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by

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ABSTRACT

The Basel II Advanced Internal Ratings (AIRB) approach is compared to capital requirements set using an equilibrium structural credit risk model. Analysis shows the AIRB approach undercapitalizes credit risk relative to regulatory targets and allows wide variation in capital requirements for a given exposure owing to ambiguity in the definitions of loss given default and exposure at default. In contrast, the Foundation Internal Ratings Based (FIRB) approach may over-capitalize credit risk relative to supervisory objectives. It is unclear how Basel II will buttress financial sector stability as it specifies the weakest risk regulatory capital standard for large complex AIRB banks.

Key Words: credit risk measurement, credit risk capital allocation, Basel II

JEL Classification: G12, G20, G21, G28

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Financial Stability and Basel II

1. Introduction

Under the June 2004 Basel II agreements, national supervisory authorities may choose among three alternative frameworks to set minimum regulatory capital for their internationally active banks.[1] One approach, the standardized approach, sets minimum capital standards using a modified version of the 1988 Basel Capital Accord that links capital requirements to external credit ratings. The remaining two approaches, the so-called Foundation (FIRB) and Advanced (AIRB) Internal Ratings Based approaches use a regulatory model to assign minimum capital requirements. The model assigns capital according to an individual credit's probability of default (*PD*), loss given default (*LGD*), maturity (*M*), and expected exposure at default (*EAD*).

The Basel II AIRB and FIRB frameworks set minimum regulatory capital requirements using a modified version the so-called Gaussian asymptotic single risk factor model of credit risk. This model, originally developed by Vasicek (1991) (hereafter, the Vasicek model), has been extended by many including Finger (1999), Schönbucher (2000) and Gordy (2003). The Vasicek model assumes that default risk is generated by Gaussian uncertainty and includes a single common source of risk and independent risk factors for each credit. *LGD* and *EAD* are specified exogenously. The model assumes bank portfolios

[1] A revised version of the June 2004 Basel II agreement appears in Basel Committee for Banking Supervision (2006b). In the U.S., banking supervisors have determined that Basel II implementation will require only the largest banks, the so-called core banks, to adopt the AIRB approach, while other banks may petition supervisors for AIRB capital treatment (so-called opt-in banks). The remaining banks (so-called general banks) will continue using the capital requirements specified in the 1988 Basel Capital Accord. Core banks are defined as institutions with total banking (and thrift) assets of $250 billion or more or total on-balance-sheet foreign exposure of $10 billion or more. General banks likely will be subject to a modified version of the 1988 Basel Accord, so-called Basel 1A, but the potential modifications have to be finalized.

are fully-diversified with respect to idiosyncratic sources of risk, and capital is needed to buffer the loss uncertainty associated with the single non-diversifiable source of risk.[2]

Under the Basel II AIRB rule, banks must estimate each credit's *PD*, *EAD* and *LGD*. Under the FIRB rule, *LGD* is a fixed regulatory parameter. In contrast to the Vasicek model, IRB capital rules specify default correlation as deterministic function of *PD* that differs according to other characteristics of the credit.[3]

In designing Basel II, the Basel Committee on Banking Supervision (BCBS) specified a target prudential standard of a 99.9 percent solvency rate of over a one-year horizon. The BCBS explained the framework's complexity and targeted prudential rigor as necessary byproduct of the complexity of large international banking organizations and the need to foreclose opportunities for regulatory arbitrage that exist under the 1988 Basel Accord.[4] Alternative Basel II approaches are calibrated so that the AIRB approach produces the lowest capital requirements to encourage banks to transition from the Standardized and FIRB approaches to the AIRB approach. Capital savings accorded under the AIRB are intended to offset the costs associated with developing and operating AIRB systems and to reflect efficiencies that are presumed to be generated by more efficient measurement of credit risk and assignment of minimum capital.

BCBS discussions suggest that Basel II capital standards will fortify the minimum bank capital requirements of internationally active banks. In contrast, the most recent Quantitative Impact Studies (QIS) rules show that large internationally active banks will benefit from large capital reductions under Basel II, especially under the AIRB approach. QIS studies also show that AIRB estimates of minimum capital requirements for positions with similar risks may vary widely across banks. The decline and variability in regulatory capital estimates raises issues regarding the calibration of the AIRB model and the

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[2] See Gordy (2003) for further discussion.

[3] The correlation function differs for corporate, sovereign and bank, small- and medium-sized enterprises, specialized lending, commercial real estate, residential mortgages, credit cards, and other retail exposures.

[4] See for example, BCBS (1998, June 1999), Jones (2000), or Myer (2001).

regulations that guide *PD*, *EAD*, and *LGD* inputs into the AIRB rule. To date, no published study has analyzed the rigor of the prudential standards that are set by the June 2006 Basel II IRB capital rules or examined the implications of their calibration and input specifications.

This paper analyzes the minimum solvency standards that are set under the June 2006 Basel II IRB approaches for corporate, sovereign, and bank credits.[5] Using a fully parameterized equilibrium structural model of credit risk, capital allocations are derived for non-revolving credit portfolios that satisfy the assumptions that underpin the FIRB and AIRB models—default correlations are driven by a single common factor, and idiosyncratic risk is fully-diversified. In a calibration exercise, the AIRB and FIRB capital requirements are compared to capital estimates calculated from a full structural credit risk model.

The calibration comparison shows that the AIRB approach substantially undercapitalizes credit risks, producing a capital shortfall that varies depending on the definitions of *EAD* and *LGD* that are used to calculate AIRB capital requirements. Under the most likely to be adopted definitions of *EAD* and *LGD*, bank default rates may exceed 5 percent when minimum capitalization rates are fully compliant with Basel II AIRB rules. AIRB capital shortfalls owe not only to use of inadequate measures of *EAD* and *LGD*, but the shortfalls also owe in part to the AIRB treatment of *LGD*. The AIRB approach fails to allocate capital for systematic risk in individual exposure *LGD*s. Positive correlation among credit *LGD*s is a characteristic of the BSM (or any other) structural model of credit risk.

In contrast to the AIRB results, the FIRB will substantially over-capitalize credit risks relative to the 99.9 percent target solvency rate. Overcapitalization is a result of the FIRB assumption of 45 percent *LGD*, an assumption that overestimates the loss rates on the credits examined in this study and over-capitalizes for the systematic risk in *LGD*s.

Under the current Basel II framework, permissible definitions for *EAD* and *LGD* can result in AIRB prudential standards that are far weaker than those set by the 1988 Basel Accord or those mandated under the alternative Basel II capital regimes. AIRB rules, moreover, are subject to interpretation that may lead to substantial variation in bank estimates

[5] The June 2006 AIRB and FIRB calibrations for corporate, sovereign and bank credits are unchanged from the calibration in the June 2004 Basel II agreements.

of the capital needed for a given credit risk exposure so all AIRB banks need not face the same prudential capital standard. The FIRB prudential standard, in contrast, is much more conservative.

Should AIRB bank capitalization levels approach the regulatory minima allowed under Basel II, the largest internationally active banking institutions will operate under a solvency standard that increases their implicit public safety net subsidy. The "tilted playing field" established under Basel II will encourage banking system assets to migrate toward AIRB banks in order to maximize the value of implicit safety net subsidies that may accrue to large systemically-important banks. This is a new form or regulatory arbitrage created by Basel II. Ultimately, the capital benefits that accrue to AIRB banks–benefits that are intended to encourage adoption of sophisticated risk management systems–may undermine the Basel II objective of buttressing the stability of the international financial system.

An outline of this paper follows. Section 2 supplies background on the development and calibration of Basel II. Section 3 summarizes the general methodology for constructing economic capital allocations. Section 4 derives capital allocations for portfolio credit risk in the context of an equilibrium structural model. Section 5 derives equilibrium credit risk capital rules under assumptions that mimic those that underlie the Basel II IRB models. Section 6 reviews the procedures for setting minimum capital requirements under the Basel IRB approaches. Section 7 discusses the calibration results and Section 8 concludes the paper.

