Time-Varying Credit Risk and Liquidity Premia in Bond and CDS Markets

Wolfgang Bühler
University of Mannheim
Chair of Finance
D-68131 Mannheim
E-Mail: w.buehler@uni-mannheim.de

Monika Trapp
University of Mannheim
Chair of Finance
D-68131 Mannheim
E-Mail: mtrapp@uni-mannheim.de

March 2008
Time-Varying Credit Risk and Liquidity Premia in Bond and CDS Markets

Abstract

In this paper, we develop a reduced-form credit risk model that incorporates illiquidity in the bond and the credit default swap (CDS) market. As CDS are derivative contracts, the effect of illiquidity on them has to be modeled differently than for bonds. In the bond market, illiquidity results in yield premia. In the CDS market, the bid-ask spread constitutes a liquidity signal. This separation has important consequences on the liquidity spill-over between the bond and the CDS market as well as the co-movement of credit risk and liquidity premia.

Our most important findings are threefold. First, we find that adding a CDS-specific liquidity component to the model has the important consequence of consistently positive credit risk and liquidity premia in bond markets. Moreover, the size of these premia relative to the yield spread is intuitively plausible. Second, our analysis of the time series of the liquidity premia shows that the bond market’s liquidity dries up as default risk increases. For the CDS market, the reaction of the liquidity premia depends on the rating class. The investment grade sector becomes more dominated by protection sellers during times of high default risk, while a lower demand pressure decreases liquidity premia in the subinvestment grade sector. Third, the liquidity of the bond market has a direct impact on the liquidity of the CDS market but not vice versa.

Keywords: yield spread, credit default swap, illiquidity, liquidity spill-over, reduced-form model

JEL classification: G 13, G 14
1 Introduction

Credit derivatives markets provide a standardized alternative to bond markets in taking on and selling off credit risk exposures. This development offers a new approach to one of the most widely explored problems in fixed income analysis - the separation of the corporate bond spread into its credit risk and liquidity component. The corporate bond spread is usually defined as the difference between the bond’s yield to maturity and a given default-free interest rate such as the swap rate or the yield on government bonds of the same maturity. Unarguably, credit risk is one of the spread’s most important determinants, but Elton et al. (2001) and Collin-Dufresne et al. (2001) provide clear empirical evidence that liquidity also has a significant impact. This evidence shows that the separation of the total bond spread into its credit risk and liquidity component constitutes a central question if an issuer’s credit risk has to be quantified. Obviously, the identification of the pure credit risk component is difficult since only the sum of the two risk premia can be observed in the market.

We contribute to the existing literature on the components of bond spreads and credit default swap (CDS) premia theoretically and empirically by exploring the idea that the bid and ask quotes for CDS premia contain information on the liquidity of the CDS market. In the theoretical part of our analysis, we extend the reduced-form credit-risk model by Longstaff et al. (2005) to incorporate illiquidity both in the bond and the CDS market. In the bond market, illiquidity results in price discounts and yield surcharges. This assumption is also made by Longstaff et al. Our extension consists of the modeling of a twofold liquidity effect on CDS premia. First, the bond-specific liquidity has a direct effect on CDS premia since the potentially illiquid bond is delivered under the CDS contract if default occurs. Therefore, the CDS premium in our model accounts for bond liquidity as a source of bond price variation. In addition to this straightforward liquidity spill-over, we include a CDS-specific liquidity which has a more intricate effect. We circumvent the question of systematic liquidity premia in CDS mid premia by modeling the ask and bid premia instead. From these, we infer a theoretical time-varying pure credit risk CDS premium which is unaffected by the CDS-specific liquidity. Our measures of pure liquidity and of the correlation between credit risk and liquidity arise as the difference between this liquidity-free CDS premium and the mid premium. As the credit risk and liquidity premia depend on the state variables, our model allows us to analyze in a consistent way the empirical relationship between time-varying bond- and CDS-specific liquidity premia. To the best of our knowledge, we are the first to explore this dynamic relationship in a model of bond and CDS liquidity. Our results on the behavior of the liquidity premia can be consistently interpreted by demand relations for credit risk between the bond and the CDS market.

In the empirical part of our analysis, we estimate the pure credit risk, the pure liquidity,
and the correlation-induced components of bond spreads and CDS premia for reference entities from a broad range of sectors and rating classes that were observed between June 1, 2001 and June 30, 2007. We then analyze the relation between the time-varying credit risk, liquidity, and correlation premia for the two markets.

Our most important findings are threefold. First, we find that adding a CDS-specific liquidity component to the model has the important consequence of consistently positive credit risk and liquidity premia in corporate bond markets. This result contrasts with those in Longstaff et al. (2005) who also obtain negative liquidity premia in corporate bond yields. In particular, we show that neglecting CDS-specific liquidity can result in negative bond liquidity premia. The average bond liquidity premia for corporate reference entities are of a similar magnitude as the liquidity risk premia which De Jong and Driessen (2005) identify for expected excess bond returns. The CDS liquidity premium is mostly positive which points to a demand pressure in the CDS market which supports the cross-sectional results of Chen et al. (2005), Bongaerts et al. (2007), and Meng and ap Gwilym (2006). Overall, we attribute 60% of the total bond spread to credit risk, 35% to liquidity, and 5% to the correlation between credit risk and liquidity. These results stand in sharp contrast to those of Elton et al. (2001), and Huang and Huang (2003) who report that the non-default component accounts for the largest percentage of the yield spread.

In the CDS market, the credit risk component constitutes 95% of the observed mid premium, the pure liquidity component 4%, and the correlation component 1%. The average liquidity premia increase for reference entities with higher credit risk in both markets. Our results indicate a remarkably higher CDS market liquidity than those of Tang and Yan (2007) who obtain an average liquidity premium of 13.2 basis points in their regression analysis which is similar to the Treasury bond liquidity premium of Longstaff (2004) and the average non-default component of Longstaff et al. (2005).

Second, our model allows us to analyze the relation between credit risk and liquidity premia in the bond and the CDS market. In a time-series analysis of the credit risk, liquidity, and correlation premia, we find that the bond market’s liquidity dries up as the reference entity’s credit risk increases. This empirical result in our reduced-form model setting supports the theoretical prediction of the structural-form model by Ericsson and Renault (2006). They assume that liquidity shocks to the bond holder are correlated with default risk. In the CDS market, the dynamics of the liquidity premia depend on the rating class. The investment grade sector becomes more dominated by protection sellers during times of high default risk. For the subinvestment grade sector, increasing credit risk coincides with a lower demand pressure for credit protection, thus decreasing CDS liquidity premia in the subinvestment grade sector. This analysis complements the cross-sectional evidence by Dunbar (2007) who calibrates a reduced-form model with credit and liquidity risk factors to CDS premia only.
Third, we extend the empirical evidence of Nashikkar et al. (2007) on the relation between the liquidity of the bond and the CDS market by disentangling the credit risk and liquidity premia. Instead of the absolute or relative CDS bid-ask spread which are affected by credit risk, our model allows us to determine comparable pure liquidity premia for the bond and the CDS market. We obtain a significant relationship between these pure liquidity premia in excess of the liquidity spill-over from the bond to the CDS market which is immanent to our model. Specifically, we demonstrate that higher liquidity premia in the bond market lead to decreasing liquidity premia in the CDS market. The relation is particularly pronounced for the subinvestment grade sector, suggesting that the CDS market becomes a more attractive substitute for taking on credit risk synthetically when the bond market is illiquid.

The remainder of the paper is structured as follows. We introduce our reduced-form model in Section 2 and derive the credit risk, liquidity, and correlation premia in Section 3. Section 4 presents the empirical results of the model calibration and a detailed analysis of the estimated time-varying premia. A stability analysis is provided in Section 5. Section 6 summarizes and concludes.

2 The Credit Risk and Liquidity Model

2.1 Specification of the Risk Structure

The first step in the model specification consists of the specification of the underlying risk structure. We assume a standard Duffie and Singleton (1997) framework in which default-free zero coupon bonds, default-risky coupon-bearing bonds and CDS are traded. The liquidity of these instruments can differ, and we choose the default-free zero coupon bonds as “liquidity numéraire” with a liquidity discount factor equal to 1. We thus avoid specifying a perfectly liquid instrument in comparison to which each illiquid instrument trades at a discount.

The default-free term structure of interest rates is driven by one risk factor, the instantaneous default free interest rate process $r(t)$. The credit risk for a specific bond issuer is characterized by a stochastic default-risk hazard rate $\lambda(t)$, which is assumed to be reflected equally in CDS premia and corporate bond prices. The process $\gamma^b(t)$ defines the liquidity intensity in the bond market. This process determines the fraction of a bond’s price due to liquidity deviations from the liquidity numéraire. In the CDS market, we use two liquidity intensities $\gamma^{\text{ask}}(t)$ and $\gamma^{\text{bid}}(t)$ to describe the individual liquidity effects for the CDS ask and bid premia. Conditional on the paths of these intensities,

$$\tilde{D}(t, \tau) = \exp \left( - \int_t^\tau r(s) \, ds \right)$$
is the discount factor for interest rates,
\[ \hat{P}(t, \tau) = \exp \left( - \int_t^\tau \lambda(s) \, ds \right) \]
equals the risk-neutral survival probability and
\[ \hat{L}^l(t, \tau) = \exp \left( - \int_t^\tau \gamma^l(s) \, ds \right) \]
is the liquidity discount factor in the bond market \((l = b)\) and the CDS market \((l = \text{ask, bid})\).

We assume that \(r\) evolves independently from the default and liquidity intensities. The model can easily be generalized to capture correlation effects between \(r\) and the other risk factors. The default intensity \(\lambda\) and liquidity intensities for the bond \((\gamma^b)\), the CDS ask premium \((\gamma^\text{ask})\) and the CDS bid premium \((\gamma^\text{bid})\) are determined by the four latent factors \(x, y^b, y^\text{ask} \) and \(y^\text{bid}\). We model \(x\) as a square root process, \(y^b, y^\text{ask}\) and \(y^\text{bid}\) as arithmetic Brownian motions. The stochastic default and liquidity intensities are described by the following four-factor model:

\[
\begin{pmatrix}
    d\lambda(t) \\
    d\gamma^b(t) \\
    d\gamma^\text{ask}(t) \\
    d\gamma^\text{bid}(t)
\end{pmatrix}
= \begin{pmatrix}
    1 & g_b & g^\text{ask} & g^\text{bid} \\
    f_b & 1 & \omega^b_{\text{ask}} & \omega^b_{\text{bid}} \\
    f^\text{ask} & \omega^b_{\text{ask}} & 1 & \omega^\text{ask}_{\text{bid}} \\
    f^\text{bid} & \omega^b_{\text{bid}} & \omega^\text{ask}_{\text{bid}} & 1
\end{pmatrix}
\begin{pmatrix}
    dx(t) \\
    dy^b(t) \\
    dy^\text{ask}(t) \\
    dy^\text{bid}(t)
\end{pmatrix}
+ \begin{pmatrix}
    \sigma \sqrt{x(t)} dW^x(t) \\
    \eta^b dW^b(t) \\
    \eta^\text{ask} dW^\text{ask}(t) \\
    \eta^\text{bid} dW^\text{bid}(t)
\end{pmatrix}
\]

with parameters \(\alpha, \beta, \mu^l, f_l, g_l, \sigma > 0\) and \(\eta^l > 0\). \(W^x\) and \(W^l\) are independent Brownian motions, \(l \in \{b, \text{ask, bid}\}\). The matrix of the factor sensitivities is assumed to have full rank.

\(f\) and \(g\) determine the correlation between \(\lambda\) and \(\gamma\). If both coefficients equal zero, credit risk and liquidity are uncorrelated. If \(f \neq 0\), credit risk directly affects liquidity, and the reverse applies if \(g \neq 0\). There are two links that determine the correlation between the liquidity intensities. First, there can be an indirect link through the impact of \(x\) via the correlation coefficients \(f\). Second, the coefficients \(\omega_{i,j}\) imply a direct link between the liquidity intensities through the latent risk factors \(y\). Economically speaking, a correlation between the liquidity intensities, which is not directly due to \(x\), allows us to determine the channel through which pure liquidity effects are transmitted from one market into the other.
2.2 Bond Market

We represent the value of a default-risky and potentially illiquid coupon-bearing bond as the expectation under a risk-neutral measure. If default occurs at time $\tau$, the bondholder recovers a fixed fraction $R$ of the face value $F$. Default can occur at any time, and recovery takes place on the first trading day following the default event. Hence, the time-$t$ price $CB(t)$ of a coupon-bearing bond with a fixed coupon $c$ paid at times $t_1, \ldots, t_n$, notional $F$, maturity in $t_n$, and recovery at times $\theta_j$ ($t \leq \theta_1 < \ldots < \theta_N \leq t_n$) is given by

$$CB(t) = c \cdot \sum_{i=1}^{n} D(t, t_i) E_t \left[ \tilde{P}(t, t_i) \tilde{L}^b(t, t_i) \right] + F \cdot D(t, t_n) E_t \left[ \tilde{P}(t, t_n) \tilde{L}^b(t, t_n) \right]$$

$$+ R \cdot F \cdot \sum_{j=1}^{N} D(t, \theta_j) E_t \left[ \Delta \tilde{P}(t, \theta_j) \tilde{L}^b(t, \theta_j) \right].$$ (2)

$E_t$ is the conditional expectation under a risk-neutral measure, and $\theta_0 := t$. Given that $r$ is independent of the other risk factors, we can compute $D(t, \tau) := E_t \left[ \tilde{D}(t, \tau) \right]$ separately from the joint expectation of the default risk factor and the liquidity factor. $\Delta \tilde{P}(t, \theta_j) := \tilde{P}(t, \theta_{j-1}) - \tilde{P}(t, \theta_j)$ denotes the probability of surviving from $t$ until $\theta_{j-1}$ and then defaulting between $\theta_{j-1}$ and $\theta_j$ conditional on the current date $t$. Equation (2) can be interpreted as the expected present value of all future bond cash-flows: the first summand gives the expected present value of the coupon payments at each coupon date. The second summand equals the expected present value of the principal payment in the last period. The last term denotes the expected present value of the recovery rate payment. Therefore, the discount factor for each future payment consists of two terms: the risk-free discount factor $D$, and the joint expectation of the default risk factor $\tilde{P}$ and the liquidity factor $\tilde{L}$.

