

The Pricing of Portfolio Credit Risk *

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Abstract

We find that *risk-neutral* asset return correlations implied by single-name credit default swap (CDS) spreads average 13%. By contrast, Moody's KMV estimate that the *physical* correlations of the same names average 24%. The CDS-implied correlations cannot account for observed prices of portfolio credit risk (ie CDS-index tranche spreads) but these prices are matched closely on the basis of the M KMV estimates. These findings underpin the two main conclusions of the paper: first, there seems to be inconsistency in the way the single-name and index markets price correlated default risk; second, there is little evidence in the sample for a correlation risk premium. In addition, we find that CDS index spreads are driven largely by the average levels of PDs and asset return correlations. While the impact of the estimated dispersion in PDs and pairwise correlations is relatively smaller, it is important for the differentiation of prices across index tranches. Furthermore, a parsimonious one-factor model of asset returns provides a good approximation for the purposes of pricing portfolio credit risk.

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1 Introduction

Portfolio credit risk has three key components: probability of default (PD), loss given default (LGD) and default correlation. From among the three, default correlation has received the least attention from academic researchers and market practitioners.¹ However, the analysis of this component of portfolio credit risk has become increasingly important owing to the recent rapid developments of innovative products in structured finance, including collateralized debt obligations (CDOs), CDO of CDOs (also known as CDO²), nth-to-default CDSs and CDS indices (see BCBS, 2004). All the numerical methods used for pricing these instruments (see Hull and White, 2004; Gibson, 2004) rely on estimates of default correlations but there is not consensus on how to obtain those estimates.

The literature has proposed three alternative approaches to estimating default correlations. The first, most direct, approach relies exclusively on default data (Daniels et al., 2005; Demey et al., 2004; Jarrow and van Deventer, 2005). Since defaults are rare events, however, the approach leads to large estimation errors, especially for portfolios consisting of investment grade entities. The second approach deduces default correlations from asset return correlations, which are estimated on the basis of the Merton (1974) framework and equity-market data. This approach delivers physical (or actual) asset return correlations, which differ from the risk-neutral correlations used for pricing to the extent that there is a premium for the risk that correlations might change in the future. Indeed, Driessen et al. (2005), who propose the third extant approach to estimating default correlations, rely on asset correlations estimated from option-prices and find that the risk-neutral correlations differ substantially from their physical counterparts.

In this paper, we adopt a new method for estimating asset return correlations, which relies on single-name credit default swap (CDS) spreads. The CDS market has developed rapidly since 2002 and has delivered several years of daily spreads associated with highly liquid contracts. Here, we focus exclusively on the CDS spreads of the companies that enter the investment-grade variety of the popular CDS index Dow Jones CDX North America 5-year (CDX.NA.IG.5Y).

Time series of the CDS spreads of these companies have implications for *risk-neutral*

¹The mainstream of the credit risk literature focuses on PD: see Duffie and Singleton (2003) for an overview. The growing literature on LGD includes Altman and Kishore (1996), Jarrow (2001) and Covitz and Han (2004).

asset return correlations, which we extract and employ as follows. We use single-name CDS spreads to derive daily time series of single-name risk-neutral PDs and, then, estimate the time path of asset returns. On the basis of the latter estimates, we calculate risk-neutral asset return correlations. Combining these correlations with the corresponding risk-neutral PDs, we conduct Monte Carlo simulations to obtain “CDS-implied” prices of portfolio credit risk. In addition, we use a copula framework and estimates of one-, two- and three-common-factor models of asset returns to derive alternative prices of portfolio credit risk. The match between these alternative prices, which are free of Monte Carlo simulation errors, and the CDS-implied prices sheds light on the number of common factors necessary to explain the joint behaviour of CDS spreads.

The Moody’s KMV estimates of *physical* asset return correlations, which are based the proprietary Global Correlation (GCorr) model (Das and Ishii, 2001; Crosbie, 2005), allow us to construct another set of prices of portfolio credit risk. We combine the GCorr correlations for the companies in CDX.NA.IG.5Y with the corresponding risk-neutral PDs, as implied by single-name CDS spreads, to obtain “GCorr-implied” prices of portfolio credit risk. Paralleling the exercise based exclusively on single-name CDS spreads, we also examine the number of common factors that are necessary to explain the GCorr estimates of asset return correlations.

The CDS-implied and GCorr-implied prices we derive can be compared directly to empirical tranche spreads of the CDS index CDX.NA.IG.5Y. We find substantial differences between the CDS-implied prices and the observed tranche spreads of the CDS index, which points to inconsistency in the pricing of correlated default risk across markets. As regards the senior (ie relatively safe) tranches for instance, the single-name CDS market implies prices that are 52% lower than the corresponding spreads observed in the data. By contrast, this deviation is roughly 5% for GCorr-implied prices, which match the data with similar precision over all index tranches. Since the GCorr model delivers physical asset return correlations, this finding suggests that the correlation risk premium is, at most, a negligible component of the prices of index tranches. The finding thus stands in contrast to the above-mentioned conclusions of Driessen et al. (2005), which are based on evidence from option prices.

We find that the main driver of CDS-implied and GCorr-implied prices is the average estimated level of pairwise correlations. For instance, the discrepancy between the average correlation implied by CDS spreads (13%) and the GCorr correlation (24%)

drives a pricing difference that fully explains the difference of the two predicted spreads for senior and super-senior tranches. Remarkably, the low average correlation embedded in single-name CDS spreads explains the bulk of the overshooting of observed equity tranche spreads and the undershooting of the senior tranche spreads by CDS-implied prices. By contrast, the average level of GCorr correlations, in conjunction with the estimated dispersion of risk-neutral PDs across names, leads to GCorr-implied prices matching the data well for all but the super-senior tranche.