2. BACKGROUND

In designing its prudential standards, the BCBS expressed an objective that includes setting minimum regulatory capital requirements to ensure a minimum bank solvency margin of 99.9 percent over a one-year horizon.[6] The rationale for setting this solvency standard has not been explained in official BCBS documents, and yet influential BCBS member studies

[6] Basel Committee on Banking Supervision, 2004, paragraph 667.

have suggested that this solvency standard is approximately consistent with the standard set by the 1988 Basel Accord.[7]

The BCBS arrived at the June 2004 IRB framework through a process of industry consultation and international compromise. There is no BCBS document that provides evidence of the solvency standards that are achieved under the alternative Basel II regimes. Instead, a series of three Quantitative Impact Studies (QIS) studies required banks to estimate the effects of alternative IRB calibrations on their minimum regulatory capital requirements. The June 2004 calibrations reflect an iterative process in which consecutive IRB formulations were modified following industry comments, internal BCBS discussions and negotiations, and review of the QIS results. Calibrations were modified with a goal of achieving capital neutrality relative to the 1988 Basel Accord (as amended) while creating incentives that encouraged banks to adopt the IRB approaches.[8]

Following the June 2004 publication of the Basel II framework, two additional QIS studies have been conducted: QIS 4 in the United States, Germany and South Africa, and QIS 5 in adopting countries in the remainder of the world. Both studies reported substantial declines in minimum capital requirements for AIRB banks relative to required capital under the 1988 Basel Accord.

The 2005 QIS 4 study included 26 U.S. institutions, all of which reported using the AIRB approach.[9] The results show, in aggregate, minimum regulatory capital for these institutions fell by 15.5 percent under the AIRB. Among these banks, the median reduction in capital was 26 percent and the median reduction in Tier I capital requirements was 31 percent. Of the few banks that experienced increased minimum capital requirements under the AIRB, the increases were driven primarily by increases in capital for consumer retail portfolios and to a lesser extent by equity exposures.

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[7] See for example Jackson, Perraudin, and Saporta (2002).

[8] The Basel Committee on Banking Supervision, October 2002, paragraphs 46-47.

[9] See the Federal Reserve Board Press release, “Summary Findings of the Fourth Quantitative Impact Study,” available at www.federalreserve.gov

In addition to large declines in capital, QIS 4 results show a high degree of dispersion in reported estimates of minimum capital requirements. Banks reported widely divergent capital estimates for their constituent portfolios (corporate, mortgage, etc.). While these differences could owe to true difference in bank risk profiles as a result of differentiation among customer bases and business strategies, additional analysis using shared national credit data indicated that banks reported widely divergent capital estimates for positions with substantially similar risk characteristics. Further analysis suggests that a significant share of the variation in QIS 4 results may be attributed to differences in bank estimates of *PD*s and *LGD*s among credits with approximately equivalent risk characteristics. For the corporate, sovereign, and bank credit portfolio, for example, QIS 4 *LGD* estimates on non-defaulted credits varied from about 15 to 55 percent across banking institutions.

The 2006 QIS 5 study included 382 banks in 32 countries outside of the U.S.[10] Of the participating banks, the largest internationally active banks—so-called Group 1 banks–posted capital declines of 7.1 percent on average under the AIRB approach. Smaller banks, so called Group 2 banks, primarily nationally focused institutions, posted much larger declines in minimum regulatory capital.[11] Within Europe,[12] Group 1 banks posted average capital declines of 8.3 percent under the AIRB. For European Group 2 banks, declines averaged 26.6 percent under the AIRB. Of the banks that experienced large declines in minimum regulatory capital requirements, the declines were attributed to bank concentrations in retail lending, especially residential mortgages. The BCBS summary of QIS 5 results does not provide a detailed analysis of the dispersion of bank minimum capital estimates. The study does however report significant variation in AIRB input values. *LGD* estimates for wholesale credits, for example, range from 10.8 to 67.6 percent across reporting banks.

[10] See, BCBS (2006a). QIS 5 AIRB capital rules include a 1.06 scaling factor that was not included in the June 2004 calibration or the instructions that guided QIS 4. The inclusion of this scaling factor means the reported capital declines will appear less severe than those reported in the U.S..

[11] BCBS (2006a).

[12] So-called CEBS (Committee of European Bank Supervisors) banks.

3. REGULATORY CAPITAL FOR CREDIT RISKS

Basel II is designed to ensure a one-year bank solvency margin of 99.9 percent; that is, Basel II minimum capital requirements are intended to ensure that over a one-year horizon, the probability that a compliant bank defaults on its financial obligations is less than 0.1 percent.

To model the Basel II capital constraint, it is assumed that capital allocations are set with a goal of maximizing leverage subject to ensuring that the bank is able to meet all the associated interest and principal payments with a minimum probability of $\alpha = 99.9$ percent. To maximize leverage, the bank is assumed to issue a single class of discount funding debt that matures at time $T = 1$ year. α = 99.9 percent is the bank's target solvency rate; $1-\alpha$ = 0.001 is the bank's *ex ante* target probability of default.

The one-year solvency standard implies a capital allocation horizon of one year. The purchased asset A, has an initial market value A_0, and a time 1 random value of $\tilde{A}_1$ with a cumulative density function $\Psi(\tilde{A}_1)$, and a probability density function $\psi(\tilde{A}_1)$.[13] Let $\Psi^{-1}(1-\alpha)$ represent the inverse of the cumulative density function of $\tilde{A}_1$ evaluated at $1-\alpha,\ \alpha \in [0,1]$. Define an α coverage VaR measure, $VaR(\alpha)$, as,

$$VaR(\alpha) = A_0 - \Psi^{-1}(1-\alpha) \tag{1}$$

$VaR(\alpha)$ is the loss amount that is exceeded by at most $(1-\alpha)$ of all potential future value realizations of $\tilde{A}_1$. Expression (1) measures value-at-risk relative to the initial market value of the asset. When credit risk losses are bounded above by A_0, $\Psi^{-1}(1-\alpha)$ is bounded below by 0.

[13] The construction of an optimal economic capital allocation is simplified when portfolios are composed of assets with non-negative market values. For purposes of this analysis, portfolio composition is restricted to include only long positions in fixed income claims that may generate losses that are bounded above by the initial market value of the credit. See Kupiec (2004a) for further discussion.

Consider a capital allocation rule that sets equity capital equal to $VaR(\alpha)$. By definition, the probability the bank will experience a loss in excess of its initial equity value is at most $100(1-\alpha)$ percent. Under the $VaR(\alpha)$ capital allocation rule, the bank must borrow $A_0 - VaR(\alpha)$ to finance the investment. If the bank borrows $A_0 - VaR(\alpha)$, it must promise to pay back *more* than $A_0 - VaR(\alpha)$ if equilibrium interest rates and credit risk compensation are positive. Because the $VaR(\alpha)$ capital allocation rule ignores time and the equilibrium returns that are required by bank creditors, the probability that the bank will default on its funding debt is greater than $1-\alpha$ if the bank's debts can only be satisfied by raising funds through the sale of $\tilde{A}_1$ at time T.

The capital allocation rule that meets the Basel II regulatory objective is: set equity capital equal to $VaR(\alpha)$ plus the bank's interest expense. An equivalent allocation is achieved by setting the par (maturity) value of the funding debt equal to $VaR(\alpha)$ and estimating the funding debt's market value at issuance. The difference between the market value of the purchased asset and the proceeds from the funding debt issue is the equity capital needed to fund the investment and satisfy the solvency rate target.

