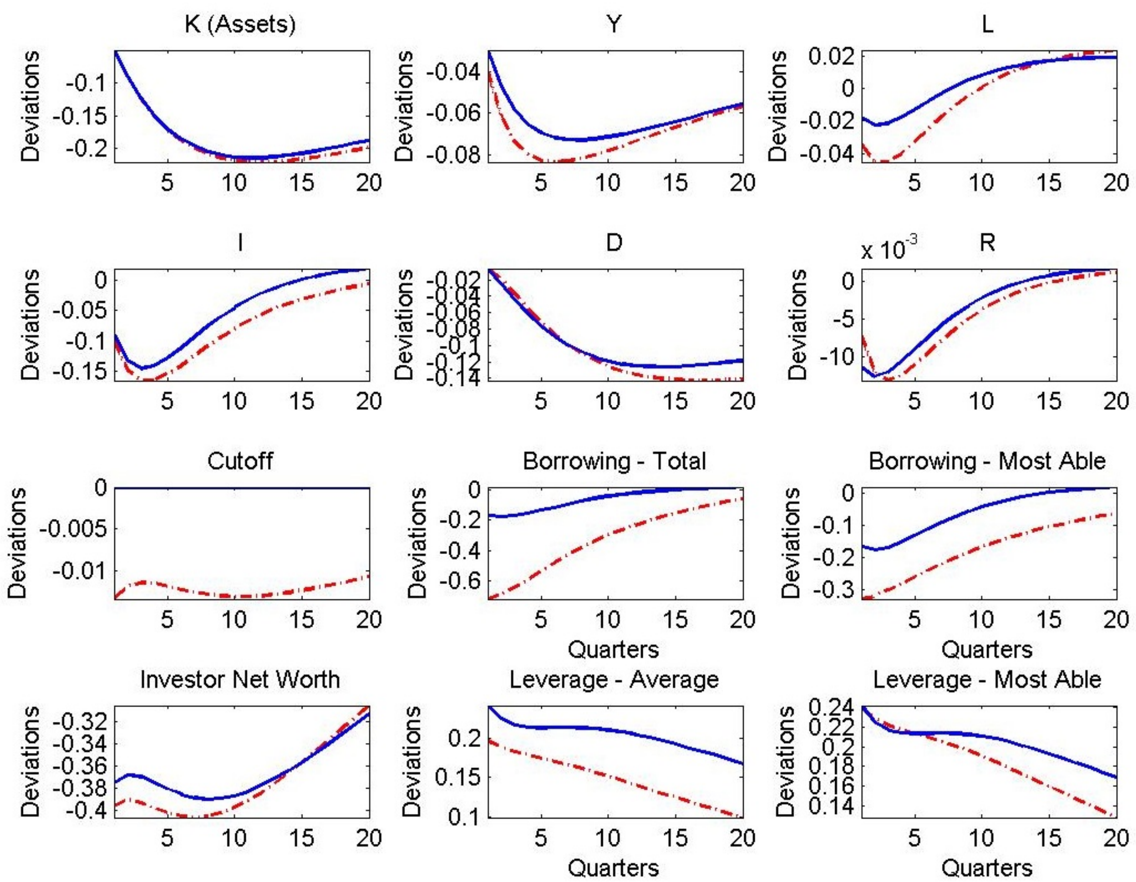
In the baseline experiment, I initiate a crisis in the model with a 5% shock to the capital quality ψ_t - the level is chosen to initiate a drop in output similar to that seen during the 2008 crisis. The shock has a persistence of 0.8, implying it returns to its steady state value after five years.

Figure 8 shows the impulse responses of both models, in terms of percentage deviations from steady state, to the initial shock. Generally, the shock makes capital more scarce, which causes banks to reduce their asset holdings, and because of the persistence of the shock, this reduced capital is reduced next period as well. This de-leveraging process ultimately produces the hump shape in capital in the figure, and drives the other responses.