We also examine the pricing impact of the average level of estimated PDs and the estimated dispersion in PDs and pairwise correlations. Importantly, a potential bias in our PD estimates cannot explain on its own the poor fit of CDS-implied prices to the data. Similarly, the dispersion in PDs and pairwise correlations falls significantly short of accounting for the discrepancy between CDS-implied and observed prices of portfolio credit risk. Nevertheless, this dispersion underpins the good performance of GCorr-implied prices across several tranches and sheds light on the so-called implied correlation smile.

The above findings are robust to a number of probable estimation errors. Alternative ways to proxy for asset return series on the basis of single-name CDS spreads have a small impact on asset return correlations and change little CDS-implied prices, even under different mappings from asset returns to default correlations. Likewise, noise from the Monte Carlo simulations has a negligible impact on the predicted tranche spreads. This is implied by our finding that a single common factor of asset returns is largely sufficient to account for both the CDS- and GCorr-implied prices. Specifically, a one-common-factor model, which allows for circumventing Monte Carlo simulations, delivers tranche spreads that deviate from their CDS-implied counterparts by 6% or less; this deviation is less than 1% for GCorr-implied spreads.

The remainder of the paper is organized as follows. Section 2 outlines the structure of the CDS index markets and explains the basics behind the pricing of index tranches. The following two sections explain different approaches to estimating prices of portfolio credit risk, as implied by the single-name CDS market (Section 3) and GCorr asset return correlations (Section 4). Section 5 describes our data and section 6 outlines our major empirical findings by comparing implied to observed prices of portfolio credit risk. Section 7 explains the driving forces behind implied prices and Section 8 examines the robustness of our empirical findings. The final section concludes.

2 The CDS index

In this paper, we consider one of the several existing products used for trading portfolio credit risk. Such products include collateralized debt obligations (CDOs), tranches of CDS indices and n-th to default CDSs.² We focus exclusively on the Dow Jones CDX North America investment grade 5-year index (CDX.NA.IG.5Y), which is reportedly the most popular CDS index. This five-year contract, which is written in standardized terms, is highly liquid in the secondary market and, thus, its trading is expected to reflect accurately the views of market participants regarding portfolio credit risk.

The portfolio underlying the CDX.NA.IG.5Y index is used to define five standardized index tranches, which are economically equivalent to the tranches of a synthetic CDO. The tranche carrying the highest level of credit risk is known as the equity tranche. If there has not been any default, the investor in this tranche (ie the protection seller) receives quarterly a fixed premium rate (known as the tranche spread) on the tranche's principal value, which is defined as 3% of the total notional principal of the index. If defaults occur, this investor is obliged to pay its counterparty (ie the protection buyer) an amount equal to the losses from default up to a maximum of 3% of the total notional principal of the index. At the same time, the principal value of the tranche, to which the premium rate is applied, is reduced accordingly to reflect the losses from default. Similarly, an investor in the so-called mezzanine tranche is responsible for losses between 3% and 7% of the total notional principal, while investors in the two senior and the super-senior tranches are responsible for losses between 7% and 10%, 10% and 15%, and 15% and 30% of the total notional principal, respectively.

2.1 The economics of tranche spreads

The main exercise of this paper consists of comparing tranche spreads observed in the data to tranche spreads implied by information on portfolio credit risk that is extracted from alternative (eg single-name CDS and equity) markets. Key components of this information are the risk-neutral PDs of the entities comprising a particular portfolio and the correlations of these entities' asset returns. Estimates of such PDs and correlations (described in the following sections) allow for calculating the probability distribution of the number of defaults in the portfolio.

²See Hull and White (2004) for a succinct description of these instruments.

Such a probability distribution, combined with data on losses given default and the risk-free rate, is what is needed to apply the methodology developed in Gibson (2004) and calculate *implied* tranche spreads for the CDX.NA.IG.5Y index.³ In most general terms, one first calculates the expected present value of the tranche principal EP_t and, then, the expected present value of contingent payments EC_t that are received by the protection buyer if defaults affect the tranche in focus. Denoting the tranche spread by s_t , the present value of the expected fee payments by the protection buyer, $s_t EP_t$, has to equal EC_t . Thus, the tranche spread is calculated as:

$$s_t = \frac{EC_t}{EP_t}$$

3 Prices of portfolio credit risk based on single-name CDS spreads

The two key components determining the spreads of CDS index tranches are (i) the set of risk-neutral PDs of the names in the index and (ii) the associated correlation matrix of asset returns. Estimates of these two components – which incorporate not only objective statistical relationships but also market views – reveal the extent to which their characteristics affect prices of portfolio credit risk. While there are well-established procedures for extracting risk-neutral PDs from CDS or bond market data, there is no consensus regarding the estimation of asset return correlations.

In this section we develop a method for pricing CDS index tranches on the basis of asset return correlations estimated from single-name CDS spreads.⁴ The procedure consists of three steps. First, we estimate a time series of *risk-neutral* PDs for each name in the CDS index, using data on CDS spreads and default recovery rates. Second, we use the so-obtained PD series to estimate the time path of asset returns for each name in the CDS index. This allows us to estimate the matrix of risk-neutral correlations of asset returns. In the third step, we use the estimated PDs and asset return correlations in a Monte Carlo exercise that delivers “CDS-implied” tranche spreads.

It is possible to avoid resorting to Monte Carlo simulations for CDS-implied tranche

³The probability distribution is calculated for different time horizons, which increase by one quarter and range from one quarter to five years.

⁴The CDS spread is widely considered as a better price of default risk than the bond spread, in that it responds more quickly to changes in credit conditions (Blanco et al., 2005; Zhu, 2004) and is less polluted by non-credit factors (Longstaff et al., 2005).

spreads by placing more structure on the asset return correlations. We do so by estimating a one-, two- and three-factor models of asset returns within a “stripped-down” and a Kalman filter setups (described below). These models have two advantages: they (i) imply tranche spreads that are free of noise stemming from Monte Carlo simulations and (ii) shed light on how many common factors are necessary to explain the joint behaviour of single-name CDS spreads. It should be kept in mind, however, that these models are polluted by errors inherent in the Kalman filter estimation.

