Distressed Debt Prices and Recovery Rate Estimation*

Xin Guo[†] Robert A. Jarrow[‡] Haizhi Lin[§]

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Abstract

This paper has two purposes. First, it uses distressed debt prices to estimate recovery rates at default. In this regard, estimates are obtained for three recovery rate models: recovery of face value, recovery of Treasury, and recovery of market value. We show that identifying the "economic" default date, as distinct from the recorded default date, is crucial to obtaining unbiased estimates. The economic default date is defined to be that date when the market prices the firm's debt as if it has defaulted. An implication is that the standard industry practice of using 30-day post default prices to compute recovery rate yields biased estimates. Second, we construct and estimate a distressed debt pricing model. We use this model to implicitly estimate the parameters of the embedded recovery rate process and to price distressed debt. Our distressed debt pricing model fits market prices well, with an average pricing error of less than one basis point.

1 Introduction

Given the current economic crisis, accentuated by the incorrect assessment of the default risk imbedded in subprime mortgages, the study of credit risk has assumed increased importance to the financial industry and regulators. In the existing literature, significant emphasis has been devoted to understanding the default process (the default probability) for a collection of credit risk entities. Less emphasis, however, has been placed on understanding the recovery rate

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†Department of IEOR, UC at Berkeley, CA 95720-1777. Email: xinguo@ieor.berkeley.edu. Tel: 510-642-3615.

‡Corresponding author. 451 Sage Hall, Johnson School of Management and Kamakura Corporation. Email: raj15@cornell.edu. Tel: 607-255-4729. Cornell University, Ithaca, NY 14853

§101 Malott Hall, Cornell University, Ithaca, NY 14853 Email: hl295@cornell.edu. Tel: 607-255-9802.

process itself. Understanding the recovery rate process is, of course, of equal importance to understanding the default likelihood in the valuation and hedging of risky debt and credit derivatives. Our paper adds to the literature on the recovery rate process.

The existing empirical literature studying recovery rates can be divided into various groups. The first group are industry papers that provide estimates of recovery rates and characterize their properties (see Moody's [20], [21], [22]). Unfortunately, although these papers provide estimates of recovery rates, they do not provide details on the estimation procedure nor do they provide a comparison of alternative estimation methods. The second group are a collection of academic papers that use these industry generated recovery rates (usually from Moody's Investor Services) to study the behavior of the recovery rates at default (see Altman, Brady, Resti and Sironi [3], Altman, Resti and Sironi [4], Acharya, Bharath, and Srinivasan [1], Covitz and Han [9], Chava, Stefanescu and Turnbull [8]). These papers provide some useful insights, but they are predicated on the validity of the industry recovery rate estimates. If the industry recovery rates are biased or misspecified,[1] then these results can not be accepted as valid. Lastly, there are a few papers that use pre-default risky debt or credit default swap (CDS) pricing models to infer the embedded recovery rate (see Bakshi, Madan, Zhang [5], Janosi, Jarrow and Yildirim [14], and Das and Hanouna [10]). These papers are not dependent on the validity of the industry recovery rate estimates. However, without the historical recovery rate estimates, there is no way to independently determine whether the implicit estimates are reasonable. For a review of the literature on recovery rates see Schuermann [24].

To our knowledge there are no academic papers that provide direct estimates of recovery rates using distressed debt prices. This is the purpose of this paper. A secondary and related purpose is to estimate and fit a model for defaulted debt prices. Again, to our knowledge, ours is the first paper to do so. Our investigation generates the following empirical insights:

1. When estimating recovery rates using cross sectional data, the recovery rate estimates are sensitive to the model used (recovery of face value (RFV), recovery of Treasury (RT), and recovery of market value (RMV)) and the date selected for estimation. Differences in recovery rate estimates between the recorded default date and 30 days subsequent are significant.

2. An investigation of the time series behavior of market debt prices around the recorded default date supports the claim that (for the majority of firms) the market anticipates the event of default before default is recorded.

3. Extending the cross-sectional recovery rate models to dynamic models (in the natural manner), we estimate the "economic" default date and show that for the majority of defaulted firms, the economic default date occurs well in advance of the recorded default date. The economic default date is defined to be that date when the market prices the firm's debt issue as if it has defaulted.

[1] Unfortunately, we will provide significant evidence below that this is indeed the case.

4. These extended cross-sectional recovery rate models provide a poor fit to distressed debt prices after the recorded default date. This is an alternative and perhaps more formal rejection of the use of debt prices 30 days after default to estimate recovery rates (RFV, RT, RMV).

5. Similar to Bakshi, Madan, Zhang [5], Janosi, Jarrow and Yildirim [14], and Das and Hanouna [10]) we formulate and estimate a new recovery rate process useful for pricing distressed debt. Our model is shown to provide a good fit to the market prices of distressed debt. We leave for subsequent research the use of our recovery rate parameters (and model) for the pricing of pre-default debt and credit derivative prices.

Related papers studying corporate bankruptcy (costs, duration) include Maksimovic and Phillips [18], Bris, Welch and Zhu [7], and Denis and Rodgers [11]. The purposes of these papers are different, however, in that they are not focused on the eventual pricing of risky debt or credit derivatives but rather on the economics and efficiency of the corporate bankruptcy process. For a paper describing the economics of the distressed debt market see Altman [2]. For a related paper modeling defaulted debt prices in a reduced form model see Guo, Jarrow, Zeng [13]. Guo, Jarrow and Zeng's model is more complex than the model implemented herein, and they provide no empirical estimation.

An outline for this paper is as follows. Section 2 provides motivation for the issues studied in the paper, while Section 3 provides the mathematical setup. Section 4 presents the analysis of cross-sectional models for recovery rates, and Section 5 studies the extended time-series models for recovery rates. Section 6 presents the results on our new recovery rate process, Section 7 provides an epilogue, and section 8 concludes.

2 Prologue

Credit risk models can be divided into two types: structural and reduced form models. Structural models, originating with Merton [19], use the management's information set when valuing risky debt. As a consequence, default can be viewed as the first hitting time of the firm's asset value to a liability determined barrier. For most models, the firm's asset value follows a continuous process, implying that at default, the value of a firm's debt does not exhibit a jump. In this circumstance one would expect to see the risky debt price randomly decline until default. Structural models have no implications for the risky debt price process subsequent to default.

Reduced form models, originating with Jarrow and Turnbull [15],[16], use the market's information set when valuing risky debt. Hence, default is modeled as the first jump time of a point process, implying that the value of a firm's debt exhibits a negative jump at default. As with structural models, reduced form models have no implications for the risky debt price process subsequent to default.

To investigate which model is most consistent with risky debt prices and to motivate the subsequent analysis, we provide some illustrations of the different paths the bond price can take when a company defaults.

Figure 1 shows the debt price evolution for a senior bond issued by Delta Airlines Inc. from October 1, 2004 through December 21, 2005. The company filed for bankruptcy under Chapter 11 on September 14, 2005. The price of the bond seems to anticipate the default and it declines more or less continuously in the eight months before default. The price of the bond does not jump at default, and after bankruptcy has been declared, the bond seems to recover from the previous losses. This debt price path is consistent with the standard structural model. Also, in this case, the price of the debt 30 days after default is (perhaps) only a slightly upward biased estimate of the bond's price at default.

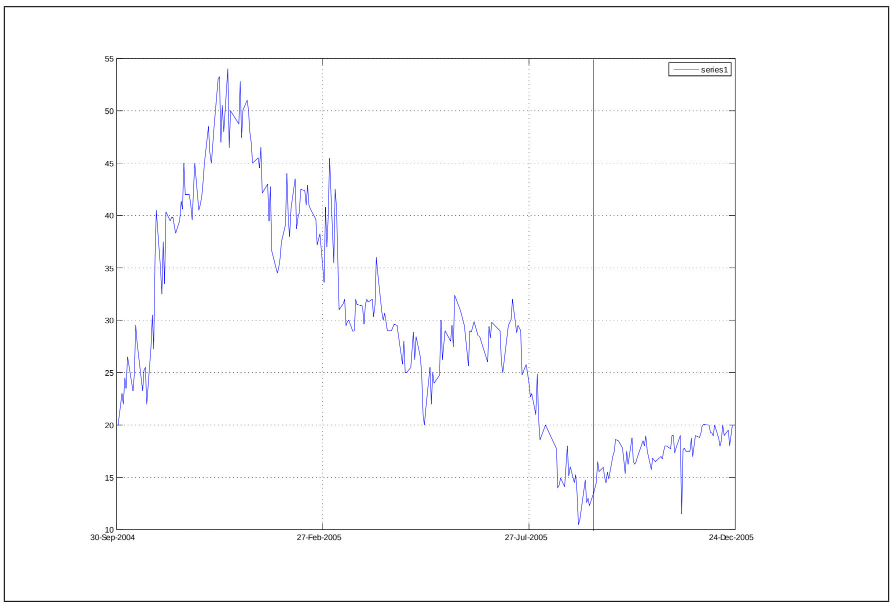

Figure 1: Delta Airlines Inc., coupon 8.3%, maturity 12/15/2029. Time series graph of debt prices as a percentage of face value ($100). The solid vertical line represents the default date.

A different situation is illustrated in Figure 2 which shows the debt prices for Trico Marine Services Inc. from October 4, 2004 through March 16, 2005. The company filed for Chapter 11 bankruptcy on December 18, 2004, the restructuring plan was approved on December 27, 2004, and the bond was reinstated on March 16, 2005. As we can see from the graph, the price does not jump on the bankruptcy filing date but remains more or less constant in a time window of weeks before and after the bankruptcy filing. The filing itself has a positive effect on the value of the bond. The price raises by approximately 50% within the restructuring period, which can be attributed to the fact that the restructuring process turns out to be successful. Although the debt's price does not jump at default (consistent with the management's information set), this price is inconsistent with the structural model because the debt price is increasing over our observation interval. This suggests that the market may have already believed that the debt had defaulted before our observation period began. Furthermore, it demonstrates that the price of the debt 30 days after default is not an unbiased estimate of the bond's price at default. Indeed, in this case, positive information on the restructuring resulted in a significant price increase.

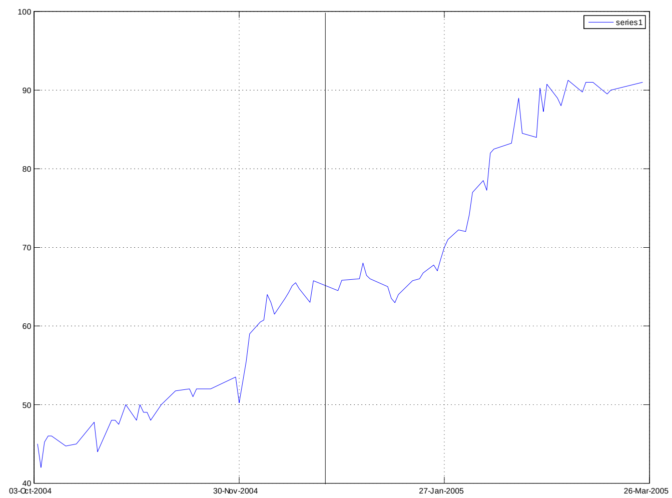

Figure 2: Trico Marine Service Inc., coupon 8.875%, maturity 5/15/2012. Time series graph of debt prices as a percentage of face value ($100). The solid vertical line represents the default date.