In both models, the story proceeds roughly the same. First, the shock destroys capital on impact, immediately reducing the stock of assets, K, invested in the economy. We can see this in the panel for capital - in both models, the fall is roughly the same. Output Y falls, and because the marginal productivity of labor falls upon impact, demand for labor falls. Wages fall as well, so the supply of labor and employment falls. Both Y and L fall in the figure, though to different minimum levels. The decrease in wages and employment forces households to save less today, leading to lower supply of deposits in both models

Second, on the bank side, because capital is held as assets by banks, the initial destruc-

Figure 8: Impulse Responses: Heterogeneous and Homogeneous Banking Sectors



Note: Impulse responses to -5% financial shock for heterogeneous banking sector (dash) and homogeneous banking sector (solid). Shock persists for 20 quarters.

tion of capital leads to a significant decrease in bank net worth - this is seen in the bottom left panel. This occurs because the initial decline in capital has second-round effects, reducing both the price of capital, which affects net worth through the value of the existing capital held by banks, and the net profits the bank receives from firms.

Because their net worth has decreased, borrowing banks must reduce their asset positions further to satisfy their borrowing constraint. As a result, investment I falls - this is depicted in the panel labeled I. Because the economy faces adjustment costs, investment falls even further in the periods just after the initial shock, as banks have to de-lever more to cover their adjustment costs.

In addition, the shock to capital quality also destroys the net worth of lender banks. This, combined with the drop in deposits supplied by households, reduces the supply of interbank loans greatly. However, the interest rate on deposits and interbank loans, R, also falls. Banks are responsible for this, preventing households from dropping the supply of deposits even further than it was already.

Though banks de-lever and reduce their assets after their net worth decreases, the leverage ratio for every bank increases in a downturn, that is, leverage ratios are countercyclical; we see this in the figure in the panel labeled average leverage. This happens because the value from holding assets increases more than the value of default on interbank loans; since capital quality shocks induce de-leveraging, the value from holding assets increases as the shock dissipates.

As outlined above, dispersion can amplify the effects of financial frictions. An implication of this is that the downturns generated in this model can be deeper than those generated in homogeneous ability model. Banks in a heterogeneous banking sector rely more on a well-functioning interbank lending market, so when interbank lending is interrupted by a financial shock, the heterogeneous banking sector sees a deeper resulting downturn.

With heterogeneous banks, because more able banks are more leveraged, the average bank size is larger than the average bank size in the homogeneous case. This means that the steady state level of investment is also larger for the heterogeneous bank model. In addition, a larger portion of that investment is funded through interbank borrowing; if the net worth decreased by the same amount in both the heterogeneous and homogeneous bank models, investment decreases more in the heterogeneous case. In the figure, we actually see that net worth decreases more in the heterogeneous case, exacerbating this difference.

Net worth decreases more in the heterogeneous banks case because the net profits from firms decrease more, which occurs because output Y falls more in the heterogeneous banks case. This itself has to do with the fact that investment falls farther - thus, the relationship between investment today and net worth tomorrow produces a feedback effect which amplifies the shock.

Heterogeneity among lenders also affects impulse responses. When faced with the same shock to net worth, more able lenders decrease their lending more than less able ones, making the average lender response in percentage terms larger in the heterogeneous banks case. Thus, the volume of interbank lending decreases by more in the homogeneous banks case.

The decrease in the interbank lending rate causes less able banks to switch from lending to borrowing on the interbank lending market. This corresponds to a decrease in the ability cutoff, shown in the panel labeled cutoff. These new borrowers act as a stabilizing force in the interbank lending market, pushing up the interest rate and the volume of interbank lending above what it would be otherwise. These new borrowers are also less able, and therefore less leveraged when they do borrow. Their entrance brings down the average leverage in the economy. No switching occurs in the homogeneous banks model, so in comparison, average leverage in the homogeneous banks model is more strongly countercyclical.

Ultimately, the farthest output deviates from its steady state value in the homogeneous economy is 15% smaller than the farthest it deviates in the heterogeneous economy. Investment sees more modest amplification of the initial shock, but note that it also takes longer to return to its steady state value.

As shown above, the banks that switch from lending to borrowing stabilize the interbank lending market, partially offsetting the negative effects of deleveraging - the extensive effect of switching opposes the intensive effect of deleveraging. As a result, even though amplification occurred for this parameterization, there are other parameterizations for which bank heterogeneity dampens downturns. Generally, amplification will occur for cases where cutoffs are already relatively low; the cutoff will not move much after a capital shock, and thus extensive switching effects are small.

5 Quantitative Effects of Rising Concentration

In this section, I consider the impact of two changes to the size distribution that resulted from the increase in US banking sector concentration over the last three decades: bank sizes became more dispersed, and the number of large banks increased relative to the number of small banks. I calibrate the model to match these changes qualitatively, and show that they tend to exacerbate downturns as well.