3.1 CDS-implied PDs

In order to uncover risk-neutral PDs from CDS spreads, we adopt the simplified framework of Duffie (1999), which incorporates the following features of CDS contracts. The counterparties in a CDS contract are the buyer of credit risk protection and the seller of that protection. The protection buyer agrees to make constant periodic premium payments to the protection seller until the contract matures or a pre-specified credit event materializes, whichever happens first. If a credit event occurs during the life of the contract, the protection seller compensates the protection buyer with the difference between the face value of the defaulted entity’s debt issue and the recovery value.

To rule out arbitrage opportunities, the present value of CDS premium payments (expressed on the left-hand side of the next equation) has to equal the present value of protection payments (on the right-hand side):

$$s \int_0^T e^{-r_t t} \Gamma_t dt = (1 - RR) \int_0^T e^{-r_t t} q_t dt$$

where r_t stands for the risk-free interest rate, s denotes the CDS premium (also known as the CDS spread), q_t denotes the instantaneous risk-neutral default probability (also known as the risk-neutral default intensity) and $\Gamma_t \equiv 1 - \int_0^t q_s ds$ is the risk-neutral survival probability until time t . In addition, the face value of the reference entity’s debt is normalized to unity and $RR \in [0, 1]$ denotes the default recovery rate.

We adopt the standard simplifying assumptions that the risk-free rate and the default intensity are constant through time. These assumptions imply a closed-form solution for the default rate:⁵

$$q = \frac{as}{a(1 - RR) + bs}$$

⁵The same formula is used in Packer and Zhu (2005).

where $a = \int_0^T e^{-rt} dt$ and $b = \int_0^T te^{-rt} dt$.

The last equation indicates that data from the single-name CDS market, which contain CDS spreads and the corresponding recovery rates, as well data on the risk-free rate allow for calculating risk-neutral default probabilities. The CDS market data are described in Section 5.2.

3.2 CDS-implied asset return correlations

In order to calculate asset return correlations, we start by constructing the time path of assets via a simple mapping from a time series of PDs, which are calculated as described in Section 3.1. Suppose that the current date is t , default can occur only on the expiration date $T > t$ and entity i faces a threshold D_i and a risk-neutral probability of $PD_{i,T-t}$. Suppose further that, under the risk-neutral measure, that entity's date- T assets $V_{i,T} = f_i * V_{i,t} + \xi_{i,T}$, where f is a positive constant and $\xi_{i,T}$ is a standard normal shock unknown at time t but realized at time T . Then we know that:

$$D_i - f_i * V_{i,t} \equiv v_{i,t} = \Phi^{-1}(PD_{i,T-t}) \quad (1)$$

where Φ^{-1} is the inverse of the standard normal CDF.

The last equation implies that the time path of $v_{i,t}$ mimics the path of the assets of entity i . Thus, the risk-neutral asset correlation between entities i and j is given by $corr(\Phi^{-1}(PD_{i,T-t}), \Phi^{-1}(PD_{j,T-t}))$. However, we do not estimate the latter population characteristic directly. The reason is that all the PD series we obtain exhibit high persistence (ie $f_i \approx 1$ for all i), which suggests that the sample correlation between $\Phi^{-1}(PD_{i,T-t})$ and $\Phi^{-1}(PD_{j,t})$ is likely to produce spurious correlation coefficients. To address this issue, we estimate the correlation of asset *returns* as⁶:

$$\rho_{ij} = corr(\Delta v_{i,t}, \Delta v_{j,t}) = corr(\Delta \Phi^{-1}(PD_{i,T-t}), \Delta \Phi^{-1}(PD_{j,T-t})) \quad (2)$$

where Δ denotes the first difference.⁷

⁶In our estimation, we do not allow for variability in asset return correlations over time. The assumption of constant asset return correlations is strong in principle but does not seem to be important in the context of our data sample. A recent study by Daniels et al. (2005) provides evidence that asset correlations change little over time. We also obtain indirect supporting evidence, which we report together with our other empirical findings.

⁷Note that ρ_{ij} equals exactly the correlation of asset returns (and $\rho_{ij} = corr(\xi_{i,T}, \xi_{j,T})$) only in the unit root scenario: $f_i = f_j = 1$. The Phillips-Perron unit root test, which allows for serial correlation in $\xi_{i,T}$, cannot reject the null hypothesis of unit root for 132 of the 136 time series in our sample. In

The construction of time paths of asset values (ie $v_{i,t}$) is based on several simplifying assumptions, which were made without loss of generality. The assumption that the shock $\xi_{i,T}$ has a zero mean and unit variance is clearly inconsequential since, for calculating correlation coefficients, it suffices to estimate any time-invariant affine transformation of asset returns. Another assumption, which seems important, is the premise that a default can occur only on the expiry date T . However, we can allow for default to occur at any point in time until date T by re-interpreting the shock $\xi_{i,T}$ as a random variable that falls below the “augmented threshold” $v_{i,t}$ with the probability that assets fall below the “true” (unobserved) default threshold between dates t and T . In turn, $v_{i,t}$ should be interpreted as a variable allowing to calculate $P_{i,T-t}$ on the basis of all the information available at date t . Indeed, these general interpretations do underlie our calculation of CDS-implied prices of portfolio credit risk.