2.3 CDS Market

A CDS is a bilateral contract which allows two counterparties to trade the credit risk of an underlying reference obligation. In the fixed leg of the swap, the counterparty which buys credit protection agrees to make periodic fee payments over the life of the swap. In return, the counterparty which sells credit protection agrees to make a payment if a “credit event” occurs for the reference obligations. This contingent payment is the floating leg of the swap.

The basic contract form can be pictured as follows. At the inception of the CDS contract, the protection buyer and seller agree on the CDS premium $s$ which the buyer pays to the seller. The premia are quoted annualized and in basis points (bp) per unit of face value of the claim to which the credit protection applies. Premium payments are made in arrears on fixed payment dates; March, June, September and December 20th have evolved as the
standard dates. If a contract is entered into on a non-standard date, the time until the next standard date is added to the quoted maturity of the contract. In case of a credit event before the maturity of the CDS, the contract automatically terminates. The buyer pays the premium accrued since the last payment date to the seller, delivers the bond on which the CDS contract is written and obtains the face value of the defaulted bond. In practice, CDS contracts are written on multiple reference obligations, they can include multiple credit events and allow the protection buyer to choose from an entire delivery basket which asset to deliver upon default or to specify an auction process for cash settlement instead of physical delivery. In our setting, we abstract from these features. The issuer simultaneously defaults on all bonds and this immediately triggers the credit event. All bonds have the same post-default price, making the cheapest-to-deliver option worthless, and settlement occurs immediately upon default.

It is not obvious whether liquidity should be included in a model for CDS premia, and if so, in which way this should be done. After all, a CDS is a derivative, not an asset, and thus not exposed to illiquidity effects caused by a fixed supply or shorting costs. Both in empirical studies and in theoretical models, see e.g. Schueler and Galletto (2003) or Longstaff et al. (2005), it is generally assumed that the CDS mid premium reflects a price which is entirely free of liquidity risk. Undoubtedly, however, the bid and ask premia reflect liquidity aspects of a CDS. From these two quotes, we will extract the unobservable, pure credit risk premium. Typically, this premium will differ from the mid premium. As a consequence, we model two values of the fixed leg of the CDS, one for the ask and one for the bid side.

The value of the fixed leg of a CDS contract at time $t$ with fixed in-arrear premium payment $s_{\text{ask}}$ at times $T_1, \ldots, T_m$, maturity $T_m$ and stochastic settlement times $\theta_j$ ($t \leq \theta_1 < \ldots < \theta_M \leq T_m$) in case of a credit event equals

$$
\text{CDS}_{\text{fix}}(t) = s_{\text{ask}} \left( \sum_{i=1}^{m} D(t, T_i) E_t \left[ \tilde{P}(t, T_i-1) \tilde{L}_{\text{ask}}(t, T_i) \right] \right.
+ \sum_{j=1}^{M} D(t, \theta_j) \delta_j E_t \left[ \Delta \tilde{P}(t, \theta_j) \tilde{L}_{\text{ask}}(t, \theta_j) \right].
$$

In equation (3), $\delta_j$ accounts for the premium fraction accrued in the interval between the last premium payment and the settlement time $\theta_j$. $\tilde{L}_{\text{ask}}$ is defined as $\tilde{L}^b$ with the bond liquidity intensity $\gamma^b$ replaced by the CDS ask liquidity intensity $\gamma^\text{ask}$. Equation (3) reflects that the payment of all ask premia $s_{\text{ask}}$ has to be discounted for the default probability as the payment at time $T_{i-1}$ only occurs with a probability $\tilde{P}(t, T_{i-1})$. The CDS-specific liquidity discount factor for the ask premium $\tilde{L}_{\text{ask}}(t, T_i)$ accounts for the possibility that part of the CDS ask premium is not due to default risk but to the fact that the protection seller demands an additional premium because of illiquidity.
The value of the floating leg, the expected discounted payment of the protection seller upon default is given by

$$
CDS_{\text{float}} (t) = \sum_{j=1}^{M} D(t, \theta_j) E_t \left[ \Delta \hat{P}(t, \theta_j) \right] - R D(t, \theta_j) E_t \left[ \Delta \hat{P}(t, \theta_j) \hat{L}^b(t, \theta_j) \right]. \quad (4)
$$

The first summand in (4) equals the discounted present value of the face value $F$, the second equals the expected discounted present value of the defaulted bond which the protection seller obtains if she sells the delivered bond. Therefore, this second summand contains the discounting factor for the bond liquidity in addition to the credit risk discounting factor. The bond liquidity affects the CDS both in the case of physical delivery and cash settlement. A less liquid bond has a lower post-default price compared to an otherwise identical bond with higher liquidity. The CDS premium is therefore higher in order to compensate the protection seller for the lower value of the bond should default occur. This effect pertains even if the CDS market is perfectly liquid.

From (3) and (4) we obtain

$$
s_{\text{ask}} (t) = \frac{F \sum_i D(t, \theta_j) E_t \left[ \Delta \hat{P}(t, \theta_j) \right]}{\sum_i D(t, \theta_j) E_t \left[ \hat{P}(t, \theta_j) \hat{L}^{\text{ask}}(t, \theta_j) \right] + \sum_j \delta_j D(t, \theta_j) E_t \left[ \Delta \hat{P}(t, \theta_j) \hat{L}^{\text{ask}}(t, \theta_j) \right]}, \quad (5)
$$

and

$$
s_{\text{bid}} (t) = \frac{F \sum_i D(t, \theta_j) E_t \left[ \Delta \hat{P}(t, \theta_j) \right]}{\sum_i D(t, \theta_j) E_t \left[ \hat{P}(t, \theta_j) \hat{L}^{\text{bid}}(t, \theta_j) \right] + \sum_j \delta_j D(t, \theta_j) E_t \left[ \Delta \hat{P}(t, \theta_j) \hat{L}^{\text{bid}}(t, \theta_j) \right]}, \quad (6)
$$

where equations (5) and (6) differ only with regard to the liquidity discount factor. Here, a short remark on the relative size of $\hat{L}^{\text{ask}}$ and $\hat{L}^{\text{bid}}$ is in order. The modeling of $\gamma^{\text{ask}}$ and $\gamma^{\text{bid}}$ in equation (1) does not guarantee that $\hat{L}^{\text{ask}} \leq \hat{L}^{\text{bid}}$ and, therefore, $s^{\text{ask}} \geq s^{\text{bid}}$. In our empirical study, we only obtain estimates that translate into this relationship.

Due to the structure of the factor model in equation (1) and the independence of $x$ and $y^t$, the expected values of $\hat{P}(t, \tau_i) \cdot \hat{L}^t(t, \tau_i)$ and $\hat{P}(t, \tau_i) \cdot \hat{L}^t(t, \tau_i+1)$ in equation (2), (5) and (6) can be represented explicitly by analytical functions which result in an affine term-structure model and which we derive in the appendix. Substituting these functions in equations (2), (5) and (6) yields the analytical solutions for the bond price $CB(t) = CB(t, x, y; f, g)$, for the CDS ask premium $s^{\text{ask}} (t) = s^{\text{ask}} (t, x, y; f, g)$, and for the CDS bid premium $s^{\text{bid}} (t) = s^{\text{bid}} (t, x, y; f, g)$, where $y = (y^b, y^{\text{ask}}, y^{\text{bid}})$, $f = (f_b, f_{\text{ask}}, f_{\text{bid}})$ and $g = (g_b, g_{\text{ask}}, g_{\text{bid}})$.
3 Measures for Credit Risk and Liquidity Premia

The model developed in Section 2.1 to 2.3 allows us to disentangle the total bond spread $bs$ into a pure default risk component $bd$, a pure liquidity component $bl$ and a correlation-induced component $bc$. By an analogous procedure based on bid and ask quotes of CDS premia, we can compute a pure credit risk component $sd$, a pure liquidity component $sl$ and a correlation-induced component $sc$. The rationale for this decomposition is most obvious for the bond. The credit risk premium $bd$ equals the yield spread that would apply if credit risk were the only priced factor (excepting, of course, $r$). In this case, the latent factor $y^b$ is identical to 0, all correlation coefficients $f$ and $g$ become irrelevant, and the credit risk intensity $\lambda$ and the latent factor $x$ coincide. The liquidity premium $bl$ equals the yield spread that would apply if liquidity were the only priced factor, i.e., $x$ is identical to 0 and the latent factor $y^b$ and the liquidity intensity $\gamma^b$ coincide. The correlation premium $bc$ then measures the additional yield spread that is incurred because the credit risk and liquidity intensities $\lambda$ and $\gamma^b$ are correlated.

Assuming a perfectly liquid bond and CDS market, the bond yield spread is directly comparable to the CDS premium if the maturity of both instruments is identical and if, in addition, the bond price equals its face value. The second condition is important to avoid the difficulties discussed by Duffie (1999), and Duffie and Liu (2001) who show that the yield spreads on fixed-coupon corporate bonds cannot be directly compared to CDS premia. Therefore, we define a bond’s pure credit risk premium $bd$ for a given value of $x$ in two steps. First, we assume that $y$ and the correlation coefficients equal 0 and determine the coupon $c_{\text{par}}$ that makes the theoretical bond price in equation (2) equal to par, i.e., $CB(x,0,t;0,0)$ equals $F$ for $c_{\text{par}}$. Second, we compute $bd$ as the yield spread over the risk-free rate for this bond:

$$CB(x,0,t;0,0) = \sum_{i=1}^{m} \frac{c_{\text{par}}}{(bd + y(t,T_i) + 1)(T_i-t)} + \frac{F}{(bd + y(t,T_m+1))(T_m-t)}$$

where $y(t,T_i) = D(t,T_i)^{-\frac{1}{T_i-t}} - 1$ equals the yield to maturity of a default-free zero bond with reference liquidity and maturity $T_i - t$.

The bond liquidity premium $bl$ follows as the premium increase in excess of $bd$ if the impact of the latent liquidity factors $y$ is included but the correlation between the credit risk and liquidity intensities equals 0, i.e., the yield spread increase for $CB(t,x,y;0,0)$. The correlation premium $bc$ then arises naturally as the difference between the total yield spread $bs$ for the bond price $CB(t,x,y;f,g)$ which includes the non-zero correlation coefficients and the sum of the credit risk and the correlation premia, $bd + bl$.

We define the credit risk, liquidity, and correlation components of a CDS analogously to the procedure in the bond market. First, we compute the pure credit risk premium $sd$
by assuming that the liquidity discount factors $L^{\text{ask}}$ or $L^{\text{bid}}$ are equal to 1. Equations (5) and (6) illustrate that in this case, $sd$ is exclusively determined by the default-free interest rates, the default probability, and the bond liquidity.

In a CDS market whose liquidity differs from the liquidity numéraire, the ask and bid premia differ from the pure credit risk premium $sd$. In line with the literature on market microstructure, it would be apparent to select the size of the bid-ask-spread as a measure of illiquidity. This is not an appropriate approach in our context for two reasons. First, a comparison of (5) and (6) shows that the bid-ask-spread is affected by credit risk as well as liquidity since a higher default probability increases the ask premium more strongly than the bid premium. Second, the bid-ask-spread, even if taken relatively to $sd$, is not comparable to our liquidity measure in the bond market. We define the liquidity premium in the CDS market $sl$ by $sl = \frac{1}{2} (s^{\text{ask}}(t,x,y;0,0) + s^{\text{bid}}(t,x,y;0,0)) - sd$; i.e., $sl$ is the difference between the theoretical mid premium for uncorrelated credit risk and liquidity intensities and the pure credit risk premium $sd$. Note that $sl$ allows for a demand related interpretation. If there is a high market pressure from the protection buyers’ side, protection sellers move the ask premium at which they are willing to trade upwards. Since the pure credit risk premium $sd$ remains at its initial value, $sl$ increases. If, on the other hand, the market pressure from the protection sellers’ side is high, protection buyers set lower bid quotes. This results in lower values of $sl$.

Our measure of CDS liquidity is thus consistent with the measure of the bond liquidity premia: if a large number of investors want to sell credit risk by selling bonds — which can be interpreted as buying credit protection — the liquidity premium in the bond market increases and vice versa.

Finally, the CDS correlation premium $sc$ equals the difference between the mid premium that includes the impact of the correlation coefficient and the theoretical, correlation-free mid premium.

\section{Empirical Analysis}

\subsection{Data}

We exclusively focus on data from the Euro area. For the current term structure of the default-free interest rates, we use the estimates provided by the Deutsche Bundesbank on a daily basis. These estimates are determined by means of the Nelson-Siegel-Svensson method from prices of German Government Bonds which represent the benchmark bonds in the Euro area for most maturities.\footnote{As an alternative, we also used the swap curve which is, on average, 10 basis points higher than the Nelson-Siegel-Svensson curve for a time-to-maturity of 5 years. Since the dynamics are almost identical, we only present the results for the German Government Bonds.} From the curve, we compute prices of default-free...
zero-coupon bonds which we assume to have the reference liquidity discount factor of 1. The recovery rate is assumed to equal 40%.