As shown above, the US banking sector has become increasingly concentrated (in asset size) over the last two decades. In this section, I consider the following question: if a less concentrated bank size distribution faced the same shock to the quality of capital, would the resulting downturns be worse? If so, by how much?

As seen in the data, increasing concentration was accompanied by an increase in the dispersion and increase in mass of the right tail of the size distribution. I will match all three qualitative changes by varying the width and the shape parameter of the ability distribution from the calibration above. An increase in the width, or the difference $\overline{\kappa} - \underline{\kappa}$, will increase the dispersion as measured by the coefficient of variation in my model, as the mean of the distribution is always 1. Since the density of the Pareto distribution reaches a peak on its left side, and an increase in the shape parameter tends to increase the size of this peak, an increase in the shape parameter will decrease the mass of the right tail of the distribution.

Banks in the model either borrow as much as they can or lend as much as they can - their equilibrium value functions are linear. Because of this, in order to generate the inequality in bank sizes observed in the data, the fraction of banks that does any borrowing should be very small. Because there is also a restriction on how low a value the friction can take, the total new investment made by all banks in the banking sector will also be very small. Under the baseline calibration, a very high inequality economy becomes dominated by the actions of lending banks, an uninteresting case from the perspective of this paper.

One partial solution to this problem is to increase the depreciation rate of capital. I change the depreciation rate from the previous section, increasing it to 10%. By doing this, we increase the new investment each borrowing bank makes, making the actions of borrowing banks more important for aggregates. This is only a partial solution in the sense that the model still cannot match the inequality we observe in the data, but it can get much closer than with the baseline calibration. In order to simplify the discussion of responses, I also drop habits and adjustment costs in this section.

I choose another ability distribution that generates more inequality than in the baseline calibration, though still not as much as what we observe in the data. The choices for these parameters, and the corresponding dispersion and skewness measures, are given in the first two rows of table 4; other parameters are the same as in the baseline calibration above. The increased inequality comes at the cost of a reduction in the induced investing fraction, which is significantly lower than in the previous section.

The next rows list the steady state concentration, dispersion, and kurtosis of the induced size distribution for each of the two cases. The change in parameters generates the same qualitative changes in the size distribution as we see in the data; dispersion increases, and kurtosis increases, as we move from the representative low concentration to high concentration case. In addition, average leverage increases, also matching the time trend in leverage in the economy. As far as concentration, the model underestimates the change in concentration as measured by the top 1% share of assets held, but overestimates the change in concentration as measured by the top 10% share.

As in the previous case, a downturn is initiated with a 5% negative shock to the quality of capital, ψ, and has a persistence of 0.8, so that it completely dissipates after five years.

Table 4: Parameters and Induced Distribution for Rising Concentration Analysis

Year	ρ	θ	$\underline{\kappa}$	$\overline{\kappa}$	c_I	δ
Low Concentration	7	0.1	0.81	2.15	0	0.1
High Concentration	6	0.1	0.835	2.8	0	0.1