3.3 Estimating common-factor models of asset returns

When pricing CDS index tranches, it is possible to use asset return correlations directly but this requires resorting to Monte Carlo simulations. The advantage of using the Monte Carlo simulation method is that it does not impose any restriction on the structure of the correlation matrix. Nevertheless, the computational burden is high and the correlations could be contaminated by data noise. To circumvent these problems we undertake two approaches to imposing structure on asset return correlations, which allows one to employ the copula method for pricing index tranches. The first, stripped down, approach requires only a correlation matrix and makes no assumptions regarding the statistical properties of common factors, which affect the asset returns of a group of borrowers, and idiosyncratic factors, which explain the behaviour of asset returns that is unaccounted for by common factors. The second approach, a straightforward application of the Kalman filter, relies on estimates of time series of asset returns, imposes distributional assumptions on the common and idiosyncratic factors and estimates the relative importance of these factors together with the dynamics of their underlying stochastic processes.

addition, setting $f_i = 1$ leads to a reasonable approximation of the dynamics of the other 4 cases.

3.3.1 A Stripped-down approach

To implement this approach for a cross section of N entities, we need the matrix of asset return correlations ρ_{ij} . Then we postulate that these correlation coefficients are underpinned by F common factors $M_t = [M_{1,t}, \dots, M_{F,t}]'$ and N idiosyncratic, or entity-specific, factors $Z_{i,t}$, which affect asset returns $\Delta v_{i,t}$ as follows

$$\Delta v_{i,t} = A_i M_t + \sqrt{1 - A_i' A_i} Z_{i,t} \quad (3)$$

where $A_i \equiv [\alpha_{i,1}, \dots, \alpha_{i,f}, \dots, \alpha_{i,F}]$ is the vector of common factor loadings, $\alpha_{i,f} \in [-1, 1]$ and $\sum_{f=1}^F \alpha_{i,f}^2 \leq 1$. All common and idiosyncratic factors are assumed to be mutually independent. We also postulate, without loss of generality, that all factors have zero means and unit standard deviations.

We estimate the loading coefficients $\alpha_{i,f}$ ($i = 1, \dots, N$, $f = 1, \dots, F$) by minimizing the mean squared difference between the factor-implied correlation and the target correlation:⁸

$$\min_{A_1 \dots A_F} \sum_{i=1}^N \sum_{j \neq i} (\rho_{ij} - A_i A_j')^2$$

3.3.2 A Kalman filter approach

To implement a Kalman filter, we use time series of asset returns, which are denoted by $\Delta v_{i,t}$ and estimated as outlined in Section 3.2. The joint behaviour of asset returns is assumed to be driven by common and idiosyncratic factors, as specified in equation (3).

Compared with the stripped-down approach, the Kalman filter specification provides a better reflection of the time series property embedded in the data, and allows for estimating the dynamics of common and idiosyncratic factors. But this is at the expense of a distributional requirement. In particular, it is necessary to assume that all the factors are distributed normally. Given that assumption, we allow for serial correlation in each common factor but assume that the idiosyncratic factors are white noise.

Greater detail on the Kalman filter maximum-likelihood estimation is relegated to

⁸For each initial guess, a local minimum can be obtained by the application of a multi-dimensional constrained optimization algorithm (Andersen et al., 2003). We implement 10,000 random initial values to ensure that the solution is a global minimum.

Appendix A. The vectors of estimated common factor loadings $\alpha_{i,f}$ ($i = 1, \dots, N$, $f = 1, \dots, F$) imply directly the pairwise correlation coefficients:

$$\text{corr}(\Delta v_{i,t}, \Delta v_{j,t}) = A_i A_j'$$

3.4 CDS-implied spreads of CDS-index tranches

We use our estimates of risk-neutral PDs and asset return correlations (based on single-name CDS spreads) to derive CDS-index tranche spreads. Without imposing structure on the asset return correlations, we need to resort to Monte Carlo simulations for estimating a key determinant of tranche spreads: the probability distribution of the number of defaults in the portfolio underlying the CDS index.⁹ By contrast, if we use a common-factor model of asset returns, we calculate this probability distribution by employing a Gaussian copula.¹⁰ Having obtained an estimate of the probability distribution of the number of defaults, we follow the approach outlined in Section 2.1 to derive CDS-implied tranche spreads.

4 GCorr-implied spreads of CDS-index tranches

Moody's KMV estimates of *physical* asset return correlations can be used to construct another set of tranche spreads. These estimates are based on the proprietary GCorr model, which delivers asset return correlations between any two names in the MKMV database. MKMV estimates the correlations in two steps. In the first step, asset returns are extracted from equity returns on the basis of an option pricing model, data on contractual liabilities and information about firms' size, industry, profitability and geographical location. The second step estimates the exposure of each entity to 120 common factors (see Das and Ishii, 2001; Crosbie, 2005): 2 global economic factors, 5 regional economic factors, 7 sector factors, 61 industry-specific factors and 45 countries-specific factors. Once this estimation is carried out, the pairwise asset return correlation can then be easily calculated from the loading coefficients on the common factors.

With the last observation in mind, we follow the approach outlined in Section 2.1 to construct GCorr-implied tranche spreads on the basis of GCorr asset return correla-

⁹The simulations exercise is described in Appendix B.1.

¹⁰The Gaussian copula exercise is outlined in Appendix B.2.

tions and risk neutral PDs.¹¹ These PDs are extracted from single-name CDS spreads and are constructed as explained in Section 3.1. Paralleling our estimates of CDS-implied tranche spreads, we also construct GCorr-implied tranche spreads on the basis of common-factor models of GCorr correlations. This exercise uses the stripped-down algorithm outlined in Section 3.3.1.

To the extent that investors require a premium for bearing correlation risk, the physical GCorr correlations should not allow for an accurate match of empirical prices of portfolio credit risk.

5 Data

The data we use in this paper can be divided in three big blocks. The first block consists of tranche spreads for the CDS index CDX.NA.IG.5Y. The second block consists of data from the single-name CDS market, which are at the root of our estimates of risk-neutral PDs and asset-return correlations. The third block consists of GCorr correlations. In addition, we obtain 5-year Treasury rates from Bloomberg in order to proxy for the risk-free rate of return (Figure 1).