All CDS and bond data is collected from Bloomberg. The daily CDS ask and bid closing premia were made available to us by an international investment bank. We compute the daily closing mid premia for CDS to compare them to the bonds’ yield spreads which are derived from Bloomberg mid yields. The research period runs from June 1, 2001 to June 30, 2007. This period covers 1,548 trading days. We restrict ourselves to using CDS premia with a reference time-to-maturity of 5 years to obtain a sample with homogenous CDS liquidity. According to the time conventions in the CDS market described in Section 2.3, we obtain the true CDS maturities by adding the distance between the quoting day and the next reference date to the quoted time-to-maturity of 5 years.

Bond data are also obtained from Bloomberg. We collect all bond mid prices for reference entities which had at least 2 bonds outstanding at some point-in-time during the observation interval. Furthermore, we drop all firms with fewer than 20 consecutive trading days on which at least two bond prices as well as the bid and ask CDS premium were available.

For each of the remaining firms, we collect the rating history from Bloomberg for the period during which we observe bond prices and CDS premia. Both the Standard&Poor’s (S&P) rating and the Moody’s rating are used and mapped on a numerical scale ranging from 1 to 50 in which 1 corresponds to an “AAA” S&P rating (“Aaa” Moody’s rating). The highest value, 50, corresponds to a “CCC+” S&P rating (“Caa1” Moody’s rating) and is the lowest rating which we observe during the observation interval. If the resulting numerical rating of S&P and Moody’s differs by 2 or more, we take the average of the two ratings. If the rating differs by 1, we choose the S&P rating and ignore the Moody’s rating. If no rating for the company can be found for at least 20 observations on consecutive trading days during which at least two bond prices as well as the bid and ask CDS premium were available, we drop the company from our sample.

The above procedure leaves us with a set of 155 firms from 8 corporate sectors and a numerical rating history that consistently lies between 1 and 50. A detailed overview is given in Table 1.

For ease of exposition, we first compute the average numerical rating of a company for all days during which there are a sufficient number of observations. We then map the numerical value to the S&P rating and use this as the column heading. Table 1 shows that the majority of firms has a time-series average rating in the investment grade segment; only 9 lie in the subinvestment grade range. The largest industry group is the sector “Financials” with a total of 54 companies which are also among the top-rated ones. Overall, Table 1 demonstrates that our sample is skewed towards financial companies and firms in the investment grade segment.
To present the time-series of bond yield spreads and CDS premia, we compute the average yield spread and CDS mid premium for each rating class at every date of our observation interval. First, we identify the rating for a particular reference entity on each day. We then compute the yield spread for each bond of that particular reference entity as the difference between its yield and that of a synthetical default-free bond with identical coupon and time-to-maturity. Next, we interpolate the resulting yield spreads to obtain a time-to-maturity of 5 years. We proceed by taking averages of the obtained yield spreads and the observed CDS mid premia for all firms with an average investment, respectively subinvestment grade rating. The resulting time series are depicted in Figure 1.

Figure 1: Average Bond Yield Spreads and Mid CDS Premia Time Series

The figure depicts the average bond yield spreads and mid CDS premia between June 1, 2001 and June 30, 2007. Average yield spreads are denoted in black, mid CDS premia in grey. The solid line is used to depict the investment grade, the dashed line to depict the subinvestment grade time series.

As we see from Figure 1, the average investment grade bond yield spreads consistently exceed the mid CDS premia. Overall, the mean investment grade yield spread has an average of 89.42 bp with a time-series standard deviation of 23.03 bp. The lowest average investment grade yield spread of 33.54 bp is attained on August 22, 2001; the highest one, which equals 178.86 bp, on February 18, 2002. Investment grade CDS premia fluctuate between 15.86 bp and 143.76 bp with a mean of 45.42 bp and a standard deviation of 27.96 bp. The difference between the two time series is partly due to the fact that the bond yield spreads, which we interpolate to the 5 year time-to-maturity, are not computed from par yields. However, the bond market’s illiquidity most likely has a more pronounced effect.
Yield spreads for the subinvestment grade sector are, as expected, clearly higher with an average of 369.62 bp and a time-series standard deviation of 188.70 bp. Overall, the subinvestment grade yield spread fluctuates between 87.60 bp and 1,320.33 bp, which are attained on March 1, 2005 and on October 9, 2002, respectively. While the average subinvestment grade CDS mid premia fluctuate above and below the yield spread with a standard deviation of 224.53 bp, the time-series average of 341.33 bp lies below that of the yield spread. Again, we suppose that the yield spreads are higher because of bond illiquidity.

4.2 Estimation Procedure

To ensure that the 9 parameters \((\alpha, \beta, \sigma, \mu^b, \eta^b, \mu^{ask}, \eta^{ask}, \mu^{bid}, \eta^{bid})\) of the 4 intensity processes, the starting values \((\lambda, \gamma^b, \gamma^{ask}, \gamma^{bid}) (t), t = 1, \ldots, 1548,\) and the correlation coefficients can be identified, we demand that the instantaneous default and liquidity intensity are equal for all bonds of the same issuer with identical seniority, but different time-to-maturity and coupon rate.\(^2\) This identification assumption renders our parameters issuer-specific.

The estimation procedure consists of two steps. Initially, we set all intensity correlation coefficients to 0. This corresponds to the case of independent credit risk and liquidity intensities. In the first step, we then choose a value for the process parameters \((\alpha, \beta, \sigma, \mu^b, \eta^b, \mu^{ask}, \eta^{ask}, \mu^{bid}, \eta^{bid})\) and determine the values \((\lambda, \gamma^b, \gamma^{ask}, \gamma^{bid}) (t), t = 1, \ldots, 1548,\) which minimize our objective function, the squared sum of the deviation between the observed and the theoretical CDS premia and bond yield spread time series at the basis point level. We repeat this procedure of iterating across process parameters and determining an optimal associated time-series of the intensities until we arrive at a minimum of the objective function in the process parameter set. In the second step, we estimate the correlation of the estimated intensity time series for the discretized version of equation (1). This provides us with the new starting point for the correlation coefficients. We then return to the first step and repeat the estimation procedure until our estimates of the correlation coefficients change by less than 0.01 in two steps.\(^3\)

Having determined the estimates of the process parameters, the intensities and the correlation coefficients, we subsequently compute the credit risk, liquidity, and correlation premia for bonds and CDS as explained in Section 3.

\(^2\)It is intuitive that bonds of the same seniority have the same default probability during the next infinitesimally small time interval. The liquidity discount, on the other hand, may well depend on the time-to-maturity and the coupon of a bond. We expect the homogeneity of the bonds of the same issuer to limit the differences. In addition, the functional form of the stochastic liquidity process results in larger liquidity premia for bonds with a longer time-to-maturity.

\(^3\)Convergence in the correlation coefficients is usually achieved in less than 10 iteration steps.
4.3 Credit Risk and Liquidity Premia: Cross-Sectional Results

Our estimates for the correlation coefficients imply that the credit risk intensity has an impact on both the bond liquidity intensity and the CDS liquidity intensities. The latent factor $x$ affects the bond liquidity intensity $\gamma^b$ significantly at the 1% significance level for 140 out of 155 firms. 138 of the significant correlation coefficients are positive with a mean value of 17.39%. The 2 negative correlation coefficients are obtained for one utility and one financial reference entity which has an AAA, respectively an AA, rating. The positive correlation suggests that the liquidity of the bond market dries up as credit risk increases; we quantify the premium impact below. The impact of $x$ on the CDS ask intensity $\gamma^{\text{ask}}$ is significant for 138, and positive for 137 firms with a mean of 26.22%. The CDS bid intensity $\gamma^{\text{bid}}$, in turn, is significantly affected by $x$ for only 66 firms with a coefficient which is negative for 37 firms. The mean values for the positive and the negative coefficient estimates are very similar at -20.73% and 20.32%, respectively. The impact of the latent factors $y^b$, $y^{\text{ask}}$ and $y^{\text{bid}}$ on the default intensity $\lambda$, on the other hand, is almost negligible: we obtain only one significant coefficient estimate for $y^b$, three for $y^{\text{ask}}$ – out of which two are positive – and two for $y^{\text{bid}}$ with a positive and a negative one. These results exemplify that credit risk premia increase liquidity premia in the bond market but not vice versa. We can also conclude that higher credit risk leads to a higher distance between the pure credit risk CDS premium and the ask premium. CDS bid premia, on the other hand, are not as unilaterally affected. We display the premium components in Table 2.

Table 2 demonstrates that the credit risk, liquidity, and correlation premia increase as the rating deteriorates. As to the AAA rating class, the pure credit risk premium in yield spreads $bd$ has an average of 6.11 bp which approximately doubles for each rating downgrade in the investment grade segment. The subinvestment grade segment exhibits values of $bd$ which are at least five times as large. Concerning the liquidity premia $bl$, the increase from the investment to the subinvestment grade segment is less steep, although we still obtain strictly positive estimates for $bl$. In addition, the minimum of $bl$ does not monotonously increase for a decreasing rating. The average correlation premia $bc$ increase in the rating up to the CCC rating class and are strictly positive except for the firms for which the estimate of $f_b$ is negative. Because of these firms, we obtain a negative average of $bc$ for the AAA rating class. On average, $bd$ accounts for 60% of the total yield spread, $bl$ for 35% and $bc$ for 5%. Comparing this decomposition with the results of Elton et al. (2001), we believe our results to be more plausible.

The CDS credit risk premia $sd$ consistently exceed $bd$ but the difference caused by the bond liquidity premia is limited. The minimal difference is attained for the AAA rating class with 0.07 bp and the maximal one for the B class with 4.97 bp. This is consistent with the increasing average level of the bond liquidity premium $bl$. 

14
The most noteworthy results of Table 2 concern the CDS pure liquidity premia $sl$ and the correlation premia $sc$. As explained in Section 3, we measure the liquidity of the CDS market by the asymmetry between the ask and the bid quotes relative to the pure credit risk premium. On average, the liquidity premium $sl$ is positive, which suggests that the CDS market is mainly ask-driven. The asymmetry increases with regard to the level of the premia as the rating deteriorates, but relative to the pure credit risk premia, the pure liquidity premia are smaller for the subinvestment grade sector. The difference is particularly pronounced for the gap between the investment and the subinvestment grade sector. In addition, we find that 19.24% of the liquidity premia in the subinvestment grade sector are negative. We take this result as an indication that the protection sellers in the subinvestment grade sector are on average less dominant than the protection buyers and, in fact, protection buyers are more dominant during certain market phases.

From an economic point of view, investors who have to maintain a given default risk level threshold or face a value-at-risk constraint demand more credit protection through CDS if a reference entity is closer to the subinvestment grade barrier. Since the liquidity of the bond market is on average lower than that of the CDS market, selling the bond is associated with a loss which can be avoided by buying the CDS. For the subinvestment grade sector, these constrained investors do not increase the demand pressure. In addition, the low liquidity of the bond market makes it more attractive to take on credit risk synthetically as a protection seller. Therefore, the demand pressure decreases and the supply pressure increases. In the next section, we will explore further how credit risk, liquidity, and correlation premia behave during times of high and low credit risk.

The average of the correlation premium $sc$ is almost negligible for the investment grade sector and grows in excess of factor 10 for the subinvestment grade sector. We conclude that while the liquidity premia in the subinvestment grade sector are low relative to the pure credit risk premia, changes in credit risk have a high impact on the correlation premium which is part of the total liquidity premium. The negative minima are due to the fact that for some firms, the correlation of both the bid and the ask liquidity with the default intensity are positive. Therefore, the CDS bid premium can at times increase more strongly than the ask premium. However, we will explore the dynamic relation between the pure credit risk, the pure liquidity and the correlation premia in greater detail in the following section. Concerning the decomposition of the total CDS premia, we observe that on average 95% of the total premium is due to $sd$, 4% to $sl$ and 1% to $sc$.

### 4.4 Credit Risk and Liquidity Premia: Time-Series Results

In this section, we first present the time series of the premia estimates for the investment and subinvestment grade sector. Next, we explore the relation between the premia across the bond and the CDS market. The section concludes with an empirical analysis of the
impact of market-wide credit and liquidity measures on the dynamics of the estimated premia time series and the comparison of these dynamics during periods with increasing and decreasing credit risk.

The estimated credit risk, liquidity, and correlation premia are depicted in Figure 2. For ease of presentation, the investment and subinvestment grade rating classes are each summarized into an individual time series.

Panels A and B of Figure 2 show that the pure credit risk premia both in yield spreads and CDS premia can hardly be distinguished both for the investment grade and the subinvestment grade sector. For the investment grade sector, there are two distinct spikes in late 2001 and late 2002 at the Enron and WorldCom defaults. The reaction of the subinvestment grade sector to the Enron default is almost negligible which may be due to the fact that our sample only consists of 2 subinvestment grade firms between June 2001 and February 2002. Overall, we observe a flattening of the pure credit risk premia time series of $bd$ and $sd$ over time with much lower average levels at the end of the observation interval. Since the premia are computed with regard to par instruments with a constant time-to-maturity of 5 years, we can directly attribute the lower premium level to a lower amount of risk.

We observe from Panel A of Figure 2 that the bond liquidity premia $bl$ exhibit a different behavior across the rating classes. During the high credit risk periods, the liquidity premia are volatile and flatten out at a higher level during the latter part of the observation interval for the investment grade sector. In the subinvestment grade sector, $bl$ is highest shortly after the high-risk periods and decreases to a lower level towards the end of the observation interval. Visual inspection of the CDS-specific liquidity premia $sl$ in Panel B of Figure 2 is more difficult since the absolute values are small. For both rating sectors, we observe a trend towards 0 as the CDS market matures. Overall, the level of $sl$ is close to 0 for the entire observation interval in the investment grade sector, but liquidity premia are higher when credit risk is high. In the subinvestment grade sector, $sl$ strongly fluctuates and becomes mostly negative when credit risk is high. This finding suggests that the demand pressure which characterizes the CDS market for investment grade debt is replaced with a supply pressure for the subinvestment grade sector.