An example consistent with the standard reduced form model is contained in Figure 3 which illustrates the bond price evolution for Winn Dixie Stores Inc. from October 1, 2004 through December 21, 2005. Default and the Chapter 11 bankruptcy filing happened simultaneously on February 21, 2005. As indicated, the price path exhibits a large jump at default where the bond loses approximately 50% of its value within a few days. Default in this case came as a complete surprise to the market. Note also that the full impact of the loss at default was not realized until much later, indicating that the debt price 30 days after default is biased low.

The last example in Figure 4 is for a senior bond issued by Northwest Airlines Inc. from October 1, 2004 through December 21, 2005. Northwest filed for bankruptcy under Chapter 11 on September 14, 2005 – the same day as Delta Airlines. Here the price drops somewhat continuously until default, where it

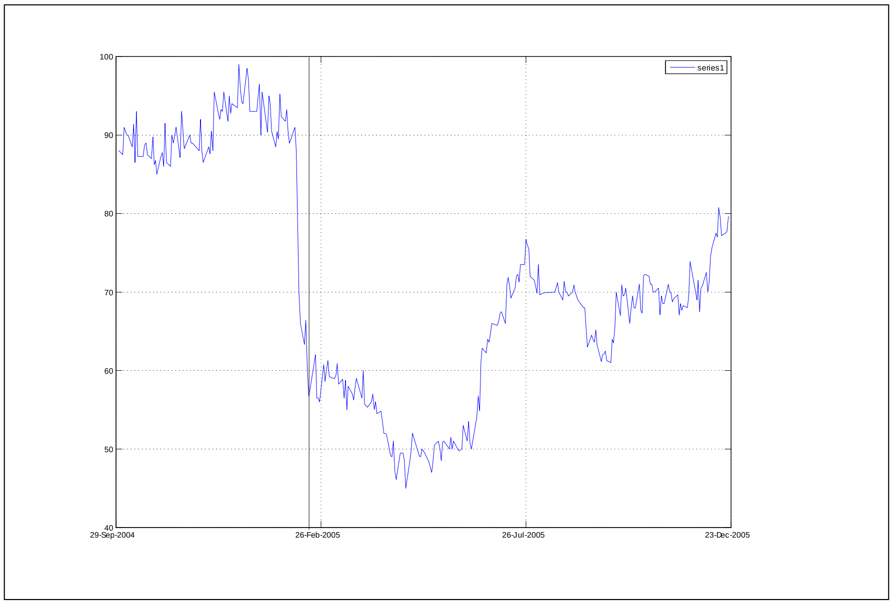

Figure 3: Winn Dixie Stores Inc., coupon 8.875%, maturity 4/1/2008. Time series graph of debt prices as a percentage of face value ($100). The solid vertical line represents the default date.

experiences an additional discrete drop. Then, the price increases randomly until our observation period ends. As given, this price process is also consistent with the standard reduced form model, but it appears that the market was not completely surprised. As with all previous illustrations, the price of the debt 30 days subsequent to default is not an unbiased estimate of its value at default.

These four illustrations demonstrate the following facts: (i) the recorded default date may not be the date when the market knows default has occurred due to the leakage of information (e.g. Trico Marine, Northwest Airlines, Delta Airlines), and (ii) the recovery rate estimate differs on the default date versus 30 days later. This difference in the recovery rate estimates may yield a potential bias in using 30 days after as a measure of the default date recovery rate. The remainder of the paper explores these and related issues.

3 Set up

We are given a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\in[0,\infty)}, P)$ satisfying the usual conditions (see Protter [23]). We denote the *recorded default date* as τ^*. The recorded default date will be given in our data set. We define the *economic default date* as the time when the market knows default has happened, denoted as time $\tau \leq \tau^*$. From an economic perspective, τ is the relevant date used for

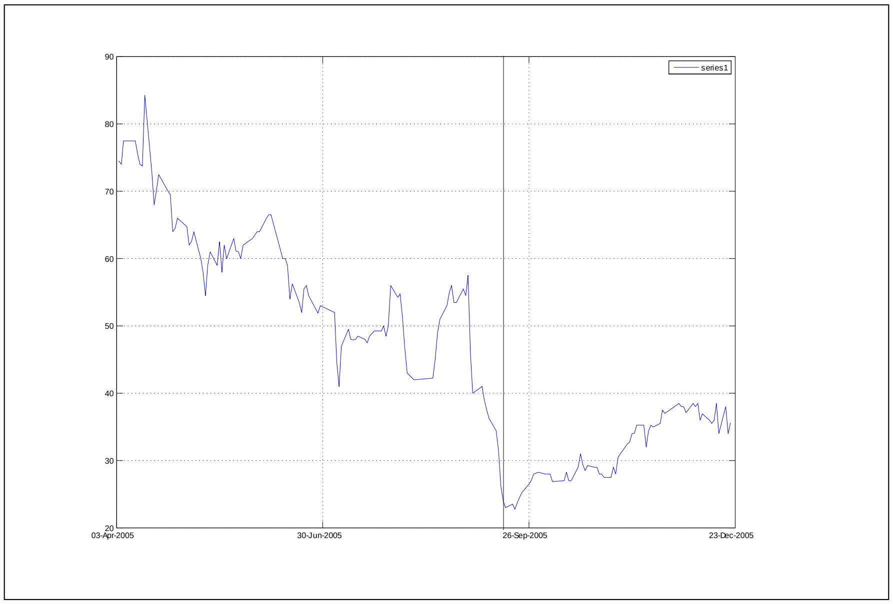

Figure 4: Northwest Airlines Corporation, coupon 9.875%, maturity 3/15/2007. Time series graph of debt prices as a percentage of face value (\$100). The solid vertical line represents the default date.

"default" in the current credit risk derivatives pricing literature.[2]

We fix a particular firm, and we let B_t denote the price of its risky debt at times $t \in [0, \infty)$, assumed to be adapted to the filtration $(\mathcal{F}_t)_{t\in[0,\infty)}$. The debt has a maturity date and coupons (floating or fixed). For our analysis, we need not be more specific about this structure. We let B_t^d denote the risky debt price after economic default at times $t \geq \tau$. For $t \geq \tau$, it follows that $B_t^d = B_t$.

For the subsequent analysis, we also need the riskless spot rate of interest, denoted r_t, and the time t price of a riskless coupon bond with the same maturity T and coupons as the risky bond under consideration, denoted $p_t(T)$. Both r_t and $p_t(T)$ are assumed to be adapted to the filtration $(\mathcal{F}_t)_{t\in[0,\infty)}$.

4 Cross-Sectional Models

There are three (existing) cross-sectional models: (i) recovery of face value (RFV) (see Moody's Investor Services [21]), (ii) recovery of Treasury (RT) (see Jarrow and Turnbull [16]), and (iii) recovery of market value (RMV) (see Duffie and Singleton [12]). These three models are designed to determine the recovery rate only on the economic default date. The purpose of these models are to

[2]Depending upon the filtration structure, both τ and τ^* may not be stopping times.

provide the necessary inputs to price both risky debt and credit derivatives prior to default.

Using cross-sectional models, the recovery rate estimation procedure is to: (i) fix a defaulted company, then (ii) fix a date $\overline{\tau}$ to observe debt prices, then (iii) estimate the recovery rate. Note that this is a single point estimate of the recovery rate per company. Then, look cross-sectionally across companies to obtain better estimators. Hence, the label "cross sectional" models.

Moody's Investor Services [20] is a standard reference for these estimates. They define default as (and implicitly the default date as when) either: (i) a missed or delayed disbursement of interest and/or principal, (ii) a filing for bankruptcy, administration, legal receivership or other legal blocks to the payment of interest or principal, or (iii) a distressed exchange where the issuer offers bondholders a new security that amounts to a diminished obligation or the exchange has the apparent purpose of helping the borrower avoid default (see [20], p. 5). Their recovery rates at default are based "on the *30-day post-default* bid prices as percent of face value, except in the case of distressed exchanges in which we use the trading prices of exchanged instruments two weeks prior to the exchange" (see [20], p. 6).[3] This "30-day" post-default date selection procedure appears to be only approximately true.[4] Unfortunately, further details on the estimation procedure are not readily available.

4.1 The Models

As mentioned, all three models are only defined on the economic default date. Their definitions are:

1. Recovery of Face Value (RFV):

$$B_\tau^d = \delta_\tau F \tag{1}$$

   where F is the face value of the debt (usually normalized to \$100), and $\delta_\tau \in [0, \infty)$ is the recovery rate. Although one could add accrued interest to the face value, this is not normally done. The justification for using the face value of the debt to measure the recovery rate is that, at default, the entire principal and accrued interest become due. This acceleration of principal, could in certain cases, yield to recovery rates of greater than one.

2. Recovery of Treasury (RT):

$$B_\tau^d = \delta_\tau p_\tau(T) \tag{2}$$

   where $\delta_\tau \in [0, \infty)$ is the recovery rate and T is the maturity of the original debt issue.

[3] Moody's also has a recovery rate estimate based on the value of the debt after resolution (see [22]). We do not analyze this alternative liquidation rate in this paper.

[4] In Moody's [21] p. 40, the qualification "roughly" 30 days is added to this definition.

3. Recovery of Market Value (RMV):

$$B^d_\tau = \delta_\tau B_{\tau-} \tag{3}$$

where $\delta_\tau \in [0, \infty)$ is the recovery rate. Note that RMV implies that the bond price jumps on the economic default date, unless $\delta_\tau = 1$.

All three recovery rates δ_τ can be random variables depending upon $\mathcal{F}_\tau$.

4.2 Data

Our bond price data is obtained from Advantage Data Inc.[5] Advantage Data Inc.'s database covers virtually the entire U.S. and international bond market. Trade data is obtained from TRACE[6], the NYSE, and AMEX. Advantage Data Inc. obtains the bond characteristics and related information from the "Mergent Fixed Income Securities Database" as well as other sources. We restrict our analysis to the U.S. bond market.

4.2.1 Debt Prices

With respect to price data, our data set consists of approximately 20 million price quotes and execution prices from almost 31 thousand different issues between December 21, 2000 and October 15, 2007**.** Mergent Fixed Income Database records default as occurring when a debt issue either violates a bond covenant (technical default), misses a coupon or principal payment, or files for bankruptcy. A grace period of 30 days must pass before default is recognized after missing a scheduled coupon payment, although this grace period can vary according to the conditions listed in the indenture agreement.[7] Initially, we start with 2574 defaulted issues, of which we have price information on 523.

