Measuring and testing for the systemically important financial institutions

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Defining and measuring the systemic importance (SI) of financial institutions (FIs): $\Delta CoVaR$.

SI of financial institutions depends on *"their potential to have a large negative impact on the financial system and the real economy."* (IMF/BIS/FSB, 2009)

$\diamond$ Co-risk measures have attracted considerable attention in both academic and policy research.

$\diamond$ Adrian and Brunnermeier (2009,2010): compare VaR of the financial system conditional on FI in distress (CoVaR) to VaR of the financial system in normal times $< 2009 >$ or the CoVaR of the financial system in normal times $< 2010 >$ (both versions extensively applied).

$\diamond$ However, statistical testing procedures to assess the significance of the findings and interpretations based on this co-risk measure "have not yet been developed".

$\diamond$ Emerging literature, Chuang, Kuan and Lin (2009), Billio, Getmansky, Lo and Pelizzon (2010), White, Kim, and Manganelli(2010).

Quantile-based Risk Measures.

- $\diamond$ $VaR_X(\tau) := inf\, \{x \in \mathbb{R} : F_X(x) \geq \ \tau\} ., \tau \in (0,1)$.
- $\diamond$ $ES_X(\tau)$ (Expected Shortfall).

Add $CoVaR_{X^{index|i(\tau_X)}}(\tau)$ to this family of measures. Where X^{index} returns on index of financial institutions (representing the system)and X^i stock return of the financial institution i (possibly the root of distress).

$$P(X^{index} \leq CoVaR_{X^{index|i(\tau_X)}}(\tau) \mid X^i = VaR_{X^i}(\tau_X)) = \tau,$$

$$\Delta CoVaR^{index|i}(\tau) = CoVaR_{X^{index|i}}(\tau) - VaR_{X^{index}}(\tau).$$

Then $\Delta CoVaR^{index|i}(\tau)$ is the marginal risk contribution (incremental VaR) of institution i; determines the SI.

CoVaR estimation.

Linear Location/Scale Model

$$X_t^{index} = K_t\delta + (\gamma K_t)\varepsilon_t,$$

Quantile (response) Function Representation

$$\begin{aligned} Q_{X^{index}|K}(\tau) &= K_t\delta + (\gamma K_t)Q_\varepsilon(\tau) \\ &= K_t\beta(\tau) \end{aligned}$$

where $\beta(\tau) = \delta + \gamma Q_\varepsilon(\tau)$.
Most applications of Adrian and Brunnermeier's methodology (Linear location-shift model, $\gamma K_t = 1$).

$$X_t^{index} = \mathbf{K}_t\delta + \varepsilon_t,$$

where $\mathbf{K}_t = [\mathbf{Z}_t, X_t^i]$.

Might be extremely restrictive model(s), more on that at the end!

Measuring the SI of FIs: application of $\Delta CoVaR$

◇ Data: daily stock returns (1986-2010) for individual FIs and index of FIs.

◇ CoVaR: conditional quantile function (CQF) (also: quantile response function).

Table: Size and $\Delta CoVaR$ of three European banks

Bank	Assets (millions)	Quantile Regression Results	$\Delta CoVaR$
A	$1,571,768$	$X^{index\|A}(0.99) = 0.026 + 0.526X^{A}(0.99)$	1.38
B	$102,185$	$X^{index\|B}(0.99) = 0.042 + 0.231X^{B}(0.99)$	1.18
C	$10,047$	$X^{index\|C}(0.99) = 0.037 + 0.028X^{C}(0.99)$	0.03

Our contribution: Testing for the SI of FIs.

◇ Conclusion: A is more SI than B and C, and B is more SI than C?

◇ Testing for the strength of the results.

Significance

$$H_0 : \Delta CoVaR^{index|i}(\tau) = 0,$$

test whether CQF differs from un-CQF for FI i

Dominance

$$H_0 : CoVaR_{X^{index|i}}(\tau) > CoVaR_{X^{index|j}}(\tau),$$

test whether CQF conditional on FI i differs from CQF conditional on FI j

Quantile treatment effects and $\Delta CoVaR$.

Two-sample treatment effects

- $\diamond$ Treatment group (CQF), with distribution G.
- $\diamond$ Control group (un-CQF), with distribution F.