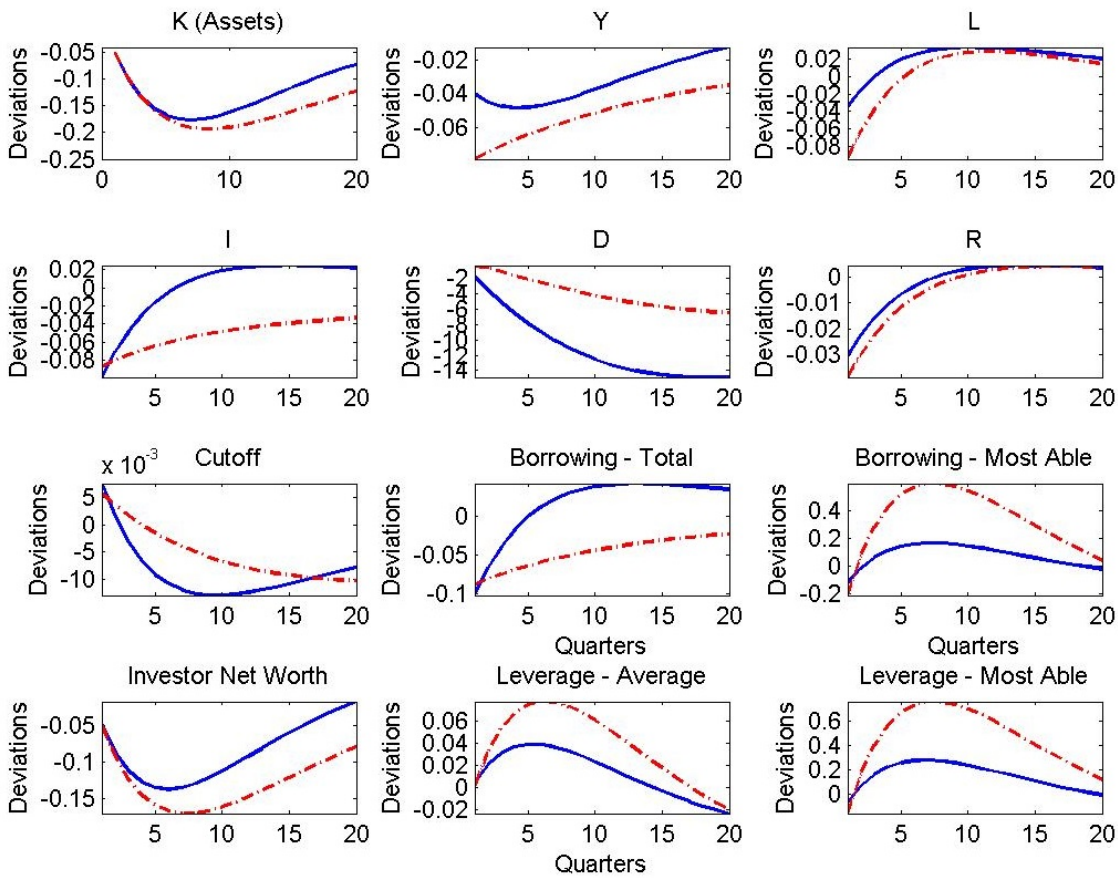
Year	Top 1%	Top 10%	GINI	Coeff of Variation	Avg Leverage
Low Concentration	0.05	0.36	0.57	1.1	6.6
High Concentration	0.15	0.58	0.72	2.1	14

The impulse responses for these cases are given in figure 9. The high concentration response is the dotted line, while the low concentration response is the solid line. Notably, output decreases more, in percentage terms, in the high concentration economy. Output in the high concentration economy falls to a minimum that is 1.66 times the minimum of the low concentration economy; thus, if the high concentration case represents the economy just before the financial crisis, the economy would have experience a downturn that was only 60% as deep.

Second, output in the high concentration economy returns to steady state much more slowly than in the low concentration economy. Though not pictured, output in the high concentration economy takes twice as many quarters to return to its steady state level than in the low concentration economy.

The difference on impact of output in the two economies is due to the initial difference in the response of employment. As shown in the top right panel, employment falls much farther in the high concentration than low concentration economy. Since the high concentration economy is more dispersed, all banks know there is a high chance of being very productive tomorrow. This makes the value of net worth in the high concentration economy higher on average today than in the low concentration economy. Thus, as capital gets scarce, even though the marginal product of labor decreases in both economies, firms want capital more in the high concentration economy, and shift their capital/labor ratio toward capital more

Figure 9: Impulse Responses: High and Low Concentration



Note: Impulse responses to -5% financial shock for high concentration (dotted, red) and low concentration (solid, blue) banking sector. Responses are expressed in percentage deviations. Shock persists for 20 quarters.

than in the high concentration economy. As a result, employment falls farther initially, and output falls with it.

Because the high concentration economy is more dispersed, the average leverage of borrowing banks is higher. As in the case of amplification over a homogeneous banking sector, these effects tend to worsen the de-leveraging borrowing banks do in response to a shock, as the more concentrated sector contains very leveraged banks.

In addition to dispersion, the mass of the right tail has also increased over time. This has two effects. First, since the number of leveraged banks increases, more banks de-leverage, and the overall effect of de-leveraging on output worsens. Second, since the number of leveraged banks increases, the same changes to the interbank lending rate produce larger swings in the ability cutoff - for a given change in interest rate, there are more banks that respond to the change.

These leverage differences are responsible for the very slow recovery of the high concentration economy. Though both economies initially drop by similar amounts, total borrowing, investor net worth, and investment in the high concentration economy stays much lower than in the high concentration economy after a year or so. This occurs despite the fact that leverage, both of the average bank and the most able, is more strongly countercyclical in the high concentration than the low concentration economy.

