Self-fulfilling Fire-sales, Bank Runs and Contagion:

Implications for Bank Capital and Regulatory Transparency*

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Abstract

The paper presents a model where fire-sales and bank runs are self-fulfilling and mutually reinforcing. When creditors anticipate low prices for a bank's asset sales, a run is triggered, which generates fire-sales and the corresponding collapses in asset prices, thus fully justifying the panics of creditors. The reasoning extends to contagion when banks have common risk exposures. As one bank fails, asset buyers become more pessimistic and perceive all banks' assets less valuable. The decline in asset prices precipitates runs onto all other banks. Hence runs and fire-sales fuel each other, a phenomenon that explains the extent of sub-prime crisis in spite of the small amount of sub-prime lending in the total credit supply. The model delivers two policy implications. (1) Since a run on a well-capitalized bank signals unusually high risk and can strongly reduce asset prices, high capital can have unintended consequences on bank illiquidity and contagion. (2) Regulators should be very careful about disclosing information on banks' common risk: While favorable opinions save illiquid banks, acknowledging a crisis will cause contagion due to buyers' pessimistic belief updating.

Keywords: Adverse selection, Illiquidity, Capital, Regulatory Transparency
JEL Classification: G21, G28

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1 Introduction

The recent banking crisis highlights the risk of illiquidity. On the one hand, market liquidity evaporated and asset prices dropped sharply. On the other hand, as funding liquidity dried up, even well capitalized banks failed to rollover their short-term debt and had to resort to central banks.

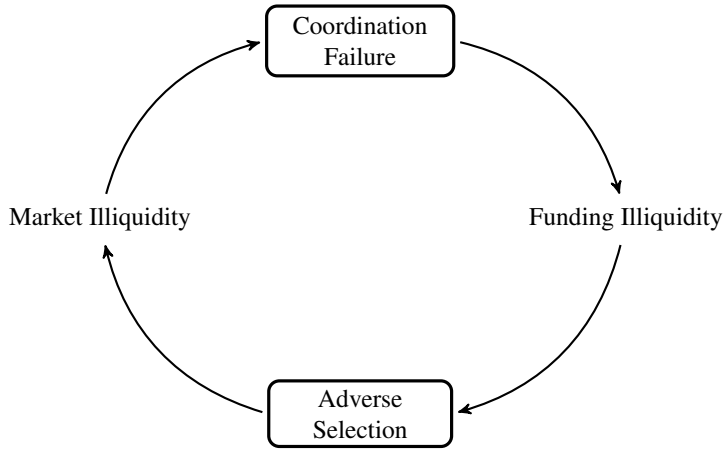
The two types of illiquidity closely link to each other. First, it has been well acknowledged that market illiquidity contributes to funding illiquidity. As market liquidity drops, the potential fire-sale losses from early liquidation make creditors panic; a coordination failure of the creditors to rollover their short-term debts can deprive a financial institution of its funding. The maturity and liquidity mismatch exposes banks to the risk of being unable to roll over their debts. A bank can be solvent but illiquid: being able to repay in full its debt if no run happens, but being liquidated early if its creditors do not roll over. The points have been emphasized by work like Morris and Shin (2000), Rochet and Vives (2004) and Goldstein and Pauzner (2005). Yet these papers ignore the feedback from bank runs to asset prices, treating separately two interconnected issues that in our opinion should be integrated.

Indeed funding illiquidity also feeds market illiquidity: bank runs can lead to fire-sales, driving down asset prices, and in extreme cases, freezing up markets. As narrated by Acharya and Roubini (2009):

> *"...the collapses on June 20, 2007, of two highly levered Bear Stearns-managed hedge funds that invested in subprime asset-backed securities (ABSs). ..., as the prices of collateralized debt obligations (CDOs) began to fall..., lenders to the funds demanded more collateral. ..., Merrill Lynch, seized $800 million of their assets and tried to auction them off. When only $100 million worth could be sold, the illiquid nature and declining value of the assets became quite evident."*

The mutually reinforcing interaction between market and funding illiquidity, with the emergence of "liquidity spirals", has been documented by Brunnermeier and Pedersen (2009) in a model where the lack of funding of informed agents leads to discrepancies of price from fundamentals. Our model provides a different insight into this issue: we emphasize the coordination failure among creditors (bank runs) and allow asymmetric information to distort asset prices. The two ingredients combined add a self-fulfilling nature to the issue and deliver new policy insights concerning capital regulation and marcroprudential communication policies.

Figure 1: A banking crisis of "twin-illiquidity"



We view the interaction between bank runs and fire sales to be information driven: resulting from asymmetric information on asset qualities. While in normal times it seems natural to assume that market discipline will be able to penalize financial intermediaries that are badly managed or excessively risky, in times of crisis accurate information is scarce and it is difficult, if not impossible, for market participants to discern whether a bank is insolvent or solvent-but-illiquid. A solvent bank, once forced by runs to sell assets, can no longer be distinguished from an insolvent one. The buyers of the bank's assets therefore face adverse selection. This single friction can set off a chain reaction: the adverse selection rationalizes low fire-sale prices; the low fire-sale prices lead to liquidation losses; the losses make creditors panic and run; and the run leads to a further decline in asset prices. As this vicious cycle repeats, a banking crisis of twin illiquidity unfolds. (See Figure 1 for an illustration.)

We present a theoretical framework where such fire-sales and bank runs/contagion happen in a self-fulfilling manner. When the buyers of a bank's assets are uninformed of a bank's asset quality, observing a run will imply low valuation from buyers' perspective. Because the uninformed buyers cannot distinguish a solvent-but-illiquid bank from an insolvent one, the information friction distorts downwards their willingness to pay. As a result, a solvent bank will not recoup a fair value for its assets on sale. The friction has several consequences. First, low asset prices fuel self-fulfilling bank runs: to avoid the fire-sale loss caused by other creditors' early withdrawals, a creditor has the incentive to run on a solvent bank. The strategic complementarity creates successful runs and illiquid banks. Second, the fire sales are self-fulfilling too. For fear of low fire-sale prices, creditors run on a solvent bank and force early liquidation.

Yet it is the run and liquidation, by pooling the solvent with the insolvent, that leads to the low fire-sale prices in the first place. In this sense, the creditors' pessimistic expectation realizes itself. Finally, it should be noted that the self-fulfilling bank runs and fire sales intertwine and constitute a vicious cycle. Driven by the adverse selection, the whole banking crisis of twin illiquidity rises as a self-fulfilling prophecy.

For the financial system, contagion happens in a similar self-fulfilling way with one more ingredient—a common risk exposure, such as to real estate bubbles, macroeconomic fundamentals or even regulatory deficiencies. As the uninformed buyers form rational expectations, they revise downwards their expectation of the common risk factor upon observing a bank going under. The reduced expectation lowers their willingness to pay for other banks' assets, which in turn precipitates runs in all other banks. It should be noted that the contagion (the run to other banks) reflects again the mutually reinforcing interaction between fire-sales and runs: anticipating the declining willingness to pay due to the lower expectation of common risk factor, the creditors of other banks panic and run, and the run confirms the worsening expectation and leads to further distressed asset prices.

As a defining feature that distinguishes the current model from the literature, we have buyers' beliefs, asset prices, bank runs, and contagion, all endogenous and jointly determined in a rational expectation equilibrium. We prove the equilibrium exists and is unique. These features of our model allow us to deliver some new policy insights. In particular, we analyze the impacts of bank capital and regulatory transparency, discussing the unintended consequences that conventionally desired policies such as capital regulation and regulatory transparency may have overlooked, thus challenging some conventional wisdoms.

First, while our paper confirms that a well-capitalized banking industry has more buffer against fire-sale losses thereby shows more resilience against systemic risk, the analysis also reveals that once asset prices are endogenous, the situation is more subtle and high capital also has unintended consequences on illiquidity and financial contagion. In particular, bank capital can negatively affect asset prices via buyers' beliefs. For an individual bank, buyers' posterior beliefs on the bank's returns deteriorate when a run happens. And the deterioration is particularly strong when the bank maintains a high capital ratio: because well-capitalized banks are able to sustain large losses, if a run happens to such a bank, the bank's losses must be unusually high. Therefore for a bank that faces a bank run, buyers' valuation of its assets

decreases in its capital level. The low willingness to pay contributes to creditors' coordination failure and makes the run likely to happen in the first place.

The argument extends to financial contagion in the presence of a common risk exposure. Asset buyers revise down their expectation of the common risk factor upon observing any bank runs. Such pessimistic belief updating reduces asset prices and leads to contagion for all other banks. The inferencing effect is especially strong when runs happen to well capitalized banks, because the failure of any of those banks suggests the common risk factor to be unusually unfavorable. Therefore, the impact of capital on financial stability can be decomposed into its pro-stability effect due to greater loss-absorbing buffers, and the counterproductive inferencing effects that come into play when adverse events such as bank runs occur. While we show that overall capital can still reduce illiquidity and systemic risk, the unintended effects should not be overlooked.

Second, the information-based run and contagion links to the debate on regulatory transparency, e.g., whether or not regulatory bodies should disclose information concerning their assistance programs. The paper views such transparency to be a double-edged sword. If the disclosed information improves the inference of the common risk, illiquid banks are saved. However, if the assistance program adds to pessimistic market inference, e.g., size greater than expected, the assistance itself will be contagious: once the severity of the problem is acknowledged, market participants further revise down the expected performance of all financial institutions', leading to greater fire-sale losses and triggering illiquidity even for fairly robust institutions.

We believe regulators have a special position in crises, because market participants tend to believe regulators have superior information. Therefore any disclosure to the public, whether it is communication or action, can have consequences for financial stability. The fall of Bear Stearns was an interesting case in this aspect. As documented in Brunnermeier (2009):

> *"...March 11, 2008, when the Federal Reserve announced its $200 billion Term Securities Lending Facility. ... However, some market participants might have (mistakenly) interpreted this move as a sign that the Fed knew that some investment bank might be in difficulty. Naturally, they pointed to the smallest, most leveraged investment bank with large mortgage exposure: Bear Stearns."*

It was unclear whether Bear Sterns was truly having financial difficulties or not. Yet as market participants believe the Fed is better informed and the action of Fed reflects the superior information, the attack began.

1.1 Related literature

Our theoretical framework is related to the literature on bank run and financial contagion. Since Diamond and Dybvig (1983) the banking literature is concerned with the financial fragility caused by runs.[1] Following their seminal contribution was a debate as to whether bank runs are due to pure panic, as implied by the sunspot equilibrium in their original paper, or unfavorable information on banks' fundamentals, as in Chari and Jagannathan (1988), Jacklin and Bhattacharya (1988) and Allen and Gale (1998).[2] The gap between the panic and fundamental view is to some extent bridged by the application of global games. Using the concept, papers such as Morris and Shin (2000), Rochet and Vives (2004) and Goldstein and Pauzner (2005) refine the multiple equilibria in Diamond and Dybvig (1983) and emphasize the role of early liquidation loss in causing bank runs. It is implied that panic happens as fundamentals weaken: an extra buffer of cash flow is needed to reassure creditors and to prevent runs. Banks that fail to provide the critical extra buffer become "solvent but illiquid".

A limitation of the existing models is that they build on the simplifying assumption of exogenous fire-sale losses.[3] So the models ignore the reinforcing effects of runs on fire-sales. In contrast, the current paper explores the relationship: as bank runs pool the assets of illiquid banks with those of the insolvent, the adverse selection causes the low asset prices and fire-sale losses.[4] Furthermore, a natural corollary of assuming exogenous fire-sale price is that funding liquidity risk is always reduced by higher capital, because the returns generated on capital add to the buffer against fire-sale losses. With endogenous fire-sale prices the current paper takes a broader view: while acknowledging the buffer effect of capital, we point out that greater capital can also contribute to illiquidity via buyers' pessimistic inference.

Three approaches can be found in the contagion literature; we now discuss each of them for their difference and relevance for the current paper.The first strand of the literature emphasizes direct linkages such as payment system, e.g., Freixas and Parigi (1998) and Freixas,

[1] It should be mentioned that some papers also consider the positive role of bank run as disciplinary device: Calomiris and Kahn (1991) and Diamond and Rajan (2001).

[2] Friedman and Schwartz (1963) provides empirical support for the panic view. Contradicting evidence in favor of the fundamental view is present in Gorton (1988), Calomiris and Gorton (1991) and Calomiris and Mason (2003).