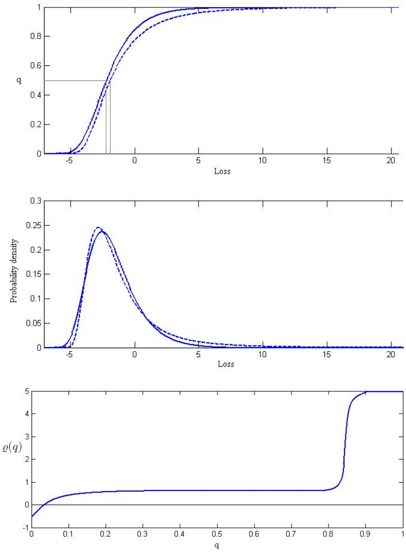
(Non-parametric) estimator of quantile treatment effects

$$\hat{\varrho}(\tau) = \hat{G}_T^{-1}(\tau) - \hat{F}_S^{-1}(\tau),$$

$\Delta CoVaR$ as a quantile treatment effect:

$$\begin{aligned} \Delta\widehat{CoVaR}^{index|i}(\tau) &= \widehat{Q}_{X^{index}|X^i}(\tau) - \widehat{Q}_{X^{index}}(\tau) \\ &= \widehat{F}^{-1}_{X^{index}|X^i}(\tau) - \widehat{F}^{-1}_{X^{index}}(\tau), \end{aligned}$$

Graphical depiction of $\Delta CoVaR$



Inference for Quantile Regression.

H_0 in both significance and dominance test involves CQF. Since CQF is linear, both tests fit in: general linear hypothesis framework:

$$H_0 : R\beta(\tau) = r(\tau), \tau \in \mathcal{T}$$

where $\beta(\tau)$ is p dimensional and q is the rank of matrix R, $(q \leq p)$. Wald (process, indexed by τ) statistic under the null, is:

$$W_T(\tau) = T\frac{(R\widehat{\beta}(\tau) - r(\tau))'(R\hat{\Omega}(\tau)R')^{-1}(R\widehat{\beta}(\tau) - r(\tau))}{(\tau(1-\tau))}$$

where $\hat{\Omega}(\tau)$ is a consistent estimator of $\Omega(\tau)$.

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Inference for Quantile Regression.

The Kolmogorov-Smirnov (KS) type statistic:

$$K_T = \sup_{\tau \in \mathcal{T}} \mid\mid \hat{W}_T(\tau) \mid\mid .$$

$$K_T' = \sup_{\tau \in [\tau_0, \tau_1]} \frac{\hat{W}_T(\tau) - \hat{W}_T(\tau_0)}{\sqrt{\tau_1 - \tau_0}}.$$

Test statistic is distribution free. Critical values: DeLong (1981) and Andrews (1993, 2003) by simulation methods, and more recently by exact methods by Estrella (2003) and Anatolyev and Kosenok (2011).

Simple Test of Significance for $\Delta CoVaR$.

$$Q_{X^{index}|X^i}(\tau) = \beta_0(\tau) + X^{i}\beta_1(\tau),$$

Theorem

Testing the hypothesis $H_0 := \beta_1(\tau) = 0$ *is equivalent to testing the hypothesis* $H_0 := \Delta CoVaR_{X^{index}|i}(\tau) = 0$, *for a given* τ.

For such simple (two-sided) test $H_0 := \beta_1(\tau) = 0$ we use Wald statistic $W_T(\tau)$.

Define R as a selection matrix $R = [0 : 1]$ and the restriction $r(\tau) = 0$.

Test of significance and dominance using quantile response function.

Theorem

From Theorem 4.1 *and let us define some continuous mapping* $g(\beta(\tau)) = \mathbf{X}\beta(\tau)$, *where this mapping defines the quantile response function, evaluated at some point in the design space.*

$$\sqrt{n}(\hat{Q}_{\mathbf{Y}|\mathbf{X}}(\tau) - Q_{\mathbf{Y}|\mathbf{X}}(\tau)) \rightarrow_d N(0, \tau(1-\tau)\mathbf{X}\Omega(\tau)\mathbf{X}')$$

Test of significance and dominance using quantile response function.