4. BASEL II CAPITAL REQUIREMENTS IN AN EQUILIBRIUM STRUCTURAL MODEL

Estimation of the equilibrium interest cost on funding debt requires the use of formal asset pricing models or an empirical approximation to value a bank's funding debt. In this analysis, we will adopt the asset pricing approach. If the risk-free term structure is flat and a firm issues only pure discount bond, and asset values follow geometric Brownian motion, under simplifying assumptions,[14] Black and Scholes (1973) and Merton (1974) (hereafter BSM) establish that the market value of a firm's debt is equal to the discounted value of the bond's par value (at the risk free rate), less the market value of a Black-Scholes put option

[14] There are no taxes, transactions are costless, short sales are possible, trading takes place continuously, if borrowers and savers have access to the debt market on identical risk-adjusted terms, and investors in asset markets act as perfect competitors.

written on the firm's assets. The put option has a maturity identical to the bond's maturity, and a strike price equal to the par value of the bond. The BSM model can be modified to produce capital allocation rules that are fully consistent with equilibrium conditions and the underlying structural features that determine the market value of bank debt and equity securities.

The Vasicek and Basel IRB models are default mode models meaning that credits are modeled as either fully performing or defaulting at the end of the model time horizon. Unlike a fully-specified economic model, these models do not have an explicit time dimension. The capital allocation horizon is implicitly set by the choice of a default probability which may vary with the time horizon. Basel II has selected a one-year time horizon for setting minimum capital requirements for credit risks.

To simplify the discussion, in the subsequent analysis we will assume that all BSM issued and purchased debt claims mature in one year. This assumption is a convenience that avoids the need to mark-to-market bank funding liabilities at the end of the one-year regulatory horizon and it is consistent with formulation of the Basel II default mode IRB models. The BSM capital allocation framework can be generalized to calculate one-year capital for longer maturity claims purchased by the bank, but there is a cost in terms of mathematical complexity that is unnecessary for purposes of this paper. The so-called market-to-market capital allocation process, where long maturity credits are valued at the end of the capital allocation horizon, is discussed in detail in Kupiec (2004a, 2004b).

Consider a bank whose only asset is a risky BSM discount bond issued by an unrelated counterparty that matures at date one. Let B_0 represent the initial market value of this bond. Assume that the bank will fund this bond with its own one-year discount debt issue, and equity. In this setting, the bank's funding debt issue is a compound option. Let $\tilde{A}_1$ and Par_P represent, respectively, the time one value of the assets that support the discount bond investment and the par value of the purchased bond. Let Par_F represent the par value of a discount bond that is issued by the bank to fund the investment. The end-of-period cash flows that accrue to the bank's debt holders are,

$$Min\left[Min\left(\tilde{A}_1, Par_P\right), Par_F\right] . \tag{2}$$

The initial equilibrium market value of the bank's discount bond issue is the discounted (at the risk free rate) expected value of the end-of-period funding debt cash flows taken with respect to the equivalent martingale probability distribution for the assets, $\tilde{A}_1^q$,

$$E\left[Min\left[Min\left(\tilde{A}_1^q, Par_P\right), Par_F\right]\right] e^{-r_f} \tag{3}$$

where

$$\tilde{A}_1^q \sim A_0\, e^{\left(r_f - \frac{\sigma^2}{2}\right) + \sigma \tilde{z}} \tag{4}$$

and $\tilde{z}$ is a standard normal random variable.

The payoff of the bank's purchased bond is, $Min\left[Par_P, \tilde{A}_1\right]$, where $\tilde{A}_1$ is the asset value at date 1, $\tilde{A}_1 \sim A_0\, e^{\left(\mu - \frac{\sigma^2}{2}\right) + \sigma \tilde{z}}$, and $\mu = r_f + \lambda\sigma$, where λ is the market price of risk. Let $\Phi(x)$ represent the cumulative standard normal distribution function evaluated at x, and let $\Phi^{-1}(\alpha)$ represent the inverse of this function for $\alpha \in [0,1]$. Because $\tilde{A}_1$ is monotonic in the realized value of $\tilde{z}$, the upper bound on the par (maturity) value of the funding debt that can be issued under the target solvency constraint is,

$$Par_F(\alpha) = \Psi^{-1}(1-\alpha) = A_0\, e^{\left[\mu - \frac{\sigma^2}{2}\right] + \sigma \Phi^{-1}(1-\alpha)} . \tag{5}$$

The initial market value of this funding debt issue is, $B_{F0}(\alpha)$,

$$B_{F0}(\alpha) = E\left[Min\left[Min\left(\tilde{A}_1^q, Par_P\right), Par_F(\alpha)\right]\right] e^{-r_f} . \tag{6}$$

and the initial equity allocation consistent with the target solvency rate α, $E(\alpha)$, is,

$$E(\alpha) = B_0 - E\left[Min\left[Min\left(\tilde{A}_1^q, Par_P\right), Par_F(\alpha)\right]\right] e^{-r_f} . \tag{7}$$

In the single asset case, when the probability of default on the purchased bond is less than or equal to $(1-\alpha)$, the bond can be financed 100 percent with bank debt without violating the solvency constraint ($Par_F(\alpha) = Par_P$). When the probability of default on the purchased bond exceeds $(1-\alpha)$, capital is required, and $Par_F(\alpha) < Par_P$. In this case, expression (7) implies a dollar capital allocation,

$$E(\alpha) = B_0 - Par_F(\alpha)e^{r_f}\Phi\left(\frac{\ln(A_c) - \ln(A_0) - \left(r_f - \frac{\sigma^2}{2}\right)}{\sigma}\right) - A_0\Phi\left(\frac{\ln(A_c) - \ln(A_0) - \left(r_f + \frac{\sigma^2}{2}\right)}{\sigma}\right). \tag{8}$$

Portfolio Capital

Except for the need to derive a probability density function for a portfolio's future value distribution under both the physical and the equivalent martingale measures, the process for setting portfolio capital requirements mirrors the calculations for a single asset. In most cases, credit portfolios do not have density functions that admit a closed-form expression for either the par value of the funding debt or its initial market value. Monte Carlo simulation is often required to estimate $VaR(\alpha)$ and the par value of the funding debt, and pricing the funding debt may require numerical evaluation of a high order integral. The next section considers portfolio capital allocation under BSM assumptions when asset price dynamics are generated by a single common factor and idiosyncratic risk is fully diversified. These assumptions reduce significantly the complexity of portfolio capital calculations.

5. BASEL II CAPITAL UNDER ASYMPTOTIC SINGLE FACTOR ASSUMPTIONS

Capital allocation calculations are simplified if a portfolio is well-diversified and asset values are driven by a single common factor in addition to individual idiosyncratic factors. Let W_M represent a standard Wiener process common in all asset price dynamics, and W_i represents an independent standard Weiner process idiosyncratic to the price dynamics of asset i. Assume that asset price dynamics for firm i are given by,

$$dA_i = \mu A_i\, dt + \sigma_M A_i\, dW_M + \sigma_i A_i\, dW_i, \tag{9}$$

$$dW_i \; dW_j = \rho_{ij} = 0, \quad \forall \, i, j.$$
$$dW_i \; dW_M = \rho_{im} = 0, \forall \, i.$$

Under these dynamics, asset prices are log normally distributed,

$$\tilde{A}_{iT} = A_{i0} \; e^{\left[r_f + \lambda \sigma_M - \frac{1}{2}\left(\sigma_M^2 + \sigma_i^2\right)\right] T + \left(\sigma_M \tilde{z}_M + \sigma_i \tilde{z}_i\right)\sqrt{T}}, \tag{10}$$

where $\tilde{z}_M$ and $\tilde{z}_i$ are independent standard normal random variables. Under the equivalent martingale change of measure, asset values at time T are also log normally distributed,

$$\tilde{A}_{iT}^q = A_{i0} \; e^{\left[r_f - \frac{1}{2}\left(\sigma_M^2 + \sigma_i^2\right)\right] T + \left(\sigma_M \tilde{z}_M + \sigma_i \tilde{z}_i\right)\sqrt{T}}. \tag{11}$$