Due to their definition and the insignificant estimates for $g$, the correlation premia are closely associated with the credit risk premia. On comparing $bl$ and $bc$, Panel A of Figure 2 shows that the pure liquidity premia lie below the correlation premia during high-risk periods and above during low-risk periods for the investment grade sector. In the subinvestment grade sector, we observe a similar result during high risk periods, e.g., at the WorldCom default in late 2002 where the correlation premia are higher than the pure liquidity premia. Overall, however, $bl$ tends to be higher than $bc$ in the lower rating
Figure 2: Estimated Credit Risk, Liquidity, and Correlation Premia Time Series

The figure depicts the estimated credit risk, liquidity, and correlation components in CDS premia and yield spreads for all rating classes. The estimates are computed with regard to a constant time-to-maturity of 5 years and a synthetical par bond.

Panel A: Bond Premia

Investment Grade Premia Bond

Subinvestment Grade Premia Bond
Panel B: CDS Premia

Investment Grade Premia CDS

Subinvestment Grade Premia CDS
classes. We interpret this as an indication that liquidity may dry up disproportionately in high credit risk phases, in particular for the investment grade sector. This agrees with the flight to quality and the flight to liquidity effects which are documented empirically by Beber et al. (2007) and the theoretical results of Vayanos (2004). The CDS correlation premia are, in contrast, almost negligible, i.e. the CDS liquidity is mostly idiosyncratic.

In order to study the dynamic interaction between the bond and the CDS market, we perform a vector autoregressive (VAR) analysis. Since the credit risk premia are computed with regard to the identical default intensity, bd and sd contain identical information except for the effect of bond liquidity on sd. The same holds true for bd and bc since the estimates of the impact of the bond liquidity intensity on the default intensity do not differ significantly from zero for almost all firms. We therefore focus on the pairwise relation between the credit risk, the liquidity, and the correlation premia across the two markets.

The augmented Dickey-Fuller test cannot reject the hypothesis of a unit root in any of the premia for 147 firms. The Johansen procedure cannot reject cointegration of the liquidity premia across the two markets for 143 reference entities. Cointegration of the credit risk premia and the correlation premia cannot be rejected for any of the 147 firms. The VAR-specifications which we use are of the form:

\[
\begin{align*}
\Delta bd_t & = \sum_{j=1}^{5} x_{1j} \Delta bd_{t-j} + \sum_{j=1}^{5} x_{2j} bd_{t-j} + \sum_{j=1}^{5} x_{3j} \Delta sd_{t-j} + \sum_{j=1}^{5} x_{4j} sd_{t-j} + \varepsilon_{1,t}, \\
\Delta sd_t & = \sum_{j=1}^{5} y_{1j} \Delta bd_{t-j} + \sum_{j=1}^{5} y_{2j} bd_{t-j} + \sum_{j=1}^{5} y_{3j} \Delta sd_{t-j} + \sum_{j=1}^{5} y_{4j} sd_{t-j} + \varepsilon_{2,t}, \\
\Delta bl_t & = \sum_{j=1}^{5} x_{1j} \Delta bl_{t-j} + \sum_{j=1}^{5} x_{2j} bl_{t-j} + \sum_{j=1}^{5} x_{3j} \Delta sl_{t-j} + \sum_{j=1}^{5} x_{4j} sl_{t-j} + \varepsilon_{3,t} \\
\Delta sl_t & = \sum_{j=1}^{5} y_{1j} \Delta bl_{t-j} + \sum_{j=1}^{5} y_{2j} bl_{t-j} + \sum_{j=1}^{5} y_{3j} \Delta sl_{t-j} + \sum_{j=1}^{5} y_{4j} sl_{t-j} + \varepsilon_{4,t}, \\
\Delta bc_t & = \sum_{j=1}^{5} x_{1j} \Delta bc_{t-j} + \sum_{j=1}^{5} x_{2j} bc_{t-j} + \sum_{j=1}^{5} x_{3j} \Delta sc_{t-j} + \sum_{j=1}^{5} x_{4j} sc_{t-j} + \varepsilon_{5,t}, \\
\Delta sc_t & = \sum_{j=1}^{5} y_{1j} \Delta bc_{t-j} + \sum_{j=1}^{5} y_{2j} bc_{t-j} + \sum_{j=1}^{5} y_{3j} \Delta sc_{t-j} + \sum_{j=1}^{5} y_{4j} sc_{t-j} + \varepsilon_{6,t},
\end{align*}
\]

where the coefficients \(x_{ij}\) and \(y_{ij}\) can differ between equations (7), (8) and (9). We demand that the parameters are identical for all, all investment grade or all subinvestment grade firms, respectively. Time lags up to degree 5 are considered to capture a weekly time interval, and the resulting parameter estimates are transformed into a single estimate using the approach of Fowler and Rorke (1983). The results of the estimation are displayed in Table 3.
As the coefficient estimates in Panel A of Table 3 indicate, $bd$ and $sd$ are mean-reverting and exhibit an overall downwards trend. The mean reversion and the downwards trend of $bd$ are stronger than those of $sd$. The sensitivity of the bond credit risk premia to changes in the CDS credit risk premia is also higher, and we attribute both these effects to the impact of the bond liquidity on $sd$. The adjusted $R^2$ of 9.95% for $bd$ and 9.02% for $sd$ is low, suggesting that the mean reversion and the trend in addition to the cross-market impact only explain a low amount of variation.

The bond and CDS liquidity premia $bl$ and $sl$ are mean reverting and have a downwards trend which is stronger for the CDS premia with a coefficient estimate of -0.31. This is evidence that CDS liquidity premia decrease as the market matures. In addition, $sl$ reacts to changes in $bl$, while the negative sign of the coefficient estimate suggests that liquidity moves in opposite directions. Even though the estimate is relatively small at -0.02, it is economically significant since bond liquidity premia are higher than CDS liquidity premia. Reversely, CDS liquidity does not seem to affect the bond market’s liquidity. The adjusted $R^2$ is about twice as high for the bond liquidity at 19.18% and three times as high for CDS premia with 27.00%, suggesting that liquidity premia are more adequately described in the VAR model than credit risk premia and that there is a significant interdependence between the markets’ liquidity.

The bond and CDS correlation premia $bc$ and $sc$ are also mean reverting and exhibit a downwards trend. Their behavior is similar to that of the credit risk premia. For the bond correlation premia, this is due to the fact that the $x$ affects $\gamma^b$ but not vice versa. If we were to model the intensities independently, $bc$ would be subsumed in $bl$. As a result, we would overestimate the pure liquidity component. The CDS correlation premium $sc$, on the other hand, is different with the VAR coefficients being close to 0 and the impact of $bc$ on $sc$ being significant at the 5% level only. The adjusted $R^2$ reflects this time-series behavior as well; for $bc$, the adjusted $R^2$ of 8.81% is close to that of $bd$, while the value of 0.97% for $sc$ is very small.

As discussed above, the time-series behavior of the premia differs between the investment and the subinvestment grade sector. The results of the VAR analysis for the investment grade sector in Panel B of Table 3 show that the dynamics of the premia are the same as in Panel A but that the size of the coefficients and the explanatory power decrease. $bd$ and $sd$ remain mean-reverting with a downwards trend, but the level of the premia in one market does not affect the premia changes in the other market significantly any more. The adjusted $R^2$ decreases by approximately 2 percentage points for both $\Delta bd$ and $\Delta sd$. The CDS liquidity premia also become less dependent on the bond liquidity premia; the coefficient estimate for the impact of $\Delta bl$ on $\Delta sl$ decreases to -0.01 and is only significant at the 10% level. As a result, the adjusted $R^2$ decreases to 25.35% while it remains virtually unaffected for $\Delta bl$. The correlation premia show an almost identical behavior in the investment grade sector as they do for the entire sample, only the adjusted
$R^2$ is slightly higher. Overall, the investment grade sector exhibits a lower connection between the premia in the bond and the CDS market. These findings suggest that the premia for investment grade firms are affected by market-specific conditions in excess of the firm-specific ones. We will further explore this possibility below.

Panel C of Table 3 shows the coefficient estimates for the subinvestment grade sector. For the credit risk premia, the coefficients and the explanatory power are more than double the size of the coefficients and the explanatory power we find for the investment grade sector. The sign of the coefficient for the impact of $\Delta bl$ on $\Delta sl$ becomes negative and the estimate itself is large at -0.08. Due to lower bond market liquidity, the CDS market becomes a more attractive substitute for taking on credit risk. In addition, the level of $bl$ negatively affects $\Delta sl$ with a coefficient estimate of -0.02, further strengthening this result. The explanatory power for $sl$ also increases to 30.87%. Changes in $sl$ have a very slight reverse impact on $\Delta bl$, but both the economic and the statistical significance of the coefficient estimate of -0.01 are limited. The correlation premia become more closely interconnected, but the explanatory power decreases for $bc$ and increases for $sc$. In comparison to the investment grade sector, the higher coefficient estimates and the higher adjusted $R^2$ imply that the bond and the CDS market for the subinvestment grade sector are more closely interconnected.

We now determine whether the time series of the credit risk, liquidity, and correlation premia are mostly driven by firm- and instrument-specific changes or whether aggregate market conditions have an additional impact. We therefore analyze the effect of market-wide credit risk and liquidity measures on the VAR dynamics. Our previous findings suggest that the impact of these measures is higher for the investment grade sector.

As a proxy for credit risk, we choose the S&P Global Corporate Bond Indices for which yield spreads are available on Bloomberg for all rating classes between AAA and B with a constant time-to-maturity of 5 years. These indices have two major drawbacks for our analysis: they are quoted in US$ and have been discontinued from May 1, 2007. Nevertheless, the 307 weekly yield observations for each index between June 1, 2001 and May 1, 2007 provide information regarding global credit risk for different rating classes.

Liquidity is proxied by the European Central Bank (ECB) financial market liquidity indicator for which daily values were made available to us by the ECB. The indicator is designed to mirror dynamic patterns in the overall liquidity of the Euro area financial market and combines information from the stock, the bond and the equity options market as well as European interest rate data. A higher value marks higher aggregate financial market liquidity.

Since the estimated premia are not stationary, we estimate a VAR similar to the one in equations (7) to (9) with the credit risk, liquidity, and correlation premia as endogenous

---

4For a detailed description of the indicator, see European Central Bank (2007).
variables and the S&P index yield spread and the liquidity indicator as exogenous variables. The results are displayed in Table 4.

As Table 4 shows, the inclusion of the aggregate credit risk and liquidity measures does not affect the dynamics of the firm-specific credit risk premia. Both $bd$ and $sd$ depend positively on market-wide credit risk and negatively on liquidity, but the increase of the adjusted $R^2$ from 9.95% to 10.91% shows that the explanatory power of the market-wide measures is small. The impact is stronger for the investment grade sector: the adjusted $R^2$ almost doubles from 7.67% to 12.65% for bonds and from 6.80% to 11.81% for CDS. For the subinvestment grade sector, the coefficient estimates are either insignificant or only significant at the 10% level. Clearly, credit risk premia in the subinvestment grade sector depend almost completely on the reference entity’s idiosyncratic default risk.

The impact of the market-wide measures on the liquidity premia is even higher than on the credit risk premia. We obtain a positive dependence on credit risk and a negative one on the liquidity measure, and the adjusted $R^2$ for $bl$ increases by almost 10 percentage points for the entire sample. For the investment grade sector, $bl$ and $sl$ both react positively to increases of credit risk and negatively to increases of liquidity, and the effect on the bond liquidity premium is more pronounced. We partly attribute this to the fact that the CDS market is, on average, rather liquid, and partly to the increasing overall liquidity in the CDS market throughout the observation interval. In the subinvestment grade sector, on the other hand, $bl$ reacts with strong increases to increases in aggregate credit risk and with very slight decreases to increases in overall market liquidity. For CDS liquidity premia, we observe a negative dependence on market-wide credit risk and a positive one on the market-wide liquidity increases. Comparing these results with our findings in Figure 2, the signs of the coefficients agree with the hypothesis that the market for CDS on subinvestment grade firms becomes less dominated by protection sellers if credit risk increases as taking on credit risk synthetically becomes more attractive. In addition, the subinvestment grade CDS market becomes more liquid during times of low overall liquidity which agrees with the flight-to-liquidity effect described by Longstaff (2004).

For the correlation premia, the impact of the aggregate market measures is almost negligible. While $bc$ is significantly affected by the credit risk and liquidity measures at the 10% level only, $sc$ is entirely unaffected. This reveals the correlation premia to be a pure measure of the firm-specific credit and liquidity risk.