We use daily closing prices for our analysis. The closing prices are based on executed trades. We further restrict our set of defaulted issues to be those where we have a price both before and after the recorded default date, and for which at least 50 prices are available. We want at least one price quote to be 30-days after recorded default. This reduces our sample to 145 bond issues. We remove bonds without maturity and/or coupon information, convertible bonds, and asset backed securities. We do not restrict our bonds further for embedded options. We will return to this issue later on in the text. This leaves 103 bonds which are grouped into 92 senior unsecured and 11 senior secured bonds from 50 different companies. It turns out that after applying all of the previously discussed filters, our final default sample consists of only

[5] See (www.advantagedata.com).

[6] TRACE (The Traded Reporting and Compliance Engine) was launched by the National Association of Securities Dealers (NASD) in April 2002. NASD lists all publicly disseminated trade information on its website (www.finra.org/marketdata).

[7] This definition is slightly different from that used by Moody's. Moody's includes more possibilities for the occurrence of a default and it excludes the 30-day grace period rule for missed coupon payments. A 90-day grace period rule is used in the New Basel II Accord (see Schuermann [24]).

those issues that eventually file for bankruptcy. Our filtering procedure has potentially induced a sample selection bias. Our results should be interpreted with this qualification in mind.

4.2.2 Recorded Default Times

For our analysis of the recorded default dates, our sample covers 2574 defaulted bonds from 1207 corporations between 1984 and 2007. Within this set of defaulted bonds, approximately 6.3% are unsecured and 93.7% are secured. Defaulted debt issues can be resolved either out-of-court through negotiations with the creditors or in-court after a filing of a bankruptcy petition. Our data includes 952 bankruptcy filings (either Chapter 11 reorganization or Chapter 7 liquidation). 76 bond issues default more than once. We treat multiple defaults as distinct. There are 118 issues that do not have complete information regarding default. This leaves 2380 bonds in our database.

We distinguish between four dates: the recorded default date, the bankruptcy filing date, the effective date, and the reinstatement date. The recorded default date is the date that default is recorded within the database. The effective date is when a bankruptcy court approved restructuring plan takes effect (Chapter 11) or when the liquidation of the company and the distribution of the proceeds is completed (Chapter 7). A bond issue is either reinstated or not. The reinstatement date is the date if it is reinstated. If reinstated, the creditors receive all missed payments and interest on the missed payments. In this case, the bond is treated as if nothing had happened. If not reinstated, then the bond issue usually gets some residual recovery rate.[8]

For approximately 80% of the defaulted issues (1902 out of 2380), default occurs because of a bankruptcy filing. To keep the default time analysis consistent with our price data analysis we only consider this subset of defaulted issues, thus, our final sample size is 1902 issues from 1014 companies.

Table 1 shows that the average time spent in Chapter 11 bankruptcy is 454 days with a standard deviation of 427 days. The time duration spent in bankruptcy documents the fact that for any particular defaulted bond issue, it can easily trade in the distressed debt market for over a year. The duration of the distressed debt market for a single issue is one indication of the importance of these markets to the resolution of financial distress (see Altman [2] for a more in depth analysis of distressed debt markets).

4.3 Estimation Results

This section estimates the different recovery rates on and around the recorded default date. We have 103 bond issues in our sample. For a subsequent test (to be discussed), we removed those issues where we do not have 30-day after

[8] We have 27 bond issues with data spanning the recorded default date to the resolution date (effective or reinstatement date) with data on the ultimate recovery values. This subset is too small to investigate the estimation of "ultimate" recovery rates.

	Mean	Std. Dev.	Median	N	λ
Chapter 7	433.22	353.20	433	9	0.80
Chapter 11	454.34	427.08	354	631	0.84

Table 1: Days between Filing Date and Effective Date categorized by Bankruptcy Filing Type. Lambda is the inverse of the average time spent in bankruptcy in years.

and/or 60-day after prices. This leaves 96 issues remaining for our investigation. In our estimation procedure, two issues need some discussion.

First, as noted earlier, we do not exclude bonds with embedded options. However, this is without loss of generality to our analysis. When a bond defaults, its convenants (and embedded options) are invalidated, and its principal and accrued interest become immediately payable. This implies that the price of a debt issue after default trades without any embedded options, even if they existed before. The recovery rate estimate, therefore, includes the loss/gain in value due to any embedded options.

Second, the quoted price of a bond equals the present value of all futures cash flows minus accrued interest. Alternatively stated, the price paid for a bond equals the quoted price plus accrued interest. After default, however, no future interest payments are due. Consequently, the quoted price equals the price paid. After the default date price quotes do not need to be adjusted for accrued interest.

4.3.1 RFV Estimates

RFV is just the closing market price of the debt (per \$100 face value) measured on the default date. Table 2 contains these estimates for all bonds in our sample. The estimated RFV on the default date is .0.4817. RFV estimates are also provided for an observation window starting thirty days before and ending thirty days after the default date. As indicated, the average price starts at \$62.19 and declines to \$42.31. As noted, the standard deviation on any given day is quite large. Given our small sample size, we do not partition our data into finer categories.

Table 2 also shows that the estimated RFV thirty days after is 0.4231. This later estimate is similar to the weighted average recovery rate of 0.422 across all bonds from 1992 - 2003 as reported in Moody's ([21], Exhibit 9). The appropriate test for the difference between the recorded default and 30-day after recovery rate is a paired t-test. We perform this test on all issues that have both at-default and exactly 30-day after prices available. There are 64 issues that meet this restriction. The *t-stat* = 2.66 with 63 degrees of freedom with a *P-value* of 0.0006. The difference is statistically significant.

The average prices across time suggest that information about default leaks prior to the default record date. Indeed, the average prices in the week preceding default are similar in magnitude to those on the default date itself.

Difference	Count	Avg. Price	Std. Dev.	Avg. Ratio
-30	23	62.19	33.04	1.3199
-20	58	48.99	28.06	1.3510
-15	46	54.70	29.46	1.2299
-10	27	66.74	28.60	1.2382
-7	60	48.31	26.77	1.1574
-5	51	40.42	25.81	1.2114
-3	29	55.33	29.50	1.0639
-2	41	44.60	29.99	1.0541
-1	61	45.05	29.55	0.9796
0	70	48.17	29.39	1
1	71	45.48	28.67	1.0292
2	63	41.27	28.85	1.0284
3	44	52.43	29.12	1.0561
5	44	48.32	31.48	1.0341
7	66	51.82	30.46	1.0743
10	45	54.62	28.83	1.0933
15	56	48.40	28.31	1.1088
20	46	53.86	32.44	1.1473
30	64	42.31	29.30	1.0779

Table 2: Recovery of Face Value Estimates from 30 days before to 30 days after Recorded Default. "Difference" is time to default in days. A negative number is before the announcement and a positive number is after. "Count" gives the number of issues with prices on that day. "Avg. Price" is the RFV estimate. "Avg. Ratio" is the ratio of the price on any day to the price on day zero.

N = 96	30-day to 0-day Ratios	60-days to 30-days Ratios
Mean	1.0967	1.0565
Median	1.0628	1.0219
Standard Deviation	0.3485	0.1939
First Quartile	0.9987	0.9988
Third Quartile	1.1735	1.1471

Table 3: Debt Price Ratio Summary Statistics both 30 days and 60 days after Default.

Table 3 extends the comparison to 60-days after the recorded default date. The 30(60)-day-post-default price for each issue is the price quote on the 30th (60th) day after recorded default or the nearest available quote. For example, if we only have prices on 28th and 33rd day after recorded default, we use one on the 28th day. As shown, the differences between 30 and 60 days are also significantly different. A simple t-test of the hypothesis that the 60-day to 30-day ratio is unity has a $t\text{-}stat = 2.86$ with 95 degrees of freedom with a *P-value* of 0.005. The null hypothesis is rejected. Tables 2 and 3 casts doubt on the use of 30 days after as an unbiased estimate of the recovery rate on the default date itself.

4.3.2 RT Estimates

Table 4 contains estimates of the RT recovery rates. These are obtained by using the market price of the debt on the default date and dividing it by the price of an otherwise identical default free bond. The price of an otherwise identical default free bond was obtained by taking the coupon and payment structure of the risky bond, and using default-free zero-coupon bond prices to compute current values. The zero-coupon bond prices were inferred from the daily par bond constant maturity yield curves available from the Federal Reserve bank (www.federalreserve.gov/releases/h15/data.htm). Details of our inference procedure are contained in an appendix to this paper. As shown, the average RT recovery rate is 0.4062. These are lower than the RFV recovery rate estimates which implies that the otherwise identical default free bonds are trading at a premium ($>$ \$100). Otherwise identical default free bonds will usually trade at a premium due to the larger coupons paid for risky debt relative to equivalent maturity Treasuries.

4.3.3 RMV Estimates

The RMV estimate is the percentage change in the price of the debt one day before the default date to the price on the default date. To compute this estimate, we collect the pre-default price of each issue by selecting its last price quote prior to the default date. A majority of our issues have prices one day before default. We require that the pre-default price be not more than 7 days before default. Among the 96 available pre-default prices, 62 are one day before

N = 96	RT Estimates
Mean	0.4062
Median	0.3452
Standard Deviation	0.2528
First Quartile	0.1692
Third Quartile	0.6374

Table 4: Recovery of Treasury Summary Statistics.

N = 96	Pre-Default	Default Date	RMV Estimates
Mean	48.4	48.6	1.0230
Median	39	38.5	1.0013
Standard Deviation	30.6	30.7	0.1824
First Quartile	21.5	22	0.9681
Third Quartile	67.55	69.375	1.0597

Table 5: Recovery of Market Value Summary Statistics. The RMV estimates are the ratio of the pre-announcent to the recorded default date prices.

default, 8 are two days before, 11 are three days before, 2 are four days before, 3 are five days before, 5 are six days before, and 5 are seven days before.

For the default date itself, if no price is available on this date, we use the next available day's price. For our sample, 70 prices are on the default date, 19 are one day after, 4 are two days after, 2 are three days after, and 1 is five days after.

Table 5 contains the RMV estimates in the last column. As seen, the average recovery rate is 1.0230 implying that, on average (and for at least 75% of the debt issues), debt prices do not jump on the default date (if anything, they increase slightly). This observation is inconsistent with the RMV model for recovery rates because it implies that, on average, the debt is "riskless." Indeed, although default occurs, there is no loss in value. This is a anomalous result, suggesting one of two possible conclusions: (i) either the RMV is a poor model for recovery rates, or (ii) the recorded default date does not equal the economic default date. We will provide evidence supporting the second of these conclusions in the next section.

5 Time-Series Models

This section extends the cross-sectional models of recovery rates to time-series models. This extension serves two purposes. First, this extension enables the estimation of the economic default date to see if it differs from the recorded default date ($\tau \leq t < \tau^*$). Second, this extension enables the pricing of distressed debt after the recorded default date ($\tau^* \leq t$). Both of these investigations are performed in this section.