[3] For example, Rochet and Vives (2004) assumes exogenous fire-sale discount and Morris and Shin (2009) assumes exogenous haircut.

[4] While the current paper justifies the low asset price by informational frictions, low asset prices can also be explained by fixed short-term cash supply—the cash-in-the-market argument pioneered by Shleifer and Vishny (1992) and Allen and Gale (1994). The main reasoning of the paper should also work through in the context of cash-in-the-market pricing.

Parigi, and Rochet (2000), or interbank market, e.g., Allen and Gale (2000). The contagion analyzed in the current paper, however, does not rely on direct linkages, and we assume for simplicity no interbank market or payment system. Second, banks' common risk exposures directly contribute to systemic risk: once the risk factor takes a downturn, all banks suffer.[5] In his influential paper Gorton (1988) documents that U.S. banks are commonly exposed to business cycles, and systemic crises correspond well to real sector slowdowns. The common exposure also underpins the results of our model. Yet as shown by the current paper, contagion is generated not only by the actual realization of common risk but also by its perception: a bank failure casts shadow on the perceived common risk; and the created negative informational externalities affect all the other banks. This observation leads to the third strand of the literature that is more directly related to our approach: the informational contagion, as exemplified by Acharya and Thakor (2011) and Oh (2012). Compared to the existing work, the current paper emphasizes the self-fulfilling nature of such contagion and the two-way feedback between runs and fire-sales.

On the policy side, the paper relates to a few papers that show increased capital requirements can increase bank risk. Emphasizing banks' portfolio choice, Koehn and Santomero (1980) and Kim and Santomero (1988) question the effectiveness of ratio regulation; and Rochet (1992) shows that improperly set risk weights can lead to risky portfolio reallocation. In an inter-temporal framework, Blum (1999) shows that when raising new capital is costly, in order to meet a future increase in capital requirement banks may be pushed to accumulate capital by risk-taking today. Martinez-Miera (2009) argues that equity increases banks' cost of funding, which leads to higher loan rates and spurs risk-taking by borrowers. As a result, banks' portfolio risk rises passively. Hakenes and Schnabel (2007) argue that a higher capital requirement erodes charter value and induces banks' active risk taking; when the higher capital requirement decreases credit supply, it also leads to borrower risk-taking via a hike in loan rate. What all these papers have in common is that they all focus on solvency risk. To the best of our knowledge, the current study is the first to show capital can contribute to illiquidity and contagion.

The discussion on disclosure policy is most related to two recent papers on the instability consequences of public bailouts: Morrison and White (2010) is concerned that a public bailout

[5]The common risk view is highlighted in a number of system-wide crises where banks are conceived to be excessively exposed to housing market conditions, e.g., the banking crisis in Japan, the savings and loans crisis in 80s and the most recent crisis in the the U.S., Spain and Ireland, etc.

can reveal regulatory deficiency and make market participants lose their confidence in all other banks under the same regulation. Wang (2010) empirically documents that after individual banks were identified in the Fed's Trouble Asset Relief Program (TARP), bank run probabilities, as reflected in CDS spread and stock market abnormal returns, rose dramatically, an outcome the author attributes to the bad news nature of public bailout. Our paper abstracts from specific policy announcements and shows that as long as market participants believe the regulator is better informed, any regulatory action and announcement concerning banks' common risk exposure may generate financial contagion.

The paper proceeds as follows. Section 2 lays out the model. Section 3 presents the bank-run model under asset market adverse selection and endogenous fire-sale price. Self-fulfilling bank runs and contagion happen as a result. Section 4 and 5 discuss the policy implications of model, in particular whether higher capital requirements can lead to greater liquidity risk and contagion, and whether regulatory bodies should disclose information on aggregate states. In section 6, we discuss briefly the implications for related policy issues such as liquidity requirements and lender of last resort policies. Section 7 concludes.

2 Model setup

We consider a three-date ($t = 0, 1, 2$) economy with three groups of players: banks, banks' creditors and uninformed buyers of banks' assets. All players are risk neutral.

At $t = 0$, banks borrow short-term and lend long-term, issuing demandable debts to creditors. It is assumed that each bank has a continuum of creditors whose mass is 1. The debt contract promises repayment r_D at $t = 2$. If a creditor early withdraws at $t = 1$, he receives qr_D, $q < 1$ reflecting the penalty for early withdrawals.

Banks lend to a portfolio of long-term risky projects that generate a random cash flow θ. For simplicity, we assume that θ follows a uniform distribution on $[\underline{\theta}_s, \overline{\theta}]$. Here $s \in \{1, 2\}$ denotes such aggregate states, e.g., housing price or other macroeconomic conditions, with $\underline{\theta}_2 > \underline{\theta}_1$, state 2 more favorable than state 1. Therefore, banks' asset values are not only affected by its idiosyncratic risk (the realization of θ) but also by a common aggregate state that determines the distribution of θ. The upper bound of the support is assumed to be the same across states, for banks hold mostly debt contracts and the best return is fixed by pre-specified interest rates.

We further make the following two assumptions on parameters.

$$r_D > \underline{\theta}_s \tag{1}$$

$$(\overline{\theta} + \underline{\theta}_1)/2 - r_D > 0 \tag{2}$$

Inequality (1) states that a bank's debt is risky in both states. And inequality (2) states that banks' role of financial intermediation is socially efficient even when the unfavorable state is $s = 1$.

Banks' assets are also long-term, taking 2 periods to mature. In particular, the assets cannot be physically liquidated before maturity. Therefore if a bank run takes place at $t = 1$, assets can only be financially liquidated and sold to outside buyers.

The potential asset buyers are uninformed and do not observe the bank's cash flow. So the market suffers from potential adverse selection concerning asset qualities. Yet the asset market is assumed to be competitive: a buyer who offers to buy at price P makes zero profit in expectation.

The long-term investment, demandable debts and asset prices distorted by adverse selection—all together generate maturity and liquidity mismatch that exposes banks to funding liquidity risk—the risk of failing because of bank runs.

2.1 Bank run and coordination failures

When a bank's asset is sold below its fundamental value $P < \theta$, i.e., a fire-sale, there is a cost of early liquidation. The demandable nature of banks' debts enables creditors to withdraw (or, equivalently, not to roll over) their credits on a short notice. The creditors who withdraw early won't be hurt by the costly early liquidation, while those who do not run may bear the fire-sale loss. As a result, distrust rises among debt holders and the coordination failure can lead to runs on solvent banks: a bank with $\theta > r_D$ can fail because of fire-sale losses and runs.

For a bank generating an intermediate range of cash flows belonging to $(r_D, \overline{\theta}]$, a bank run game of complete information has two strict equilibria: all creditors running on the bank or nobody running. To refine the equilibria, we take the global games approach pioneered by Carlsson and Van Damme (1993) and study games with incomplete information where there is no common knowledge on θ. We assume that by the beginning of $t = 1$, both aggregate (s) and idiosyncratic (θ) risk have been realized, but the information is not fully revealed to players. In

particular, at $t = 1$ buyers observe neither s or θ's realization. On the other hand, each creditor, instead of perfectly foreseeing the future cash flow θ, observes a noisy signal $s_i = \theta + \epsilon_i$, where the noise ϵ_i is drawn from a continuous distribution with a support that is sufficiently small. Based on the information, the creditors play a bank-run game with each others. Each creditor has two actions: to wait until maturity or to early withdraw. If the bank does not fail at $t = 2$, depositors who wait receive the promised repayment r_D. A creditor can otherwise withdraw early to receive qr_D, if the bank does not fail at $t = 1$. The lack of information will lead to coordination failure among debt holders, and bank runs follow. For simplicity, we assume that the legal cost for bankruptcy is sufficiently high such that once a bank is bankrupt, either because of insolvency or illiquidity, either at $t = 1$ or $t = 2$, its debt holders receives zero payoffs.

To absorb the fire-sale losses and to reassure its debt holders not to run, a bank has to be more than merely solvent. It implies a critical cash flow level $\hat{\theta} > r_D$ for a bank to survive runs. In particular, the critical $\hat{\theta}$ relates to fire-sale price P: a lower asset price implies a greater fire-sale cost and a greater chance of coordination failure.

2.2 Adverse selection and bank runs

We assume that buyers move after creditors. Creditors, who can be lending to different banks, make their individual decision simultaneously on whether to run or not the bank to which they lend. Once the runs happen and the bank assets are on sale, the uninformed buyers move to bid on the assets.

The sequence of moves gives the following information structures: buyers in the market observe only whether a bank faces a run or not, and do not observe the bank's cash flow θ or the aggregate state s. But they do know that the creditors observe noisy signals of the bank's cash flow and decide whether to run or not upon the observed signals. The price the uninformed buyers offer therefore conditions on whether a bank run happens, or in the case of multiple bank runs, the number of runs. For example, if two banks exist in the economy, their information set is an element in $\{0, 1, 2\}$. They infer from the observed outcome banks' asset qualities, forming a rational expectation that a bank's cash flow falls below $\hat{\theta}$ when a run is observed.

As $\hat{\theta}$ is affected by P, there will be two-way feedback between fire-sale prices and runs. Information asymmetry in the market makes a "solvent-but-illiquid" bank indistinguishable from an insolvent one. The adverse selection distorts down the price and leads to self-fulfilling

bank-runs: a bank encounters a run because of the low price P and it is because of the bank run that P is downward distorted.

2.3 Belief updating on states and contagion

We model contagion in the context of two banks, which are ex ante identical and differ only ex post in the realization of their asset returns. A bank may face no liquidity risk if the other bank does not fail, but will fail when the other fails. The latter case defines contagion.

The priori belief is assumed to be that s takes the value of 1 or 2 with equal probability. Upon observing the number of bank runs, the uninformed buyers update their beliefs on s according to Bayes' rule. The belief updating on aggregate state relates to contagion. As banks have common risk exposure, asset buyers update their beliefs about the aggregate state upon observing more bank failures. Their willingness to pay for banks assets is reduced as both bank fail, since that indicates that $s = 1$ is more likely.

The contagion is also self-fulfilling via asset prices. As more banks liquidate and asset prices drop, anticipating the further reduced fire-sale prices, a bank's creditors are more likely to run on the bank. It is the bank runs, however, that lead to lower valuation of the bank and lead to runs in the first place.

Therefore two factors can contribute to low asset prices. First, once a bank liquidates its assets, it cannot be distinguished from the insolvent. The adverse selection distorts downward asset price. Second, liquidation also leads to inferences that an unfavorable state is more likely, and the willingness to pay for the bank's asset drops.

2.4 Public policies

The theoretical framework allows us to study public polices such as capital requirements and regulatory transparency. We model capital regulation in its most simplistic form: a regulator can put restriction on banks' total debt obligation r_D. It is further assumed that banks always set their capital ratio binding. We then examine the change of various risk following the change in capital regulation.

Concerning the disclosure policy of regulators, it is assumed that a regulator has superior information about the aggregate state s and can choose whether to disclose that to market participants. For simplicity, we assume that legislation allows the regulator to perfectly commit

to disclose or suppress information. To concentrate on the effects of disclosure, we consider a simplistic case where the regulator makes neither type I nor type II errors: the aggregate state s is observed perfectly. We then evaluate whether possible information disclosure is welfare-enhancing.

Once the regulator commits to disclose information, the information concerning the aggregate state will be released before market trading. The information set of asset buyers changes correspondingly. In contrast to the belief updating upon observing a number of bank runs, now buyers know with certainty the aggregate state. So the price no longer conditions on the number of runs but on the announcement that the regulator makes.

2.5 Timing

Depending on whether the regulator discloses information concerning aggregate states, the timing of the game is summarized in the following figures, where the events at $t = 1$ happen sequentially.

Figure 1. Timing of the game: without public disclosure:

t = 0	t = 1	t = 2
Banks issue demandable debt and lend.	1. s and θ are realized. 2. Each creditor receives a noisy signal of θ. 3. Upon signals, creditors decide to run or not. 4. Observing runs, buyers offer price P.	Returns are realized.

Figure 2. Timing of the game: with public disclosure:

t = 0	t = 0.5	t = 1	t = 2
Banks issue demandable debt and lend.	1. State s is realized. 2. The regulator announces the realization of s.	1. θ is realized. Creditors receive noisy signals. 2. Upon signals, creditors decide to run or not. 3. Depending on the announcement, buyers offer price P_1 or P_2.	Returns are realized.