To conclude the empirical analysis, we explore how the relation between the premia across the two markets is affected by increasing and decreasing credit risk. We measure the integration of the premia by the cointegration coefficient and the speed of adjustment by the coefficient of the error correction term in a vector error correction model (VECM).
We rewrite equations (7) to (9) in the following form:

\[ \Delta bd_t = x_1 \cdot (bd_{t-5} + \rho sd_{t-5}) + \sum_{j=1}^{5} x_{2j} \cdot \Delta bd_{t-j} + \sum_{j=1}^{5} x_{3j} \cdot \Delta sd_{t-j} + \varepsilon_{1,t}, \]

\[ \Delta sd_t = y_1 \cdot (bd_{t-5} + \rho sd_{t-5}) + \sum_{j=1}^{5} y_{2j} \cdot \Delta bd_{t-j} + \sum_{j=1}^{5} y_{3j} \cdot \Delta sd_{t-j} + \varepsilon_{2,t}, \]

\[ \Delta bl_t = x_1 \cdot (bl_{t-5} + \rho sl_{t-5}) + \sum_{j=1}^{5} x_{2j} \cdot \Delta bl_{t-j} + \sum_{j=1}^{5} x_{3j} \cdot \Delta sl_{t-j} + \varepsilon_{3,t}, \]

\[ \Delta sl_t = y_1 \cdot (bl_{t-5} + \rho sl_{t-5}) + \sum_{j=1}^{5} y_{2j} \cdot \Delta bl_{t-j} + \sum_{j=1}^{5} y_{3j} \cdot \Delta sl_{t-j} + \varepsilon_{4,t}, \]

\[ \Delta bc_t = x_1 \cdot (bc_{t-5} + \rho sc_{t-j}) + \sum_{j=1}^{5} x_{2j} \cdot \Delta bc_{t-j} + \sum_{j=1}^{5} x_{3j} \cdot \Delta sc_{t-j} + \varepsilon_{5,t}, \]

\[ \Delta sc_t = y_1 \cdot (bc_{t-5} + \rho sc_{t-j}) + \sum_{j=1}^{5} y_{2j} \cdot \Delta bc_{t-j} + \sum_{j=1}^{5} y_{3j} \cdot \Delta sc_{t-j} + \varepsilon_{6,t}, \]

where \( \rho \) is the cointegration coefficient and \( x_1 \) and \( y_1 \) are the coefficients of the error correction term. As above, \( \rho, x_i \) and \( y_i \) can differ between equations (10), (11) and (12), but \( \rho \) is identical for both equations in (10), (11) and (12). Lags of up to 5 days are considered. We estimate the above equations for increasing and decreasing risk phases. Increasing risk phases are defined as time intervals with four consecutive weekly increases in the S&P yield spread for the rating class to which a company belonged during that interval. Decreasing risk phases are analogously defined as intervals with four consecutive weekly decreases. Overall, we obtain 21 four-week intervals with increasing and 17 with decreasing risk for which we perform a VECM analysis of the premia at the daily level. As above, we demand that the coefficient estimates are identical for all firms, respectively all investment grade or all subinvestment grade firms, during the increasing, respectively decreasing, risk phases. The results of the estimation are given in Table 5.

Table 5 illustrates that the relation between \( bd \) and \( sd \) is stable across the increasing and decreasing risk phases. The cointegration coefficient estimate is close to -1 which shows that credit risk premia in both markets move jointly in spite of the impact of bond liquidity on \( sd \). The coefficient estimates for the error correction term, on the other hand, are higher during phases of increasing credit risk. This result implies that the effect of the

---

5We can directly transform equations (7) to (9) into equations (10) to (12). For convenience, we only analyze the cointegration vector and the error correction term coefficient since the focus lies on the difference between cointegration and efficiency in different market conditions.

6Alternatively, we have used the JP Morgan Aggregate Index Europe asset swap rates and the ICMA European Corporate Bond All Maturities Yield Index for which Bloomberg provides daily values to define the increasing and the decreasing risk phases. The results were virtually identical.
bond liquidity on sd becomes less important for deteriorating market conditions which is further supported by the higher adjusted $R^2$ for periods with increasing risk.

Contrary to the credit risk premia, the connection between bl and sl differs across periods with increasing and decreasing risk. During increasing risk phases, the cointegration coefficient is positive, hence the liquidity premia tend to move in opposite directions. For the subinvestment grade sector this also applies when credit risk decreases. Investment grade liquidity premia, on the other hand, move in the same direction when credit risk decreases, which agrees with our graphical results in Figure 2. Comparing the error correction coefficients, we observe that bl is affected more strongly in decreasing and sl in increasing risk phases. For sl, this is true both for the investment grade and the subinvestment grade sector but bl is not significantly affected by the error correction term in the investment grade sector. This is further evidence that the liquidity of the investment grade bond market is unaffected by that of the CDS market while the reverse is not true. For the subinvestment grade sector, liquidity premium deviations in one market have a consistently reverse effect on the other market’s liquidity. As shown before, the bond market reacts less strongly than the CDS market. In particular when risk decreases, the sensitivity of the bond market decreases and that of the CDS market increases on an absolute level.

The negative estimate for the cointegration coefficient of bc and sc is consistent with our earlier finding that correlation premia are mostly due to the effect of the credit risk intensity on the liquidity intensity. Interestingly, the absolute value of the cointegration coefficient is higher when credit risk decreases. We take this as a sign that the dynamics of the CDS bid and ask premia become more dissimilar in increasing risk phases, therefore a smaller fraction of the CDS premia can be attributed to the impact of the default intensity on the liquidity intensity. Our above results suggest that protection sellers may increase ask premia disproportionately when credit risk increases which we cannot fully capture by means of our time-invariant correlation coefficient.

5 Stability of Credit Risk and Liquidity Premia

In this section, we perform a stability analysis of the estimated credit risk, liquidity, and correlation premia for bonds and CDS. For this purpose, we first explore how bond premia react if we ignore liquidity in the CDS market. We then compare the premia which are obtained if the default and liquidity intensities are estimated from CDS ask or bid quotes only, to the estimate which uses both observations.
5.1 The Effect of Excluding CDS Illiquidity

First, we explore whether the strictly positive bond liquidity premia $bl$ are a result of including stochastic liquidity in CDS ask and bid premia or simply a property of our data set. To do so, we propose the following modification of our model: first, we shift our focus to the CDS mid premia in our estimation procedure since there is no theoretically compelling reason why $sd$ must differ systematically from the mid premium. Second, we re-estimate the default and bond liquidity intensity time-series under the restriction $\gamma^c = \mu^c = \eta^c = 0$, $c = \text{ask, bid}$. This is basically the approach by Longstaff et al. (2005) and suggests that the CDS market is the liquidity numéraire. Lastly, we compute $bd$, $bl$ and $bc$ as explained above and compare them to the results from the initial estimation which includes liquidity in CDS premia. Since the effect only pertains when bonds are liquid relative to the CDS, we present the estimated time-series for a representative company, the Dutch communications company The Nielsen Company (formerly VNU Group B.V.).

The results are displayed in Figure 3.

Figure 3: The Effect of Excluding Stochastic CDS Liquidity

The figure depicts the bond credit risk, liquidity, and correlation premia estimated for the communications company Nielsen when stochastic CDS illiquidity is ignored (dotted lines) and included (solid lines).

As we see in Figure 3, the estimated default risk premium in the bond yield spread $bd$ has similar dynamics, regardless of whether CDS liquidity is included or not, but there are also clear differences. Overall, when stochastic CDS liquidity is excluded, $bd$ is higher,

7The Nielsen Company is active in marketing and media information, business publications and trade shows in over 100 countries and has a total of 42,000 employees.
fluctuating between 33.23 bp and 510.83 bp with a mean of 120.67 bp and a standard deviation of 92.68 bp. When stochastic CDS liquidity is included, $bd$ lies between 32.88 bp and 394.95 bp, the mean equals 97.42 bp and the standard deviation 67.51 bp. The differences are especially noteworthy during the beginning of our observation interval when the CDS market was less liquid.

Conversely, the bond liquidity premium $bl$ that results from excluding stochastic CDS liquidity is consistently lower than when CDS liquidity is modeled with a mean of 3.44 bp versus 22.71 bp, a minimum of -88.27 bp (5.08 bp), a maximum of 75.28 bp (73.40 bp) and a standard deviation of 29.95 bp (14.82 bp). The correlation premia $bc$ are similar in both cases: we obtain a mean value of 7.65 bp and a standard deviation of 4.15 bp when CDS liquidity is excluded and 10.45 bp with a standard deviation of 5.79 when CDS liquidity is accounted for. Even if we did not separate the pure liquidity premium and the correlation premium, we would still obtain a negative sum of the two components during the beginning of the observation interval.

These results suggest that neglecting stochastic CDS liquidity can yield overestimates of liquidity in the bond market and, for the above time-series, result in bond price surcharges instead of discounts. At the same time, the default risk is overestimated when the bond liquidity premia become negative, and this results in overestimates of a company’s default probability. Since neglecting CDS liquidity attributes yield differences between the bond and the CDS market directly to bond liquidity, the effect is especially pronounced when the bond liquidity is high relative to the CDS liquidity. As the CDS market matures, the erroneous results of neglecting CDS liquidity become less striking as long as the net liquidity premium in the bond market remains positive.

5.2 Estimation from Ask or Bid CDS Premia

In Section 4, we use the CDS ask and bid premia simultaneously to extract the pure credit risk, the liquidity, and the correlation component from CDS premia and bond yield spreads. However, only the sum of these components can be observed in practice and our estimate may differ significantly from the true values. As a robustness test, we repeat the firm-specific estimation procedure described in Section 4.2 once using only CDS ask premia and once using only CDS bid premia instead of both. We then compare the resulting estimates of $bd$, $bl$, $bc$, $sd$, $sl$ and $sc$ with those we obtained for the entire sample. In particular, we compute the mean, the standard deviation and the mean absolute difference between the estimates which are obtained using only one CDS premium and the estimates which use both simultaneously. The results are displayed in Table 6.

Table 6 shows that the estimates of the credit risk, liquidity, and correlation premia are almost identical regardless of which CDS premia are used in the estimation. On average,
the mean estimate of \( bd \) from the CDS ask premium of 44.15 bp falls below the one using both premia by 0.23 bp, but the similar standard deviation and the mean absolute deviation of 0.48 bp imply that the sign is not indicative of a systematic error. The same is true for the estimate which uses only the bid premia with a mean difference between the credit risk premia of 0.40 bp and a mean absolute error of 0.55 bp. For the bond liquidity premia \( bl \), we obtain the reverse result, the mean estimates which only use ask premia are slightly higher and the ones using only bid premia are slightly lower. The difference, however, does not appear to be systematic which is supported by the absolute mean deviation of 1.37 bp and 1.42 bp. A similar pattern as for \( bd \) applies for \( bc \). The results for the CDS credit risk premia \( sd \) are also similar to those for the bond. Again, the use of ask premia leads to a very slight underestimation of the credit risk premia while bid premia yield slightly higher values. The largest differences are caused for the CDS liquidity and correlation premia. This agrees with the earlier result that the correlation of the CDS ask and bid liquidity intensities with the default intensity differs. For ask premia, we find lower pure liquidity premia and higher correlation premia. Bid premia for which the liquidity intensity is less strongly correlated with the default intensity yield higher liquidity premia and lower correlation premia.

Overall, we find that the choice of ask or bid premia in the CDS market does not significantly affect the size and the dynamics of the estimated premia. Since these premia are not directly observable in the market, we take the robust behavior of the estimates as a sign that our estimation does not result in a systematic deviation from the true premia.

### 6 Summary and Conclusion

The purpose of our paper was to develop a credit risk model that simultaneously accounts for stochastic liquidity in bond and in CDS markets. While there is broad agreement in the literature that modeling liquidity in bond prices is an important issue both in structural and in reduced-form models, CDS markets are often assumed to be perfectly liquid. Therefore, default probabilities or, in the context of reduced-form models, default intensities, are estimated directly from observed CDS mid premia and the results are used to measure the size of the default component in corporate bond prices and yield spreads.

In our paper, we develop a model that explicitly allows for stochastic liquidity in CDS ask and bid premia. As CDS are derivatives and not assets, it is not clear whether illiquidity should consistently increase mid premia or whether it ought only to result in larger bid-ask-spreads. We avoid this issue by directly modeling the CDS ask and bid premium. This approach allows for closed-form bond prices and CDS premia which are affected by the identical default risk but are subject to different liquidity risk factors. Specifically, we determine a theoretical, liquidity-free yield spread for par bonds and CDS
premia unaffected by the CDS-specific liquidity. These credit risk premia can be compared to the corresponding liquidity and correlation premia and the CDS mid premium as well as to the yield spread. The latter two can be - and our empirical analysis shows them to be - affected by illiquidity and the correlation between credit risk and liquidity.

We estimate the model using bond mid prices and CDS ask and bid premia for companies with ratings between AAA and CCC from a broad range of sectors. As the default-free liquidity numéraire, we use the market for German government bonds. Our results show that the bond and CDS markets as a whole reacted strongly to the Enron and WorldCom defaults. The estimated credit risk component in CDS premia and bond credit spreads is almost identical. In the bond market, it amounts to 60% of the observed credit spreads while 95% of the CDS mid premia are due to pure credit risk. The pure liquidity premia constitute 35% of the credit spread and 4% of the CDS premia, and the correlation component amounts to 5% and 1%, respectively. We also find that the period of the highest credit risk coincides with a period of low liquidity in the bond market, both with regard to the pure liquidity and to the correlation premium. Liquidity in the CDS market exhibits a less straightforward behavior, but the estimate of the pure credit risk component in the CDS market becomes more biased towards the bid in times of high credit risk for investment-grade CDS. Economically, this suggests that protection sellers demand an additional premium in excess of the “fair” credit risk premium when setting their ask quotes. In the subinvestment grade sample, the evidence is mixed. On the one hand, the subinvestment grade CDS market shows a higher average liquidity and correlation premium than the investment grade CDS market. On the other hand, the pure liquidity premia become negative as credit risk rises for badly rated debt. This implies that investors increasingly use the CDS market to take on synthetic credit risk as the liquidity of the bond market dries up.

Throughout our observation interval from June 1, 2001 to June 30, 2007, we observe a declining default risk, an increasing bond market liquidity and an increase in liquidity in conjunction with a more symmetrical distribution of liquidity premia in the CDS market. From an economic perspective, this agrees with the evolution and the standardization of the CDS market. The asymmetrical distribution of the liquidity premia between protection buyers and sellers in the CDS market indicates that CDS mid premia are not an appropriate measure of the pure credit risk component. In a firm-specific analysis, we demonstrate that restricting stochastic liquidity to the bond market can result in price surcharges and yield discounts for corporate bonds. This effect, however, becomes less apparent as CDS market liquidity evolves.