5.1 Data on tranche spreads

The CDX.NA.IG.5Y index consists of 125 investment-grade North American entities that represent major industrial sectors and are actively traded in the single-name CDS market as well. Each entity has the same share in the total notional principal of the index. The index was introduced on November 13, 2003, and has been updated semi-annually to reflect events such as defaults, rating changes and mergers or acquisitions. These updates have resulted in four releases of the index.

The tranche spreads we use in this paper are provided by JP Morgan Chase. The data include daily spreads for five tranches (from equity to super-senior) of the “on-the-run” CDX.NA.IG.5Y index. We consider the first three releases of the index that were launched on 13 November 2003, 23 March 2004 and 21 September 2004, respectively. Owing to credit and market events causing exits from and entries into the index, we consider 136 constituent names in total.

¹¹In addition, we use the LGD estimates described in Section 5.2.

5.2 Data from the single-name CDS market

The single-name CDS data are provided by Markit, which has constructed a network of industry-leading partners who contribute information across several thousand credits on a daily basis. Using the contributed quotes, Markit calculates the daily CDS spreads for each credit in its database as well as the daily recovery rates used to price the contract. In the light of the composition and contractual terms of the CDX.NA.IG.5Y index, we use a times series of 5-year senior unsecured CDS spreads associated with the no-restructuring clause (see ISDA, 2003) and denominated in US dollars. We consider each of the 136 reference entities that belonged to the CDS index at any point in time between 13 November 2003 and March 20 2005. In order to work with time series of equal length, we use single-name CDS spreads from April 24, 2003 to September 27, 2005 (for a total of 634 business days).

The default recovery rates provided by Markit vary little both in the cross-section and over time (see Table 1 and Figure 1). Considering the cross section of time averages, we obtain the 1st and 99th percentiles of recovery rates to be at 36.8% and 40.3% respectively. Likewise, the daily average recovery rates fluctuate within a narrow band: between 37% and 40%. In order to eliminate potential noise in these data, we set the recovery rate to be the same across entities on each day and smooth the time series of recovery rates via an HP filter (Figure 1).¹²

5.3 Data from Moody's KMV

Moody's KMV update monthly their estimates of the GCorr model. Each estimate provides the physical correlation of asset returns for the firms in Moody's KMV rating universe. We use the March 2005 estimate of the GCorr pairwise correlations for the 136 firms that belonged to the CDX.NA.IG.5Y index at any point in time between 13 November 2003 and March 20 2005.

6 Empirical findings

In this section we discuss the CDS- and GCorr-implied prices of portfolio credit risk, which we calculate as described in Sections 3 and 4. The two sets of prices consist of implied spreads for the CDX.NA.IG.5Y index and can be compared directly to the

¹²We set the HP filter parameter λ to 64000.

actual (ie observed) tranche spreads of this index. Our main conclusions are based on Figure 2, which plots the time series of the alternative spreads, one tranche at a time, and Table 2, which provides summary statistics.

The observed, CDS- and GCorr-implied tranche spreads exhibit similar patterns over time. In particular, the observed tranche spreads increase during the first months of 2004 and are on a downward path thereafter. This pattern is mirrored closely by the implied spreads, with the exception of June and July 2004. Importantly, spreads across *all* index tranches exhibit similar time paths. This suggests that these paths are unlikely to be driven by changes in market perceptions regarding asset return correlations, as such changes have opposite effects on the spreads of equity and senior tranches. (This point is explained further below.) Instead, the intertemporal pattern of tranche spreads seems to be driven mostly by risk-neutral PDs, which decline in late 2003, rise in early 2004 and decline after September 2004 (see Figure 3).

A comparison across the levels of spreads unveils significant discrepancies between observed and CDS-implied spreads. To be sure, the implications of the single-name CDS market seem to be largely in line with the data for the equity and mezzanine tranches, at which the average pricing discrepancies are 8.1% and 1.6%, respectively, of the observed spreads. At the same time, however, CDS-implied spreads undershoot substantially the actual spreads for the senior and super-senior tranches: by 43, 30 and 12 basis points (or, in relative terms, by 38%, 66% and 94%), respectively.

By contrast, GCorr-implied spreads match the data closely across four of the five tranches. On average over time, these spreads deviate from the corresponding observed spreads by less than 9% for the equity, mezzanine and two senior tranches. The super-senior tranche provides an exception to the general picture with a discrepancy of 54%, which is, nonetheless, twice as small as the corresponding discrepancy between observed and CDS-implied super-senior tranche spreads.

The above comparison across the levels of observed and implied spreads suggests inconsistency in the way the index and single-name CDS markets incorporate correlated credit risk into prices. This conclusion is based on (i) the relative success with which CDS- and GCorr-implied spreads match the data, (ii) the fact that any difference between the two sets of implied spreads is due, by construction, to differences in the underlying asset return correlations and (iii) the premise that GCorr correlations reflect accurately perceptions of asset return correlations in the index market. If that premise

is true, then the poor match between CDS-implied and observed spreads is evidence of market segmentation, whereby the focus on single-name default risk may lead to independent pricing across names, which reflects poorly probabilities of joint defaults. The premise need not be true, however, if non-credit factors (eg administrative costs or a liquidity premium), which do not enter the calculation of implied spreads, inflate observed spreads. While non-credit factors are indeed likely to have a substantial effect at the super-senior tranche, which carries very low credit risk, it seems a stretch to claim that they are a major driving force for the spreads of the riskier senior tranches. Instead, the poor performance of the CDS-implied spreads at the senior tranches may reflect the fact that the calculation of these spreads does not incorporate information regarding the pricing of protection against catastrophic events (ie when 12% or more of the investment-grade entities in the index default). Such information pertains to the tails of assets' risk-neutral distributions and its extraction is beyond the scope of this paper.