Under these price dynamics, the correlation between geometric asset returns is,

$$Corr\left[\frac{1}{T}\ln\left(\frac{\tilde{A}_{it}}{A_{i0}}\right), \frac{1}{T}\ln\left(\frac{\tilde{A}_{jt}}{A_{j0}}\right)\right] = \frac{\sigma_M^2}{\left(\sigma_M^2 + \sigma_i^{\,2}\right)^{\frac{1}{2}}\left(\sigma_M^2 + \sigma_j^{\,2}\right)^{\frac{1}{2}}}, \; \forall i, j. \tag{12}$$

If the model is further specialized so that the volatilities of assets' idiosyncratic factors are assumed identical, $\sigma_i = \sigma_j = \bar{\sigma}, \forall \, i, j,$ the pair-wise asset return correlations are,

$$\rho = Corr\left[\frac{1}{T}\ln\left(\frac{\tilde{A}_{it}}{A_{i0}}\right), \frac{1}{T}\ln\left(\frac{\tilde{A}_{jt}}{A_{j0}}\right)\right] = \frac{\sigma_M^2}{\sigma_M^2 + \bar{\sigma}^2} \quad \forall i, j. \tag{13}$$

Asset Return Distributions

The *T*-period return on BSM risky bond i that is held to its maturity date T is,

$$\tilde{M}_{iT} = \frac{1}{B_{i0}}\left(Min\left(\tilde{A}_{iT}, Par_i\right)\right) - 1. \tag{14}$$

$\tilde{M}_{iT}$ is bounded in the interval $[-1, a]$, where a is a finite constant. When return realizations are in the range, $-1 < M_{iT} < 0$, M_{iT} represents the loss rate on the bond held to maturity.

For realizations in the range, $0 < M_{iT} < \frac{Par_i}{B_{i0}} - 1$, the bond has defaulted on its promised payment terms, but the bond has still generated a positive return. A fully performing bond posts a return equal to $\frac{Par_i}{B_{i0}} - 1 < a$ which is finite by assumption.

A bond's physical return distribution (14) has an associated equivalent martingale return distribution,

$$\tilde{M}_{iT}^{q} = \frac{1}{B_{i0}}\left(Min\left(\tilde{A}_{iT}^{q}, Par_i\right)\right) - 1 . \tag{15}$$

By construction, expressions (14) and (15) have identical support.

Asymptotic Portfolio Return Distribution

The *T*-period return on a portfolio of N risky individual credits, ${}_P\tilde{M}_T$, is

$${}_P\tilde{M}_T \equiv \frac{\sum_{i=1}^{N} \tilde{M}_{iT} B_{i0}}{\sum_{i=1}^{n} B_{i0}} \tag{16}$$

Let ${}_P\tilde{M}_T \big| z_M$ represent the portfolio return conditional on a realization of the common market factor, $\tilde{z}_M = z_M$,

$${}_P\tilde{M}_T \big| z_M = \frac{\sum_{i=1}^{N} \tilde{M}_{iT} \big| z_M \cdot B_{i0}}{\sum_{i=1}^{n} B_{i0}} \tag{17}$$

If $\psi\left(\tilde{M}_{iT} \middle| z_M\right)$ represents the conditional return density function, under the single common factor assumption, $\psi\left(\tilde{M}_{iT} \middle| z_M\right)$ and $\psi\left(\tilde{M}_{jT} \middle| z_M\right)$ are independent for $\forall\, i \neq j.$[15]

Consider a portfolio composed of equal investments in N individual bonds that share identical *ex ante* credit risk profiles. That is, assume that the bonds in the portfolio are identical regarding par value $\{Par_i = Par_j, \forall\, i, j\}$, maturity $\{T\}$, and volatility characteristics, $\{\sigma_i = \sigma_j = \overline{\sigma}, \forall\, i, j\}$. The conditional returns of these bonds are independent and identically distributed with a finite mean. As the number of bonds in the portfolio, N, grows without bound, the Strong Law of Large Numbers requires,

$$\lim_{N\to\infty}\left[{}_P\tilde{M}_T \mid z_M\right] = \lim_{N\to\infty}\left[\frac{\sum_{i=1}^{N}\tilde{M}_{iT}\middle| z_M}{n}\right] \xrightarrow[a.s]{} E\left[\psi\left(\tilde{M}_{iT}\middle| z_M\right)\right] \;\forall\, z_M \tag{18}$$

The notation $a.s.$ indicates "almost sure" convergence (convergence with probability one). Under the BSM single factor assumptions, expression (18) becomes,

$$\begin{aligned}\lim_{N\to\infty}\left[{}_P\tilde{M}_T\middle| z_M\right] &= \frac{Par_i}{B_{i0}}\left[1-\Phi\left(w_{iT}(z_M)\right)\right] \\ &\quad + \frac{Q(z_M)}{B_{i0}}\left[1-\Phi\left(-w_{iT}(z_M)+\gamma_{iT}\right)\right]-1\end{aligned} \tag{19}$$

where,

[15] Independence in this non-Gaussian setting requires that an observation of the return to bond j be uninformative regarding the conditional distribution function for bond i, $\Pr\left(\tilde{M}_{it} \mid z_M\right) < a = \Pr\left(\left(\tilde{M}_{it} \mid z_M\right) < a \text{ given that } \tilde{M}_{jt} = M_{jt}\right), \forall a, i \neq j.$ This condition is satisfied under the single common factor model assumption.

$$\mu_{iT}(z_M) = \ln[A_{i0}] + \left[r_f + \lambda\sigma_M - \frac{1}{2}\left(\sigma_M^2 + \sigma_i^2\right) \right] T + z_M \sigma_M \sqrt{T}$$

$$\gamma_{iT} = \sigma_i \sqrt{T}$$

$$w_{iT}(z_M) = \frac{\ln[Par_i] - \mu_{iT}(z_M)}{\gamma_{iT}}$$

$$Q(z_M) = e^{\mu_{iT}(z_M) + \frac{\gamma_{iT}^2}{2}}$$

where $\Phi(x)$ represents the cumulative standard normal distribution function evaluated at x.

Similarly, the equivalent martingale portfolio conditional return distribution is given by,

$$\begin{aligned} \underset{N\to\infty}{Lim}\left[{}_P\tilde{M}_T^q \middle| z_M \right] &= \frac{Par_i}{B_{i0}} \left[1 - \Phi\left(w_{iT}^q(z_M)\right)\right] \\ &+ \frac{Q^q(z_M)}{B_{i0}} \left[1 - \Phi\left(-w_{iT}^q(z_M) + \gamma_{iT}\right)\right] - 1 \end{aligned} \tag{20}$$

where,

$$\mu_{iT}^q(z_M) = \ln[A_{i0}] + \left[r_f - \frac{1}{2}\left(\sigma_M^2 + \sigma_i^2\right) \right] T + z_M \sigma_M \sqrt{T}$$

$$\gamma_{iT} = \sigma_i \sqrt{T}$$

$$w_{iT}^q(z_M) = \frac{\ln[Par_i] - \mu_{iT}^q(z_M)}{\gamma_{iT}}$$

$$Q^q(z_M) = e^{\mu_{iT}^q(z_M) + \frac{\gamma_{iT}^2}{2}}$$

Expressions (19) and (20) represent, respectively, the inverse of the conditional probability distribution functions for the physical and equivalent martingale portfolio returns.

Uncertainty in these return distributions are driven by the single common factor, $\tilde{z}_M$,as idiosyncratic risks have been completely diversified.

Optimal Portfolio Capital Allocation in a Default Mode Approach

The default mode approach used in Basel II assumes credits are held to maturity and either they fully perform or they default. Over the one-year capitalization horizon adopted in Basel II, a default mode model is only appropriate for one-year maturity credits. The Basel II IRB capital rules adjust for maturity differences using an ad hoc factor that modifies the one-year default mode capital requirements according to a credit's duration. Shorter duration credits get capital reductions and longer duration credits get assigned increased capital relative to a one-year default mode capital estimate. In the approach that follows, it is assumed that all credits have an identical maturity of $T = 1$ years, and we do not consider the performance of the maturity adjustment factor.