Another contributing factor to the deeper downturn in the high concentration economy is the smaller extensive effect of banks switching from lending to borrowing. The net worth of banks in the high concentration economy decreases less than in the low concentration economy, while the volume of interbank borrowing decreases more. Both the supply and demand for interbank loans decreases. At first, this restricts demand, but because supply falls farther, there is a net positive effect on the value banks can receive from interbank lending in both cases. This negative effect drives more banks into switching from lending to borrowing, pushing down the cutoff in the low concentration economy farther, dampening the negative effects of de-leveraging relative to the high concentration economy.

6 Conclusion

In this paper, I construct a macroeconomic model with a heterogeneous banking sector, and show that heterogeneity has consequences for downturns. In particular, mean-independent changes in the distribution of bank sizes can mirror the effects of financial frictions, and as a result, changes to the bank size distribution qualitatively similar to those that occurred with the rising banking sector concentration observed in the US make potential downturns more severe today.

Though this paper only considers positive questions, we can still discuss the role of policy responses[20] The model in this paper puts us in the unique position to consider targeted macro-prudential policies in a general macroeconomic framework. One such policy taken up by the US Treasury and the Fed was embodied in broad targeted asset purchases like the TARP, which in some cases equated to the government taking up part-ownership positions in some banks. In the context of this model, this would reduce the friction target banks face. If the economy is in really bad shape, however, the economy starts at a high level of friction, extensive effects can dominate. Loosening the friction will increase the interbank interest rate, drawing some banks into interbank lending - however, the banks that that this lending goes to are still so constrained that they don't expand their investment enough. They get an extra dollar of loan, but at a higher interest rate, which is not offset by the change in the friction for high levels of friction. In this case, offering a targeted program which doesn't affect the interbank market would improve welfare more than one which ignored this.

Despite the stylized assumptions in the model, the environment is rich, and I see several avenues worth pursuing in future work. First, the model has difficulty generating the extreme bank size inequality we see in the data. This problem is partly due to the sharp transition from lending to borrowing banks make as their ability increases. If banks calculated their net worth as a strictly convex, rather than linear, combination of deposits, assets and interbank loans, their value functions would also be convex in these three arguments, and banks would smoothly transition from lending to borrowing as ability increased. With this, we could more easily generate high inequality.

Second, dropping assumption 2 will cause bank size next period to depend on its size this period. This will induce an endogenously evolving size distribution, which may itself uncover some interesting dynamics. For example, if ability tomorrow doesn't depend on ability today, but size carries to next period, large banks can use their assets as a buffer against poor ability draws, but small banks would be more sensitive to such changes. As a result, the average value of interbank lending would not adjust as much as in the case of this model.

Third, if we modify the model by restricting the loans banks make to a subset of all banks, we can create an environment where network effects are important. Relationships matter in interbank lending (and other short-term debt, e.g. repo), with banks borrowing from a partner in one period only to lend to the same partner in the next. Bank A's failure

[20]The policy responses discussed above are implemented in a companion paper, still in progress.

will matter more for bank B if the two were lending partners. As a result, some network structures of lending relationships will transmit downturns better than others, just as a more concentrated banking sector seems to transmit crises better in this model.

This project is a small step in a larger agenda of analyzing the macroeconomic implications of industry-wide trends (not just size differences) in banking. So far, research in this field has not placed much emphasis on the characteristics of individual banks and the dynamic consequences from changes in those characteristics. Research in this vein may be especially informative for the conduct of unconventional monetary policy.

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A Equilibrium Conditions

In this section, I describe the full system of equations that describes equilibrium. Before proceeding, make a few simplifications to the model. First, clearing the interbank market automatically clears the market for deposits. Call B the set of ability types with representative banks that borrow, and L the set of all types that lend. The sum of all interbank lending by types in L should equal the sum of all interbank borrowing by types in B:

$$\begin{aligned}\int_{\kappa\in B} b(\kappa)p(\kappa)d\kappa &= \int_{\kappa\in B} \bar{b}(\kappa)p(\kappa)d\kappa = \int_{\kappa\in B} \left(\phi_t(\kappa)n_t(\kappa) - d_t(\kappa)\right)d\kappa \\ &= -\int_{\kappa\in L} b(\kappa)p(\kappa)d\kappa = \int_{\kappa\in L} \left(n_t(\kappa) + d_t - Q_t(\kappa)k_t(\kappa)\right)d\kappa \end{aligned} \tag{51}$$