3 Self-fulfilling bank runs and contagion

The analysis unfolds in 3 steps: first, for a *given* asset price P, subsection 3.1 characterizes a critical cash flow for a bank to be solvent and liquid . Second, we endogenize the price in a competitive market with adverse selection on asset qualities. To illustrate the main reasoning, in subsection 3.2 we first solve the model for its most simplified case, with only one bank and one state. And subsection 3.3 extends this case to the full-fledged model of two banks and two states, where we show that the equilibrium price continues to exist and be unique. Finally, we show in subsection 3.4 that there is always the risk of contagion: a bank will stay solvent and liquid if the other bank has no run, but fails because of illiquidity when the other bank has a bank run.

3.1 Run and coordination failure

We examine pure liquidity risk for solvent banks, i.e., banks with $\theta > r_D$. Such a bank can fail because of a run either at $t = 1$ or $t = 2$. In the former case, the liquidation value of all assets is insufficient to repay early withdrawals. In the latter case, while partial liquidation generates sufficient cash to pay early withdrawals at $t = 1$, the cash flow generated by the residual portfolio is insufficient to pay creditors who wait until $t = 2$. Once a bank's cash flow is insufficient to repay its debt, either at $t = 1$ or $t = 2$, the bank declares bankruptcy and incurs a bankruptcy cost. For simplicity, we assume the cost is sufficiently high such that a bank has no value after bankruptcy.

Denote by L the fraction of creditors who run on the bank. The bank will fail at $t = 1$ if and only if $P \leq Lqr_D$, or

$$L \geq \frac{P}{qr_D} \equiv L'. \tag{3}$$

Note that three factors contribute to a successful run: (i) a sufficiently large number of early withdrawals, (ii) low market price for assets on sale and (iii) a high debt obligation.

A bank that survives $t = 1$ withdrawals will have to partially liquidate its portfolio. Denote $f = Lqr_D/P$ the fraction of asset sold to meet the $t = 1$ debt obligation. The bank will fail at $t = 2$ if and only if $(1 - f)\theta \leq (1 - L)r_D$, or

$$L \geq \frac{P(\theta - r_D)}{(q\theta - P)r_D} \equiv L''. \tag{4}$$

Such a $t = 2$ failure happens because the partial early liquidation incurs the cost of fire sale: when a sufficiently large number of creditors run and the bank is forced to liquidate a significant share of assets, the remaining assets will be insufficient to generate cash flows to meet $t = 2$ debt obligation. The creditors who withdraw early at $t = 1$ therefore impose negative externalities on creditors who choose to wait.

In obtaining inequality (4), we use condition $\theta > P/q$, which is guaranteed if

$$qr_D > P. \tag{5}$$

Condition (5) states that it is more difficult for a bank to meet its debt obligation when run happens. Also note that condition (5) implies $L' > L''$. Compared to $t = 2$ failure, a $t = 1$ failure requires more creditors to run on the bank.

Whether a creditor wants to run on the bank depends on the choices of other creditors. In particular, it is not in the interest of a creditor to wait when a sufficiently large number of other creditors withdraw early. It is essential to characterize the belief as to the mass of creditors who will withdraw. When players take a switching strategy: to run the bank if the observed signal is smaller than s^* and to wait otherwise, the belief on the mass of creditors who will withdraw early then boils down to the belief on the mass of creditors who observe a signal lower than s^*. Consider a creditor i who happens to observe exactly s^*: denote by M the fraction of creditors whom creditor i believes observe a higher signal than his. The Laplacian property of global games implies that $M \sim U(0, 1)$.[6]

Consequently, conditional on observing signal s^* the probability of $t = 1$ survival is

$$Prob(L \leq L'|s = s^*) = Prob(1 - M \leq L'|s = s^*) = L'.$$

Similarly, the probability of $t = 2$ survival is

$$Prob(L \leq L''|s = s^*) = Prob(1 - M \leq L''|s = s^*) = L''.$$

For $L' > L''$, we will have $Prob(L \leq L''|s = s^*) < Prob(L \leq L'|s = s^*)$, for a bank that fails at $t = 1$ necessarily fails at $t = 2$.

[6]The argument is due to Morris and Shin (2001). We reproduce the proof in appendix.

Depending on the amount of early withdrawals, the payoffs for playing "run" and "wait" are tabulated as follows.

	$L \in (L', 1]$	$L \in (L'', L']$	$L \in [0, L'']$
run	0	qr_D	qr_D
wait	0	0	r_D

When running on the bank, the creditor receives qr_D with $Prob(L \leq L'|s = s^*)$. By waiting, she receives r_D with a lower probability $Prob(L \leq L''|s = s^*)$. Therefore, in playing the bank run game, a creditor trades off between a higher chance of receiving non-zero payoff and the higher payoff from waiting.

To be consistent with the definition of switching strategy, the creditor who observes the critical signal s^* should be indifferent between running on the bank or not.

$$Prob(1 - M \leq L'|s = s^*) \cdot q = Prob(1 - M \leq L''|s = s^*)$$

The indifference condition implies the following critical cash flow $\hat{\theta}$ for a given P.

$$\hat{\theta}_P = \frac{r_D - P}{1 - q}, \tag{6}$$

For a given asset price P, a run successfully happens to banks with $\theta < \hat{\theta}_P$.

It can also be verified that condition (5) is always satisfied in equilibrium. To see why, rewrite expression (6) as $P = r_D - (1 - q)\theta$. It can be shown that

$$\frac{\partial}{\partial q}\left(\frac{P}{q}\right) = \frac{\theta - r_D}{q^2} > 0,$$

so that P/q is maximized at $q = 1$. Furthermore, $P/q = r_D$ when $q = 1$. Therefore, we always have

$$P/q < r_D$$

and condition (5) is always satisfied in equilibrium.

The result is summarized in the proposition below. In the appendix we give an alternative proof that also characterizes explicitly the belief updating of players.

Proposition 1. *A solvent bank with $\theta > r_D$ can be illiquid: it is able to repay in full its debt if no creditor runs but it will fail in a bank run as creditors withdraw early. There exists a critical*

cash flow $\hat{\theta}_P = (r_D - P)/(1-q)$ *such that a run will be triggered once a bank's cash flow is below* $\hat{\theta}_P$.

Note that the critical cash flow is decreasing in P: a lower asset price leads to a higher chance of illiquidity. As adverse selection distorts price in the asset market, bank runs can happen to solvent banks.

3.2 Self-fulfilling bank run: an illustration

To illustrate the main intuition, we start with a special case where there is only one bank and one state, i.e., $\underline{\theta}_1 = \underline{\theta}_2 = \underline{\theta}$. There is no inference for aggregate states and there is no contagion.

In a competitive market, the uninformed asset buyers break even. A candidate equilibrium price P must satisfy two conditions. First, buyers do not make positive profit in expectation. That is, the following zero-profit condition must hold.

$$P = E[\theta|\theta < \hat{\theta}_P] = \frac{1}{2}\left(\frac{r_D - P}{1-q} + \underline{\theta}\right) \tag{7}$$

Second, a buyer should not be able to profit by unilaterally bidding a higher price. Therefore, in equilibrium a buyer's expected payoff, $E(\theta|\theta < \hat{\theta}_P) - P$, should not increase in P, which naturally holds as the expected payoff takes the form

$$\pi = \frac{1}{2}\left(\frac{r_D - P}{1-q} + \underline{\theta}\right) - P.$$

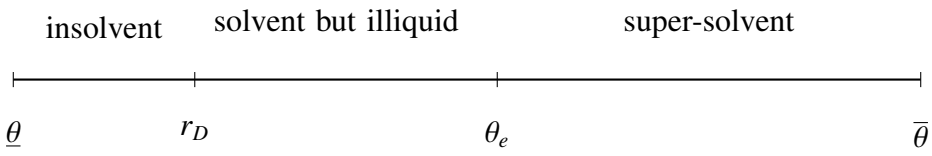
An equilibrium is therefore characterized by a pair $\{\theta_e, P_e\}$: (1) P_e satisfies the above two conditions in a competitive market where adverse selection prevails and qualities vary from $\underline{\theta}$ to θ_{P_e}; and (2) θ_e is the corresponding critical flow for P_e, as specified by equation (6). Combining the equilibrium conditions (6) and (7), we have a system of two equations in P_e and θ_e, which yields

$$P_e = \frac{1}{3-2q}[r_D + (1-q)\underline{\theta}].$$

And correspondingly,

$$\theta_e = \frac{1}{3-2q}[r_D + (r_D - \underline{\theta})].$$

As $q \nearrow 1$, P_e converges to r_D and θ_e converges to $r_D + (r_D - \underline{\theta}) > r_D$, as represented graphically below.



A bank with $\theta \in (r_D, \theta_e]$ is solvent but illiquid: it can fully repay its debt obligation at $t = 2$ if no bank run happens, but will fail at $t = 1$ because of coordination failure among its creditors. Such run is self-fulfilling: the adverse selection and low price lead to the panic among creditors; but a solvent but illiquid bank is pooled with the insolvent just because the creditors run and the bank has to liquidate its assets. The risk of illiquidity can be measured by the relative length of the interval, $(\theta_e - r_D)/(\overline{\theta} - \underline{\theta})$.

Another two observations can be made. First, the price reflects adverse selection: by offering r_D, a buyer makes a loss when the bank is insolvent, and profits when the bank is only illiquid, but on average breaks even. Second, the adverse selection is reflected in the critical level of the cash flow. As $\underline{\theta}$ decreases, θ_e rises: a low $\underline{\theta}$ aggravates the adverse selection in the asset market and makes the bank more likely to be illiquid.

3.3 Self-fulfilling bank runs: full model

Now we turn to more complex cases where there are different states and the uninformed buyers rationally update their beliefs. The difference from the example above is that now the asset price not only reflects adverse selection in asset qualities, but also reflects the updated belief on the aggregate state s. For example, observing two bank runs can indicate that the realization of the aggregate state is likely to be low, i.e., $\underline{\theta}_s = \underline{\theta}_1$, which further lowers buyers' valuation of banks' assets.

3.3.1 Case: 1 bank run observed

We start with the case where a bank run happens successfully to a single bank and the other bank survives. Upon observing this outcome, uninformed asset buyers update their beliefs of aggregate state s according to Bayes' rule. Let θ^* denote the critical cash flow for a bank to survive given that the endogenous asset price is conditional on a single bank failure. And ω_s,

$s \in \{1, 2\}$ denotes the posterior probability of states s.

$$\begin{aligned}
\omega_1(\theta^*) \equiv Prob(s = 1|\text{single failure}) &= \frac{(\theta^* - \underline{\theta}_1)(\overline{\theta} - \theta^*)}{(\theta^* - \underline{\theta}_1)(\overline{\theta} - \theta^*) + (\theta^* - \underline{\theta}_2)(\overline{\theta} - \theta^*)} \\
&= \frac{(\theta^* - \underline{\theta}_1)}{(\theta^* - \underline{\theta}_1) \ + (\theta^* - \underline{\theta}_2)} \\
\omega_2(\theta^*) \equiv Prob(s = 2|\text{single failure}) &= \frac{(\theta^* - \underline{\theta}_2)(\overline{\theta} - \theta^*)}{(\theta^* - \underline{\theta}_1)(\overline{\theta} - \theta^*) + (\theta^* - \underline{\theta}_2)(\overline{\theta} - \theta^*)} \\
&= \frac{(\theta^* - \underline{\theta}_2)}{(\theta^* - \underline{\theta}_1) \ + (\theta^* - \underline{\theta}_2)}
\end{aligned}$$

We use notation $\omega_s(\theta^*)$ to emphasize that the posterior belief depends on the endogenous price P and θ^*, showing the two-way feedback between asset prices and beliefs.