To the extent that GCorr correlations reflect accurately market perceptions of asset return correlations, the close match between GCorr-implied and observed index spreads suggests that the correlation risk premium is quite small in the CDS index market. The GCorr model delivers *physical* asset return correlations, which should imply too low index spreads if the index market prices in a compensation for uncertainty in these correlations. As suggested by Table 2 and Figure 2, this does not seem to be the case, in sharp contrast to the conclusions of Driessen et al. (2005), who find strong evidence for a correlation risk premium in the option market. Section 7.2 provides further support of our results by comparing GCorr correlations to the level of asset-return correlations necessary for matching exactly observed spreads.

7 Explaining the implied tranche spreads

While the CDS-implied spreads do not match closely the observed spreads of the CDX.NA.IG.5Y index, especially at senior tranches, the match is improved considerably when one considers GCorr-implied spreads. In this section, we attempt to explain these results by focusing on the two main inputs into the pricing of portfolio credit risk: individual PDs and asset return correlations.

7.1 Impact of the average PD

Our first exercise is to examine the pricing implications of bias in our estimates of risk-neutral PDs. Thus, we calculate implied spreads for different average levels of PDs (keeping the dispersion in PDs, as well as all the other parameters, as originally estimated) and illustrate the results in Figure 4. Since higher PDs are tantamount to increased credit risk, higher PDs lead unambiguously to higher spreads for all index tranches. In quantitative terms, moderate bias in PDs can have a sizeable impact on tranche spreads: for example, a 5% change in the average PD causes the mezzanine tranche spread to change by roughly 9%. At the same time, errors in the estimate of the average PD due to outliers have negligible pricing implications: setting the average PD to equal the median of the individual PDs¹³ increases all tranche spreads by less than 2%.

Most importantly, however, a bias in our estimates of risk-neutral PDs cannot account for the deviations of CDS- or GCorr-implied spreads from the data at *all* tranches. This is so because, on the one hand, all tranche spreads increase in the average PD, while, on the other hand, CDS-implied (GCorr-implied) spreads overshoot observed spreads for the equity and mezzanine (mezzanine) tranches but undershoot for the other tranches. Thus, eliminating a hypothetical bias in our PD estimates in order to match exactly observed spreads for a particular index tranche would lead to a larger pricing discrepancy at another tranche.

7.2 Impact of the average correlation

We also examine the pricing implications of a potential bias in the estimates of asset return correlations. To this end, we recalculate the CDS- and GCorr-implied index spreads for alternative average correlation coefficients, keeping all the other parameters as originally estimated and illustrate the results in Figure 5.¹⁴

Figure 5 illustrates the standard qualitative result that a change in correlations that lowers the spread for a given tranche increases the spread for other tranches. The intuition behind this result has been discussed in numerous papers (see for example

¹³Table 1 provides descriptive statistics of our estimates of PDs.

¹⁴This exercise removes the cross sectional dispersion in correlation coefficients, because there is no clear way to change average correlations without affecting the structure of the correlation matrix. It will be shown later (section 7.4) that this abstraction only has second-order pricing implications.

Belsham et al., 2005; Amato and Gyntelberg, 2005) and can be seen by considering two extreme cases: a fully diversified portfolio (correlation of 0) and a portfolio of perfectly correlated entities (correlation of 1). A switch from the first to the second portfolio increases the probability of defaults *en masse* but also increases the probability of no defaults. Thus, such a switch lowers the spread for the equity tranche (which is relevant only for the first defaults) but increases the spread for the senior tranches (which are relevant at high default rates). By contrast, when one moves from a well-diversified to a non-diversified portfolio and considers the mezzanine tranche, the two forces at work counteract each other and the overall impact is ambiguous. This is what Figure 5 illustrates.

Quite importantly, the bulk of the differences between CDS- and GCorr-implied spreads is explained by the differences in the underlying average correlations. Assuming no cross-sectional dispersion in pairwise correlations but increasing their level from 13% (the average CDS-implied correlation) to 24% (the average GCorr-implied correlation) lowers the equity tranche spread by 390 basis points, which equals 137% of the difference between CDS- and GCorr-implied equity tranche spreads. This share stands at 248%, 137%, 114% and 93% for the other tranches, from mezzanine to super-senior, respectively.

In addition, the sensitivity of tranche spreads to the level of asset return correlations has a direct bearing on the existence of a correlation risk premium in the CDS-index market. To see this, one needs to first observe that, as portrayed in Figure 5, perturbations from realistic asset correlation levels (i.e. below 60%) have unambiguous impacts on the spreads for the equity, senior and super-senior tranches. Consequently, the pricing of correlation risk can explain why the GCorr-implied spreads tend to undershoot the observed spreads for these tranches, provided that the GCorr correlations reflect accurately market perceptions of *physical* distributions.¹⁵

To quantify the correlation risk premium, we focus on one tranche at a time and deduce the constant correlation coefficient that implies an exact match of the average observed spread, while all the other parameters are kept unchanged. It turns out that, for the equity, the two senior and the super-senior tranches, that correlation coefficient

¹⁵By contrast, the impact of a correlation risk premium on the mezzanine tranche spread is ambiguous, as the relationship between the correlation level and the spread for this tranche is not monotone.

equals 20%, 23%, 26% and 32%, respectively.¹⁶ The distance between these values and the average GCorr correlation of 24% provides a measure of the correlation risk premium. Thus, this premium is seen to be much smaller than the one deduced by Driessen et al. (2005) who calculate an 18-percentage-point difference between the risk-neutral and physical correlations on the basis of option-market data.