Two different (at least one column is different) design matrices **X** and **Z** (two different continuous treatment effects applied to the same population Y. The respective empirical quantile response functions are a follows:

$$\hat{Q}_{\mathbf{Y}|\mathbf{X}}(\tau) = \mathbf{X}\hat{\beta}^{x}_{T}(\tau)$$

and

$$\hat{Q}_{\mathbf{Y}|\mathbf{Z}}(\tau) = \mathbf{Z}\hat{\beta}^{z}_{T}(\tau)$$

Test of significance and dominance using quantile response function.

Without loss of generality, we consider equal amount of observations T through out the design space. Therefore, we have the following parametric empirical process:

$$\begin{aligned} W_T(\tau) &= \sqrt{T}(\hat{Q}_{\mathbf{Y}|\mathbf{X}}(\tau) - \hat{Q}_{\mathbf{Y}|\mathbf{Z}}(\tau)) \\ &= \sqrt{T}(\tilde{\mathbf{X}}\hat{\beta}^x_T(\tau) - \tilde{\mathbf{Z}}\hat{\beta}^z_T(\tau)) \end{aligned}$$

Where $\tilde{\mathbf{X}}$ and $\tilde{\mathbf{Z}}$ implies the quantile response function is evaluated at any point of the design space (centroid $(\bar{\mathbf{X}}, \bar{\mathbf{Z}})$ or an extreme quantile of interest).

Recall hypothesis test and statistic

Significance: Two-sided.

$$H_0 : \Delta CoVaR^{index|i}(\tau) = 0,$$

Dominance: One-sided.

$$H_0 : CoVaR_{X^{index|i}}(\tau) > CoVaR_{X^{index|j}}(\tau),$$

Recall hypothesis test and statistic

Statistic

$$W_T(\tau) = T\,\frac{(R\widehat{\beta}(\tau) - r(\tau))'(R\hat{\Omega}(\tau)R')^{-1}(R\widehat{\beta}(\tau) - r(\tau))}{(\tau(1-\tau))}$$

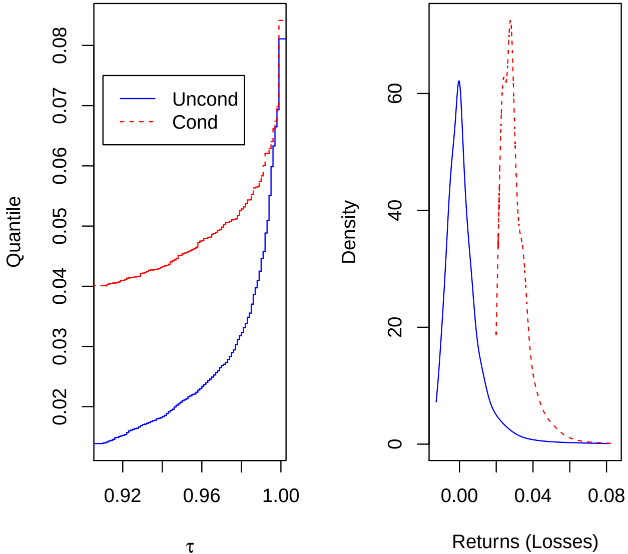
Hypothesis	Significance	Dominance
R	$[\tilde{\mathbf{X}}^i, -1]$	$[\tilde{\mathbf{X}}, -\tilde{\mathbf{Z}}]$
$\hat{\beta}(\tau)$	$[\hat{\beta}^i(\tau), Q_{X^{index}}(\tau)]$	$[\hat{\beta}^i(\tau), \hat{\beta}^j(\tau)]$
r	0	0

Testing for the SI of FIs: significance

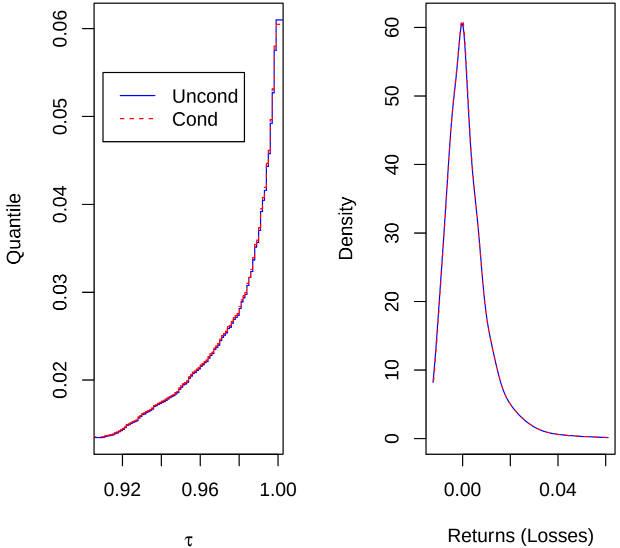
Table: Testing for Significance (p-values)