Bidding a price P, buyers have expected payoff $E_s(\theta|\theta < \hat{\theta}_P)$ for the unit asset purchased. Under a uniform distribution, we have

$$E_s(\theta|\theta < \hat{\theta}_P) = \frac{\hat{\theta}_P + \underline{\theta}_s}{2}.$$

The expected payoff, on the one hand depends on the adverse selection, and on the other hand depends on the realized aggregate state, as the subscript s indicates. In a competitive market buyers make zero profit, implying the following necessary condition for possible equilibrium prices,

$$P = \omega_1(\hat{\theta}_P) \cdot E_1(\theta|\theta < \hat{\theta}_P) + \omega_2(\hat{\theta}_P) \cdot E_2(\theta|\theta < \hat{\theta}_P) \equiv \phi(P). \tag{8}$$

For the ease of presentation, we denote by π_s the expected net gain (loss) of asset purchases in aggregate state s.

$$\pi_s(P) \equiv E_s(\theta|\theta < \hat{\theta}_P) - P = \frac{\hat{\theta}_P + \underline{\theta}_s}{2} - P = \frac{1}{2}(\frac{r_D - P}{1 - q} + \underline{\theta}_s) - P$$

The last equality makes use of equation (6). And the equilibrium condition can be represented by

$$\omega_1(P^*) \cdot \pi_1(P^*) + \omega_2(P^*) \cdot \pi_2(P^*) = 0. \tag{9}$$

Recall again equation (6), the linear relationship between the critical θ and market price P, the necessary condition (9) can be rewritten in terms of critical θ.

$$\omega_1(\theta^*) \cdot \pi_1(\theta^*) + \omega_2(\theta^*) \cdot \pi_2(\theta^*) = 0. \tag{10}$$

Define an auxiliary function $F_1(\theta) = \omega_1(\theta) \cdot \pi_1(\theta) + \omega_2(\theta) \cdot \pi_2(\theta)$. A competitive equilibrium indicates a buyer cannot profit by by unilaterally raising P (and therefore decreasing θ_P). Mathematically it requires $F_1(\theta)$ to be monotonically increasing in θ, a result that we show in the lemma below.

Lemma 1. *$F_1(\theta)$ monotonically increases in θ, meaning a buyer's expected payoff monotonically decreases in his bid P.*

Proof. See appendix B.1. □

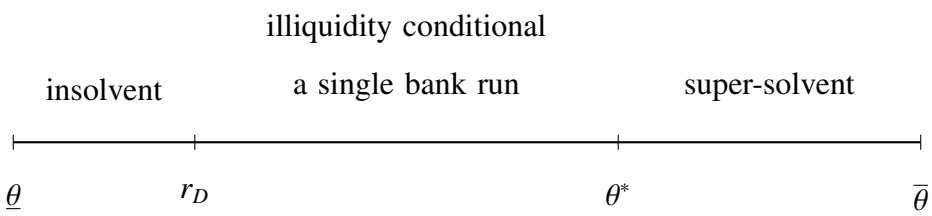
Intuitively, bidding a higher P affects buyers' expected payoffs via two ways. First, it directly reduces the payoff. On top of that, it also alleviates the liquidity problem faced by banks, making fewer banks sell for liquidity reasons. As a result, more banks sell for insolvency and the average quality of assets drops.

We now examine the existence of solvent but illiquid banks. The existence of such equilibria, as equation (9) implies, boils down to finding fixed points for function $\phi(P)$ on the interval $(r_D, \overline{\theta}]$. We show the fixed point does not only exists but is also unique.

Proposition 2. *There exists an unique $\theta^* > r_D$. When one bank run is observed, a bank of cash flow $\theta \in (r_D, \theta^*]$ is solvent but illiquid, being able to pay its debt obligation r_D if no one runs the bank, but failing if a run and fire-sales happen.*

Proof. See appendix B.2. □

Graphically, we have:



3.3.2 Case: 2 bank run observed

We now turn to the case where 2 bank failures are observed. The analysis resembles the case before. Define θ^{**} to be the critical cash flow for a bank to survive given the endogenous

asset price is conditional on two bank failures. And denote the posterior beliefs by ω'_s. As two bank failures are observed, creditors update their beliefs according to Bayes' rule.

$$\omega'_1(\theta^{**}) \equiv Prob(s = 1|\text{two failures}) = \frac{(\theta^{**} - \underline{\theta}_1)^2}{(\theta^{**} - \underline{\theta}_1)^2 + (\theta^{**} - \underline{\theta}_2)^2}$$
$$\omega'_2(\theta^{**}) \equiv Prob(s = 2|\text{two failures}) = \frac{(\theta^{**} - \underline{\theta}_2)^2}{(\theta^{**} - \underline{\theta}_1)^2 + (\theta^{**} - \underline{\theta}_2)^2}$$

The zero-profit condition (10) now becomes

$$\omega'_1(\theta^{**}) \cdot \pi_1(\theta^{**}) + \omega'_2(\theta^{**}) \cdot \pi_2(\theta^{**}) = 0. \tag{11}$$

To check that a buyer cannot profit by unilaterally bidding a higher price, define an auxiliary function $F_2(\theta)$:

$$F_2(\theta) = \omega'_1(\theta) \cdot \pi_1(\theta) + \omega'_2(\theta) \cdot \pi_2(\theta).$$

It again can be shown that $F_2(\theta)$ is monotonically increasing. Now we show the existence and uniqueness of θ^*.

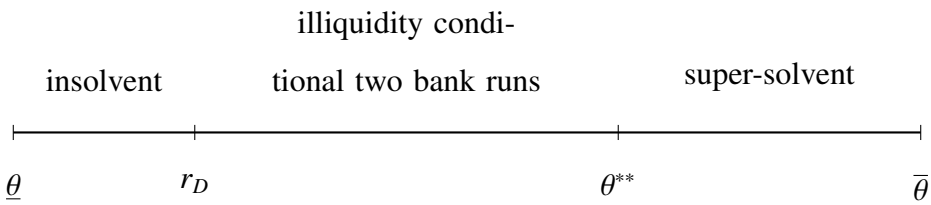
Lemma 2. *$F_2(\theta)$ monotonically increases in θ, meaning a buyer's expected payoff monotonically decreases in his bid P.*

Proof. See appendix B.1. □

Proposition 3. *There exists an unique $\theta^{**} > r_D$. When two bank runs are observed, a bank of cash flow $\theta \in (r_D, \theta^{**}]$ is solvent but illiquid.*

Proof. See appendix B.3. □

Graphically, we have:



3.4 Contagion

$\theta^{**} > \theta^*$ will imply potential contagion: a bank with cash flow between θ^* and θ^{**} is exposed to the risk of contagion, which happensss in a self-fulfilling manner. Consider two banks i and

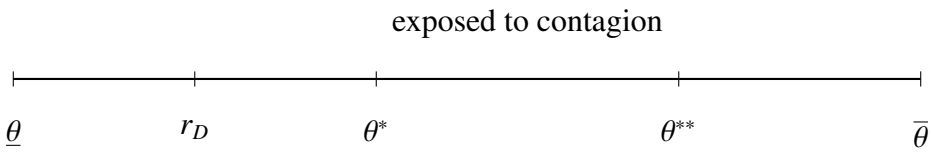
ii. Bank i generates cash flow $\theta \in (\theta^*, \theta^{**}]$. According to the definition of θ^{**}, this bank will survive if and only if bank *ii* has $\theta > \theta^{**}$, but will fail otherwise. For example, if bank *ii* generates $\theta < \theta^*$, bank i will be liquidated as a result of a successful bank run, which would not happen if the other bank is doing well or is simply absent. Moreover, if both banks generate $\theta \in (\theta^*, \theta^{**}]$, both will fail. The common market price and belief updating create a channel for contagion through bank runs. Such contagion is also self-fulfilling: the creditors of bank i anticipate a further lower fire-sale price if they run on the bank and such a low price generates coordination failure and runs.

Intuitively, θ^{**} will be greater than θ^*, because to escape a bank run, an extra buffer is needed to offset the further reduced fire-sale price: upon observing two bank failures, buyers revise down their beliefs concerning aggregate states and banks' asset values. Now we show this rigorously.

Proposition 4. *With $\theta^{**} > \theta^*$, a bank of cash flow $\theta \in (\theta^*, \theta^{**}]$ is subject to the risk of contagion: being able to survive if the other bank generates $\theta > \theta^{**}$ but suffering a bank run otherwise.*

Proof. See appendix B.4. □

Graphically, we have:



We present the model in a simultaneous setup where creditors at both banks need to decide at the same time whether to run or not. The result however will remain the same if we assume sequential moves. For example, if creditors of bank 2 move after observing the outcome of bank 1's bank run game, the critical cash flow will rise to θ^{**} if bank 1 fails and will remain at θ^* if the other bank does not fail. We use the simultaneous setup to emphasize that the role of the two ex ante identical banks is the same in terms of contagion. A run on bank 2 can also negatively affect bank 1.

4 Capital regulation

It is an entrenched belief that higher capital helps reduce funding liquidity risk and prevent financial contagion. We show, however, once asset prices are endogenous, the situation is more complicated and capital can have unintended consequences on illiquidity risk and financial contagion. Compared to the case where capital is low, a bank failure under high capital adequacy is much worse news concerning bank asset qualities and banks' common risk exposure. The pessimistic inference will put downward pressure on asset prices and undermine capital's role in reducing runs and contagion. With the stylized model we identify the unintended illiquidity consequence of capital and show that failures of better capitalized banks result in stronger financial contagion. However, because bank capital reduces the total risk of each individual bank, overall, higher bank capital still contributes to financial stability by reducing the chance of systemic crises.

4.1 Capital and illiquidity

Under exogenous asset prices, a natural corollary of proposition 1 is that a higher capital requirement always helps to reduce funding liquidity risk. The cash flows generated by capital serve as an extra buffer against fire-sale losses. Debt value is better protected and creditors have less incentives to run, a channel that we call "buffer effect". To verify this, denote the illiquidity risk by $IL \equiv \hat{\theta}_P - r_D$ and note that

$$\frac{\partial IL}{\partial r_D} = \frac{q}{1-q} > 0 \tag{12}$$

When higher capital requirement pushes down r_D, funding liquidity risk always decreases.

With endogenous asset prices, the situation is more complex. Once investors rationally update their beliefs of banks asset qualities, a higher capital level also contributes to illiquidity via endogenous fire-sale prices. The intuition is as follows. In terms of inferring the realization of θ, a bank run presents a more negative news when it happens to a well-capitalized bank than when it happens to a poorly capitalized bank. Due to the deteriorating inference, buyers' willingness to pay drops more sharply upon observing runs to well-capitalized banks.

We show this point with the most simplified version of the model in section 3.2. To verify the argument, notice that the endogenous price P_e is increasing in r_D,

$$\frac{\partial P_e}{\partial r_D} = \frac{1}{3-2q} > 0.$$

The higher the capital (low r_D), the lower the willingness to pay. The market liquidity of a bank's assets is affected by the bank's capital and liquidity positions. The low price in turn generates panic among debt holders and leads to funding liquidity risk, a mechanism we dub "inference effect".

The overall impact of capital on funding liquidity risk depends on the relative strength of the "margin" and "inference" effects. Yet as long as the asset price is endogenous, the "inference effect" will always be present; and capital requirements are less able to contain liquidity risk. The point can be verified by comparing the comparative statics under endogenous price with that under exogenous price, i.e., expression (12). Notice that under endogenous price

$$\frac{\partial IL_e}{\partial r_D} = \frac{\partial}{\partial r_D}\left(\frac{2r_D - \underline{\theta}}{3-2q} - r_D\right) = \frac{2}{3-2q} - 1 = \frac{-1+2q}{3-2q}. \tag{13}$$

It is straightforward to verify that

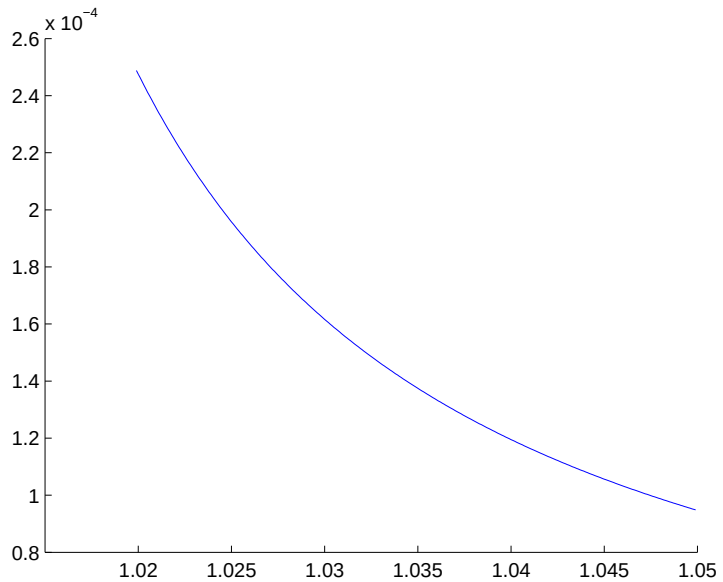
$$\frac{q}{1-q} > \frac{-1+2q}{3-2q}.$$

Once the asset price is endogenous, funding liquidity risk diminishes at a lower rate as capital requirement rises. Buyers' belief updating limits the role of capital in containing funding liquidity risk. We summarize the results in the following proposition.