5.1 Extension

Each of the existing models can be represented (abstractly) as

$$B_\tau^d = m \cdot \delta_\tau \tag{4}$$

where

$$m = \begin{cases} F & if & RFV \\ p_\tau(T) & if & RT \\ B_{\tau-} & if & RMV \end{cases}.$$

The extension to times $\tau < t \leq \tau^*$ is very natural. Assuming that the recovery payment is made at time τ, or equivalently, that the risky debt position is sold at this time, then the payment obtained is worth

$$B_t^d = m \cdot \delta_\tau e^{\int_\tau^t r_s ds} \tag{5}$$

at a future date $t \geq \tau$. This future value is obtained by investing the proceeds from the sale of the risky debt at time t into a money market account earning the default free spot rate of interest. In an arbitrage-free setting, note that B_t^d is worth B_τ^d at time τ.[9] Hence, this pricing model is consistent with standard asset pricing theory.

For subsequent usage, we write this as:

$$B_t^d = B_{\tau^*}^d e^{\int_{\tau^*}^t r_s ds} \ for \ \ t \geq \tau.^{10} \tag{6}$$

It should be noted that this model also applies to times after the recorded default date ($\tau^* \leq t$). And, as formulated, the extended cross-sectional model is seen to be independent of the recovery rate model selected because it only depends on the debt price $B_{\tau^*}^d$. This was the purpose for deriving this alternative representation.

5.2 The Economic Default Date Estimates

We first estimate the economic default date. Recall that the economic default date is that time $\tau \leq \tau^*$ when the market knows default has happened. We estimate the economic default date using debt prices via the expression:

$$\widehat{\tau} = \inf_{t \leq \tau^*} \{t : B_t \leq B_{\tau^*}^d e^{-\int_t^{\tau^*} r_s ds}\}. \tag{7}$$

In words, $\widehat{\tau}$ is the first time that risky debt is priced as if default has happened. Note that our estimator $\widehat{\tau}$ depends on τ^* and uses the information available up to the recorded default date τ^* which includes the debt's price $B_{\tau^*}^d$ and the realized spot rates $\{r_s : s \leq \tau^*\}$.

[9] Indeed, $E\left(B_t^d e^{-\int_\tau^t r_s ds} | \mathcal{F}_\tau\right) = E\left(m \cdot \delta_\tau | \mathcal{F}_\tau\right) = B_\tau^d$ where $E(\cdot)$ represents expectation under the martingale measure.

[10] If $t < \tau^*$ then $e^{\int_{\tau^*}^t r_s ds} = e^{-\int_t^{\tau^*} r_s ds}$.

Recall that after default, the debt trades without any coupon payments nor embedded options, so no adjustments are necessary for these considerations. In particular, the quoted price is not adjusted for accrued interest. For the realized spot rate, we use the 3 month Treasury yield as available from the Federal Reserve bank (www.federalreserve.gov/releases/h15/data.htm). Note that this estimator is independent of the different recovery rate models RFV, RT, RMV.

We (somewhat arbitrarily) restrict our estimator to lie in the interval $[\tau^* - (180\ days), \tau^*]$. A hundred and eighty days is selected as an upper bound on the distance between the economic and recorded default dates because many debt issues have semi-annual interest payments.

Figure 5 contains a histogram of the time between economic and recorded default. Of the 96 debt issues, 73 trigger economic default strictly before the recorded default date. This difference is significant. Of the 73 issues, 13 hit the upper bound and have the economic default date 180 days before recorded default. Given that the economic and recorded default dates differ, we next

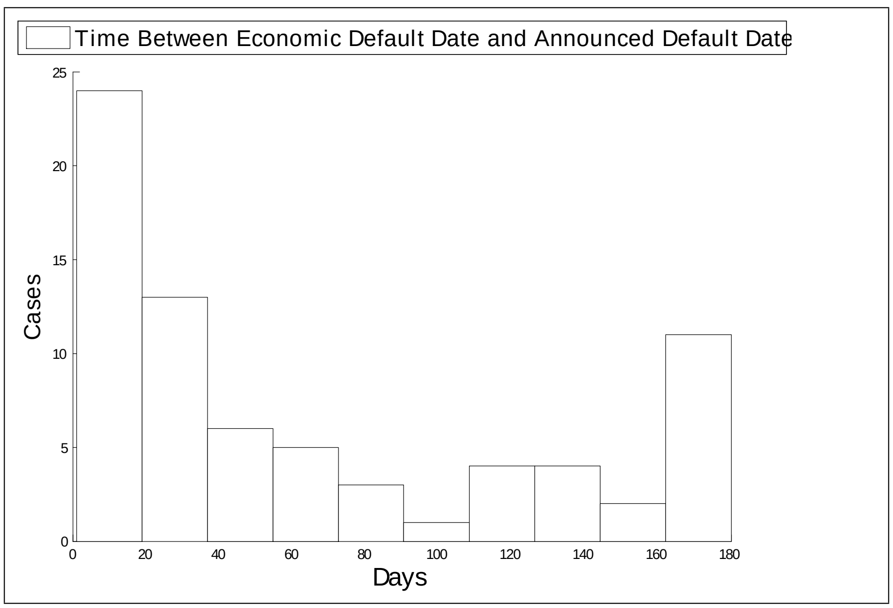

Figure 5: Histogram of the Time Between the Economic and Recorded Default Dates. The number of nonzero differences is 73.

explore if this difference has a significant impact on the recovery rate estimates.

5.3 Revised Recovery Rate Estimates

Given our estimate of the "economic" default date for each of the 73 issues where economic and recorded defaults differ, we re-estimate the recovery rate

N = 73	Economic Default	Recorded Default
Mean	0.4879	0.5283
Median	0.45	0.5782
Standard Deviation	0.3044	0.3151
First Quartile	0.2	0.2225
Third Quartile	0.76	0.8425

Table 6: Comparison of the Recovery of Face Value Estimates for the Economic and Recorded Default Dates.

N = 73	Economic Default	Recorded Default
Mean	0.3970	0.4335
Median	0.3291	0.4776
Standard Deviation	0.2461	0.2600
First Quartile	0.1578	0.1803
Third Quartile	0.6031	0.6610

Table 7: Comparison of the Recovery of Treasury Estimates for the Economic and Recorded Default Dates.

for the three different models using the estimator:

$$\widehat{\delta}_\tau = \begin{cases} \frac{B_\tau}{F} & if \quad RFV \\ \frac{B_\tau}{p(\tau,T)} & if \quad RT \\ \frac{B_\tau}{B_{\tau-}} & if \quad RMV \end{cases} .$$

5.3.1 RFV Estimates

Table 6 contains a comparison of the RFV recovery rate estimates on the economic default date versus the recorded default date.

A paired t-test rejects the null hypothesis that these two recovery rates are equal with a *t-stat* $= -7.10$ with 72 degrees of freedom and a *P-value* essentially zero.

To illustrate the differences between these estimates, Figure 6 plots the density function for the different recovery rate estimates.[11] As evidenced by this analysis, the specification of the default date makes a significant difference to the estimated recovery rate.

5.3.2 RT Estimates

Table 7 contains a comparison of the RT recovery rate estimates on the economic default date versus the recorded default date.

[11] All density functions estimated in this paper are based on "ksdensity" in Matlab. This function computes the density using a kernel-smoothing method based on normal kernels.

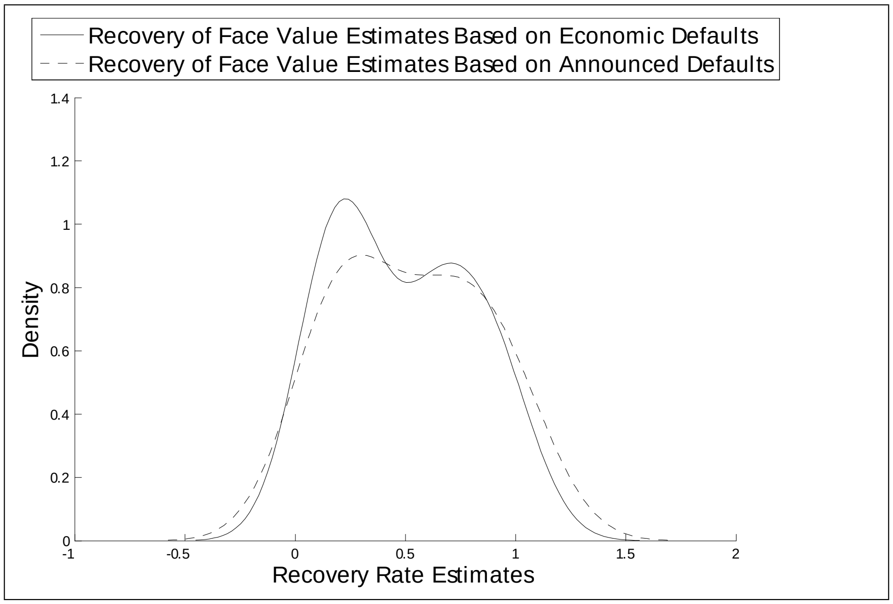

Figure 6: Density Function for the Recovery of Face Value estimates based on the Economic and Recorded Default dates.

A paired t-test rejects the null hypothesis that these two recovery rates are equal with a *t-stat* $= -6.65$ with 72 degrees of freedom and a *P-value* essentially zero.

To illustrate the differences between these estimates, Figure 7 plots the density function for the different recovery rate estimates. Note that Figures 6 and 7 differ only in the denominator, hence, would one expect that the densities would have a similar appearance.

5.3.3 RMV Estimates

Table 8 contains a comparison of the RMV recovery rate estimates on the economic default date versus the recorded default date.

The average recovery rate on the economic default date is significantly less than one. Indeed, a paired *t - test* yields a *t-stat* $= -6.98$ with 72 degrees of freedom and a *P-value* essentially zero. This implies that debt prices jump on the economic default date with an average percentage drop of 0.1686.

In contrast, on the recorded default date the debt price experiences a positive change, on average. Hence, the use of reduced form models (and the RMV) necessitates the use of the economic default date and not the reported default date for estimating the recovery rate. Otherwise, as noted in the previous section, the debt would be "riskless."

Figure 8 contains a comparison of the density functions for these estimates.

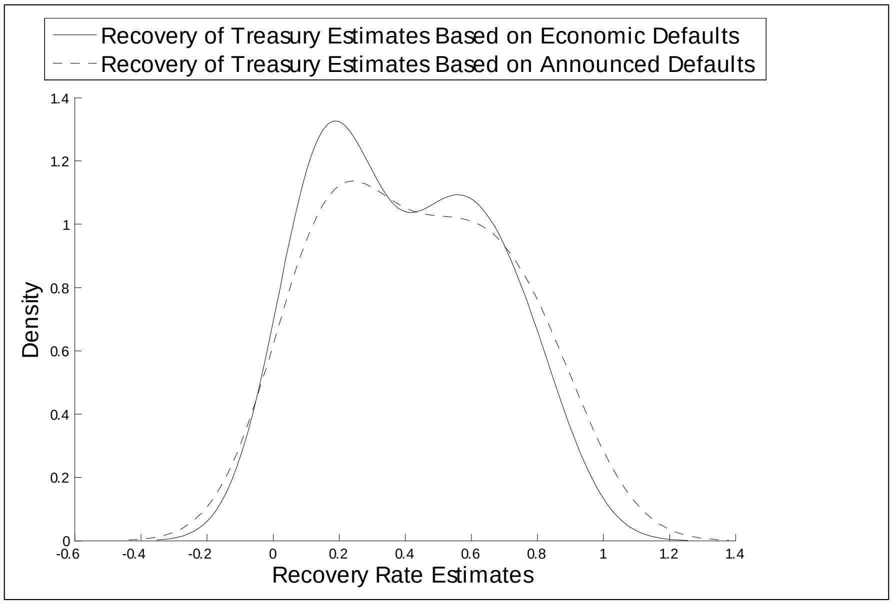

Figure 7: Density Function for the Recovery of Treasury estimates based on the Economic and Recorded Default dates.