The portfolio return distribution is monotonic in z_M , and so the capital allocation calculations need only involve the conditional portfolio return distributions. When expressed as a proportion of the investment portfolio's initial market value, the optimal par value of funding debt can be determined by setting $z_M = \Phi^{-1}(1-\alpha)$ and using expression (19) to solve for the end-of-horizon portfolio critical value,

$$par_F^P(\alpha) = \left(\begin{array}{l} \dfrac{Par_i}{B_{i0}} \left[1 - \Phi\left(w_{i1}\left(\Phi^{-1}(1-\alpha)\right)\right)\right] \\ \quad + \dfrac{Q(z_M)}{B_{i0}} \left[1 - \Phi\left(-w_{i1}\left(\Phi^{-1}(1-\alpha)\right) + \gamma_{i1}\right)\right] \end{array} \right) \tag{21}$$

To determine the market value of the funding debt, it is necessary to solve for the value of z_M that determines the default threshold under the risk neutral measure, $\hat{z}_M$,

$$\hat{z}_M = \Phi^{-1}(1-\alpha) + \lambda \tag{22}$$

$\hat{z}_M$ is one of the limits of integration needed to calculate the expected discounted payoff of the funding debt using the risk neutral measure. Expressed as a proportion of the investment portfolio's initial market value, the initial market value of the funding issue, $b_{F0}^P(\alpha)$, is ,

$$b_{F0}^P(\alpha) = e^{-r_f}\left(\begin{array}{l} \int_{-\infty}^{\hat{z}_M}\left[\frac{Par_i}{B_{i0}}\left[1-\Phi\left(w_{i1}^q(z_M)\right)\right]\right]\phi(z_M)dz_M \\ +\int_{-\infty}^{\hat{z}_M}\left[\frac{Q^q(z_M)}{B_{i0}}\left[1-\Phi\left(-w_{i1}^q(z_M)+\gamma_{i1}\right)\right]\right]\phi(z_M)dz_M \\ +\, par_F^P(\alpha)\left[1-\Phi(\hat{z}_M)\right] \end{array}\right) \tag{23}$$

where $\phi(x)$ is the standard normal density function evaluated at x . The economic capital allocation for the portfolio, expressed as a proportion of the portfolio's initial market value, $K_{BSM}^P(\alpha)$ is,

$$K_{BSM}^P(\alpha) = 1 - e^{-r_f}\left(\begin{array}{l} \int_{-\infty}^{\hat{z}_M}\left[\frac{Par_i}{B_{i0}}\left[1-\Phi\left(w_{i1}^q(z_M)\right)\right]\right]\phi(z_M)dz_M \\ \quad +\int_{-\infty}^{\hat{z}_M}\left[\frac{Q^q(z_M)}{B_{i0}}\left[1-\Phi\left(-w_{i1}^q(z_M)+\gamma_{i1}\right)\right]\right]\phi(z_M)dz_M \\ \quad\quad +\, par_F^P(\alpha)\left[1-\Phi(\hat{z}_M)\right] \end{array}\right) \tag{24}$$

The dollar value capital required is $\sum_{i=1}^{n} B_{i0}\, K_{BSM}^P(\alpha)$.

The BSM capital formula is for an asymptotic portfolio in which idiosyncratic risk is fully diversified. When an additional credit is added to this portfolio, the marginal capital required to maintain the target solvency margin is equal to the portfolio's average capitalization rate (expression (24)) multiplied by the market value of the marginal credit added to the portfolio. Expression (24) represents the minimum regulatory capitalization rate for both the average and the marginal credit in an asymptotic portfolio when credit risks are priced to satisfy BSM equilibrium conditions and capital is set to achieve a 99.9 percent solvency rate.

6. Minimum Capital Requirements Under the Basel II IRB Approaches

The June 2006 formula for calculating AIRB capital requirements for corporate, sovereign and bank exposures are $EAD \cdot K$, where K , is given by[16]

$$K = \left[LGD \times \Phi\left[\frac{1}{\sqrt{1-R}} \Phi^{-1}(PD) + \sqrt{\frac{R}{1-R}} \Phi^{-1}(.999) \right] - PD \times LGD \right] \left(\frac{1+(M-2.5)b}{1-1.5b} \right) \qquad (25)$$

where, $R = 0.12\left(\frac{1-e^{-50PD}}{1-e^{-50}}\right) + 0.24\left(1 - \frac{1-e^{-50PD}}{1-e^{-50}}\right)$, $b = (0.11852 - .05478\,Ln(PD))^2$,

PD is a credit's probability of default expressed as a percentage, LGD is a credit's expected loss given default expressed as a percentage, M is the credit's maturity measured in years, and K represents the percentage capital requirement per dollar of EAD exposure. When $M = 1$, the maturity adjustment, $\frac{1+(M-2.5)b}{1-1.5b} = 1$. If for any credit, $K<0$, regulatory

[16] See BCBS (June 2006b), page 64.

capital requirements are set to zero. FIRB capital requirements are calculated by using the AIRB capital requirement formula with *LGD* set at 45 percent.

Basel II IRB Model Inputs

Basel II guidelines include a discussion of the methods and data requirements that are acceptable for estimating the inputs for the IRB capital rules (maturity, *PD*, *EAD*, and *LGD*). In contrast to Basel IRB methods, the BSM approach for setting capital is parameterized by selecting a credit's maturity, par value, underlying assets' initial value, underlying assets' volatilities, the risk free rate of interest, and the market price of risk. For the calibration analysis that follows, it is useful to review the Basel guidance regarding the definitions for the IRB capital rule input values and to relate these credit risk characteristics to the underlying factors that drive the BSM capital rule.

Maturity (M)

BCBS (June 2006b) paragraph 320 defines *M* as, "the greater of one year and the remaining effective maturity in years," where the remaining effective maturity is the time weighted-average of the instrument's cash flows, $Min\left(\frac{\sum_{\forall t} t \cdot CF_t}{\sum_{\forall t} CF_t}, 5 \text{ years} \right)$, where CF_t is the instrument cash flow *t* periods into the future, where *t* is measured in years. The subsequent analysis will be restricted to one-year discount bonds. As such, $M = 1$, and the maturity adjustment factor does not enter into the subsequent calibration analysis.

Probability of default (PD)

For corporate and bank exposures, Basel II defines *PD* as the greater of a one-year *PD* estimate or 0.03%. One-year *PD* must be estimated using at least 5 years of data. For purposes of the BSM model, *PD* is defined by the BSM asset price dynamics. The value of a BSM bond may vary over its life, but it may only default at maturity. Under the maintained stochastic assumptions, the physical probability that a one-year BSM bond defaults at maturity is, $PD = \Phi\left(z_i^{df}\right)$ where,

$$z_i^{df} = \left(\frac{Log\left(Par_i\right) - Log\left(A_{i0}\right) - \left(r_f + \lambda\sigma_M - \frac{\sigma_M^2 + \bar{\sigma}_i^2}{2} \right)}{\sqrt{\sigma_M^2 + \bar{\sigma}_i^2}} \right) \tag{27}$$

Exposure at default (EAD)

Under Basel II IRB guidelines, *EAD* and *LGD* are inter-related; *LGD* is measured as a percentage loss relative to *EAD*. According to paragraph 474 of BCBS (June 2006b)

> "*EAD* for an on-balance sheet or off-balance sheet item is defined as the expected gross exposure of the facility upon default of the obligor. For on-balance sheet items, banks must estimate *EAD* at no less than the current drawn amount, subject to recognizing the effects of on-balance sheet netting as specified in the foundation approach."

The U.S. Basel II NPR (p.123) states, "*EAD* for the on-balance sheet component of a wholesale or retail exposure means (i) the bank's carrying value for the exposure (including accrued but unpaid interest and fees)…" Thus, for simple loans or bonds without any additional attached line of credit, Basel II requires that *EAD* must be at least as large as the current carrying value of the asset at the time that minimum regulatory capital is calculated.