Proposition 5. *Bank capital in containing illiquidity risk is compromised by lowering the fire-sale prices. In particular, with higher capital requirements the risk of illiquidity falls more slowly under endogenous fire-sale prices than under exogenous prices.*

In this illustration, the buffer effect is dominating ($\partial IL_e/\partial r_D > 0$ for q close to 1). So overall capital reduces illiquidity. Since capital also helps to reduce insolvency risk, banks' total credit risk (pure insolvency plus pure illiquidity) decreases with higher bank capital.

Figure 2: Bank capital and the risk of contagion



4.2 Capital and systemic risk

In this subsection, we move from individual bank risk to financial contagion and the risk of systemic crisis. Because the failure of a well capitalized bank raises concerns about banks' common risk exposure, capital also has unintended contribution to the risk of financial contagion. We drive a sufficient condition under which increasing banking capital leads to a higher exposure to contagion. More precisely, by measuring contagion by $\theta^{**} - \theta^{*}$, we show that the difference decreases in a banks' debt obligation r_D.

Proposition 6. *If* $\max\left\{\frac{\theta^{**}-\underline{\theta}_1}{\theta^{*}-\underline{\theta}_1}, \frac{\theta^{**}-\underline{\theta}_2}{\theta^{*}-\underline{\theta}_2}\right\} < \sqrt{2}$*, the exposure to financial contagion, as measured by* $\theta^{**} - \theta^{*}$*, increases with capital.*

Proof. See Appendix B.5. □

In Figure 2, we illustrate the result numerically: increasing banking capital leads to higher exposure to contagion. With banks' exposure to contagion measured by $\theta^{**} - \theta^{*}$ (on the y-axis), we plot the difference against banks' total liability r_D (on the x-axis). As regulation commands banks to hold more capital (lower debt obligation r_D), the risk of contagion increases.

In this stylized model, a systemic crisis occurs when both banks fail. The probability of a systemic crisis is $[Prob(\theta < \theta^{**})]^2$. To see the point, note that conditional on one bank's cash

flow falls below θ^{**}, the other bank will fail if (1) its cash flow falls below θ^{*}, in which case the bank fails because of individual insolvency/illiquidity, or (2) if its cash flow falls between θ^{*} and θ^{**}, in which the banks fails because of financial contagion. While capital increases the exposure of financial contagion, i.e., increasing $Prob(\theta^{*} < \theta < \theta^{**})$, we show more capital helps to decrease the risk of a systemic crisis by reducing individual bank risk, i.e., reducing $Prob(\theta < \theta^{*})$. Formally, we have the following theorem.

Proposition 7. *Higher capital requirement reduces systemic risk.* $\partial[Prob(\theta < \theta^{**})]^2/\partial r_D > 0$.

Proof. See appendix B.6. □

Overall, the result suggests that even if the bank capital has the unintended inferencing effect in terms of contagion, it still helps to reduce bank illiquidity and systemic risk.

5 Disclosure policy

The reasoning of the model naturally links to the establishment of an early warning system. While there has been some concern that the warning system could make type I or type II errors, the current paper shows even if a regulator makes no error at all, the regulators should be cautious in sounding the alarm. Making public announcements of high aggregate risk can be counter-productive, leading to illiquidity and failing financial institutions, which otherwise would not occur.

5.1 Critical cash flow under policy intervention

In this section, we consider the case where a regulator can observe the realization of the true states and can credibly transmit the information to the public. As the regulator commits to disclose the aggregate state to market participants, the uninformed buyers derive their posterior beliefs directly from the announcement. The perception of the aggregate state no longer depends on the number of observed bank runs. Upon receiving the regulatory announcement, the buyers can therefore offer prices contingent on the the true states of world: P_G in the good state and P_B in the bad state. Correspondingly, the critical cash flow for a bank to be liquid depends on the announcement and the realized state. Denote θ_B^* and θ_G^* to be the critical levels in state 1 and 2 respectively.

While the information on the aggregate state improves, the buyers still cannot distinguish an illiquid bank from an insolvent one. Therefore adverse selection remains. In a competitive market where buyers break even, we have

$$P_G = E(\theta|\theta < \theta_G) = \frac{\theta_G + \underline{\theta}_2}{2}, \qquad \text{and} \qquad P_B = E(\theta|\theta < \theta_B) = \frac{\theta_B + \underline{\theta}_1}{2}.$$

As adverse selection distorts prices, self-fulfilling runs persist. But the regulatory announcement changes the information available and affects the equilibrium critical cash flows.

Proposition 8. *Under regulatory announcements there exist* $\theta_G^* = (2r_D - \underline{\theta}_2)/(3 - 2q)$ *and* $\theta_B^* = (2r_D - \underline{\theta}_1)/(3 - 2q)$. *Banks with cash flows* $\theta \in (r_D, \theta_B^*]$ *are solvent but illiquid if* $s = 1$ *is announced. And banks with cash flows* $\theta \in (r_D, \theta_G^*]$ *are solvent but illiquid if* $s = 2$ *is announced.*

Proof. See appendix B.7. □

Notice that the regulatory announcement also eliminates inference as a source of contagion: instead of two critical cash flows depending on the number of observed bank runs, there is a single critical cash flow depending on the realized state.

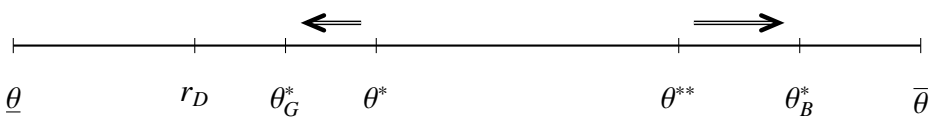
5.2 The effect of policy intervention

Intuitively, $\theta_G^* < \theta^*$ and $\theta_B^* > \theta^{**}$. Even if having observed two bank runs, buyers cannot be certain that $s = 1$. But as as long as the regulatory announcement is more accurate, buyers will lower their valuation of assets further. Similarly, making a favorable announcement will save certain banks from illiquidity, as market participants are reassured. We now show this rigorously.

Proposition 9. *For q sufficiently close to* 1, $\theta_G^* < \theta^*$ *and* $\theta_B^* > \theta^{**}$. *The regulatory announcement reduces illiquidity if* $s = 2$ *but increases it if* $s = 1$.

Proof. See appendix B.8. □

Graphically, we have:



The disclosed information, when favorable, reduces adverse selection: banks with $\theta \in (\theta_G^*, \theta^*]$ are saved from bank runs. But to acknowledge a crisis in the bad state will exacerbate liquidity problems: a solvent bank is more likely to suffer from illiquidity when market participants are more aware that the whole economy is in the bad state. In particular, banks with $\theta \in (\theta^{**}, \theta_B^*]$ will be confronted with runs. Therefore, in determining whether to commit to disclose information, the regulator faces a trade-off: if the state is good, it saves banks from illiquidity; if the state is bad, transparency will create even more panic and runs by pushing asset prices further down.

Whether it is optimal to commit to the regulatory transparency depends on its costs and benefits. For simplicity, we concentrate on the social cost of systemic banking crises. The cost of a single bank failure is much lower in a tranquil time than in a systemic crisis. We therefore normalize the social cost of bank failure to 0 if there is only one bank failure. The assumption captures the idea that without a systemic crisis, the failing bank's assets can be acquired by other financial institutions that can still utilize the assets to maximum capacity, so that the social cost is minimal. Thus we assume social costs only in the cases where both bank fail. Denoting by C_G and C_B the social cost in good and bad states respectively, we further assume $C_B > C_G$: the costs of bank failures are more prominent in economic downturns.

From an ex ante perspective, the expected social cost of bank failures can be formulated as follows: with the information disclosure, the expect cost faced by government is:

$$\frac{1}{2}\frac{(\theta_G^* - r_D)^2}{(\bar{\theta} - \underline{\theta}_2)^2} \cdot 2C_G + \frac{1}{2}\frac{(\theta_B^* - r_D)^2}{(\bar{\theta} - \underline{\theta}_1)^2} \cdot 2C_B.$$

Without the information disclosure, the expected cost faced by government is:

$$\frac{1}{2}\frac{(\theta^{**} - r_D)^2}{(\bar{\theta} - \underline{\theta}_2)^2} \cdot 2C_G + \frac{1}{2}\frac{(\theta^{**} - r_D)^2}{(\bar{\theta} - \underline{\theta}_1)^2} \cdot 2C_B$$

By Proposition 8, we have

$$\frac{(\theta^{**} - r_D)^2}{(\bar{\theta} - \underline{\theta}_2)^2} > \frac{(\theta_G^* - r_D)^2}{(\bar{\theta} - \underline{\theta}_2)^2} \qquad \text{and} \qquad \frac{(\theta^{**} - r_D)^2}{(\bar{\theta} - \underline{\theta}_1)^2} < \frac{(\theta_B^* - r_D)^2}{(\bar{\theta} - \underline{\theta}_1)^2}.$$

Thus there is a tradeoff across states in disclosing information. When a systemic banking crisis creates greater social costs in the bad state than in the good state, transparency is suboptimal.

Proposition 10. *There exists a critical $\hat{C}$: for $\frac{C_B}{C_G} > \hat{C}$, regulatory disclosure leads to greater social costs due to financial instability.*

Proof. See appendix B.9. □

The paper indicates that depending on whether the information is concerning individual banks or the aggregate states, disclose policy can have dramatically different consequences on financial stability. Information on an individual bank's health, once available, reduces adverse selection and contains illiquidity risk. The disclosure of aggregate states, in contrast, is a double-edged sword: helping to contain liquidity risk in good states but exacerbating it in bad states.

6 More policy discussion

We now discuss two more policy issues that are potentially related to the model. Some caveats are discussed too, but we believe with proper modification of the baseline model, the discussed policies and caveats can be addressed and lead to future research.

6.1 Capital and liquidity requirement

The conventional analysis predicts that bank capital provides a buffer against fire-sale losses and therefore contains liquidity risk and informational contagion. So it seems that capital requirement are tools that can both reduce solvency and liquidity risk. And there has been some discussion on how capital and liquidity requirement can substitute for each other, e.g., Schanz (2012).

In our opinion, the two regulations are rather complements rather than substitutes. On the one hand, liquid assets in practice generate low returns. The reduced profitability cripples banks' capability to accumulate capital. Therefore high liquidity requirements are likely to increase solvency risk. On the other hand, high capital requirements can trigger more pessimistic beliefs once a run happens, with the distressed prices leading to illiquidity and contagion. Therefore, in our opinion, concerning solvency and liquidity risk, each regulation fits one purpose best and carries potential drawbacks for the other. In this respect, the proposal of Basel III to set separate capital and liquidity requirements should be a step in the right direction.

6.2 Lender of last resort

With accurate information scarce in crises, both market participants and regulators face the challenge and difficulty in discerning whether a bank is insolvent or illiquid. If a regulator is no better informed about individual bank health, this information constraint is likely to limit its role as a lender of last resort (LOLR). In particular, because of the winner's curse, untargeted LOLR support will incur losses by assisting the truly insolvent banks. By offering prices higher than what markets offer, a LOLR faces a trade-off between incurring losses in the asset purchase and saving the solvent banks from illiquidity and contagion.

On the other hand, since the uncertainty of individual bank health is the source of adverse selection, any information on that, if available, will reduce illiquidity: a well functioning asset market will align asset prices with fundamentals, automatically solve the funding liquidity problems of the solvent banks and will shut down the insolvent ones. This will then leave little room for LOLR policies. Therefore, whether the information on individual bank health is available and whether such information can be credibly conveyed to the market crucially define the role of LOLR: without such information, LOLR policies risk making losses, and with such information, LOLR can do little to help: as long as there is no aggregate shortage of cash, just disclosing the information will suffice.