As exhibited, the density functions are quite different depending upon the selection of the default date utilized, supporting the previous conclusions based on Table 8.

5.4 Tests of the Extended Debt Price Model

As mentioned previously, the extended model provides a model for distressed debt prices after economic default. This section tests the quality of this model in matching distressed debt market prices using the estimated τ and $\widehat{\delta}_\tau$ from the previous section. Given observation error, the distressed debt pricing model

N = 73	Economic Default	Recorded Default
Mean	0.8314	1.0653
Median	0.9094	1.0217
Standard Deviation	0.1775	0.1729
First Quartile	0.7267	0.9976
Third Quartile	0.9649	1.0854

Table 8: Comparison of the Recovery of Treasury Estimates for the Economic and Recorded Default Dates.

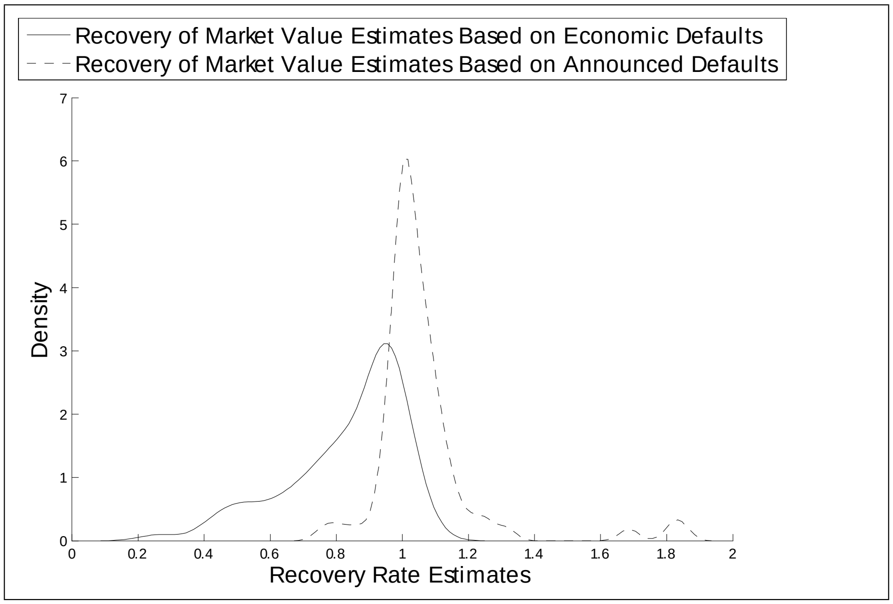

Figure 8: Density Function for the Recovery of Market Value estimates based on the Economic and Recorded Default dates.

can be written as

$$B_t^d = m \cdot \widehat{\delta}_\tau e^{\int_\tau^t r_s ds} + \epsilon_t \quad for \quad t \geq \tau. \tag{8}$$

The model will be considered "good" if the residuals have zero mean, and are independent and identically distributed (i.i.d.). Note that the distressed debt pricing model is independent of the recovery rate specification (RFV, RT, RMV) because $m\widehat{\delta}_\tau = B_\tau$ and our estimate for τ does not depend on the model for m. This model is consistent with stochastic interest rates and all three recovery rates being random variables depending upon $\mathcal{F}_\tau$.

Table 9 provides the summary statistics of the pricing errors across all issues and dates. As indicated, the mean pricing error is quite large at 17.81% of the debt's face value with a standard deviation of 28.06%.

To test if the pricing errors have zero mean and are i.i.d., we run for each bond issue the time-series regression $\epsilon_t = \alpha + \beta t$ and test if $\alpha = 0$ and $\beta = 0$. We first run the regression for those issues where the economic and recorded default dates differ. For 62 out of 73 such issues, we reject the null hypothesis that $\alpha = 0$ and $\beta = 0$ with a significance level of 0.01 (for 56 of them we have negligible P-values). The rejection of $\alpha = 0$ implies that the model over estimates the market prices and the rejection of $\beta = 0$ implies that the residuals are not i.i.d. For 55 out of the 73 issues, $\beta > 0$, indicating that the residuals are increasing after the economic default date.

N = 20,942	
Mean	17.81
Median	9.31
Standard Deviation	28.06
First Quartile	0.11
Third Quartile	30.00

Table 9: Summary Statistics for the Percentage Pricing Errors of the Extended Model.

We also ran the regressions for all 103 bond issues in our sample. For 87 we reject the null hypothesis that $\alpha = 0$ and $\beta = 0$ with a significance level of 0.01 (for 79 we have negligible P-values). Moreover, regressions based on 77 out of 103 issues produce positive slopes. Again the mean zero and i.i.d. hypotheses are rejected.

The extended recovery rate model does not fit market prices well and can be rejected for the majority of debt issues considered. These tests provide an additional statistical rejection of using the 30-day after recovery rate as an unbiased estimate of the recovery rate at economic default. Indeed, the 30-day recovery rate estimate is only valid if the extended recovery rate model is valid, and one can easily reject the extended model.

The rejection of the extended model should not come a surprise. The extended model values the distressed debt after default as if it were liquidated at the default time (see the discussion before expression (6)). This value ignores the information generated by the bankruptcy and/or restructuring process as time evolves. In the financial resolution, if positive information results, the distressed debt price should increase ($\beta > 0$ in the regression). Expression (6) does not capture this effect. If negative information continues to occur, the debt price will continue to decline. Expression (6) does not capture this effect either. It is unlikely that the information generated by the financial resolution process results in the debt price evolving randomly around the time τ liquidation value (adjusted for the spot rate). This is the only effect that expression (6) captures.

The rejection of the extended model also implies that our estimates of the economic default date may be misspecified. Indeed, our estimator in expression (7) depends on a model for the defaulted debt price B_t^d. And, we just rejected this model for valuing distressed debt. This observation motivates the need for a distressed debt model that reflects the information generated by the bankruptcy and/or restructuring process.

6 A Recovery Rate Model

The recovery rate model that we utilize in this paper is a simplification of a more general model, as will be evidenced below.[12] We selected the subsequent model based on the limitations of our database. However, as more data becomes available, more complex recovery rate models can be estimated. In our database, once a firm defaults, it eventually files for bankruptcy. Hence, we only model the resolution of the bankruptcy filing, either Chapter 11 (restructuring) or Chapter 7 (liquidation).

6.1 The Model

Recall that we are given a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\in[0,\infty)}, P)$. The subsequent modeling only refers to times after the economic default date, i.e. $t \geq \tau$. We use this fact below without further reference. Let τ_0 represent the time to resolution of bankruptcy, assumed to be a $(\mathcal{F}_t)_{t\in[\tau,\infty)}$ stopping time. We let τ_0 have an exponential distribution with parameter λ. Hence, the point process $1_{\{\tau_0 \leq\ t\}}$ follows a Poisson process with constant intensity λ.

At the bankruptcy resolution time τ_0, we assume that the debt receives a $\mathcal{F}_{\tau_0}$ random dollar payoff equal to $m \cdot \delta_{\tau_0} \geq 0$ where $m \in \mathbb{R}$ is a normalization that gives δ_{τ_0} the interpretation of being a recovery rate (a percentage) instead of a dollar amount. For the subsequent estimation, we consider three normalizations corresponding to the three cross-sectional recovery rates previously studied:

$$m = \begin{cases} F & if & RFV \\ p_\tau(T) & if & RT \\ B_{\tau-} & if & RMV \end{cases}.$$

Assuming that distressed debt trades in the standard continuous time arbitrage free setting (see Bielecki and Rutkowski [6] for the standard construction), the value of the distressed debt at time t is

$$\begin{aligned} B_t^d &= mE\left(\delta_{\tau_0} e^{-\int_t^{\tau_0} r_s ds} \,|\mathcal{F}_t\right) \\ &= mE\left(\int_t^\infty \delta_s e^{-\int_t^s r_u du} \lambda e^{-\lambda(s-t)} ds \,|\mathcal{F}_t\right) \qquad (9) \end{aligned}$$

where $E(\cdot)$ represents expectation under an equivalent martingale probability measure.

This expression has a nice interpretation. It is the expected discounted recovery rate δ_s received at time s, occurring with probability $\lambda e^{-\lambda(s-t)}$, and summed across all times s. The martingale measure implicitly incorporates risk aversion into the present value computation.

In our estimation, in order not to separately estimate both a default free spot rate process and a recovery rate process, we consider the joint process

$$R_s \equiv \delta_s e^{-\int_\tau^s r_u du}. \qquad (10)$$

[12] The more general model using Cox processes and multiple default resolution outcomes is contained in Lin [17].

We will call this the "modified recovery rate" process. Note that R_s implicitly depends on the economic default date τ. We will need to address this dependence in our estimation. Then, we can rewrite expression (9) as:

$$B_t^d = me^{\int_\tau^t r_u du} \int_t^\infty E\left(R_s \left| \mathcal{F}_t \right.\right) \lambda e^{-\lambda(s-t)} ds. \tag{11}$$

Last, we assume that R_s follows an Ornstein-Uhlenbeck process:[13]

$$dR_t = a(b - R_t)dt + \sigma dW_t \tag{12}$$

where a, b, σ are constants and W_t is a standard Brownian motion defined on our filtered probability space *under the martingale measure*. Then,

$$E\left(R_s \left| \mathcal{F}_t \right.\right) = R_t e^{-a(s-t)} + b(1 - e^{-a(s-t)}). \tag{13}$$

The Ornstein-Uhlenbeck process is selected for two reasons: (i) the conditional expected value of the modified recovery rate does not explode as time goes to infinity, and (ii) changes in the modified recovery rate process over discrete intervals have a linear structure and are normally distributed. This later property facilitates our subsequent estimation procedure.

Substitution of expression (13) into expression (11) yields our final pricing model:

$$B_t^d = me^{\int_\tau^t r_u du} \left[R_t \frac{\lambda}{(a+\lambda)} + \frac{ba}{(a+\lambda)} \right]. \tag{14}$$

In this expression, the modified recovery rate process $(R_t)_{t \in [\tau,\infty)}$ is unobservable and it needs to be estimated along with the parameters of the process.