With this in hand, we can rewrite the interbank lending market condition in terms of aggregates:

$$D_t = \int_{\kappa^*}^{\overline{\kappa}} p(\kappa)\phi_t(\kappa)((Z_t + (1-\delta)Q_t^i)K_t(\sigma+\xi) - \sigma R_{t-1}D_{t-1})d\kappa$$
$$- \int_{\underline{\kappa}}^{\kappa^*} p(\kappa)((Z_t + (1-\delta)Q_t(a))K_t(\sigma+\xi) - \sigma R_{t-1}D_{t-1})d\kappa$$
$$+ \int_{\underline{\kappa}}^{\kappa^*} p(\kappa)\frac{Q_t(a)}{\kappa(a)}(1-\delta)K_t d\kappa \quad (52)$$

Where D_t is aggregate deposits and K_t is the aggregate capital stock.

In order to pin down investment, we need to consider the market for new capital alone. We know that the aggregate investment and existing capital held by firms on borrower islands is equal to $I_t + (1-\delta)K_t \int_{\kappa^*}^{\overline{\kappa}} p(\kappa)$, and that capital is demanded in the form of interbank borrowing and deposits. Noting that borrowing islands will borrow the maximum possible, we can replace the borrowing constraint into the flow of funds equation:

$$s_t(\kappa) = i_t(\kappa) + k_t(\kappa) = \frac{\kappa}{Q_t(\kappa)}(n_t(\kappa) + d_t(\kappa) + b_t(\kappa))$$
$$= \frac{\kappa}{Q_t^i}(1+\phi_t(\kappa))n_t(\kappa) \;\; \forall \kappa \in [\kappa^*, \overline{\kappa}] \quad (53)$$

Finally, integrate both sides over the set B to get the condition in terms of aggregates:

$$Q_t^i\left(I_t + (1-\delta)K_t \int_{\kappa^*}^{\overline{\kappa}} p(\kappa)\right) = \int_{\kappa^*}^{\overline{\kappa}} \kappa(1+\phi_t(\kappa))n_t(\kappa)d\kappa \quad (54)$$

A.1 Full System

HH/Firms

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{55}$$

$$K_t = \psi_t(I_{t-1} + (1-\delta)K_{t-1}) \tag{56}$$

$$Y_t = C_t + (1 + f(\frac{I_t}{I_{t-1}}))I_t \tag{57}$$

$$1 = E_t \Lambda_{t,t+1} R_t \tag{58}$$

$$u_{Ct} = (C_t - \gamma C_{t-1})^{-1} - \beta\gamma(C_{t+1} - \gamma C_t)^{-1} \tag{59}$$

$$\Lambda_{t,t+1} = \beta \frac{u_{Ct+1}}{u_{Ct}} \tag{60}$$

$$Z_t = \alpha A_t (\frac{L_t}{K_t})^{1-\alpha} \tag{61}$$

$$Q_t^i = 1 + f(\frac{I_t}{I_{t-1}}) + (\frac{I_t}{I_{t-1}}) f'(\frac{I_t}{I_{t-1}}) - E_t \Lambda_{t,t+1} (\frac{I_{t+1}}{I_t})^2 f'(\frac{I_{t+1}}{I_t}) \tag{62}$$

$$\chi L_t^{\varphi} = (1-\alpha)\frac{Y_t}{L_t} E_t u_{Ct} \tag{63}$$

To drop habit formation from the model, set $\gamma = 0$. To drop adjustment costs from the model, set $f(\frac{I_t}{I_{t-1}}) = 0$ everywhere.

Exogenous Shock Processes

$$A_t = \rho_A A_{t-1} + \epsilon_{At} \tag{64}$$

$$\psi_t = \rho_\psi \psi_{t-1} + \epsilon_{\psi t} \tag{65}$$

Bank Optimization

$$Q_t(\kappa) = \frac{\kappa}{\kappa_t^c} Q_t^i \ \ \forall \kappa \in L \tag{66}$$

$$\nu_t = \nu_{bt} \tag{67}$$

$$\nu_{bt} = \frac{\kappa_t^c}{Q_t^i} \nu_{st} \tag{68}$$

$$\lambda_t(\kappa) = \frac{(\kappa - \kappa_t^c)\nu_{st}}{\kappa\theta Q_t^i - (\kappa - \kappa_t^c)\nu_{st}} \ \ \forall \kappa \in B \tag{69}$$