FI	$\Delta CoVaR$	$H_0 : \beta(0.99) = 0$	$H_0 : \Delta CoVaR(0.99) = 0$
A	1.38	0.000	0.000
B	1.18	0.039	0.000
C	0.03	0.782	0.424

Testing for the SI of FI A: significance



Testing for the SI of FI C: significance

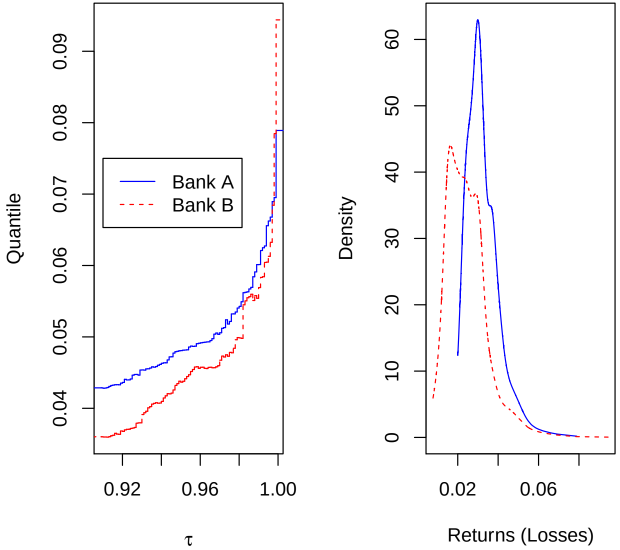


Testing for the SI of FIs: dominance

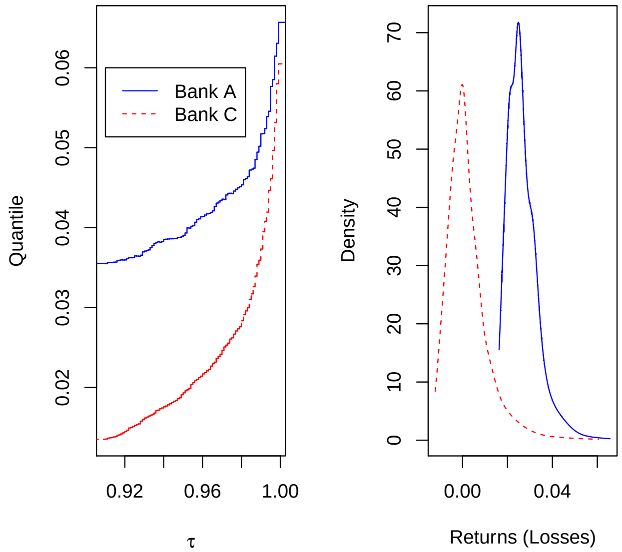
Table: Testing for Dominance (p-values)

FI	$\Delta CoVaR$	$[\tau_0, \tau_1] = [0.90, 0.99]$	$[\tau_0, \tau_1] = [0.10, 0.99]$
AB	1.38	0.000	0.913
AC	1.18	0.000	0.874
BC	0.03	0.000	0.482

Testing for the SI of FI A and B: dominance



Testing for the SI of FI A and C: dominance



Concluding remarks

$\diamond$ $\Delta CoVaR$ is interesting tool for measuring SI, but statistical testing is required before interpreting results.

$\diamond$ We develop such tests in linear quantile regression framework. This linear framework (location-shift model and location/scale model) is restrictive.

$\diamond$ work in progress.

- $\diamond$ Power of the test.
- $\diamond$ At some point when $\tau \to 1$, the convergence of the statistic breaks down, Chernozhukov (2000).
- $\diamond$ Test for stochastic dominance at the extremum for a general class of (models) conditional and unconditional quantile functions.

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