To understand this valuation formula, we rewrite it as:

$$B_t^d = m \left[\delta_t \frac{\lambda}{(a+\lambda)} + \frac{ba}{(a+\lambda)} e^{\int_\tau^t r_u du} \right]. \tag{15}$$

Comparing this to the extended distressed debt model in expression (5), we see that the "invested value at time t" of the recovery payment received at time τ, $\left(m\delta_\tau e^{\int_\tau^t r_u du} \right)$ is replaced by $(m\delta_t)$. This difference and the remaining adjustments in expression (15) yields our stochastic extension of expression (5). This extension, in contrast to expression (5), accounts for the information flows across time concerning the resolution of the bankruptcy process and the eventual recovery payment received.

6.2 Estimation Methodology

Our estimation methodology is conceptually a two step procedure. The first step estimates the parameters of the process, and the second step estimates the economic default date.

[13] Note that depending upon the selection of m this process will only differ by a scale factor.

6.2.1 Given τ, Estimating $(\lambda, a, b, \sigma, (\delta_t)_{t\geq\tau})$

We discretize time $t = 0, 1, 2, ..$ to perform the estimation. Time 0 corresponds to the economic default date τ. We can rewrite our model so as to utilize a Kalman filter estimation procedure. For this identification, the observable is the discounted (and normalized) market price of the debt issue, $\left(\frac{B_t^d}{m} e^{-\int_\tau^t r_u du}\right)$. Given expression (14), the measurement equation is

$$\frac{B_t^d}{m} e^{-\int_\tau^t r_u du} = A_t + H_t R_t + \epsilon_t \tag{16}$$

where $\epsilon_t \sim N(0, \rho)$ is assumed to be a serially uncorrelated observation error.

The transition equation relates to the unobservable modified recovery rate. By expression (12) we obtain

$$R_t = C_t + F_t R_{t-1} + \eta_t \tag{17}$$

where $\eta_t \sim N(0, \frac{\sigma^2}{2a}(1 - 2e^{-a})$. The identifications of the coefficients are:

$$\begin{aligned} A_t &\equiv \frac{ba}{(a+\lambda)}, \\ H_t &\equiv \frac{\lambda}{(a+\lambda)}, \\ C_t &\equiv b(1 - e^{-a}), \quad \text{and} \\ F_t &\equiv e^{-a}. \end{aligned}$$

We use a maximum likelihood procedure to estimate this system of equations (for further details see Lin [17]). The Kalman filter estimates the process $(R_t)_{t\geq\tau}$ and the parameters $(\lambda, a, b, \sigma, \rho)$. It is a simple transformation using expression (10) to obtain $(\delta_t)_{t\geq\tau}$.

6.2.2 Given $(\lambda, a, b, \sigma, (\delta_t)_{t\geq\tau})$, Estimating τ

Unfortunately, our Kalman filter estimates for $(\lambda, a, b, \sigma, (\delta_t)_{t\geq\tau}; \rho)$ depend on knowing τ. Indeed, recall that R_t depends on τ. But, τ is unknown and it needs to be jointly estimated as well. To overcome this difficulty, we use an iterative procedure whose limit yields our final estimates.

Step 1. Since the economic default date is unknown, we first fix $\tau = \tau^*$, the recorded default date. We know that the recorded default date is an upper bound on the economic default date. Then, we obtain our estimates $(\lambda, a, b, \sigma, (\delta_t)_{t\geq\tau}; \rho)_1$. Note that our estimates are subscripted by the step in the estimation procedure.

Next, we compute an improved estimate of τ, i.e.

$$\tau_2 = \inf_{\tau^*-180\leq t\leq\tau^*} \{t : B_t \leq B_{\tau^*}^d e^{-\int_t^{\tau^*} r_u du} e^{a(\tau^*-t)} + mb(1 - e^{a(\tau^*-t)})\} \tag{18}$$

where we impose an exogenous lower bound of $\tau^* - 180$ days on the estimator.

The proof of this estimator is contained in the appendix. Note that the first term in this estimator is an extension of that contained in expression (7).

By construction $\tau^* - 180 \leq \tau_2 \leq \tau_1 = \tau^*$.

Step 2. Set $\tau = \tau_2$, then repeat step 1 to obtain $(\lambda, a, b, \sigma, (\delta_t)_{t \geq \tau}; \rho)_2$. Next, compute

$$\tau_3 = \inf_{\tau^* - 180 \leq t \leq \tau_2} \{t : B_t \leq B^d_{\tau_2} e^{-\int_t^{\tau_2} r_u du} e^{a(\tau_2 - t)} + mb(1 - e^{a(\tau_2 - t)})\}$$

where we keep the lower bound as $\tau^* - 180$.

By construction, $\tau^* - 180 \leq \tau_3 \leq \tau_2 \leq \tau_1 = \tau^*$.

Continue this process. After step n, we have $(\lambda, a, b, \sigma, (\delta_t)_{t \geq \tau}; \rho)_n$ and τ_n. By construction, τ_n is a decreasing sequence which is bounded below, hence it has a limit. The limit is our estimate of τ. This τ yields our final estimates $(\lambda, a, b, \sigma, (\delta_t)_{t \geq \tau}; \rho)$. To implement this procedure, we stop iterating when $|\tau_n - \tau_{n-1}| < 1$ (one day).

6.3 Results

This section presents the results of our implementation. In theory, the recovery rate process and its parameters $(\lambda, a, b, \sigma, (\delta_t)_{t \geq \tau}; \rho)$ are uniquely identified by the system (16) and (17) using maximum likelihood estimation. In practice, however, the likelihood function is quite flat, yielding slow convergence and unstable estimates. To understand why this is true, note that in expression (16) both (λ, a) and $(\kappa\lambda, \kappa a)$ where $\kappa > 0$ yield the same debt price. Hence, the parameters (λ, a) are identifiable only up to a scale factor via expression (16).

In theory, expression (17) should be able to resolve this non-uniqueness. In practice, however, this does occur. To see the difficulty, we rewrite expression (17) using a first order approximation for the exponential function:

$$R_t - R_{t-1} \approx a\,(b - R_{t-1}) + \eta_t. \tag{19}$$

One observes that multiplying a by κ on the right side only affects the *change in* R_t, not R_t itself. Hence, the impact on R_t is second order. Furthermore, for κ near one, joint estimation of the error term η_t (and its variance) makes the identification of the scale factor even more difficult.

To overcome this instability, we independently estimated the resolution intensity λ using our bankruptcy data (the estimate of $\lambda = 0.84$ appears in Table 2 for Chapter 11 bankruptcy filings), and only estimated the remaining quantities $(a, b, \sigma, (\delta_t)_{t \geq \tau}; \rho)$ using the Kalman filter. Estimating λ independently uniquely identifies a in expression (16).

We estimated the system for the RFV model ($m = 100$) and report only those parameter estimates below. An exception occurs when we present our results for the recovery rates on the economic default date, where we present the estimates for all three models (RFV, RT, RMV).

6.3.1 The Economic Default Date τ

In the joint estimation of $(a, b, \sigma, (\delta_t)_{t \geq \tau}\,; \rho)$ and τ, our iterative estimation procedure converged in two steps. Figure 9 contains our estimates of τ represented as the number of days before the recorded default date. As evidenced, the economic and recorded default dates differ. 82 of 103 issues have the economic default date before the recorded default date. Only 14 issues have the economic default date equal to the upper bound of 180 days.

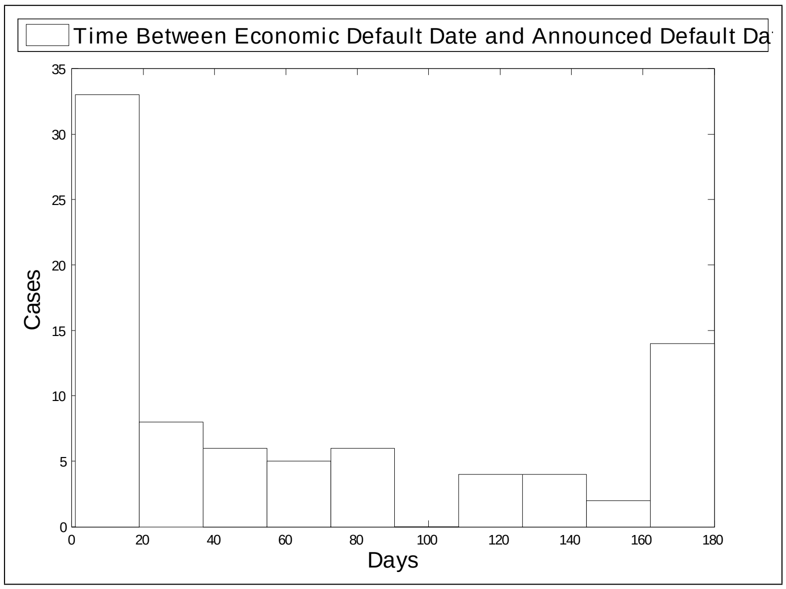

Figure 9: Histogram of the Time Between the Economic and Recorded Default Dates. The number of nonzero differences is 82.

6.3.2 The Parameters (a, b, σ, ρ)

Table 10 contains the parameter estimates for (a, b, σ, ρ). As shown, the average speed of mean reversion is $\overline{a} = 5.03$ and the average long-term modified recovery rate R_t is $\overline{b} = 0.5558$. The average variance of the O-U process and observation error are $\overline{\sigma} = 1.33$ and $\overline{\rho} = 0.009232$, respectively. These estimates appear reasonable. The median, quantiles, min and max are also contained in Table 10. As indicated, the distributions of the estimates for b and ρ across issues are reasonably tight, while the estimates for a and σ exhibit more variation.

N = 21,083	a	b	σ	ρ
Mean	5.03	0.5558	1.33	0.009232
Median	2.04	0.6183	0.23	0.008507
Standard Deviation	14.10	0.2778	5.97	0.001082
First Quartile	0.65	0.3052	0.047	0.0008339
Third Quartile	4.28	0.7966	0.84	0.01319

Table 10: Summary Statistics for the Parameter Estimates of the O-U Process for the Modified Recovery Rate Process in Expression (12).

6.3.3 The Recovery Rate Process $(\delta_t)_{t \geq \tau}$

Figure 10 contains the average RFV recovery rate estimates $(\delta_t)_{t \geq \tau}$ across time, starting at the economic default date and ending one year subsequent. Due to the limitations of our database, we do not have an equal number of issues on each day that we have price quotes. Hence, each daily average in Figure 10 is based on a different numbers of issues. Nonetheless, this graph is still informative. It shows that the average distressed debt issue's recovery rate increases as the bankruptcy process is resolved. This figure documents the large average returns potentially obtained from investing in distressed debt.