$$\lambda_t(\kappa) = 0 \ \ \forall \kappa \in L \tag{70}$$

$$\Omega_t(\kappa) = 1 - \sigma + \sigma \nu_{bt}(1 + \lambda_t(\kappa)) \tag{71}$$

$$\phi_t(\kappa) = \frac{\kappa(\nu_{st} - \theta Q_t^i)}{\nu_{bt} Q_t^i - \kappa(\nu_{st} - \theta Q_t^i)} \tag{72}$$

$$\nu_{bt} = E_t R_t \Lambda_{t,t+1}(1 - \sigma + \sigma \nu_{bt+1}) + E_t R_t \Lambda_{t,t+1} \sigma \nu_{bt+1} G_0 \tag{73}$$

$$\begin{aligned}
\nu_{st} = E_t \Lambda_{t,t+1} \Psi_{t+1} [&(1 - \sigma + \sigma \nu_{bt+1} + \sigma \nu_{bt+1} G_0) Z_{t+1} \\
&+ (1 - \sigma + \sigma \nu_{bt+1})(1 - \delta) Q_{t+1}^i (\int_{\kappa_{t+1}^c}^{\overline{\kappa}} p(\kappa) d\kappa) + \sigma \nu_{bt+1} G_0 (1 - \delta) Q_{t+1}^i \\
&+ (1 - \sigma + \sigma \nu_{bt+1})(\int_{\underline{\kappa}}^{\kappa_{t+1}^c} p(\kappa) \kappa d\kappa)(1 - \delta) Q_{t+1}^n]
\end{aligned} \tag{74}$$

$$G_0 = \int_{\kappa_{t+1}^c}^{\overline{\kappa}} p(\kappa) \lambda_{t+1}(\kappa) \, d\kappa \tag{75}$$

Securities Market

$$Q_t^i (I_t + (1 - \delta) K_t \int_{\kappa_t^c}^{\overline{\kappa}} p(\kappa) d\kappa) = ((Z_t + (1 - \delta) Q_t^i)(\sigma + \xi) K_t - \sigma R_{t-1} D_{t-1}) G_1 \tag{76}$$

$$G_1 = \int_{\kappa_t^c}^{\overline{\kappa}} p(\kappa) \kappa (1 + \phi_t(\kappa)) \, d\kappa \tag{77}$$

Deposit Market

$$
\begin{aligned}
D_t = (Z_t + (1-\delta)Q_t^i)(\sigma+\xi)K_t - \sigma R_{t-1}D_{t-1})G_2 \\
- (Z_t - \sigma R_{t-1}D_{t-1})(\int_{\underline{\kappa}}^{\kappa_t^c} p(\kappa)d\kappa) \\
- (1-\delta)Q_t^n(\sigma+\xi)K_t(\int_{\underline{\kappa}}^{\kappa_t^c} p(\kappa)\kappa d\kappa) + Q_t^n(1-\delta)K_t(\int_{\underline{\kappa}}^{\kappa_t^c} p(\kappa)d\kappa)
\end{aligned} \tag{78}
$$

$$
G_2 = \int_{\kappa_t^c}^{\overline{\kappa}} p(\kappa)\phi_t(\kappa)\, d\kappa \tag{79}
$$

A.2 Steady State

HH

$$
I = \delta K \tag{80}
$$

$$
C = [A(\frac{L}{K})^{1-\alpha} - \delta]K \tag{81}
$$

$$
\chi L^{\varphi} = (1-\alpha)A(\frac{L}{K})^{-\alpha}\frac{1-\beta\gamma}{1-\gamma}\frac{1}{C} \tag{82}
$$

$$
L = (\frac{Z}{\alpha A})^{\frac{1}{1-\alpha}}K \tag{83}
$$

$$
R = \frac{1}{\beta} \tag{84}
$$

$$
\Lambda = \beta \tag{85}
$$

$$
Q^i = 1 \tag{86}
$$

Bank Optimization

$$
\lambda(\kappa) = \frac{(\kappa-\kappa^c)\nu_s}{\kappa\theta - (\kappa-\kappa^c)\nu_s} \;\; \forall \kappa \in B \tag{87}
$$