In an economy where aggregate liquidity shortages exist though, liquidity injection by central banks is perfectly justified, as the cash-in-the-market models predict. Yet if the liquidity problem is caused by asymmetric information, instead of the aggregate shortage of cash, public policies need to be geared to address the market failure of information friction. The effectiveness of LOLR policies is then significantly reduced.

6.3 Caveats and future research

Before finally concluding, we want to point out a few caveats of the model that also invite future research. First of all is the banks' passive role: the assumption of passive banks abstracts banks' risk-taking from the analysis and makes the model unable to fully examine the impact of market discipline. While short-term debts are believed to exert discipline on banks' risk taking, the current paper shows a combination between such maturity mismatch and lack of information presents a threat to financial stability. As market discipline relies crucially on accurate information, under incomplete information it is no longer the panacea that it was sup-

posed to be in the Basel II framework. Yet it is less clear whether information based illiquidity and contagion will reduce or strengthen banks' risk-taking. On the one hand, the extra risk can make banks more cautious in taking risk. On the other hand, in the presence of contagion, a bank's risk-taking presents negative externalities to the other banks, which implies excessive risk-taking as banks make private decisions.

Furthermore, the model assumes exogenous bank portfolios and leverage. Intuitively, information based contagion will diminish when banks hold different portfolios. Therefore to avoid this source of systemic risk, effective regulation should discourage bank herding. The open question is how macro-prudential regulations should be designed in the light of such information based illiquidity and contagion.

Last, in the model bank debt structure is assumed to be given, consisting solely of uninsured short-term debts and binding capital requirements—both simplifying assumptions rather than realities. For commercial banks, the liability structure often involves a mix between insured deposits and uninsured debt. Banks also hold economic capital in excess of their regulatory requirements and actively manage their leverage in response to various risks. It is challenging yet interesting to examine how the information based risks lead banks to adjust their liability structure and how the changes in the liability feeds back.

7 Concluding remarks

In this paper, we investigate the relationship between fire-sales and bank runs. We present a model where fire-sale prices and bank runs, driven by asymmetric information and buyers' belief updating, are endogenously determined in rational expectations equilibrium. Furthermore, we extend the model to incorporate contagion when there is a common risk exposure. We draw several results from our analysis. First, fire-sales and bank runs are self-fulfilling and mutually reinforcing: when creditors anticipate low prices for a bank's asset sales, a run will be triggered, which generates fire-sales and the corresponding collapse in prices, thus fully justifying creditors' strategies. Second, as a bank fails, asset buyers lower their expectation of common risk factor and perceive banks' asset to be less valuable: the declining asset price will precipitate runs at all other banks.

Based on the model, we draw policy implications regarding capital and regulatory transparency. We show that, high capital overall makes banking industry more resilient against

systemic crises. And complementary to conventional wisdom, capital can also have side effects on both illiquidity and contagion because buyers' inference via endogenous fire sale prices. A run presents more negative news, both for idiosyncratic and common risk factor, when it happens to a well-capitalized bank. Asset buyers' perceived asset quality will deteriorate further compared to the case where runs are on poorly capitalized bank. As buyers' willingness to pay drops more sharply, it is more likely that creditors panic such that funding liquidity dries up and contagion starts. We also show that regulatory transparency is a double-edged sword: it saves illiquid banks when the disclosure is favorable, but amplifies illiquidity and contagion problem when the disclosure deteriorates market beliefs. When systemic crisis is more costly than individual bank failures, the desirability of revealing aggregate risk is open to question.

Appendix A The bank run game

Appendix A.1 Laplacian properties

In the model, the noisy signal received by representative creditor i has the structure

$$x_i = \theta + \epsilon_i.$$

We assume ϵ_i follows a continuous distribution with c.d.f. G.

Denote the critical signal for creditor i to switch from "wait" to "run" by x^*. And upon observing x^*, the creditor i believes a M fraction of creditors observing signals higher than hers. We prove $M \sim U(0, 1)$.

Proof. For the continuous distribution G, the fraction of creditors who observe signal higher than x^* equals the probability that a creditor j's signal $x_j > x^*$. Then, we have

$$\begin{aligned} M &= Prob(x_j > x^* | x_i = x^*) = Prob(\theta + \epsilon_j > x^* | x_i = x^*) \\ &= Prob(\epsilon_j > x^* - \theta | x_i = x^*) \\ &= 1 - G(x^* - \theta) \end{aligned}$$

The randomness of M is rooted in the fact that by observing $x_i = x^*$, creditor i is uncertain about the realization of θ. As the *perceived* value of θ is random, so is the *perceived* M. Now

we derive the distribution function of M. For $\hat{M} \in [0, 1]$, we have

$$\begin{aligned}
Prob(M < \hat{M}|x_i = x^*) &= Prob(1 - G(x^* - \theta) < \hat{M}|x_i = x^*) \\
&= Prob(\theta < x^* - G^{-1}(1 - \hat{M})|x_i = x^*) \\
&= Prob(x^* - \epsilon_j < x^* - G^{-1}(1 - \hat{M})|x_i = s^*) \\
&= Prob(\epsilon_j > G^{-1}(1 - \hat{M})|x_i = x^*) \\
&= 1 - G(G^{-1}(1 - \hat{M})) \\
&= \hat{M}
\end{aligned}$$

Note that $M = 1 - G(x^* - \theta) \in [0, 1]$. Therefore for $\hat{M} < 0$, $Prob(M < \hat{M}) = 0$; and for $\hat{M} > 1$, $Prob(M < \hat{M}) = 1$. We prove M follows a uniform distribution on $[0, 1]$. □

Appendix A.2 Uniform distribution of noise

In this section, we will reproduce our derivation of equilibrium in section 3.1 following a strict global game approach. In section 3.1, the fire sale price of the asset with return θ is denoted by P. we will instead assume $P = \theta/(1 + \lambda)$, then λ performs as the role of fire sale discount.

Denote the fraction of creditors who run on the bank by L. The bank will fail at $t = 1$ if $Lqr_D > \theta/(1 + \lambda)$. Thus, we get one critical value of cash flow θ^*:

$$\theta < (1 + \lambda)L \cdot qr_D \equiv \theta^*. \tag{A.14}$$

Similarly, the bank will survive $t = 1$ bank run but fail at $t = 2$ if $(1 - L)qr_D > (1 - f)\theta$. Here f denotes the fraction of partial liquidation carried out on date-1. Specifically,

$$f = (1 + \lambda)\frac{Lqr_D}{\theta}.$$

We can derive the critical level cash flow for $t = 2$ failure

$$\theta < (1 - L)r_D + (1 + \lambda)Lqr_D \equiv \theta^{**}. \tag{A.15}$$

Depending on the value of θ, the payoffs for "run" and "wait" are tabulated as follows.

	$\theta < \theta^*$	$\theta^* < \theta < \theta^{**}$	$\theta^{**} < \theta$
run	0	qr_D	qr_D
wait	0	0	r_D

The same logic prevails again. When running the bank, a creditor receives qr_D with $Prob(\theta > \theta^*)$. By waiting, she receives r_D with a lower probability $Prob(\theta > \theta^{**})$.

A creditor chooses her action based on the private noisy signal $x_i = \theta + \epsilon_i$, where the noise follows a uniform distribution on $[-\sigma, \sigma]$. The resulting ex post distribution of θ conditional on observing x_i is a uniform distribution on $[x_i - \sigma, x_i + \sigma]$. We focus on the case where σ approaches 0: the private signal is as accurate as desired but it is still imperfect and underpins the lack of common knowledge on θ.

Again, we assume players play a switching strategy: to run on the bank if the observed signal is smaller than x^* and to wait otherwise. Equilibrium is characterized by a set of three variables $\{x^*, \theta^*, \theta^{**}\}$, such that (1) upon observing the critical signal x^*, a creditor is indifferent between running on the bank or not, and (2) conditional on the realization of θ, creditors act according to the equilibrium switching strategy, which leads to outcomes consistent with the definition of θ^* and θ^{**}.

$$\begin{cases} Pr(\theta > \theta^* | x^*) qr_D = Pr(\theta > \theta^{**} | x^*) r_D \\ \theta^* = (1 + \lambda) L(\theta^*, x^*) qr_D \\ \theta^{**} = [1 - L(\theta^{**}, x^*)] r_D + (1 + \lambda) L(\theta^{**}, x^*) qr_D \end{cases}$$

The first equation is the indifference condition. And the second and the third equations correspond to the regime change and define θ^* and θ^{**}. The distributional assumption yields a system of linear equations in x^*, θ^* and θ^{**}.

$$\begin{cases} \frac{x^* - \theta^* + \sigma}{2\sigma} \cdot q = \frac{x^* - \theta^{**} + \sigma}{2\sigma} \\ \theta^* = (1 + \lambda) \frac{x^* - \theta^* + \sigma}{2\sigma} qr_D \\ \theta^{**} = [1 - \frac{x^* - \theta^{**} + \sigma}{2\sigma}] \cdot r_D + (1 + \lambda) \frac{x^* - \theta^{**} + \sigma}{2\sigma} \cdot qr_D \end{cases}$$

Solving the system, the 2nd and 3rd equations yield

$$\begin{cases} \theta^* = \frac{x^* + \sigma}{\frac{2\sigma}{(1+\lambda) qr_D} + 1} \\ \theta^{**} = \frac{(\sigma - x^*) r_D + (1+\lambda)(x^* + \sigma) qr_D}{2\sigma - r_D + (1+\lambda) qr_D} \end{cases}$$

Plug the expression of θ^* and θ^{**} into the 1st equation. The left hand side becomes

$$\frac{x^* + \sigma}{2\sigma + (1+\lambda)qr_D} \cdot q.$$

The right hand side becomes

$$\frac{x^* + \sigma - r_D}{2\sigma - r_D + (1+\lambda)qr_D}.$$

When the two are equal, x^* obtains. We focus on the limit case where $\sigma \to 0$. It yields solution

$$x^* = \frac{1+\lambda}{1 + (1+\lambda)(1-q)} \cdot r_D$$

Also note that

$$\lim_{\sigma \to 0} \theta^* = x^* \quad \text{and} \quad \lim_{\sigma \to 0} \theta^{**} = x^*.$$

We therefore derive a single critical level of $\hat{\theta}$ such that a bank run successfully happens at $t = 1$ and otherwise the bank survives.

$$\hat{\theta} = \mu r_D \,,\, \mu = \frac{1+\lambda}{1 + (1+\lambda)(1-q)} \tag{A.16}$$

To derive our critical cash flow $\hat{\theta}$ in section 3.1, notice that $1 + \lambda = \theta/p$, $\hat{\theta}$ is then given by:

$$\hat{\theta} = \frac{\hat{\theta}/P}{1 + (\hat{\theta}/P)(1-q)} \cdot r_D.$$

Finally, we have $\hat{\theta} = \frac{r_D - P}{1-q}$ exactly gives us the same result as in section 3.1.