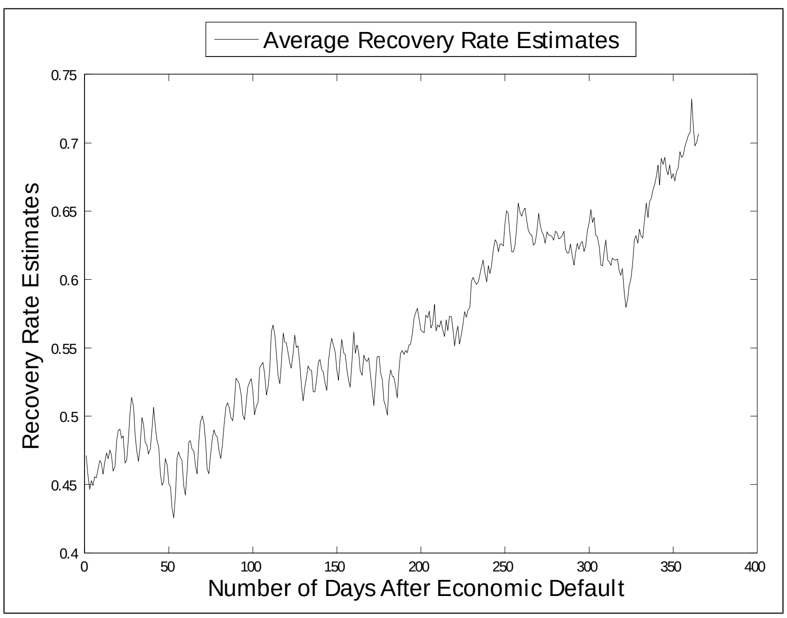

Figure 10: Time Series Graph of the Average Recovery Rates obtained from the Kalman Filter Estimation after the Economic Default Date.

Table 11 contains the economic default date recovery rate estimates (δ_τ)

	RFV		RT		RMV	
N = 69	EM	KFM	EM	KFM	EM	KFM
Mean	0.4814	0.5068*	0.3919	0.4134*	0.8272	0.8868**
Median	0.4006	0.55	0.3204	0.4698	0.9	0.9299
Standard Deviation	0.3029	0.2985	0.2442	0.2423	0.1780	0.1255
First Quartile	0.2	0.23	0.1578	0.1830	0.7267	0.8074
Third Quartile	0.71	0.785	0.6020	0.6150	0.95770	0.9743

Table 11: Summary Statistics for the Different Recovery Value Estimates at the Economic Default Date. EM is the extended model, KFM is the Kalman Filter model. A paired t-test for the difference in means between EM and KFM yields: * t-stat of -1.96 (p-value 0.054) and ** t-stat of -3.12 (p-value of 0.00).

N = 21,083	Pricing Errors
Mean	-0.0018
Median	0.0000
Standard Deviation	0.4611
First Quartile	$-.0124$
Third Quartile	0.0153

Table 12: Summary Statistics for Distressed Debt Pricing Errors in Dollars using the Recovery Rate Model given by Expression (15).

for the three different recovery rate models (RFV, RT, RMV). To obtain the estimates for the RT and RMV models, we perform a scale adjustment to the RFV estimates for δ_τ. Across both the extended and Kalman filter models, 69 common issues have economic default prior to recorded default. We report the summary statistics for these common issues. As indicated, a paired t-test for the difference in means is provided. In all cases the Kalman filter recovery rate estimates are significantly greater. As documented, the RFV estimate using the Kalman filter is 0.5068. This estimate is significantly greater than the 0.4231 RFV estimate thirty days after recorded default from Table 2 (or the weighted average recovery rate of 0.422 across all bonds from 1992 - 2003 as reported in Moody's ([21], Exhibit 9)).

6.3.4 The Pricing Errors $\left(m\epsilon_t \exp\left(\int_\tau^t r_u du\right)\right)$

The pricing errors are contained in Table 12. As shown, the average pricing error across both time and all debt issues is -0.0018 or $-.18$ bp with a standard deviation of 0.4611. These pricing errors are quite small.

To test if the pricing errors have zero mean and are i.i.d., for each bond issue we fit the time series regression $\epsilon_t = \alpha + \beta t$ and test if $\alpha = 0$ and $\beta = 0$. For 83 out of 103 issues, we fail to reject the null hypothesis that $\alpha = 0$ and $\beta = 0$ with significance level 0.01. This evidence is consistent with our pricing errors being unbiased and not time dependent. Last, we also perform a Durbin-

Watson autocorrelation test. For 62 out of the 103 issues, we fail to reject the null hypothesis that $corr(\epsilon_t, \epsilon_{t-1}) = 0$ with significance level 0.01. This is consistent with independence.

On average, our pricing model appears to fit distressed debt market prices well. For those debt issues in the tail of the pricing error distribution, perhaps a generalization of our O-U process would provide a better fit. Such an extension awaits subsequent research.

7 Epilogue

It is informative to consider the four examples that we used to motivate our study and see if our distressed debt pricing model clarifies the path of the market prices around the recorded default date.

Figure 11 shows the debt prices for Delta Airlines Inc. which filed for bankruptcy on September 14, 2005. Our model indicates that the economic default date occurs only 12 days prior to the recorded default date. Visually, this is intuitively plausible. The estimates for the economic default date recovery rates are 0.124 using RFV, 0.08523 using RT, and 0.9185 using RMV.

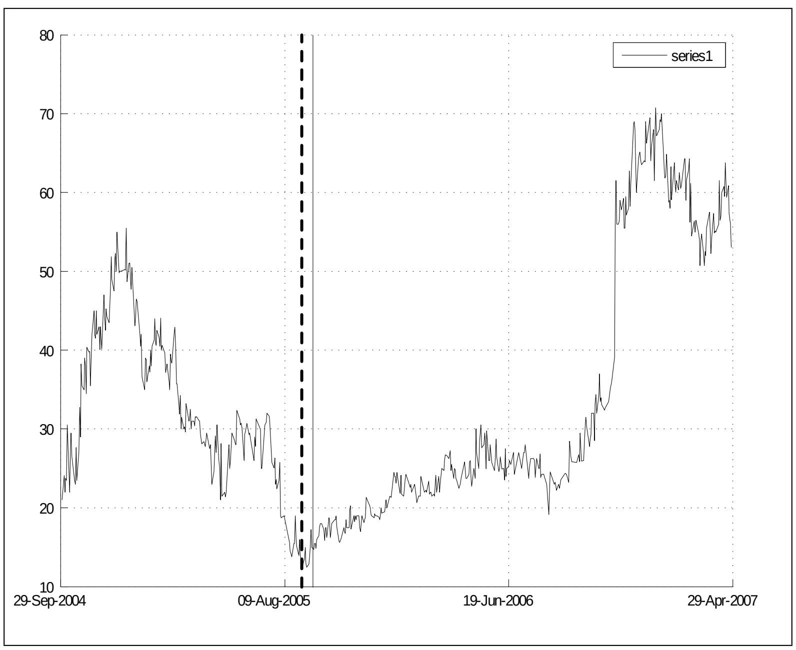

Figure 11: Delta Airlines. Debt prices as a percentage of face value ($100). The dashed line is the economic default date. The solid line is the recorded default date.

Recall that Trico Marine Service Inc.'s recorded default date of December 18, 2004, as represented in Figure 12, didn't appear consistent with the price path. Our model confirms this inconsistency and it identifies the economic default as October 4, 2004, before the positive price movements began. The recovery rate estimates are 0.45 using RFV and 0.3291 using RT. The RMV estimate is not available because the first price observation is the economic default date.

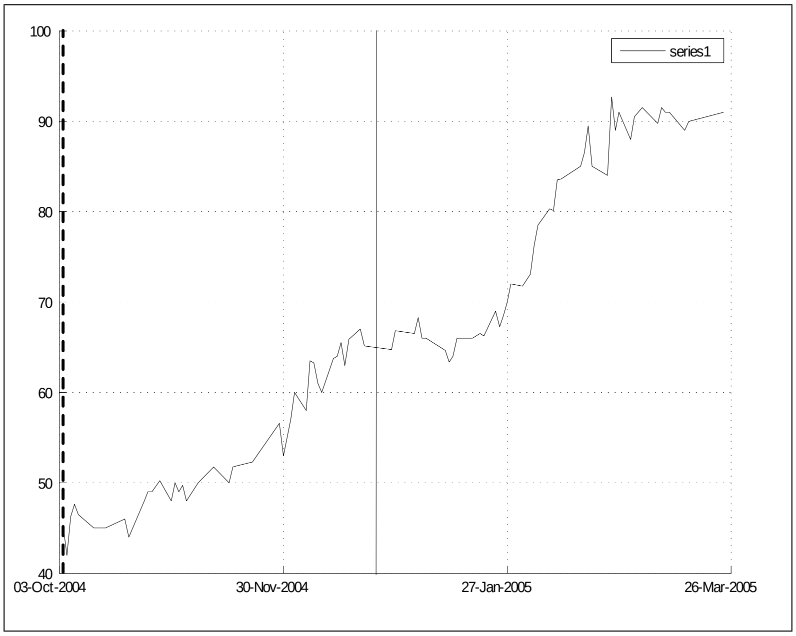

Figure 12: Trico Marine Service Inc. Debt prices as a percentage of face value (\$100). The dashed line is the economic default date. The solid line is the recorded default date.

Winn Dixie Stores Inc. is contained in Figure 13. Winn Dixie Stores Inc. filed for bankruptcy on February 21, 2005. Our model indicates economic default only a few days before on February 16, 2005**.** Again, this is visually plausible. The recovery rate estimates are 0.6036 using RFV, 0.5073 using RT, and 0.9094 using RMV.

Finally, Figure 14 is for Northwest Airlines Inc. which filed for bankruptcy on September 14, 2005. Although its recorded default is the same day as Delta Airlines, our model indicates economic default on May 5, 2004**.** Our estimate suggests that the market recognized default significantly earlier than it was reported, and that information regarding financial resolution got worse before it improved. The recovery rate estimates are 0.605 using RFV, 0.5402 using RT and 0.9378 using RMV.

In all of these cases, the economic default date helps us understand the debt price evolution. In addition, consistent with the reduced form credit risk model, all of the debt prices exhibit a negative jump on the economic default date

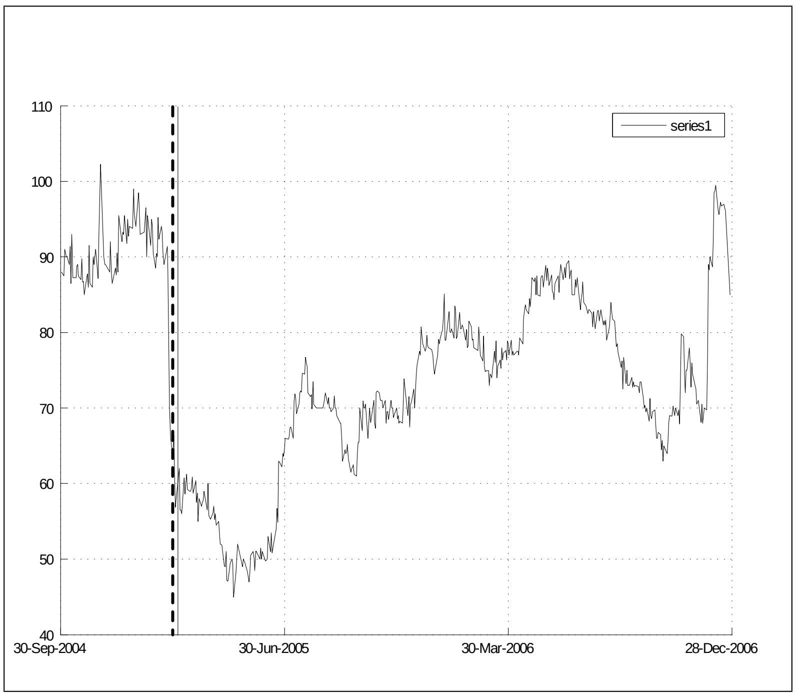

Figure 13: Winn Dixie Stores. Debt prices as a percentage of face value ($100). The dashed line is the economic default date. The solid line is the recorded default date.