We find evidence of an empirical relation between the liquidity of the bond and the CDS market in excess of the liquidity spill-over which is immanent to our model. Specifically, we observe that a change in bond liquidity affects CDS liquidity premia in the investment grade sector in the opposite direction but that the reverse effect does not apply. In
the subinvestment grade sector, the effect is bilateral but remains more pronounced for CDS, suggesting that the CDS market becomes a more attractive substitute for taking on credit risk synthetically when the bond market is illiquid. Our results also imply that the investment grade bond and CDS markets are less integrated than the subinvestment grade markets. Overall market conditions, on the other hand, affect the investment grade sector more strongly. Lastly, our analysis reveals that the relation between the liquidity premia across the two markets differs during periods of increasing and decreasing default risk.

An issue not addressed in this paper is that CDS contracts are usually designed to allow for a number of bonds deliverable upon the default of a given reference asset. Before default, however, it is not clear which bond will be cheapest to deliver. Jankowitsch et al. (2007) show that this choice option of the protection buyer is also priced in CDS premia. As a second extension of our model, it is also possible to add information from the stock market. Blanco et al. (2005) and Norden and Weber (2004) have explored the information spillover between stock, CDS and bond markets, and their results suggest that incorporating stock market information may facilitate the estimation of the default intensity. It may be interesting to explore whether including this information in a reduced-form model will render explicit liquidity-modeling in CDS premia unnecessary or whether, as our results suggest, our proposed extension of the existing reduced-form models is vital if information from the CDS market is used.
References


Appendix A. Analytical Solutions for the Discount Factors

The dynamics of the default and liquidity intensities are defined as follows. First, we define the latent risk factors \( x \) and \( y^l \) through the following system of stochastic differential equations:

\[
\begin{pmatrix}
    dx(t) \\
    dy^b(t) \\
    dy^{\text{ask}}(t) \\
    dy^{\text{bid}}(t)
\end{pmatrix} =
\begin{pmatrix}
    \alpha - \beta x(t) \\
    \mu^b \\
    \mu^{\text{ask}} \\
    \mu^{\text{bid}}
\end{pmatrix} dt +
\begin{pmatrix}
    \sigma x(t) dW_x(t) \\
    \eta^b dW_{y^b}(t) \\
    \eta^{\text{ask}} dW_{y^{\text{ask}}}(t) \\
    \eta^{\text{bid}} dW_{y^{\text{bid}}}(t)
\end{pmatrix},
\tag{13}
\]

with constants \( \alpha, \beta, \sigma, \mu^l, \eta^l \) and Brownian motions \( W_x \) and \( W_{y^l}, l \in \{b, \text{ask}, \text{bid}\} \). The Brownian motions governing \( x \) and \( y^l, l \in \{b, \text{ask}, \text{bid}\} \) are independent. The intensities \( \lambda \) and \( \gamma \) are then defined as linear combinations of the latent factors:

\[
\begin{pmatrix}
    \lambda(t) \\
    \gamma^b(t) \\
    \gamma^{\text{ask}}(t) \\
    \gamma^{\text{bid}}(t)
\end{pmatrix} =
\begin{pmatrix}
    x(t) + g_b \cdot y^b(t) + g^{\text{ask}} y^{\text{ask}}(t) + g^{\text{bid}} y^{\text{bid}}(t) \\
    f_b \cdot x(t) + y^b(t) + \omega_{b, \text{ask}} y^{\text{ask}}(t) + \omega_{b, \text{bid}} y^{\text{bid}}(t) \\
    f^{\text{ask}} \cdot dx(t) + \omega_{b, \text{ask}} y^{\text{ask}}(t) + y^{\text{ask}}(t) + \omega_{b, \text{bid}} y^{\text{bid}}(t) \\
    f^{\text{bid}} \cdot dx(t) + \omega_{b, \text{bid}} y^{\text{bid}}(t) + \omega_{a, \text{bid}} y^{\text{bid}}(t) + y^{\text{bid}}(t)
\end{pmatrix}.
\]

We only show the derivation for \( E_t \left[ \tilde{P}(t, \tau) \tilde{L}^b(t, \tau) \right] \), the other pricing factors are derived in the same way. The joint expectation for the discount factors \( \tilde{P}(t, \tau) \) and \( \tilde{L}^b(t, \tau) \) is given by:

\[
E_t \left[ \tilde{P}(t, \tau) \cdot \tilde{L}^b(t, \tau) \right] = E_t \left[ \exp \left( -\int_t^\tau \lambda(s) + \gamma^b(s) ds \right) \right]
\]

\[
= E_t \left[ \exp \left( -\int_t^\tau (1 + f_b) x(s) + (1 + g_b) y^b(s) + (g^{\text{ask}} + \omega_{b, \text{ask}}) y^{\text{ask}}(s) + (g^{\text{bid}} + \omega_{b, \text{bid}}) y^{\text{bid}}(s) ds \right) \right]
\]

\[
= E_t \left[ \exp \left( -\int_t^\tau (1 + f_b) x(s) ds \right) \right] \cdot E_t \left[ \exp \left( -\int_t^\tau (1 + g_b) y^b(s) ds \right) \right]
\]

\[
\cdot E_t \left[ \exp \left( -\int_t^\tau (g^{\text{ask}} + \omega_{b, \text{ask}}) y^{\text{ask}}(s) ds \right) \right] \cdot E_t \left[ \exp \left( -\int_t^\tau (g^{\text{bid}} + \omega_{b, \text{bid}}) y^{\text{bid}}(s) ds \right) \right]
\]

\[
=: P(t, \tau, x; 1 + f_b) \cdot L(t, \tau, y^b; 1 + g_b) \cdot L(t, \tau, y^{\text{ask}}; g^{\text{ask}} + \omega_{b, \text{ask}}) \cdot L(t, \tau, y^{\text{bid}}; g^{\text{bid}} + \omega_{b, \text{bid}}),
\]

where \( P^b \) and \( L^b \) are the solutions referenced in section 2.3. The dynamics of the scaled latent risk factors \( (1 + f_b) x, \ (1 + g_b) y^b, \ (g^{\text{ask}} + \omega_{b, \text{ask}}) y^{\text{ask}} \) and \( (g^{\text{bid}} + \omega_{b, \text{bid}}) y^{\text{bid}} \) are
identical to those of the original latent factors with the process parameters adjusted:

\[
\begin{align*}
    d (1 + f_b) x (t) &= \left[ (1 + f_b) \alpha - \beta (1 + f_b) x (t) \right] dt + \sqrt{1 + f_b} \sigma \sqrt{(1 + f_b) x (t)} \, dW_x (t), \\
    d (1 + g_b) y^b (t) &= (1 + g_b) \mu^b \, dt + (1 + g_b) \eta^b \, dW_{y^b} (t), \\
    d (g_{ask} + \omega_{b, ask}) y^{ask} (t) &= (g_{ask} + \omega_{b, ask}) \mu^{ask} \, dt + (g_{ask} + \omega_{b, ask}) \eta^{ask} \, dW_{y^{ask}} (t), \\
    d (g_{bid} + \omega_{b, bid}) y^{bid} (t) &= (g_{bid} + \omega_{b, bid}) \mu^{bid} \, dt + (g_{bid} + \omega_{b, bid}) \eta^{bid} \, dW_{y^{bid}} (t),
\end{align*}
\]

with initial values \((1 + f_b) x_0\), \((1 + g_b) y^b_0\), \((g_{ask} + \omega_{b, ask}) y^{ask}_0\) and \((g_{bid} + \omega_{b, bid}) y^{bid}_0\).

Thus, the following well-known analytical solutions arise:

\[
\begin{align*}
    P (t, \tau, x; k) &:= a_1 (t, \tau; k) \cdot \exp \left[ a_2 (t, \tau; k) \, k \, x (t) \right], \\
    L (t, \tau, y^l; k) &:= a'_3 (t, \tau; k) \cdot \exp \left[ a'_4 (t, \tau; k) \, k \, y (t) \right],
\end{align*}
\]

where

\[
\begin{align*}
    a_1 (t, \tau; k) &= \left( \frac{1 - \kappa (k)}{1 - \kappa (k) \exp [\phi (k) (\tau - t)]} \right)^{\frac{2 \alpha}{\sigma^2}} \exp \left[ \frac{\alpha (\beta + \phi (k)) (\tau - t)}{\sigma^2} \right], \\
    a_2 (t, \tau; k) &= \frac{\phi (k) - \beta}{\sigma^2 k} + \frac{2 \phi (k)}{\sigma^2 k (\kappa (k) \exp [\phi (k) (\tau - t)] - 1)}, \\
    a'_3 (t, \tau; k) &= \exp \left[ \frac{k^2 \eta^2 (\tau - t)^3}{6} + \frac{k \mu^l (\tau - t)^2}{6} \right], \\
    a'_4 (t, \tau; k) &= \tau - t, \\
    \phi (k) &= \sqrt{2 \sigma^2 k + \beta^2}, \quad \kappa (k) = \frac{\beta + \phi (k)}{\beta - \phi (k)}.
\end{align*}
\]

The bond and CDS pricing equations also contain \(E_t \left[ \tilde{P} (t, \tau_{i-1}) \cdot \tilde{L}^b (t, \tau_i) \right] \). Since \(\lambda\) and \(\gamma\) are correlated, we also have to determine the expectation of this non-simultaneous discount factor. Wlg, assume that \(\tau_{i-1} = \tau_1\) and \(\tau_i = \tau_2, \tau_1 \leq \tau_2\). Then, the definition of \(\tilde{P}\) and \(\tilde{L}^b\)
implies that

\[
\tilde{P}(t, \tau_1) \cdot \tilde{L}^b(t, \tau_2) = \exp \left( - \int_t^{\tau_1} x(s) + g_b y^b(s) + g_{\text{ask}} y^{\text{ask}}(s) + g_{\text{bid}} y^{\text{bid}}(s) \, ds \right.
\]

\[
- \int_t^{\tau_2} \hat{f}_b x(s) + y^b(s) + \omega_{b,\text{ask}} y^{\text{ask}}(s) + \omega_{b,\text{bid}} y^{\text{bid}}(s) \, ds \right)
\]

\[
\exp \left( - \int_t^{\tau_1} (1 + f_b) x(s) \, ds - \int_{\tau_1}^{\tau_2} \hat{f}_b x(s) \, ds \right.
\]

\[
- \int_t^{\tau_1} (1 + g_b) y^b(s) \, ds - \int_{\tau_1}^{\tau_2} y^b(s) \, ds
\]

\[
- \int_t^{\tau_1} (g_{\text{ask}} + \omega_{b,\text{ask}}) y^{\text{ask}}(s) \, ds - \int_{\tau_1}^{\tau_2} \omega_{b,\text{ask}} y^{\text{ask}}(s) \, ds
\]

\[
- \int_t^{\tau_1} (g_{\text{bid}} + \omega_{b,\text{bid}}) y^{\text{bid}}(s) \, ds - \int_{\tau_1}^{\tau_2} \omega_{b,\text{bid}} y^{\text{bid}}(s) \, ds \right).
\]

The independence of the latent factors allows us to split up the joint expectation as

\[
E_t \left[ \tilde{P}(t, \tau_1) \cdot \tilde{L}^b(t, \tau_2) \right] = E_t \left[ \begin{array}{c}
\exp \left( - \int_t^{\tau_1} (1 + f_b) x(s) \, ds \right) \exp \left( - \int_{\tau_1}^{\tau_2} \hat{f}_b x(s) \, ds \right) \\
\end{array} \right]
\]

\[
:= P(t, \tau_1, \tau_2; f_b)
\]

\[
\cdot E_t \left[ \begin{array}{c}
\exp \left( - \int_t^{\tau_1} (1 + g_b) y^b(s) \, ds \right) \exp \left( - \int_{\tau_1}^{\tau_2} y^b(s) \, ds \right) \\
\end{array} \right]
\]

\[
\cdot E_t \left[ \begin{array}{c}
\exp \left( - \int_t^{\tau_1} (g_{\text{ask}} + \omega_{b,\text{ask}}) y^{\text{ask}}(s) \, ds \right) \exp \left( - \int_{\tau_1}^{\tau_2} \omega_{b,\text{ask}} y^{\text{ask}}(s) \, ds \right) \\
\end{array} \right]
\]

\[
\cdot E_t \left[ \begin{array}{c}
\exp \left( - \int_t^{\tau_1} (g_{\text{bid}} + \omega_{b,\text{bid}}) y^{\text{bid}}(s) \, ds \right) \exp \left( - \int_{\tau_1}^{\tau_2} \omega_{b,\text{bid}} y^{\text{bid}}(s) \, ds \right) \\
\end{array} \right]
\]

\[
:= L^b(t, \tau_1, \tau_2, y^b)
\]