For simple loan or bond positions, Basel II regulations do not include any further discussion; in particular, there is no discussion or guidance that suggests that *EAD* must be larger than the current carrying value of a simple fully drawn loan.

Loss given default (LGD)

Basel II requires *LGD* be measured a percentage of the *EAD*, with the following minimum requirements for *LGD* under the AIRB (BCBS, 2006b, paragraph 468):

> "A bank must estimate an *LGD* for each facility that aims to reflect economic downturn conditions where necessary to capture the relevant risks. This *LGD* cannot be less than the long-run default-weighted average loss rate given default calculated based on the average economic loss of all observed defaults within the data source for that type of facility."

The Basel II guidelines also do not set a lower bound on the *LGD* that banks can use in the AIRB approach for corporate credits.

Under the BSM model, the expected value of a one-year BSM bond's payoff at maturity, given that the bond defaults is,

$$E\left[Min\left(A_{i1}, Par_i\right) \mid A_{i1} < Par_i\right] = \frac{1}{\Phi\left(z_i^{df}\right)} \int_{-\infty}^{z_i^{df}} A_{i0} e^{\left(r_f + \lambda\sigma_M - \frac{1}{2}\left(\sigma_M^2 + \sigma_i^2\right)\right) + \sqrt{\left(\sigma_M^2 + \sigma_i^2\right)} z} \phi(z) dz \tag{28}$$

Expression (28) can be used to define *LGD*, but there is significant latitude as to how loss given default might be measured under Basel II. When *LGD* is calculated as a loss relative to a credit's current exposure (initial market value), LGD^{CE}, is,

$$LGD^{CE} = 1 - \frac{1}{B_{i0}\Phi\left(z_i^{df}\right)} \int_{-\infty}^{z_i^{df}} A_{i0} e^{\left(r_f + \lambda\sigma_M - \frac{1}{2}\left(\sigma_M^2 + \sigma_i^2\right)\right) + \sqrt{\left(\sigma_M^2 + \sigma_i^2\right)} z} \phi(z) dz \,. \tag{29}$$

Expression (29) is a current exposure measure of calculating *LGD* because it measures the default loss relative to the current exposure at the time that the capital allocation is being estimated.

Another measure of loss given default is the payment shortfall relative to the promised maturity value should a credit default. Define the future exposure measure of loss given default, LGD^{FE}, as loss measured relative to the full contract value at the end of the capital allocation horizon,

$$LGD^{FE} = 1 - \frac{1}{Par_i \Phi\left(z_i^{df}\right)} \int_{-\infty}^{z_i^{df}} A_{i0} e^{\left(r_f + \lambda\sigma_M - \frac{1}{2}\left(\sigma_M^2 + \sigma_i^2\right)\right)T + \sqrt{T\left(\sigma_M^2 + \sigma_i^2\right)}z} \phi(z) dz \tag{30}$$

Clearly, $LGD^{FE} > LGD^{CE}$.

Within the context of the Basel II framework, there is no theoretical preference for either measure of *LGD* as these models deal exclusively with the default process and are silent on the losses generated in default. As the current exposure method for calculating *LGD* is clearly acceptable approach under Basel II, and since LGD^{CE} also produces the lowest minimum regulatory capital requirements, it is reasonable to assume that LGD^{CE} will be the preferred definition among AIRB banks.

In the BSM capital allocation model (expression (24)), *LGD* is fully endogenous. Individual credit's *LGD*'s are random and individual credit's *LGD* realizations are correlated owing to the common factor that in part determines each credit's value in default. Expression (24) fully accounts for *LGD* correlation, and there is no need for calculating an explicit *LGD* estimate when calculating capital needs using the BSM model.

Table 1: Calibration Assumptions

risk free rate	$r_f = .05$
market price of risk	$\lambda = .10$
market factor volatility	$\sigma_M = .10$
Firm specific volatility	$\bar{\sigma}_i = .20$
Initial market value of assets	$A_0 = 100$
correlation between asset returns	$\rho = .20$

7. Implied Bank Default Risk under the Basel II IRB Approaches

In this section, economic capital allocations prescribed by the BSM model are compared to the minimum regulatory capital requirements set by the Basel II IRB approaches. The comparison is made for asymptotic portfolios with a wide range of risk characteristics. An underlying set of assumptions regarding asset price dynamics are maintained throughout the analysis; they are listed in Table 1. All individual credits are assumed to have identical firm specific risk factor volatilities of 20 percent. The common BSM factor has a volatility of 10 percent. The market-wide price of risk for exposure to the common factor is 10 percent. The risk free rate is 5 percent. These factor volatilities imply an underlying geometric asset return correlation of 20 percent.[17]

[17] When the bond *PD*s in Table 2 are input into the A-IRB approach, the correlation parameter, R, ranges from 13.6 (par value 70) to 22.7 percent (par value 55).

Capital requirements are calculated for 16 different asymptotic portfolios using the BSM model and two alternative specifications for the AIRB and FIRB approaches. The AIRB and FIRB capital calculations differ according to the definitions used for *EAD* and *LGD*. The 16 portfolios used in this comparison differ according to the credit risk characteristics of their constituent credits. Each portfolio is composed of credits with identical *ex ante* risk characteristics: all credits in an asymptotic portfolio have the same initial value, par value, initial value of supporting assets, and underlying asset volatility characteristics. The market-wide price of risk and the risk free interest rate are fixed for all portfolios analyzed.

Portfolios risk characteristics are altered by varying the par value of their constituent BSM discount bonds. Holding other things constant, a higher bond par value implies a greater *PD* and larger *LGD*. Consistent with Basel II requirements, the analysis focuses on a one-year horizon. We assume all credits mature in one-year to avoid the need to analyze the maturity adjustment. In this comparison, Basel II minimum capital requirements include expected loss since there is no scope for loan loss reserves in this one-period setting. The modified Basel II definition of K used in the comparison is,

$$K = \left[LGD \times \Phi\left[\frac{1}{\sqrt{1-R}} \Phi^{-1}(PD) + \sqrt{\frac{R}{1-R}} \Phi^{-1}(.999) \right] \right] \tag{31}$$

where R is specified using the regulatory correlation function given in expression (25).

The credit risk characteristics of the individual bonds that are used to construct the asymptotic portfolios are reported in the rows of Table 2. Individual bond *PD*s range from 23 basis points—for a bond with par values of 55, to 3.99 percent for a bond with a par value of

70. The LGD^{CE} characteristics (measured from initial market value) range from 1.40 percent to 3.28 percent. When *LGD* is measured on a future value basis, LGD^{FV} ranges from 6.22 to 8.34 percent. While the BSM model produces only modest *LGD*s relative to historical estimates of default losses on rated corporate bonds, the AIRB rule explicitly accounts for *LGD*, so *a priori*, there is no reason to expect that any specific set of *LGD* values may compromise the performance of the AIRB approach.

The results of the comparison are reported in Table 3 and plotted in Figure 1. The results show, depending on how *EAD* and *LGD* are defined, AIRB capital requirements may take on a wide range of values. The future exposure definition of *LGD* produces the largest AIRB capital measures, but these capital requirements still fall short of the capital needed to achieve a 99.9 percent solvency rate for all but the safest credits analyzed. When capital requirements are calculated using the AIRB approach and the current exposure measure of *LGD*, the true capital needed to achieve the 99.9 percent target solvency rate may be more than 5 times larger than the minimum capital set by the AIRB approach.

The AIRB capital shortfall arises in part because the capital rule is derived under the assumption that *LGD*s are not stochastic. Kupiec (2006) shows that when individual credit's *LGD*s are stochastic, and *LGD*s are correlated as they are for example in the BSM model, the AIRB rule will understate capital requirements. When *LGD*s are correlated, a portfolio has an additional source of systematic risk that is not diversified away in an asymptotic portfolio. This source of credit risk exposure requires additional capital that is not accounted for in the AIRB capital framework.