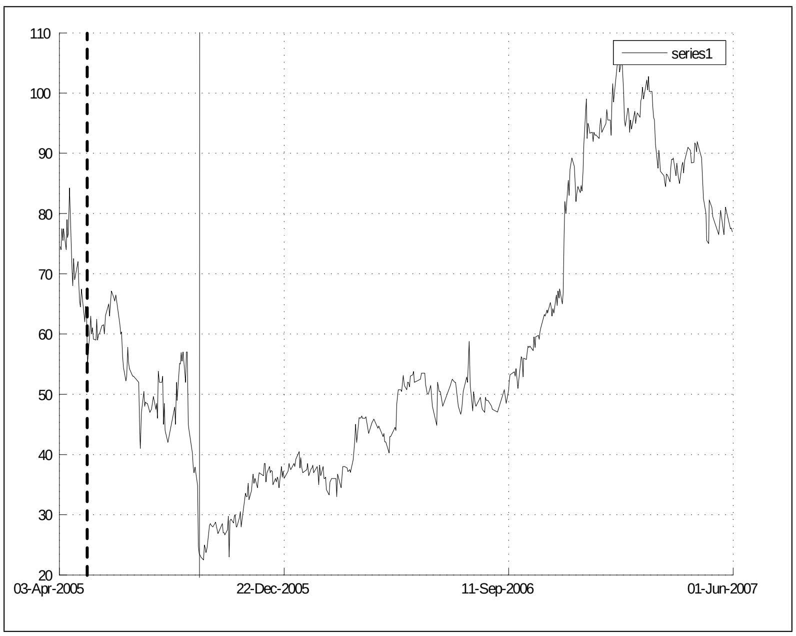

Figure 14: Northwest Airlines. Debt prices as a percentage of face value ($100). The dashed line is the economic default date. The solid line is the recorded default date.

(the RMV estimates are less than one). And finally, these examples illustrate the importance of determining the economic default date for the estimation of recovery rates since they are significantly different from the estimates obtained on the recorded default date.

8 Conclusion

This paper uses distressed debt prices to estimate three recovery rate models: recovery of face value, recovery of Treasury, and recovery of market value. We show that identifying the "economic" default date, as distinct from the recorded default date, is crucial to obtaining unbiased estimates. For most debt issues, the economic default date occurs far in advance of the reported default date. An implication is that the standard industry practice of using 30-day post default prices to compute recovery rate yields biased estimates. This result, unfortunately, reveals that the empirical studies investigating the economic characteristics of industry based recovery rates are using biased data (see Altman, Brady, Resti and Sironi [3], Altman, Resti and Sironi [4], Acharya, Bharath, and Srinivasan [1], Covitz and Han [9], Chava, Stefanescu and Turnbull [8]). It is an open question if the qualitative conclusions of these investigations are affected by this bias.

Second, we construct and estimate a distressed debt pricing model. We use this model to implicitly estimate the parameters of the embedded recovery rate process and to price distressed debt. Our distressed debt pricing model fits market prices well, with an average pricing error of less than one basis point. Our pricing model is simple and future research is needed to generalize our model on multiple dimensions. We hope that this initial investigation motivates continuing research into the estimation of the recovery rate process and the improved pricing of pre-default debt and credit derivatives.

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Appendix

A1. Computation of the Zero-coupon Bond Prices

We used the daily par bond yield curves available from the Federal Reserve bank (see http://www.federalreserve.gov/releases/h15/data.htm) daily from 2000 to 2007. Given are 1-month, 3-month, 6-month, 1-year, 2-year, 3-year, 5-year, 7-year, 10-year, 20-year and 30-year rates on every business day, except from Feb. 2002 to Feb. 2006 where 30-year data is not available. Let the yields be denoted $y(t)$ for $t = 1/12, 1/4, 1/2, 1, 2, 3, 5, 7, 10, 20$ years. $y(t) \in (0, 1)$, a percentage. For maturities $t = 1/12, 1/4, 1/2, 1$ the Treasuries are bills and for $t = 2, 3, 5, 7, 10, 20$ they are Treasury notes and bonds paying coupons.

Let $P(T)$ denote the price of a default free zero coupon bond with maturity at time T (assume that we are standing at time 0). The formulas for $y(T)$ are:

$$P(T) = e^{-y(T) \cdot T} \tag{20}$$

for $T = 1/12, 1/4, 1/2, 1$.

To get the coupon bond formula, we note that

$$100 = \sum_{j=1/2}^{T} \frac{\frac{y(T).100}{2}}{(1 + \frac{y(T)}{2})^{2 \cdot j}} + \frac{100}{(1 + \frac{y(T)}{2})^{2 \cdot T}} \tag{21}$$

for $T = 2, 3, 5, 7, 10, 20$. This formula assumes semi-annual compounding and that the yields are determined by the bond being priced at par. The index of summation is by units of 1/2. In terms of zero coupon bond prices, we have

$$100 = \sum_{j=1/2}^{T} \frac{y(T).100}{2} P(j) + 100 \cdot P(T). \tag{22}$$

Let $a \le s \le b$ be a time interval, and suppose that we are given $f(a)$ and $f(b)$ where these correspond to the continuously compounded forward rates, defined by

$$P(t) = e^{-\int_o^t f(s)ds}. \tag{23}$$

We want to determine $f(s)$. We assume linear interpolation is appropriate for $s \ge 1/12$, thus

$$f(s) = f(a) + \frac{f(b) - f(a)}{b - a} \cdot (s - a). \tag{24}$$

The forward rate curve is assumed to be constant for $0 \le t \le 1/12$, i.e.

$$f(t) = f(1/12) \text{ for } 0 \le t \le 12. \tag{25}$$

From the definition of a forward rate we get

$$P(t) = P(a)e^{-\int_a^t f(s)ds}. \tag{26}$$

Hence,

$$P(t) = e^{-f(1/12)\cdot t} \text{ for } 0 \le t \le 1/12. \tag{27}$$

$$P(t) = P(a)e^{-\int_a^t [f(a) + \frac{f(b)-f(a)}{b-a}\cdot(s-a)]ds} \text{ for } t \ge 12 \text{ and } t \in [a, b]. \tag{28}$$

Algebra gives

$$P(t) = P(a)e^{-f(a)(t-a) - \frac{f(b)-f(a)}{b-a}\cdot\frac{(t-a)^2}{2}} \text{ for } t \ge 12 \text{ and } t \in [a, b]. \tag{29}$$

The recursive solution is:

1. For $t = 1/12$, $f(1/12) = y(1/12)$.

2. For $t = 1/4, 1/2, 1$, use (20) to obtain $P(t)$ for all t. Given $P(t)$ for all t use expression (28) starting from smallest t to the largest t to determine $f(t)$. The last computation gives $f(1)$.

3. For $t = 2, 3, 5, 7, 10, 20$, we start with $P(j)$ for $j = 1/2, 1, ..., a$ (increments by 1/2), and $y(b)$ for b the smallest number in this set of times $> a$. The goal is to compute $f(b)$. Note from the previous recursive step we have $f(a)$.

   To do this we use expression (22) to get

$$\begin{aligned} 100 \quad = \quad & \sum_{j=1/2}^{a} \frac{y(b).100}{2} P(j) \\ & + \sum_{j=a+1/2}^{b-1/2} \frac{y(b).100}{2} P(j) + [100 + \frac{y(b)\cdot 100}{2}]P(b). \end{aligned} \tag{30}$$

   In this equation, use expression (29) for $P(j)$ with $j = a + 1/2, ..., b$. The only unknown is $f(b)$. We solve this equation.

A2. Derivation of Expression (18).

Given $\tau_1 = \tau^*$, our estimates $(\lambda, a, b, \sigma, (\delta_t)_{t \geq \tau}; \rho)_1$ depend on the information $\{(r_t)_{t\geq 0}, (B_t)_{t\geq 0}\}$ and $\{(R_t)_{t\geq \tau_1}\}$. This implies that to get our estimator for τ, we will compute the initial value of the O-U process (12) looking backwards in time, starting from τ_1. Let time t be a candidate for the economic default date. Let $\widehat{E}(\cdot)$ denote our expectation of R_t looking backwards in time, starting at τ_1, using the information sets discussed above. Then, using expression (13) we obtain:

$$\widehat{E}\left(R_t\right) = R_{\tau_1} e^{a(\tau_1 - t)} + b(1 - e^{a(\tau_1 - t)}) \text{ for } t \leq \tau_1. \tag{31}$$

Our distressed debt price, looking backwards in time, is

$$B_t^d = m\left[\widehat{E}\left(R_t\right)\frac{\lambda}{(a+\lambda)} + \frac{ba}{(a+\lambda)}\right] \tag{32}$$

where we substitute our estimator for R_t into expression (14). Recall that time t is a candidate for the economic default date. Note that at time $\tau_1 > t$,

$$\frac{B_{\tau_1}^d}{m} e^{-\int_t^{\tau_1} r_u du} = \left[R_{\tau_1}\frac{\lambda}{(a+\lambda)} + \frac{ba}{(a+\lambda)}\right]. \tag{33}$$

Substitution of expression (31) into expression (32) yields

$$\begin{aligned} B_t^d &= m\left[\left(R_{\tau_1} e^{a(\tau_1 - t)} + b(1 - e^{a(\tau_1 - t)})\right)\frac{\lambda}{(a+\lambda)} + \frac{ba}{(a+\lambda)}\right] \\ &= m\left[\begin{array}{c} e^{a(\tau_1 - t)}\left(R_{\tau_1}\frac{\lambda}{(a+\lambda)} + \frac{ba}{(a+\lambda)}\right) \\ -e^{a(\tau_1 - t)}\frac{ba}{(a+\lambda)} + b(1 - e^{a(\tau_1 - t)})\frac{\lambda}{(a+\lambda)} + \frac{ba}{(a+\lambda)} \end{array}\right] \end{aligned}$$

Substitution of expression (33) into this last expression and simplification yields

$$B_t^d = e^{a(\tau_1 - t)} B_{\tau_1}^d e^{-\int_t^{\tau_1} r_u du} + mb(1 - e^{a(\tau_1 - t)}). \tag{34}$$

Our estimator for τ is the first time t such that B_t (observed market price) is less than B_t^d in expression (34).