Appendix B Proof of propositions

Appendix B.1 The monotonicity of $F_1(\theta)$ and $F_2(\theta)$

Proof. We start with $F_1(\theta)$. First, we write explicitly function $F_1(\theta)$ as:

$$\begin{aligned} F_1(\theta) &= \frac{\theta - \underline{\theta}_1}{(\theta - \underline{\theta}_1) + (\theta - \underline{\theta}_2)} \frac{\theta + \underline{\theta}_1}{2} + \frac{\theta - \underline{\theta}_2}{(\theta - \underline{\theta}_1) + (\theta - \underline{\theta}_2)} \frac{\theta + \underline{\theta}_2}{2} - [r_D - (1-q)\theta] \\ &= \frac{1}{2} \frac{2\theta^2 - (\underline{\theta}_1^2 + \underline{\theta}_2^2)}{2\theta - (\underline{\theta}_1 + \underline{\theta}_2)} + (1-q)\theta - r_D \end{aligned}$$

To check the monotonicity of $F_1(\theta)$, we take the derivative:

$$\begin{aligned}
\frac{dF_1(\theta)}{d\theta} &= \frac{1}{2}\frac{4\theta[2\theta-(\underline{\theta}_1+\underline{\theta}_2)]-2[2\theta^2-(\underline{\theta}_1^2+\underline{\theta}_2^2)]}{[2\theta-(\underline{\theta}_1+\underline{\theta}_2)]^2}+(1-q)\\
&= \frac{1}{2}\frac{4\theta^2-4(\underline{\theta}_1+\underline{\theta}_2)\theta+2(\underline{\theta}_1^2+\underline{\theta}_2^2)}{[2\theta-(\underline{\theta}_1+\underline{\theta}_2)]^2}+(1-q)\\
&= \frac{1}{2}\frac{[4\theta^2-4(\underline{\theta}_1+\underline{\theta}_2)\theta+(\underline{\theta}_1+\underline{\theta}_2)^2]+2(\underline{\theta}_1^2+\underline{\theta}_2^2)-(\underline{\theta}_1+\underline{\theta}_2)^2}{[2\theta-(\underline{\theta}_1+\underline{\theta}_2)]^2}+(1-q)\\
&= \frac{1}{2}\frac{[2\theta-(\underline{\theta}_1+\underline{\theta}_2)]^2+(\underline{\theta}_1^2+\underline{\theta}_2^2-2\underline{\theta}_1\underline{\theta}_2)}{[2\theta-(\underline{\theta}_1+\underline{\theta}_2)]^2}+(1-q)\\
&= \frac{1}{2}\frac{[2\theta-(\underline{\theta}_1+\underline{\theta}_2)]^2+(\underline{\theta}_1-\underline{\theta}_2)^2}{[2\theta-(\underline{\theta}_1+\underline{\theta}_2)]^2}+(1-q)>0
\end{aligned}$$

To prove the monotonicity of $F_2(\theta)$, we write explicitly function $F_2(\theta)$ as:

$$F_2(\theta)=\frac{1}{2}\left[\frac{(\theta-\underline{\theta}_1)^2(\theta+\underline{\theta}_1)}{(\theta-\underline{\theta}_1)^2+(\theta-\underline{\theta}_2)^2}+\frac{(\theta-\underline{\theta}_2)^2(\theta+\underline{\theta}_2)}{(\theta-\underline{\theta}_1)^2+(\theta-\underline{\theta}_2)^2}\right]+(1-q)\theta-r_D \tag{B.17}$$

Again, we take the derivative of $F_2(\theta)$ respect to θ. The derivative to θ of the first term in the parenthesis is:

$$\frac{2(\theta+\underline{\theta}_1)[(\theta-\underline{\theta}_2)^2(\theta-\underline{\theta}_1)-(\theta-\underline{\theta}_1)^2(\theta-\underline{\theta}_2)]+(\theta-\underline{\theta}_1)^4+(\theta-\underline{\theta}_1)^2(\theta-\underline{\theta}_2)^2}{[(\theta-\underline{\theta}_1)^2+(\theta-\underline{\theta}_2)^2]^2}$$

The derivative to θ of the second term in the parenthesis is:

$$\frac{2(\theta+\underline{\theta}_2)[(\theta-\underline{\theta}_1)^2(\theta-\underline{\theta}_2)-(\theta-\underline{\theta}_2)^2(\theta-\underline{\theta}_1)]+(\theta-\underline{\theta}_2)^4+(\theta-\underline{\theta}_1)^2(\theta-\underline{\theta}_2)^2}{[(\theta-\underline{\theta}_1)^2+(\theta-\underline{\theta}_2)^2]^2}$$

Notice that

$$\begin{aligned}
&2(\theta+\underline{\theta}_1)[(\theta-\underline{\theta}_2)^2(\theta-\underline{\theta}_1)-(\theta-\underline{\theta}_1)^2(\theta-\underline{\theta}_2)]\\
&+2(\theta+\underline{\theta}_2)[(\theta-\underline{\theta}_1)^2(\theta-\underline{\theta}_2)-(\theta-\underline{\theta}_2)^2(\theta-\underline{\theta}_1)]\\
&=2(\underline{\theta}_2-\underline{\theta}_1)^2(\theta-\underline{\theta}_1)(\theta-\underline{\theta}_2)>0
\end{aligned}$$

We have:

$$\frac{dF_2(\theta)}{d\theta}=\frac{2(\underline{\theta}_2-\underline{\theta}_1)^2(\theta-\underline{\theta}_1)(\theta-\underline{\theta}_2)+(\theta-\underline{\theta}_1)^4+(\theta-\underline{\theta}_2)^4+2(\theta-\underline{\theta}_1)^2(\theta-\underline{\theta}_2)^2}{2[(\theta-\underline{\theta}_1)^2+(\theta-\underline{\theta}_2)^2]^2}+(1-q)>0$$

□

Appendix B.2 The existence and uniqueness of θ^*

Proof. We focus on possible $\theta^* \in [r_D, \overline{\theta}]$ and prove by continuity the existence of such $\theta^* \in [r_D, \overline{\theta}]$. We prove by continuity that there exists θ^* such that $F_1(\theta^*) = 0$.

First value the function $F_1(\theta)$ at $\theta = r_D$. Notice that

$$\omega_1(r_D) = \frac{r_D - \underline{\theta}_1}{(r_D - \underline{\theta}_1) + (r_D - \underline{\theta}_2)} > 0 \quad \text{and} \quad \omega_2(r_D) = \frac{r_D - \underline{\theta}_2}{(r_D - \underline{\theta}_1) + (r_D - \underline{\theta}_2)} > 0.$$

Moreover, as q sufficiently close to 1, it holds that

$$\pi_1(r_D) = \frac{r_D + \underline{\theta}_1}{2} - qr_D < 0 \quad \text{and} \quad \pi_2(r_D) = \frac{r_D + \underline{\theta}_2}{2} - qr_D < 0.$$

Therefore, we have $F_1(r_D) < 0$.

Now we examine $F_1(\theta)$ at $\theta = \overline{\theta}$. Similarly, we have

$$\omega_1(\overline{\theta}) = \frac{\overline{\theta} - \underline{\theta}_1}{(\overline{\theta} - \underline{\theta}_1) + (\overline{\theta} - \underline{\theta}_2)} > 0, \quad \text{and} \quad \omega_2(\overline{\theta}) = \frac{\overline{\theta} - \underline{\theta}_2}{(\overline{\theta} - \underline{\theta}_1) + (\overline{\theta} - \underline{\theta}_2)} > 0.$$

And under our assumption: $(\overline{\theta} - r_D) + (\underline{\theta}_1 - r_D) > 0$, it holds that

$$\pi_1(\overline{\theta}) = \frac{\overline{\theta} + \underline{\theta}_1}{2} - r_D + (1-q)\overline{\theta} > 0 \quad \text{and} \quad \pi_2(\overline{\theta}) = \frac{\overline{\theta} + \underline{\theta}_2}{2} - r_D + (1-q)\overline{\theta} > 0.$$

Therefore, we have: $F_1(\overline{\theta}) > 0$.

By the continuity of function $F_1(\theta)$, there exists $\theta^* \in [r_D, \overline{\theta}]$ such that $F_1(\theta^*) = 0$.

Finally, recall lemma 1 that $F_1(\theta)$ monotonically increases in θ. The uniqueness of θ^* is guaranteed by the monotonicity of $F_1(\theta)$. Given the one-to-one relationship between θ^* and P^*, the equilibrium $\{\theta^*, P^*\}$ is existent and unique. □

Appendix B.3 The existence and uniqueness of θ^{**}

Proof. We follow the same approach as the proof of the above proposition. Notice that:

$$\omega_1'(r_D) = \frac{(r_D - \underline{\theta}_1)^2}{(r_D - \underline{\theta}_1)^2 + (r_D - \underline{\theta}_2)^2} > 0, \qquad \omega_2'(r_D) = \frac{(r_D - \underline{\theta}_2)^2}{(r_D - \underline{\theta}_1)^2 + (r_D - \underline{\theta}_2)^2} > 0.$$

Moreover,

$$\omega_1'(\bar{\theta}) = \frac{(\bar{\theta} - \underline{\theta}_1)^2}{(\bar{\theta} - \underline{\theta}_1)^2 + (\bar{\theta} - \underline{\theta}_2)^2} > 0, \qquad \omega_2'(\bar{\theta}) = \frac{(\bar{\theta} - \underline{\theta}_2)^2}{(\bar{\theta} - \underline{\theta}_1)^2 + (\bar{\theta} - \underline{\theta}_2)^2} > 0$$

The sign of function $F_2(\theta)$ depends on $\pi_1(\theta)$ and $\pi_2(\theta)$. We have already showed that: $\pi_1(r_D) < 0, \pi_1(\bar{\theta}) > 0$ and $\pi_2(r_D) < 0, \pi_2(\bar{\theta}) > 0$. Thus we can again claim:

$$F_2(r_D) < 0 \qquad \text{and} \qquad F_2(\bar{\theta}) > 0.$$

By the continuity of $F_2(\theta)$, there exists a $\theta^{**} \in [r_D, \bar{\theta}]$ satisfying $F_2(\theta^{**}) = 0$.

Since F_2 is monotonically increasing in θ, the uniqueness of this θ^{**} is again guaranteed. Given the one-to-one relationship between θ^{**} and P^{**}, the equilibrium $\{\theta^{**}, P^{**}\}$ exists and is unique. □

Appendix B.4 Risk of contagion

Proof. The proof hinges on the monotonicity of two ratios

$$\frac{\omega_1'(\theta)}{\omega_2'(\theta)} \qquad \text{and} \qquad \frac{\pi_2(\theta)}{\pi_1(\theta)}.$$

The first is a conditional likelihood ratio and the second is a payoff ratio. It can be shown both ratios are strictly monotonically decreasing in θ. Furthermore, notice that for $\omega_1(\theta)/\omega_2(\theta) > 1$, we have

$$\frac{\omega_1(\theta)}{\omega_2(\theta)} < \left[\frac{\omega_1(\theta)}{\omega_2(\theta)}\right]^2 = \frac{\omega_1'(\theta)}{\omega_2'(\theta)} \tag{B.18}$$

Now we prove by contradiction. Suppose $\theta^* > \theta^{**}$. By the monotonicity of $\pi_2(\theta)/\pi_1(\theta)$, we will have

$$\frac{\pi_2(\theta^*)}{\pi_1(\theta^*)} < \frac{\pi_2(\theta^{**})}{\pi_1(\theta^{**})}.$$

By the equilibrium conditions (10) and (11), we have

$$\frac{\pi_2(\theta^*)}{\pi_1(\theta^*)} = -\frac{\omega_1(\theta^*)}{\omega_2(\theta^*)} \qquad \text{and} \qquad \frac{\pi_2(\theta^{**})}{\pi_1(\theta^{**})} = -\frac{\omega_1'(\theta^{**})}{\omega_2'(\theta^{**})},$$

which implies

$$\frac{\omega_1'(\theta^{**})}{\omega_2'(\theta^{**})} < \frac{\omega_1(\theta^*)}{\omega_2(\theta^*)}.$$

By condition (B.18), we know

$$\frac{\omega_1'(\theta^{**})}{\omega_2'(\theta^{**})} < \frac{\omega_1(\theta^*)}{\omega_2(\theta^*)} < \frac{\omega_1'(\theta^*)}{\omega_2'(\theta^*)}.$$

But this contradicts the monotonicity of $\omega_1'(\theta)/\omega_2'(\theta)$. Therefore, we prove $\theta^{**} > \theta^*$. □

Appendix B.5 Bank capital and contagion

Proof. θ^{**} and θ^* are the unique solutions of equations $F_2(\theta^{**}) = 0$ and $F_1(\theta^*) = 0$. Applying the implicit function theorem, we have

$$\frac{d\theta^{**}}{dr_D} = -\frac{dF_2(\theta^{**})/dr_D}{dF_2(\theta^{**})/d\theta^{**}} \qquad \frac{d\theta^*}{dr_D} = -\frac{dF_1(\theta^*)/dr_D}{dF_1(\theta^{**})/d\theta^*}.$$

We can calculate

$$\frac{dF_2(\theta^{**})}{d\theta^{**}} = \frac{1}{2} + \frac{(\underline{\theta}_1 - \underline{\theta}_2)^2(\theta^{**} - \underline{\theta}_1)(\theta^{**} - \underline{\theta}_2)}{[(\theta^{**} - \underline{\theta}_1)^2 + (\theta^{**} - \underline{\theta}_2)^2]^2}$$

and

$$\frac{dF_1(\theta^*)}{d\theta^*} = \frac{1}{2} + \frac{(\underline{\theta}_1 - \underline{\theta}_2)^2}{2[(\theta^* - \underline{\theta}_1) + (\theta^* - \underline{\theta}_2)]^2}.$$