Table 2: Credit Risk Characteristics of 1-Year Credits

par value	initial market value	probability of default in percent	expected value given default	in percent: loss given default from initial value	in percent: loss given default from par value	in percent: yield to maturity
55	52.31	0.23	51.58	1.40	6.22	5.142
56	53.26	0.30	52.45	1.53	6.35	5.145
57	54.2	0.38	53.31	1.64	6.47	5.166
58	55.15	0.48	54.17	1.78	6.60	5.168
59	56.1	0.59	55.03	1.91	6.73	5.169
60	57.04	0.73	55.88	2.03	6.87	5.189
61	57.98	0.90	56.73	2.16	7.00	5.209
62	58.92	1.09	57.57	2.29	7.14	5.227
63	59.86	1.31	58.41	2.42	7.28	5.246
64	60.8	1.57	59.25	2.55	7.43	5.263
65	61.73	1.86	60.08	2.68	7.57	5.297
66	62.66	2.20	60.90	2.80	7.72	5.330
67	63.59	2.57	61.73	2.93	7.87	5.362
68	64.51	3.00	62.54	3.05	8.03	5.410
69	65.43	3.47	63.35	3.17	8.18	5.456
70	66.34	3.99	64.16	3.28	8.34	5.517

Source: Author's estimates of individual bond *PD* and *LGD* measures calculated using Black-Scholes-Merton model relationships and the assumptions in Table 1.

As Figure 1 shows, owing to the 45 percent *LGD* assumption, the FIRB approach dramatically increases capital requirements relative to the AIRB approach. For credits with probabilities of default less than about 1.6 percent, FIRB capital requirements provide relief relative to the 8 percent capital required by the 1988 Basel Accord. Notwithstanding capital reductions for some credits, for the portfolios examined in this analysis, the FIRB will set capital requirements that are many times larger than are needed to achieve the regulatory target default rate of 0.1 percent. Under the FIRB approach, use of the future exposure *EAD* measure further compounds the capital surplus

Table 3: Estimates of Portfolio Capital Requirements for 1-year Credits

bond par value	probability of default in percent	99.9 percent BSM capital in percent	FIRB capital current exposure EAD in percent	FIRB capital future exposure EAD in percent	AIRB capital current exposure LGD, EAD in percent	AIRB capital future exposure LGD,EAD in percent
55	0.233	0.396	2.733	2.874	0.086	0.400
56	0.298	0.487	3.241	3.407	0.110	0.479
57	0.379	0.593	3.747	3.941	0.136	0.565
58	0.476	0.715	4.297	4.519	0.169	0.660
59	0.593	0.854	4.822	5.072	0.206	0.761
60	0.732	1.011	5.400	5.680	0.245	0.869
61	0.896	1.187	5.999	6.312	0.287	0.980
62	1.088	1.384	6.573	6.917	0.334	1.097
63	1.311	1.601	7.146	7.521	0.384	1.217
64	1.568	1.839	7.733	8.140	0.439	1.343
65	1.862	2.098	8.307	8.747	0.494	1.472
66	2.196	2.379	8.906	9.381	0.554	1.608
67	2.574	2.681	9.499	10.009	0.619	1.751
68	2.997	3.005	10.138	10.687	0.687	1.906
69	3.469	3.348	10.797	11.386	0.761	2.069
70	3.992	3.712	11.497	12.131	0.839	2.249

Source: Author's estimates of alternative estimates of the capital required to achieve a 99.9 percent solvency margin for asymptotic portfolios composed of individual bonds with *PD* and *LGD* characteristics reported in Table 2. Basel II FIRB and AIRB capital requirements include expected losses as well as unexpected losses.

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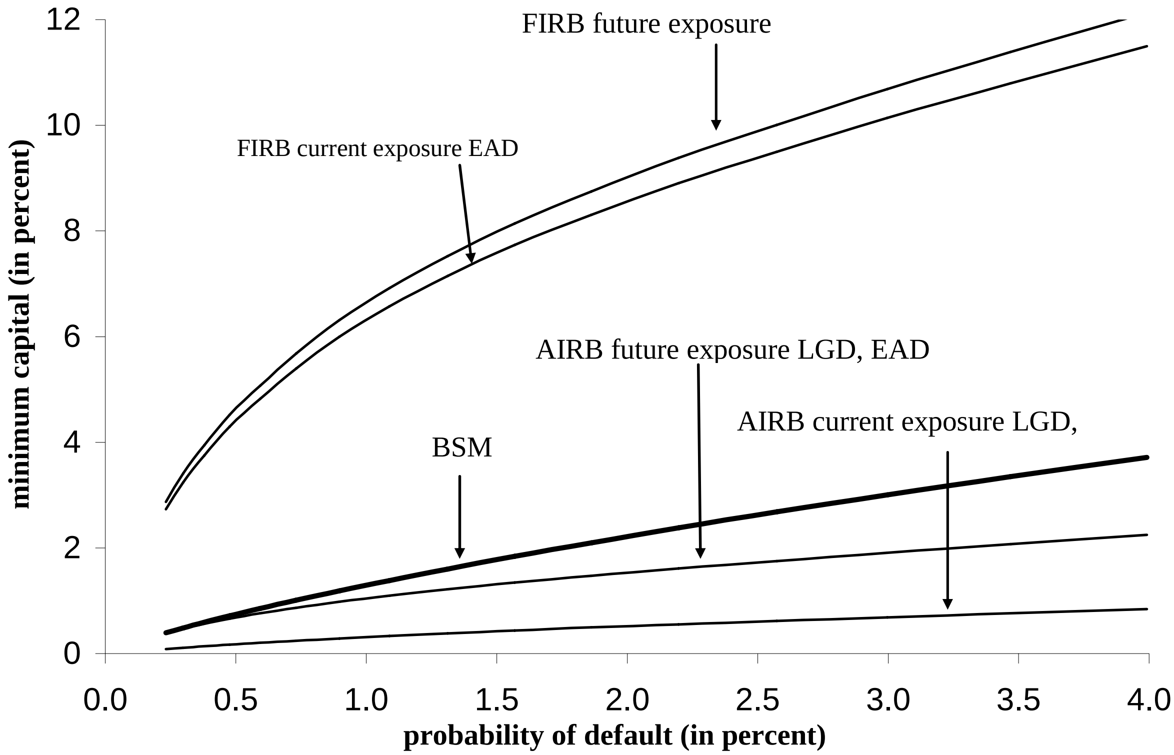
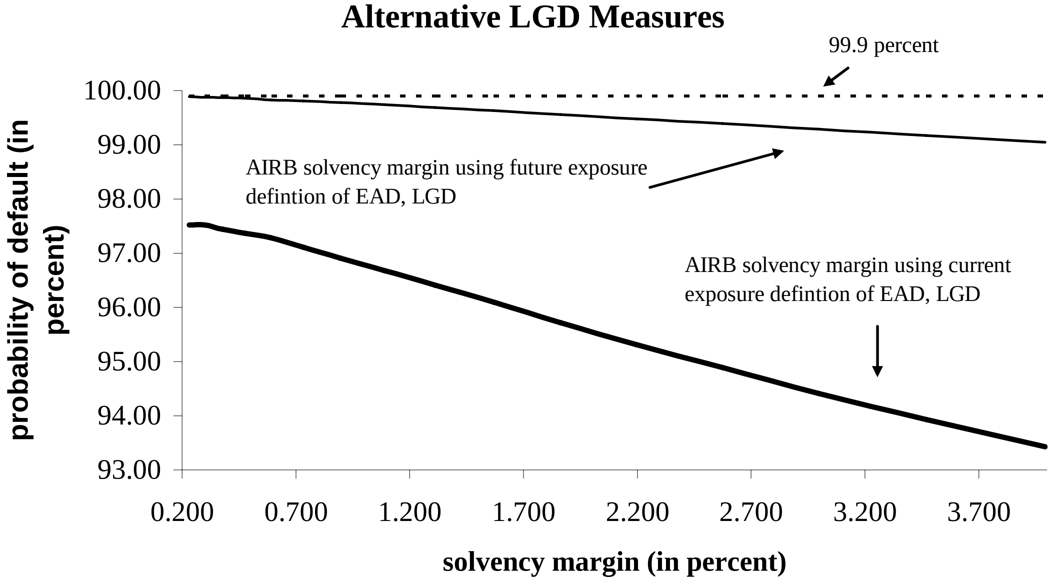
Figure 1: Comparison of Alternative Minimum Capital Estimates for 1-Year Credits