Applying the law of iterated expectation, we obtain

\[
E_t \left[ \begin{array}{c}
\exp \left( - \int_t^{\tau_1} (1 + f_b) x(s) \, ds \right) \exp \left( - \int_{\tau_1}^{\tau_2} \hat{f}_b x(s) \, ds \right) \\
\end{array} \right]
\]

\[
= E_t \left[ \begin{array}{c}
\exp \left( - \int_t^{\tau_1} (1 + f_b) x(s) \, ds \right) E_{\tau_1} \left[ \exp \left( - \int_{\tau_1}^{\tau_2} \hat{f}_b x(s) \, ds \right) \right] \\
\end{array} \right]
\]

\[
= a_1(\tau_1, \tau_2; f_b) E_t \left[ \exp \left( -a_2(\tau_1, \tau_2; f_b) \int_{\tau_1}^{\tau_2} x(s) \, ds \right) \right] \exp \left( - \int_t^{\tau_1} (1 + f_b) x(s) \, ds \right)
\]

which, by the moment-generating function of \( x \), has the following exponential-affine solution

\[
P(t, \tau_1, \tau_2, x; f_b, 1 + f_b) = a_1(\tau_1, \tau_2; f_b) b_1(t, \tau_1, \tau_2; f_b, 1 + f_b) \exp \left[ -b_2(t, \tau_1, \tau_2; f_b, 1 + f_b) x(t) \right],
\]
where $\phi, a_1,$ and $a_2$ are defined as above and

\[
b_1(t, \tau_1, \tau_2; k_1, k_2) = \frac{2 \phi(k_2) \exp \left[ \frac{\tau_1 - t}{2} (\phi(k_2) + \beta) \right]}{\sigma^2 a_2(\tau_1, \tau_2; k_1) k_1 \left( \exp \left[ \phi(k_2) (\tau_1 - t) \right] - 1 \right) + \phi(k_2) - \beta + \exp \left[ \phi(k_2) (\tau_1 - t) \right] (\phi(k_2) + \beta)},
\]

\[
b_2(t, \tau_1, \tau_2; k_1, k_2) = \frac{a_2(\tau_1, \tau_2; k_1) k_1 [\phi(k_2) + \beta + \exp [\phi(k_2) (\tau_1 - t)] (\phi(k_2) - \beta)] + 2k_2 (\exp \left[ \phi(k_2) (\tau_1 - t) \right] - 1)}{\sigma^2 a_2(\tau_1, \tau_2; k_1) k_1 \left( \exp \left[ \phi(k_2) (\tau_1 - t) \right] - 1 \right) + \phi(k_2) - \beta + \exp \left[ \phi(k_2) (\tau_1 - t) \right] (\phi(k_2) + \beta)}.
\]

Simultaneously, we obtain

\[
E_t \left[ \exp \left( - \int_t^{\tau_1} k_1 y^l(s) \, ds \right) \right] E_t \left[ \exp \left( - \int_{\tau_1}^{\tau_2} k_2 y^l(s) \, ds \right) \right] = a_3(t, \tau_2; k_2) E_t \left[ \exp \left( - a_4(t, \tau_2; k_2) k_2 y^l(\tau_1) \right) \right] \exp \left( - \int_t^{\tau_1} k_1 y^l(s) \, ds \right),
\]

which has the exponential-affine solution

\[
L^l(t, \tau_1, \tau_2, y^l; k_1, k_2) = a_3(t, \tau_2; k_2) b_3^l(t, \tau_1, \tau_2; k_1, k_2) \exp \left( -b_4^l(t, \tau_1, \tau_2; k_1, k_2) y^l(t) \right),
\]

where $a_3^l$ and $a_4^l$ are defined as above and

\[
b_3^l(t, \tau_1, \tau_2; k_1, k_2) = \exp \left[ \frac{\eta^2 k_1^2}{6} (\tau_1 - t)^3 + \frac{\eta^2 k_2 a_4(t, \tau_2; k_2) - \mu}{2} k_1 (\tau_1 - t)^2 \right. \\
\left. + \left( \frac{\eta^2 k_2 a_4(t, \tau_2; k_2) - \mu}{2} a_4(t, \tau_2; k_2) k_2 (\tau_1 - t) \right) \right],
\]

\[
b_4^l(t, \tau_1, \tau_2; k_1, k_2) = a_4^l(t, \tau_2; k_2) k_2 + k_1 (\tau_1 - t).
\]
Table 1: Reference Entities by Rating Class and Industry Sector

The table presents the number of reference entities in each rating class and industry group. Ratings are averages for the reference entity over time when both CDS premia and at least 2 bond yields were observed. The last columns and rows show the number of observed mid bond yields and mid CDS premia. The number of synthetical bond yields matched to the CDS contract maturity equals the number of CDS observations and is therefore suppressed.

<table>
<thead>
<tr>
<th>Industry Sector</th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>All</th>
<th># Obs. Bonds</th>
<th># Obs. CDS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic Materials</td>
<td>-</td>
<td>2</td>
<td>4</td>
<td>7</td>
<td>2</td>
<td>1</td>
<td>16</td>
<td>33,393</td>
<td>13,079</td>
</tr>
<tr>
<td>Communication</td>
<td>-</td>
<td>1</td>
<td>7</td>
<td>8</td>
<td>3</td>
<td>-</td>
<td>19</td>
<td>73,211</td>
<td>20,481</td>
</tr>
<tr>
<td>Cycl. Cons. Goods</td>
<td>-</td>
<td>2</td>
<td>3</td>
<td>9</td>
<td>2</td>
<td>-</td>
<td>16</td>
<td>47,497</td>
<td>15,634</td>
</tr>
<tr>
<td>Diversified</td>
<td>-</td>
<td>-</td>
<td>2</td>
<td>2</td>
<td>-</td>
<td>-</td>
<td>4</td>
<td>6,536</td>
<td>3,096</td>
</tr>
<tr>
<td>Financial</td>
<td>-</td>
<td>22</td>
<td>28</td>
<td>4</td>
<td>-</td>
<td>-</td>
<td>54</td>
<td>175,870</td>
<td>38,046</td>
</tr>
<tr>
<td>Industrial</td>
<td>-</td>
<td>-</td>
<td>4</td>
<td>5</td>
<td>-</td>
<td>-</td>
<td>9</td>
<td>40,624</td>
<td>9,531</td>
</tr>
<tr>
<td>Noncycl. Cons. Goods</td>
<td>-</td>
<td>-</td>
<td>5</td>
<td>8</td>
<td>1</td>
<td>-</td>
<td>14</td>
<td>40,519</td>
<td>12,319</td>
</tr>
<tr>
<td>Utility</td>
<td>1</td>
<td>5</td>
<td>13</td>
<td>4</td>
<td>-</td>
<td>-</td>
<td>23</td>
<td>79,604</td>
<td>19,036</td>
</tr>
<tr>
<td>All</td>
<td>1</td>
<td>32</td>
<td>66</td>
<td>47</td>
<td>8</td>
<td>1</td>
<td>155</td>
<td>497,254</td>
<td>131,222</td>
</tr>
</tbody>
</table>
Table 2: Estimated Credit Risk, Liquidity, and Correlation Premia

The table presents the mean, standard deviation, minimum and maximum for the pure credit risk, the pure liquidity and the correlation premia components for each rating class. All values are in basis points.

<table>
<thead>
<tr>
<th></th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>bd</td>
<td>6.11</td>
<td>13.05</td>
<td>28.46</td>
<td>55.18</td>
<td>246.54</td>
<td>345.04</td>
<td>256.62</td>
<td>44.38</td>
</tr>
<tr>
<td>Std. Dev. (bd)</td>
<td>4.46</td>
<td>10.60</td>
<td>28.35</td>
<td>64.67</td>
<td>237.99</td>
<td>163.16</td>
<td>33.71</td>
<td>83.46</td>
</tr>
<tr>
<td>min (bd)</td>
<td>0.96</td>
<td>1.38</td>
<td>2.96</td>
<td>2.96</td>
<td>33.86</td>
<td>32.60</td>
<td>113.59</td>
<td>0.96</td>
</tr>
<tr>
<td>max (bd)</td>
<td>52.91</td>
<td>260.85</td>
<td>352.11</td>
<td>1,214.39</td>
<td>1,807.09</td>
<td>1,126.95</td>
<td>386.33</td>
<td>1,807.09</td>
</tr>
<tr>
<td>bl</td>
<td>0.67</td>
<td>12.16</td>
<td>24.92</td>
<td>32.63</td>
<td>50.65</td>
<td>61.85</td>
<td>3.09</td>
<td>26.36</td>
</tr>
<tr>
<td>Std. Dev. (bl)</td>
<td>3.12</td>
<td>30.12</td>
<td>43.95</td>
<td>55.77</td>
<td>58.19</td>
<td>3.48</td>
<td>46.20</td>
<td></td>
</tr>
<tr>
<td>min (bl)</td>
<td>0.55</td>
<td>1.41</td>
<td>3.02</td>
<td>3.09</td>
<td>1.51</td>
<td>1.48</td>
<td>1.50</td>
<td>0.55</td>
</tr>
<tr>
<td>max (bl)</td>
<td>30.78</td>
<td>567.08</td>
<td>495.96</td>
<td>349.34</td>
<td>451.59</td>
<td>296.79</td>
<td>10.44</td>
<td>567.08</td>
</tr>
<tr>
<td>bc</td>
<td>-0.02</td>
<td>0.25</td>
<td>2.93</td>
<td>8.25</td>
<td>16.89</td>
<td>19.51</td>
<td>1.96</td>
<td>3.59</td>
</tr>
<tr>
<td>Std. Dev. (bc)</td>
<td>0.03</td>
<td>4.92</td>
<td>6.44</td>
<td>16.01</td>
<td>44.38</td>
<td>14.17</td>
<td>1.98</td>
<td>12.04</td>
</tr>
<tr>
<td>min (bc)</td>
<td>-1.28</td>
<td>-0.61</td>
<td>1.01</td>
<td>1.08</td>
<td>0.72</td>
<td>0.38</td>
<td>0.38</td>
<td>1.28</td>
</tr>
<tr>
<td>max (bc)</td>
<td>0.54</td>
<td>97.21</td>
<td>120.79</td>
<td>251.51</td>
<td>353.46</td>
<td>133.98</td>
<td>5.09</td>
<td>353.46</td>
</tr>
<tr>
<td>sd</td>
<td>6.18</td>
<td>13.45</td>
<td>28.98</td>
<td>56.33</td>
<td>249.52</td>
<td>349.97</td>
<td>258.83</td>
<td>44.76</td>
</tr>
<tr>
<td>Std. Dev. (sd)</td>
<td>4.42</td>
<td>12.88</td>
<td>28.35</td>
<td>65.59</td>
<td>237.99</td>
<td>163.16</td>
<td>33.71</td>
<td>83.46</td>
</tr>
<tr>
<td>min (sd)</td>
<td>5.12</td>
<td>4.24</td>
<td>4.65</td>
<td>4.58</td>
<td>34.92</td>
<td>33.97</td>
<td>115.92</td>
<td>4.24</td>
</tr>
<tr>
<td>max (sd)</td>
<td>52.21</td>
<td>279.05</td>
<td>356.83</td>
<td>1,281.93</td>
<td>1,948.43</td>
<td>1,175.58</td>
<td>397.29</td>
<td>1,948.43</td>
</tr>
<tr>
<td>sl</td>
<td>0.16</td>
<td>1.64</td>
<td>1.79</td>
<td>2.21</td>
<td>4.31</td>
<td>9.00</td>
<td>8.77</td>
<td>1.94</td>
</tr>
<tr>
<td>Std. Dev. (sl)</td>
<td>0.30</td>
<td>6.47</td>
<td>2.32</td>
<td>8.23</td>
<td>40.46</td>
<td>43.38</td>
<td>12.05</td>
<td>10.74</td>
</tr>
<tr>
<td>min (sl)</td>
<td>-0.61</td>
<td>0.48</td>
<td>0.92</td>
<td>-2.82</td>
<td>-153.82</td>
<td>-152.59</td>
<td>-6.86</td>
<td>-153.82</td>
</tr>
<tr>
<td>max (sl)</td>
<td>1.69</td>
<td>3.43</td>
<td>8.97</td>
<td>27.66</td>
<td>123.06</td>
<td>194.25</td>
<td>96.38</td>
<td>194.25</td>
</tr>
<tr>
<td>sc</td>
<td>-0.14</td>
<td>0.46</td>
<td>0.19</td>
<td>0.45</td>
<td>5.41</td>
<td>8.48</td>
<td>5.57</td>
<td>0.41</td>
</tr>
<tr>
<td>Std. Dev. (sc)</td>
<td>0.34</td>
<td>8.83</td>
<td>0.49</td>
<td>1.49</td>
<td>10.53</td>
<td>7.96</td>
<td>1.30</td>
<td>4.91</td>
</tr>
<tr>
<td>min (sc)</td>
<td>-1.26</td>
<td>-0.89</td>
<td>-6.68</td>
<td>-1.24</td>
<td>-3.19</td>
<td>-1.42</td>
<td>-1.38</td>
<td>-6.68</td>
</tr>
<tr>
<td>max (sc)</td>
<td>0.49</td>
<td>4.86</td>
<td>5.35</td>
<td>51.83</td>
<td>98.85</td>
<td>43.77</td>
<td>18.17</td>
<td>98.85</td>
</tr>
</tbody>
</table>
Table 3: The Dynamic Relationship of Credit Risk, Liquidity, and Correlation Premia

The table presents the estimated coefficients for the vector autoregressive model in equations (7), (8) and (9). ***, ** and * denote significance at the 1%, 5% and 10% level. Coefficients are given for premia in basis points, the adjusted $R^2$ is in percentage points.

<table>
<thead>
<tr>
<th></th>
<th>Panel A: All</th>
<th></th>
<th>Panel B: Investment Grade</th>
<th></th>
<th>Panel C: Subinvestment Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Delta bd_{-1}$</td>
<td>$\Delta sd_{-1}$</td>
<td>$\Delta bl_{-1}$</td>
<td>$\Delta sl_{-1}$</td>
<td>$\Delta bc_{-1}$</td>
</tr>
<tr>
<td>$\Delta bd_{-1}$</td>
<td>-0.96***</td>
<td>0.34***</td>
<td>-0.44***</td>
<td>-0.02***</td>
<td>-0.30***</td>
</tr>
<tr>
<td>$\Delta sd_{-1}$</td>
<td>0.71***</td>
<td>-0.66***</td>
<td>-0.01</td>
<td>-0.58***</td>
<td>-0.30***</td>
</tr>
<tr>
<td>$\Delta bl_{-1}$</td>
<td>-0.44***</td>
<td>-0.02***</td>
<td>-0.01</td>
<td>0.00</td>
<td>-0.01***</td>
</tr>
<tr>
<td>$\Delta sl_{-1}$</td>
<td>-0.01</td>
<td>-0.58***</td>
<td>0.01</td>
<td>-0.31***</td>
<td>0.00</td>
</tr>
<tr>
<td>$\Delta bc_{-1}$</td>
<td>-0.30***</td>
<td>0.13***</td>
<td>-0.03***</td>
<td>0.00</td>
<td>-0.01***</td>
</tr>
<tr>
<td>$\Delta sc_{-1}$</td>
<td>-0.06***</td>
<td>0.03***</td>
<td>-0.04***</td>
<td>0.00</td>
<td>-0.01***</td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>9.95</td>
<td>9.02</td>
<td>19.18</td>
<td>27.00</td>
<td>8.81</td>
</tr>
</tbody>
</table>
Table 4: Impact of Aggregate Credit Risk and Liquidity Measures

The table presents the estimated coefficients for the VAR with exogenous variables. The S&P rating class subindex yield spread is used to proxy for credit risk, the ECB financial market liquidity indicator for liquidity. ***, ** and * denote significance at the 1%, 5% and 10% level. The adjusted $R^2$ is given in percentage points.