$$
\lambda(a) = 0 \;\; \forall a \in L \tag{88}
$$

$$
\phi(\kappa) = \frac{\kappa(\nu_s - \theta)}{\nu_b - \kappa(\nu_s - \theta)} \tag{89}
$$

$$
Q^n = \frac{1}{\kappa^c} \tag{90}
$$

$$
\frac{1}{\kappa^c} = \nu_s(1 - \frac{\sigma}{1-\sigma}\overline{G_0}) \tag{91}
$$

$$
\nu_b = \kappa^c \nu_s \tag{92}
$$

$$Z = \frac{\nu_s}{\beta\nu_b} - \frac{1-\delta}{\nu_b}\left((1-\sigma+\sigma\nu_b)(\int_{\kappa^c}^{\overline{\kappa}} p(\kappa)d\kappa) + \sigma\nu_b\overline{G_0}\right) - \frac{(1-\delta)Q^n}{\nu_b}\left((1-\sigma+\sigma\nu_b)(\int_{\underline{\kappa}}^{\kappa^c} p(\kappa)\kappa d\kappa)\right) \quad (93)$$

Securities Market

$$\frac{\sigma}{\beta}\frac{D}{K} = (Z+1-\delta)(\sigma+\xi) - \frac{\delta+(1-\delta)(\int_{\kappa^c}^{\overline{\kappa}} p(\kappa)d\kappa)}{\overline{G_1}} \quad (94)$$

Deposit Market

$$\frac{N^i}{K} = (Z+1-\delta)(\sigma+\xi) - \frac{\sigma}{\beta}\frac{D}{K} \quad (95)$$

$$\frac{D}{K} = \frac{N^i}{K}\overline{G_2} + Q^n(1-\delta)\left(\int_{\underline{\kappa}}^{\kappa^c} p(\kappa)d\kappa\right) - \left(Z(\sigma+\xi) - \frac{\sigma}{\beta}\frac{D}{K}\right)\left(\int_{\underline{\kappa}}^{\kappa^c} p(\kappa)d\kappa\right) - (1-\delta)Q^n(\sigma+\xi)\left(\int_{\underline{\kappa}}^{\kappa^c} p(\kappa)\kappa d\kappa\right) \quad (96)$$

$$G_0 = \int_{\kappa_{t+1}^c}^{\overline{\kappa}} \frac{p(\kappa)(\kappa-\kappa_{t+1}^c)\nu_{st+1}}{\kappa\theta Q_{t+1}^i - (\kappa-\kappa_{t+1}^c)\nu_{st+1}} d\kappa \quad (97)$$

$$G_1 = \int_{\kappa_t^c}^{\overline{\kappa}} \frac{p(\kappa)\kappa\kappa_t^c\nu_{st}}{\kappa_t^c\nu_{st} - \kappa\nu_{st} + \kappa\theta Q_t^i} d\kappa \quad (98)$$

$$G_2 = \int_{\kappa_t^c}^{\overline{\kappa}} \frac{p(\kappa)\kappa(\nu_{st} - \theta Q_t^i)}{\kappa_t^c\nu_{st} - \kappa\nu_{st} + \kappa\theta Q_t^i} d\kappa \quad (99)$$

$$\overline{G_0} = \int_{\kappa^c}^{\overline{\kappa}} \frac{p(\kappa)(\kappa-\kappa^c)\nu_s}{\kappa\theta - (\kappa-\kappa^c)\nu_s} d\kappa \quad (100)$$

$$\overline{G_1} = \int_{\kappa^c}^{\overline{\kappa}} \frac{p(\kappa)\kappa\kappa^c\nu_s}{\kappa^c\nu_s - \kappa\nu_s + \kappa\theta} d\kappa \quad (101)$$

$$\overline{G_2} = \int_{\kappa^c}^{\overline{\kappa}} \frac{p(\kappa)\kappa(\nu_s - \theta)}{\kappa^c\nu_s - \kappa\nu_s + \kappa\theta} d\kappa \quad (102)$$

To solve the model, I first guess a productivity cutoff κ^c. Then the value from interbank lending $\nu_b = \kappa^c \nu_s$, and the equation for interbank lending above becomes an equation in one variable, which we can solve for parameters. We can then use the equation for ν_s to get Z, and use the investing islands securities market equation to get $\frac{D}{K}$. With this in hand, we can test the deposit market clearing condition, by calculating both the right and left hand sides of equation (21).