The BSM capital allocation rule can be inverted to recover the implied probability of default that will set under the AIRB capital rule. Figure 2 plots the solvency margins set by the AIRB framework when capital is set using both the current exposure and the future exposures measures for *EAD* and *LGD*. The solvency margin set by the AIRB rule depends on the risk attributes of the credit and the definitions used for the *EAD* and *LGD* inputs. For the credits examined in this analysis, using the current exposure measure for *LGD*, the AIRB solvency margin ranges from 97.5 percent to 93.4 percent. Irrespective of the *LGD* measure, the AIRB solvency margin declines as credit risk increases. The decline in the solvency margins owes in part to the regulatory specification for the correlation (R in expression (25)) that must be input into the IRB capital rules. Basel II uses a regulatory function to set the default correlation that is used in the IRB formula. This function assigns lower correlations to higher *PD* credits, which means that minimum capital requirements are reduced as *PD* increases. This built-in correlation feature results in regulatory correlation assignments that are above 20 percent for very low *PD* credits, and below 20 percent correlation for high *PD* credits. These assumptions produce the effective solvency margin pattern illustrated in Figure 2 where high *PD* credits are substantially undercapitalized relative to regulatory solvency targets.

Figure 2:Approximate Solvency Margins Under Basel II and Alternative LGD Measures



Discussion

No published study has attempted to measure the rigor of the prudential standard that will be set by Basel II capital regulations. Notwithstanding the BCBS's stated goal of promoting a prudential standard consistent with a 99.9 percent bank solvency rate, the Basel II Quantitative Impact Studies (QIS) and subsequent IRB model calibration adjustments have not focused on producing calibrations consistent with any specific target solvency margin. Basel II deliberations have produced IRB model calibrations that create incentives to promote AIRB adoption, but at the cost of significant diminution in the prudential standards that will apply to credit risk exposures in AIRB banks.

To gain perspective on the solvency standard set under the AIRB approach, consider for a moment that the AIRB approach default rates plotted in Figure 2 are approximately equal to the failure rate experienced by U.S. savings and loan institutions during the height of the 1980s S&L crisis. In 1988, the failure rate among insured savings and loans was 6.4 percent.[18] Over the 1980-1994 period, the annual compound average default rate of banks insured by the Federal Deposit Insurance Corporation was less than 1.2 percent, and even in the worst year (1988) of a period that has been characterized as a "banking crisis," the default rate on FDIC insured banks never exceed 2 percent.[19]

The use of the BSM model as a benchmark of comparison merits discussion because the model has well-known empirical shortcomings. Econometric studies suggest that, on average, the BSM model overprices corporate bonds (i.e., underestimates required bond

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[18] "History of the Eighties—Lessons for the Future," p. 168.

[19] See , "History of the Eighties—Lessons for the Future," p. 479.

yields). Empirical evidence indicates that the BSM bias is related to maturity and credit quality.[20] BSM overpricing errors are more severe on short-term high quality credits. In the context of this capital calibration exercise, the observed pattern of bias implies that the BSM model analysis in this paper will likely understate the true amount of capital that is required to support a credit risky portfolio because the bank's funding debt is likely to be overpriced by the BSM framework. Recognizing the shortcomings of the BSM model, true economic capital allocations are likely larger than the BSM model estimates suggest, and true AIRB capital shortfalls are likely more severe than indicated.

To the extent that banks enjoy safety-net engendered subsidies that are attenuated by minimum regulatory capital requirements, the Basel II IRB calibrations engender incentives that will encourage banking system assets to migrate toward AIRB banks. Asset migration could be achieved through consolidation or through an increase in the number of banks that are granted regulatory approval for the AIRB approach. If regulatory hurdles and the fixed costs associated with adopting AIRB compatible systems are high, only the largest banks will favor the AIRB approach. Absent liberal regulatory approval policies or declines in the cost of AIRB-compliant data and systems, strong economic incentives are in place to encourage industry consolidation into institutions that gain AIRB regulatory approval.

Basel II has created a new set of incentives that promote the migration of assets into AIRB banks to take advantage of a richer available safety net subsidy. The migration of assets into institutions that face a reduced prudential standard is a new form of regulatory

[20] See for example, Jones, Mason, and Rosenfeld (1984), Ogden (1987), or Eom, Helwege and Huang (2004).

arbitrage that is created by Basel II. This new avenue for regulatory arbitrage may lead to increased financial sector risks as even fully compliant AIRB banks are permitted to have high potential default rates unless market discipline on other regulations prohibit the realization of the full capital relief granted by the AIRB approach.[21]

8. CONCLUSIONS

Compared to capital requirements calculated using a full equilibrium structural model of credit risk, the Basel II AIRB approach substantially understates the capital that is required to achieve the regulatory target of a 99.9 percent bank solvency rate. Estimates suggest that AIRB banks may have default rates in excess of 5 percent on their corporate sovereign and bank credit portfolios and still meet the minimum risk-based regulatory capital requirements promulgated by the June 2006 the AIRB approach. In contrast, the FIRB approach may require far more capital than is necessary to meet the regulatory target solvency standard.

The analysis in this paper highlights important ambiguities in the Basel II definitions of *EAD* and *LGD*, and these ambiguities may lead to significant variation in the capital standards that apply across IRB banks. Current Basel II regulations are unnecessarily vague regarding the definitions of *EAD* and *LGD*. Different interpretations of these concepts can lead to vastly different minimum regulatory capital requirements under the AIRB approach.

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[21] For example, the leverage constraint under the U.S. system of prompt corrective action (12 U.S.C. Section 1831) may become the binding minimum regulatory capital requirement for many U.S. AIRB banks. See for example, the testimony of Donald E. Powell, Chairman Federal Deposit Insurance Corporation, on the Development of the New Basel Capital Accords before the Committee on Banking, Housing, and Urban Affairs. November 10, 2005, available at: http://www.fdic.gov/news/news/speeches/archives/2005/chairman/spnov1005.html

Regardless of the definitions employed, the AIRB framework must still be recalibrated to produce an increase in minimum capital requirements if the 99.9 percent solvency margin target is to be respected. The AIRB rule is not formulated to account for the systematic risk in *LGD* that arises naturally in any well-specified structural model of credit risk that includes a common risk factor. The recalibration of the AIRB rule must include a capital allowance for the systematic risk in *LGD* and require reformulated definitions of *EAD* and *LGD* that remove ambiguity and firmly establish the characteristics of the inputs used in the AIRB capital rule.[22]

Under the current Basel II formulation, banks that adopt the AIRB approach will gain substantial regulatory capital relief without a commensurate reduction in their potential risk profile. FIRB banks, in contrast, will face a much stricter prudential standard. This dichotomy creates strong economic incentives for banking system assets to migrate into AIRB banks. Since the analysis suggests that AIRB banks potentially carry higher default risk absent safety net support, the migration of banking system assets toward AIRB regulatory capital treatment is unlikely to enhance financial stability. Given the prudential weaknesses associated with the AIRB approach, the adoption of Basel II in its current form need not promote improved capital allocation practices in banks or reduce risk in the international banking system.

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[22] Kupiec (2006) also shows that the AIRB must include capital for the systematic risk in *EAD*s on portfolios of revolving credits. The analysis in this paper has not considered revolving credits, but instead has taken *EAD* to be fixed as it is in the case of a term loan or discount bond.

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