If $\max\{\frac{\theta^{**}-\underline{\theta}_1}{\theta^*-\underline{\theta}_1}, \frac{\theta^{**}-\underline{\theta}_2}{\theta^*-\underline{\theta}_2}\} < \sqrt{2}$, it holds that

$$(\theta^{**} - \underline{\theta}_1) + (\theta^{**} - \underline{\theta}_2) < \sqrt{2}[(\theta^* - \underline{\theta}_1) + (\theta^* - \underline{\theta}_2)].$$

Notice for

$$\sqrt{\frac{\theta^{**} - \underline{\theta}_2}{\theta^{**} - \underline{\theta}_1}} < 1 < \sqrt{\frac{\theta^{**} - \underline{\theta}_1}{\theta^{**} - \underline{\theta}_2}},$$

we can find a combination of $\underline{\theta}_1, \underline{\theta}_2$, such that:

$$(\theta^{**} - \underline{\theta}_1)\sqrt{\frac{\theta^{**} - \underline{\theta}_1}{\theta^{**} - \underline{\theta}_2}} + (\theta^{**} - \underline{\theta}_2)\sqrt{\frac{\theta^{**} - \underline{\theta}_2}{\theta^{**} - \underline{\theta}_1}} = (\theta^{**} - \underline{\theta}_1) + (\theta^{**} - \underline{\theta}_2)$$

As a result,

$$(\theta^{**} - \underline{\theta}_1)\sqrt{\frac{\theta^{**} - \underline{\theta}_1}{\theta^{**} - \underline{\theta}_2}} + (\theta^{**} - \underline{\theta}_2)\sqrt{\frac{\theta^{**} - \underline{\theta}_2}{\theta^{**} - \underline{\theta}_1}} < \sqrt{2}[(\theta^* - \underline{\theta}_1) + (\theta^* - \underline{\theta}_2)].$$

Taking square of both sides, we get:

$$\frac{[(\theta^{**}-\underline{\theta}_1)^2+(\theta^{**}-\underline{\theta}_2)^2]^2}{(\theta^{**}-\underline{\theta}_1)(\theta^{**}-\underline{\theta}_2)} < 2[(\theta^{*}-\underline{\theta}_1)+(\theta^{*}-\underline{\theta}_2)]^2,$$

which implies

$$\frac{dF_2(\theta^{**})}{d\theta^{**}} > \frac{dF_1(\theta^{*})}{d\theta^{*}}.$$

Recall further that $dF_2(\theta^{**})/dr_D = dF_1(\theta^{*})/dr_D = -1$. We prove

$$\frac{d(\theta^{**}-\theta^{*})}{dr_D} < 0.$$

□

Appendix B.6 Capital reduces systemic risk

Proof. First recall that:

$$\begin{aligned} Pr(\theta < \theta^{**}) &= Pr(\theta < \theta^{**}|s=1)Pr(s=1) + Pr(\theta < \theta^{**}|s=2)Pr(s=2) \\ &= \frac{1}{2}\frac{\theta^{**}-\underline{\theta}_1}{\overline{\theta}-\underline{\theta}_1} + \frac{1}{2}\frac{\theta^{**}-\underline{\theta}_2}{\overline{\theta}-\underline{\theta}_2} \end{aligned}$$

Then the systemic risk can be expressed as the probability (2 banks fail together):

$$[Pr(\theta < \theta^{**})]^2 = \frac{1}{4}\left(\frac{\theta^{**}-\underline{\theta}_1}{\overline{\theta}-\underline{\theta}_1} + \frac{\theta^{**}-\underline{\theta}_2}{\overline{\theta}-\underline{\theta}_2}\right)^2$$

As a result:

$$\frac{\partial[Pr(\theta < \theta^{**})]^2}{\partial r_D} = \frac{1}{2}(\frac{\theta^{**}-\underline{\theta}_1}{\overline{\theta}-\underline{\theta}_1} + \frac{\theta^{**}-\underline{\theta}_2}{\overline{\theta}-\underline{\theta}_2})\frac{\partial}{\partial r_D}\left(\frac{\theta^{**}-\underline{\theta}_1}{\overline{\theta}-\underline{\theta}_1} + \frac{\theta^{**}-\underline{\theta}_2}{\overline{\theta}-\underline{\theta}_2}\right)$$

Start with the first partial derivative.

$$\frac{\partial}{\partial r_D}\left(\frac{\theta^{**}-\underline{\theta}_1}{\overline{\theta}-\underline{\theta}_1}\right) = \frac{\partial\theta^{**}/\partial r_D}{\overline{\theta}-\underline{\theta}_1}$$

By implicit function theorem, θ^{**} is the unique solution of $F_2(\theta) = 0$. Thus we have:

$$\frac{\partial\theta^{**}}{\partial r_D} = -\frac{\partial F_2(\theta^{**})/\partial r_D}{\partial F_2(\theta^{**})/\partial\theta^{**}}$$

where:

$$F_2(\theta^{**}) = \frac{1}{2}\left[\frac{(\theta^{**}-\underline{\theta}_1)^2(\theta^{**}+\underline{\theta}_1)}{(\theta^{**}-\underline{\theta}_1)^2+(\theta^{**}-\underline{\theta}_2)^2} + \frac{(\theta^{**}-\underline{\theta}_2)^2(\theta^{**}+\underline{\theta}_2)}{(\theta^{**}-\underline{\theta}_1)^2+(\theta^{**}-\underline{\theta}_2)^2}\right] + (1-q)\theta^{**} - r_D$$

Consequently, we have: $\frac{\partial F_2(\theta^{**})}{\partial r_D} = -1$ and by the monotonicity of $F_2(\theta)$ which we showed $\frac{\partial F_2(\theta^{**})}{\partial \theta^{**}} > 0$. We proved that $\frac{\partial}{\partial r_D}\left(\frac{\theta^{**}-\underline{\theta}_1}{\bar{\theta}-\underline{\theta}_1}\right) > 0$. For $\frac{\partial}{\partial r_D}\left(\frac{\theta^{**}-\underline{\theta}_2}{\bar{\theta}-\underline{\theta}_2}\right)$, the same argument applies, and the derivative is again larger than 0. Thus, we conclude:

$$[Pr(\theta < \theta^{**})]^2 > 0$$

higher capital requirement (lower r_D) lowers systemic risk (lower the probability $[Pr(\theta < \theta^{**})]^2$). □

Appendix B.7 The effect of regulatory transparency on bank run

Proof. We solve here only for the equilibrium in state $s = 2$. The equilibrium under $s = 1$ can be solved with the same procedure. The equilibrium is determined by a system of two equations:

$$\begin{cases} \theta_G^* = (r_D - P_G)/(1-q) \\ P_G = (\theta_G^* + \underline{\theta}_2)/2 \end{cases}$$

Solving the system of equations, we have the equilibrium critical cash flow and the endogenous fire-sale price:

$$\theta_G^* = \frac{2r_D - \underline{\theta}_2}{3-2q} \qquad \text{and} \qquad P_G = \frac{r_D - (1-q)\underline{\theta}_2}{3-2q}$$

It is straightforward to check that for q sufficiently close to 1, $\theta_G^* > r_D$. □

Appendix B.8 Regulatory transparency and illiquidity

Proof. We start by proving $\theta_G^* < \theta^*$. Recall that $F_1(\theta^*) = 0$ and $F_1(\theta)$ is monotonically increasing. So $\theta_G^* < \theta^*$ will hold if and only if $F_1(\theta_G^*) < 0$. Also notice that $\theta_G^* < 2r_D - \underline{\theta}_2$. It is therefore sufficient to prove $F_1(2r_D - \underline{\theta}_2) < 0$. To see so, write $F_1(\theta)$ explicitly

$$F_1(\theta) = \frac{1}{2}\frac{2\theta^2 - (\underline{\theta}_1^2 + \underline{\theta}_2^2) - 2r_D[2\theta - (\underline{\theta}_1 + \underline{\theta}_2)]}{2\theta - (\underline{\theta}_1 + \underline{\theta}_2)} + (1-q)\theta$$

and evaluate it at $\theta = 2r_D - \underline{\theta}_2$. The second term vanishes as $q \to 1$. The first term of the denominator can be written as:

$$4r_D - (3\underline{\theta}_2 + \underline{\theta}_1),$$

which is positive, since $r_D > \underline{\theta}_1$ and $r_D > \underline{\theta}_2$,. And the numerator, after some rearrangement, can be written as:

$$-(\underline{\theta}_2 - \underline{\theta}_1)[2r_D - (\underline{\theta}_1 + \underline{\theta}_2)] < 0.$$

It follows that

$$F_1(\theta_G^*) < F_1(2r_D - \underline{\theta}_2) < F_1(\theta^*) = 0$$

and we have proved $\theta_G^* < \theta^*$.

To prove $\theta_B^* > \theta^{**}$, again recall that θ^{**} is the solution of $F_2(\theta^{**}) = 0$ and $F_2(\theta)$ is monotonically increasing. Write $F_2(\theta)$ explicitly

$$F_2(\theta) = \frac{1}{2}\frac{(\theta - \underline{\theta}_1)^2(\theta + \underline{\theta}_1) + (\theta + \underline{\theta}_2)^2(\theta + \underline{\theta}_2)2r_D[(\theta - \underline{\theta}_1)^2 + (\theta - \underline{\theta}_2)^2]}{(\theta - \underline{\theta}_1)^2 + (\theta - \underline{\theta}_2)^2} + (1 - q)\theta$$

and evaluate it at $\theta = 2r_D - \underline{\theta}_1$. The second term vanishes as we consider q approaching 1. Concerning the first term, it is obvious that the denominator is positive. And the numerator can be written as

$$(2r_D - \underline{\theta}_1 - \underline{\theta}_2)^2(\underline{\theta}_2 - \underline{\theta}_1) > 0.$$

We therefore have $F_2(2r_D - \underline{\theta}_1) > F_2(\theta^{**})$.

Notice that there exists ϵ such that

$$2r_D - \underline{\theta}_1 - \epsilon < \theta_B^* < 2r_D - \underline{\theta}_1.$$

And as q approaches 1, then for any ϵ close enough to 0, the above inequality is satisfied. We have $F_2(2r_D - \underline{\theta}_1 - \epsilon) \to F_2(2r_D - \underline{\theta}_1)$. By the continuity of $F_2(\theta)$, we have

$$F_2(2r_D - \underline{\theta}_1 - \epsilon) > F_2(\theta^{**}).$$

It then follows that:

$$F_2(\theta^{**}) < F_2(2r_D - \underline{\theta}_1 - \epsilon) < F_2(\theta_G^*).$$

Given the monotonicity of $F_2(\theta)$, $\theta^{**} < \theta_G^*$ is proved. □

Appendix B.9 Socially undesirable disclosure

Proof. Regulatory transparency is not desirable if and only if it generates a higher social cost:

$$\frac{(\theta_G^* - r_D)^2}{(\bar{\theta} - \underline{\theta}_2)^2} \cdot C_G + \frac{(\theta_B^* - r_D)^2}{(\bar{\theta} - \underline{\theta}_1)^2} \cdot C_B > \frac{(\theta^{**} - r_D)^2}{(\bar{\theta} - \underline{\theta}_2)^2} \cdot C_G + \frac{(\theta^{**} - r_D)^2}{(\bar{\theta} - \underline{\theta}_1)^2} \cdot C_B.$$

The inequality gives the critical C_B/C_G ratio, beyond which the regulatory transparency is suboptimal:

$$C = \frac{C_B}{C_G} > \frac{(\theta^{**} - r_D)^2 - (\theta_G^* - r_D)^2}{(\theta_B^* - r_D)^2 - (\theta^{**} - r_D)^2} \cdot \frac{(\bar{\theta} - \underline{\theta}_1)^2}{(\bar{\theta} - \underline{\theta}_2)^2} \equiv \hat{C}.$$

If $\theta^{**} - \theta_G^* > \theta_B^* - \theta^{**}$, we will have $\hat{C} > 1$. □

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