7.3 A Lesson from implied correlations

The analysis in Sections 7.1 and 7.2 reveals that the *levels* of PDs and asset return correlations are the main drivers of the close fit between observed and GCorr-implied spreads for the equity, mezzanine and senior tranches. As seen above, the level of estimated PDs is crucial for avoiding a *consistent* bias in implied spreads across tranches, while the level of correlations allows for a close match between implied spreads and data for *each* particular tranche.

One should note, however, that in making these observations we have abstracted from the pricing implications of the cross-sectional dispersion in PDs and correlation coefficients. To test whether we have abstracted from an empirically important point, we consider the well-known “implied correlations”, which are deduced (following Hull and White, 2004) from observed index spreads on the assumption that PDs and pairwise correlations do not vary in the cross section. For each tranche and date in our sample, an implied correlation is defined as the correlation that delivers the same spread as the observed one, assuming that all PDs equal the estimated average for that day.¹⁷ As portrayed by Figure 6, implied correlations decrease (from 18% to 10%, on average) as one switches from the equity to the mezzanine tranche and then increase (to 21.6%, 23.7% and 30.2%) with the seniority of the tranche: ie they exhibit the standard smile found in the literature.¹⁸

If cross-sectional dispersions in PDs and correlations do indeed have negligible pricing implications, then the average GCorr correlation should be close to the implied

¹⁶The difference between the average GCorr and the “exact match” correlations is highest for the super-senior tranche. This comes as no surprise given that the GCorr-implied spreads provide the poorest match of the data exactly for that tranche, which, as argued above, is likely to be influenced disproportionately by non-credit factors.

¹⁷Since the relationship between correlations and tranche spreads need not be monotone, there might be multiple or no solutions for implied correlations. In our sample, such a problem arises only for the mezzanine tranche. When there is no solution, we do not report an implied correlation. When there are multiple solutions, we pick the one that limits the volatility of the implied correlations over time.

¹⁸See Amato and Gyntelberg (2005).

correlations for all tranches. The implied correlation for the mezzanine tranche illustrates most starkly that this is not the case. In the summer of 2004, for example, the implied correlation for that tranche is lower than the GCorr correlation by about 20 percentage points: a difference that should change the tranche spread by 140 basis points according to the sensitivity results reported in Figure 5. By contrast, the difference between observed and GCorr-implied spreads is much smaller, at about 85 basis points. This result suggests that dispersion in PDs and asset return correlations, which have been ignored so far in the analysis of empirical findings, might have important pricing implications.

7.4 Impact of dispersion in PDs and correlations

Estimates of correlation coefficients and risk-neutral PDs exhibit substantial variation in the cross section. As shown in Table 1, which provides summary statistics of averages over time, the standard deviation of PDs in the cross section equals 50 bps. In addition, the entity carrying the highest level of single-name credit risk has an average PD that is more than 10 times larger than the PD of the least risky entity. In turn, pairwise correlation coefficients vary between -0.5692 and 0.7962, when implied by the single-name CDS market, and between 0.0464 and 0.65, when implied by the GCorr model.

Hull and White (2004) report that dispersion of PDs and correlations coefficients could have significant pricing implications for the CDS index tranches. We quantify these implications in the context of our data set by perturbing the CDS-implied and GCorr-implied spreads (reported in Figure 2) in three different ways. First, we calculate implied spreads by setting all individual PDs to their cross-section average in each day but keep all the other parameters intact. Second, we repeat the first exercise with pairwise correlations taking the place of individual PDs. Third, we calculate implied spreads after eliminating the cross-sectional variation in both PDs and asset return correlations. The results are plotted in Figure 7 and summarized in Table 4.B-4.D for the GCorr-implied spreads. To save space, we report the results for CDS-implied spreads only briefly in Table 4.B-4.D.

The dispersion in PDs and correlations has a smaller pricing effect than the levels of these parameters but does help to explain further the close match between GCorr-implied and observed tranche spreads. In particular, removing dispersion in PDs and/or correlations falls significantly short of explaining the poor match between these spreads

and the data. Likewise, the dispersion in GCorr correlation has a negligible impact on GCorr-implied spreads. Interestingly, however, removing the dispersion in individual PDs worsens the fit of GCorr implied spreads for all tranches, except for the super-senior one. This worsening can be as high as 300% for the more risky senior tranche in mid-2004.

As explained by Hull and White (2004), dispersion in PDs affects tranche spreads via two channels. To understand the first channel, it is useful to think of a portfolio consisting of two independent entities. In this setup, changing the difference between the two PDs while keeping their average constant is analogous to changing the area of a rectangle while keeping the total length of its sides constant. Just as the area of the rectangle is maximized when its sides are equal, the probability of joint defaults is maximized when the two PDs are the same. This logic can be extended to any number of entities in the portfolio to see that dispersion in PDs lowers the probability of defaults *en masse*. In addition, it is easily seen that the probability of no defaults is independent of the dispersion in PDs. The bottom line is that, when a CDS index consists of independent entities, increasing the dispersion in their PDs would tend to raise tranche spreads, with the impact increasing in the seniority of the tranche.

The second channel, via which dispersion in PDs affects prices, is seen most clearly if one considers a portfolio of perfectly correlated entities. In such a setting, the probability of at least one default equals the highest PD in the cross section, whereas the probability of defaults *en masse* depends positively on the lower PDs in the cross section. Thus, increasing the dispersion in PDs renders the equity tranche riskier and the senior tranches less risky but can have ambiguous effects for “intermediate” tranches.

The cyan lines in Figure 7 illustrate the combined implications of these two channels. Consistent with the provided intuition, dispersion in PD raises the spread for the equity tranche but lowers the spreads for all the other tranches.

The pricing implications of dispersion in PDs prompt us to reconsider the so-called correlation-smile puzzle, which is illustrated in Figure 6 and is based on the assumption that PDs do not vary in the cross section. There is a puzzle, because, contrary to what that figure illustrates, it seems strange that different index tranches should be priced on the basis of different asset return correlations. Acknowledging that the puzzle can, in principle, be due to market segmentation across tranches or to non-credit factors, we propose an alternative explanation via Figure 8. We construct that figure in two steps.