<table>
<thead>
<tr>
<th></th>
<th>$\Delta b_{-1}$</th>
<th>$\Delta s_{-1}$</th>
<th>$\Delta b_1$</th>
<th>$\Delta s_1$</th>
<th>$\Delta bc$</th>
<th>$\Delta sc$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: All</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.95***</td>
<td>0.59***</td>
<td>-0.44***</td>
<td>-0.01***</td>
<td>-0.30***</td>
<td>0.01***</td>
</tr>
<tr>
<td></td>
<td>0.70***</td>
<td>-0.33***</td>
<td>-0.01</td>
<td>-0.58***</td>
<td>0.12***</td>
<td>-0.03***</td>
</tr>
<tr>
<td></td>
<td>-0.06***</td>
<td>0.03***</td>
<td>-0.04***</td>
<td>0.00</td>
<td>-0.01***</td>
<td>0.01***</td>
</tr>
<tr>
<td></td>
<td>0.05***</td>
<td>-0.03***</td>
<td>0.01*</td>
<td>-0.32***</td>
<td>0.00</td>
<td>-0.02***</td>
</tr>
<tr>
<td>Credit Risk</td>
<td>0.11***</td>
<td>0.10***</td>
<td>0.09***</td>
<td>0.10***</td>
<td>0.01*</td>
<td>0.00</td>
</tr>
<tr>
<td>Liquidity</td>
<td>-0.18***</td>
<td>-0.18***</td>
<td>-0.17***</td>
<td>-0.16***</td>
<td>-0.01*</td>
<td>0.00</td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>10.91</td>
<td>9.99</td>
<td>28.38</td>
<td>31.10</td>
<td>8.87</td>
<td>0.98</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$\Delta b_{-1}$</th>
<th>$\Delta s_{-1}$</th>
<th>$\Delta b_1$</th>
<th>$\Delta s_1$</th>
<th>$\Delta bc$</th>
<th>$\Delta sc$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel B: Investment Grade</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.40***</td>
<td>0.18***</td>
<td>-0.45***</td>
<td>0.00</td>
<td>-0.32***</td>
<td>0.01***</td>
</tr>
<tr>
<td></td>
<td>0.14***</td>
<td>-0.08***</td>
<td>-0.01</td>
<td>-0.54***</td>
<td>0.04***</td>
<td>-0.02***</td>
</tr>
<tr>
<td></td>
<td>-0.02***</td>
<td>0.00</td>
<td>-0.03***</td>
<td>0.00</td>
<td>-0.01***</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>0.00</td>
<td>-0.01***</td>
<td>-0.01</td>
<td>-0.14***</td>
<td>0.00</td>
<td>-0.02***</td>
</tr>
<tr>
<td>Credit Risk</td>
<td>0.07***</td>
<td>0.08***</td>
<td>0.08***</td>
<td>0.05***</td>
<td>0.01*</td>
<td>0.00</td>
</tr>
<tr>
<td>Liquidity</td>
<td>-0.20***</td>
<td>-0.20***</td>
<td>-0.15***</td>
<td>-0.07***</td>
<td>-0.01*</td>
<td>0.00</td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>12.65</td>
<td>11.82</td>
<td>31.56</td>
<td>35.45</td>
<td>10.40</td>
<td>1.04</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$\Delta b_{-1}$</th>
<th>$\Delta s_{-1}$</th>
<th>$\Delta b_1$</th>
<th>$\Delta s_1$</th>
<th>$\Delta bc$</th>
<th>$\Delta sc$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel C: Subinvestment Grade</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-2.55***</td>
<td>1.79***</td>
<td>-0.46***</td>
<td>-0.08***</td>
<td>-0.30***</td>
<td>0.03***</td>
</tr>
<tr>
<td></td>
<td>2.29***</td>
<td>-1.51***</td>
<td>0.01</td>
<td>-0.64***</td>
<td>0.51***</td>
<td>-0.18***</td>
</tr>
<tr>
<td></td>
<td>-2.31***</td>
<td>1.86</td>
<td>-0.08***</td>
<td>-0.02***</td>
<td>-0.02***</td>
<td>0.01*</td>
</tr>
<tr>
<td></td>
<td>2.34***</td>
<td>-1.88***</td>
<td>0.03</td>
<td>-0.45***</td>
<td>0.00</td>
<td>-0.03***</td>
</tr>
<tr>
<td>Credit Risk</td>
<td>0.74*</td>
<td>0.64</td>
<td>0.22***</td>
<td>-0.29***</td>
<td>0.04</td>
<td>0.00</td>
</tr>
<tr>
<td>Liquidity</td>
<td>-0.17</td>
<td>-0.48</td>
<td>-0.02**</td>
<td>0.63**</td>
<td>-0.22</td>
<td>-0.04</td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>18.67</td>
<td>16.59</td>
<td>21.96</td>
<td>30.89</td>
<td>5.03</td>
<td>2.09</td>
</tr>
</tbody>
</table>
Table 5: Dynamic Relation between Credit Risk, Liquidity, and Correlation Premia for Increasing and Decreasing Credit Risk Phases

The table presents the estimated coefficients for the VECM in the error correction form. ***, ** and * denote significance at the 1%, 5% and 10% level. The adjusted $R^2$ is given in percentage points.

<table>
<thead>
<tr>
<th></th>
<th>Increasing Risk Phase</th>
<th>Decreasing Risk Phase</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Credit Risk Premia</td>
<td>Liquidity Premia</td>
<td>Correlation Premia</td>
</tr>
<tr>
<td>Coint. Coef.</td>
<td>-1.02***</td>
<td>66.21***</td>
<td>-5.50***</td>
</tr>
<tr>
<td>ECT Coef. for b</td>
<td>-1.82***</td>
<td>-0.00***</td>
<td>-0.01***</td>
</tr>
<tr>
<td>$\Delta b_{-1}$</td>
<td>-2.49***</td>
<td>-0.29***</td>
<td>-0.33***</td>
</tr>
<tr>
<td>$\Delta s_{-1}$</td>
<td>2.35***</td>
<td>0.09***</td>
<td>1.14***</td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>15.47</td>
<td>8.91</td>
<td>7.82</td>
</tr>
<tr>
<td>ECT Coef. for s</td>
<td>-1.35***</td>
<td>-0.01***</td>
<td>0.01***</td>
</tr>
<tr>
<td>$\Delta b_{-1}$</td>
<td>-1.72***</td>
<td>-0.02***</td>
<td>0.02***</td>
</tr>
<tr>
<td>$\Delta s_{-1}$</td>
<td>1.56***</td>
<td>-0.73***</td>
<td>-0.06***</td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>11.42</td>
<td>35.63***</td>
<td>1.05</td>
</tr>
</tbody>
</table>

Panel A: All

<table>
<thead>
<tr>
<th></th>
<th>Increasing Risk Phase</th>
<th>Decreasing Risk Phase</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Credit Risk Premia</td>
<td>Liquidity Premia</td>
<td>Correlation Premia</td>
</tr>
<tr>
<td>Coint. Coef.</td>
<td>-1.02***</td>
<td>68.96***</td>
<td>-9.15***</td>
</tr>
<tr>
<td>ECT Coef. for b</td>
<td>-0.14**</td>
<td>0.00</td>
<td>-0.04***</td>
</tr>
<tr>
<td>$\Delta b_{-1}$</td>
<td>-1.01***</td>
<td>-0.28***</td>
<td>-0.36***</td>
</tr>
<tr>
<td>$\Delta s_{-1}$</td>
<td>0.77***</td>
<td>-0.06***</td>
<td>2.49***</td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>7.25</td>
<td>7.60</td>
<td>16.59</td>
</tr>
<tr>
<td>ECT Coef. for s</td>
<td>0.07***</td>
<td>-0.05***</td>
<td>0.01***</td>
</tr>
<tr>
<td>$\Delta b_{-1}$</td>
<td>-0.46***</td>
<td>-0.01**</td>
<td>0.01***</td>
</tr>
<tr>
<td>$\Delta s_{-1}$</td>
<td>0.22***</td>
<td>-0.63***</td>
<td>-0.41***</td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>6.52</td>
<td>30.33</td>
<td>17.87</td>
</tr>
</tbody>
</table>

Panel B: Investment Grade

<table>
<thead>
<tr>
<th></th>
<th>Increasing Risk Phase</th>
<th>Decreasing Risk Phase</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Credit Risk Premia</td>
<td>Liquidity Premia</td>
<td>Correlation Premia</td>
</tr>
<tr>
<td>Coint. Coef.</td>
<td>-1.02***</td>
<td>2.99***</td>
<td>-4.74***</td>
</tr>
<tr>
<td>ECT Coef. for b</td>
<td>-2.92***</td>
<td>-0.07***</td>
<td>0.08</td>
</tr>
<tr>
<td>$\Delta b_{-1}$</td>
<td>-3.00***</td>
<td>-0.37***</td>
<td>0.08</td>
</tr>
<tr>
<td>$\Delta s_{-1}$</td>
<td>2.91***</td>
<td>0.15***</td>
<td>-0.46</td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>23.09</td>
<td>15.67</td>
<td>0.28</td>
</tr>
<tr>
<td>ECT Coef. for s</td>
<td>-2.46***</td>
<td>-0.27***</td>
<td>0.01***</td>
</tr>
<tr>
<td>$\Delta b_{-1}$</td>
<td>-2.14***</td>
<td>0.06</td>
<td>0.05</td>
</tr>
<tr>
<td>$\Delta s_{-1}$</td>
<td>2.03***</td>
<td>-0.77***</td>
<td>-0.18</td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>17.17</td>
<td>38.39</td>
<td>1.30</td>
</tr>
</tbody>
</table>
Table 6: Estimated Credit Risk, Liquidity, and Correlation Premia using CDS Ask or Bid Premia

The table presents the sample mean and standard deviation as well as the mean absolute difference between the credit risk, liquidity, and correlation premia estimated from CDS bid and ask premia simultaneously (column 2), using only CDS ask premia (column 3) and using only CDS bid premia (column 4). All values are in basis points.

<table>
<thead>
<tr>
<th></th>
<th>Bid and Ask</th>
<th>Ask Only</th>
<th>Bid Only</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>bd</strong></td>
<td>44.38</td>
<td>44.15</td>
<td>44.78</td>
</tr>
<tr>
<td>Std. Dev. <strong>(bd)</strong></td>
<td>83.46</td>
<td>84.76</td>
<td>84.99</td>
</tr>
<tr>
<td>Mean Abs. Difference</td>
<td>–</td>
<td>0.48</td>
<td>0.55</td>
</tr>
<tr>
<td><strong>bl</strong></td>
<td>26.36</td>
<td>26.98</td>
<td>25.69</td>
</tr>
<tr>
<td>Std. Dev. <strong>(bl)</strong></td>
<td>46.20</td>
<td>47.90</td>
<td>48.02</td>
</tr>
<tr>
<td>Mean Abs. Difference</td>
<td>–</td>
<td>1.37</td>
<td>1.42</td>
</tr>
<tr>
<td><strong>bc</strong></td>
<td>3.59</td>
<td>3.48</td>
<td>3.69</td>
</tr>
<tr>
<td>Std. Dev. <strong>(bc)</strong></td>
<td>12.04</td>
<td>12.90</td>
<td>13.02</td>
</tr>
<tr>
<td>Mean Abs. Difference</td>
<td>–</td>
<td>0.16</td>
<td>0.14</td>
</tr>
<tr>
<td><strong>sd</strong></td>
<td>44.76</td>
<td>44.20</td>
<td>45.33</td>
</tr>
<tr>
<td>Std. Dev. <strong>(sd)</strong></td>
<td>82.96</td>
<td>83.01</td>
<td>82.16</td>
</tr>
<tr>
<td>Mean Abs. Difference</td>
<td>–</td>
<td>0.88</td>
<td>0.85</td>
</tr>
<tr>
<td><strong>sl</strong></td>
<td>1.94</td>
<td>1.50</td>
<td>2.34</td>
</tr>
<tr>
<td>Std. Dev. <strong>(sl)</strong></td>
<td>10.74</td>
<td>8.25</td>
<td>8.29</td>
</tr>
<tr>
<td>Mean Abs. Difference</td>
<td>–</td>
<td>0.87</td>
<td>0.85</td>
</tr>
<tr>
<td><strong>sc</strong></td>
<td>0.41</td>
<td>1.01</td>
<td>0.28</td>
</tr>
<tr>
<td>Std. Dev. <strong>(sc)</strong></td>
<td>4.91</td>
<td>6.23</td>
<td>3.82</td>
</tr>
<tr>
<td>Mean Abs. Difference</td>
<td>–</td>
<td>0.73</td>
<td>0.38</td>
</tr>
</tbody>
</table>