In the first step we extract five sets of index tranches from the time series plotted in Figure 6: each set is for June 30, 2004; one set consists of the observed tranche spreads, one consists of CDS-implied tranche spreads when both PDs and correlations vary in the cross section, and the remaining three sets correspond to scenarios in which variability in PDs and/or correlations is shut off. In the second step, we calculate the implied correlation coefficients, assuming that each spread is priced on the basis of PDs and correlations that do not vary in the cross section.

The main message of Figure 8 is that differences in implied correlation coefficients across tranches may be largely due to the unjustified underlying assumption that PDs and correlation coefficients do not change across the entities in the portfolio. It is obvious from the methodological descriptions in Section 4 that the GCorr-implied spreads rely on the same set of correlation coefficients for all tranches. Nevertheless, assuming that these correlation coefficients and the associated PDs do not change in the cross section produces the implied correlation smile depicted with a blue line, which comes close to the correlation smile implied by the observed tranche spreads (in red line). Spreads calculated after shutting off either the variability of PDs or that of correlations also lead to implied correlation smiles (the dashed green and black lines), indicating that dispersion in both sets of parameters has important pricing implications for the mezzanine tranche.

8 Robustness checks

The rapid growth of credit derivatives markets has spurred the development of various numerical methods for pricing purposes. Such methods include the copula method, which exploits the fact that asset return correlations are driven by firms' exposures to common factors, and a multi-period Monte Carlo simulation, which allows for default at any point in time before the contract's maturity date. In this section, we examine whether our empirical findings (reported above) could paint a misleading picture if market participants adopt alternative numerical methods in the pricing process.

8.1 The common factor structure of asset return correlation

It is standard practice for market practitioners to adopt a common-factor model in order to price structured finance products, including CDS indices, nth-to-default CDS

and CDOs. When the dependence of asset values on common factors is estimated, the Gaussian copula (Appendix B.3) provides an efficient algorithm for calculating prices of portfolio credit risk.

To employ this algorithm, one needs to decide on the number of common factors and determine the coefficient with which each asset value loads on each common factors. We use a Kalman filter framework (Section 3.3.2) to estimate the unobserved common factors and the associated loading coefficients in the single-name CDS market, and employ the stripped-down specification (Section 3.3.1) to extract the factor loading structure underlying the GCorr correlations.¹⁹ Then, we use the factor-loading structure to determine tranche spreads, which sheds light on the relevance of common-factor models for pricing purposes.

Our Kalman filter results, based on the CDS data between April 24, 2003 and September 27, 2005, are summarized in Table 3.A-3.C. In addition, Figure 9 plots the histogram of the differences between (i) the correlations implied directly by single-name CDS spreads and used in the main part of the paper and (ii) the correlations based on one-, two- and three-common-factor models. For each of the three CDS indices, the one-factor model performs well in matching average correlations but tends to under-estimate the dispersion in correlation coefficients and fails to explain the skewness and kurtosis. This suggests that one factor alone is not capable of fully generating the heterogeneity of constituent entities in the CDS market. When the number of common factor increases, the result improves substantially. In particular, the 3-factor model appears to perform well in explaining both the mean and the higher moments of correlation coefficients: the mean squared deviation decreases from 0.083 for the 1-factor model to 0.06 for the 3-factor model.

As regards GCorr estimates, the one-factor model appears to perform extremely well in explaining pairwise correlations (Table 3.D). This model matches exactly the mean and standard deviation of pairwise correlations and leads to only slight deviations from higher moments. The two-factor model performs even better, but the benefit is only marginal.

As far as pricing implications are concerned (Table 4.E), common-factor models

¹⁹The stripped-down and Kalman filter specifications lead to virtually identical conclusions regarding CDS-implied spreads. To avoid redundancy, we do not report the “stripped down” estimates implied by the single-name CDS market.

do not alter the empirical findings reported in Section 6. Most strikingly, even the tranche spreads implied by the one-factor models are extremely close to the baseline values. Thus, despite the capacity of multi-factor models to explain better the higher moments of correlation distributions, this improvement has negligible implications for pricing. This is consistent with our previous finding that the dispersion in correlation coefficients plays a very small role for tranche spreads. In sum, the evidence shows that it is reasonable for market participants to use a one-common-factor model when pricing portfolio credit risk.²⁰

8.2 Alternative simulations of defaults

Our calculations of CDS-implied tranche spreads are based on simulations of default that have been criticized in the literature. In particular, we have followed the logic of the copula method, which postulates that a default is triggered by the *single* draw of a random variable (representing the borrower’s assets) falling below a particular threshold. Alternatively, however, a default can be simulated in a *multi-period* setting under the assumption that it is triggered the first time asset values cross a threshold. As pointed out by Duffie and Singleton (2003), the alternative specification may lead to different probabilities of joint defaults and, thus, different prices of portfolio credit risk. In this section, we examine the relevance of this observation for our sample.

For the multi-period simulations, we generate 10 intra-day observations (i.e. 13200 intervals in 5 years). Owing to the computational burden, we calculate the tranche spreads every 20 business days during the period between November 21, 2003 and March 18, 2005. Figure 11 and Table 4.F report the simulation results.

Overall, the multi-period simulation does generate pricing differences, particularly for the mezzanine and senior tranches. Nevertheless, incorporating these pricing differences pushes the CDS-implied tranche spreads even further away from the observed spreads.

There are two reasons behind the price differentials between the one-period and multi-period simulations. First, the joint default distribution tends to be different. As shown in Figure 12, the multi-period method raises the probability of a small number of defaults and lowers the probability of a large number of defaults (consistent with the

²⁰The generality of this conclusion may be subject to further investigation, owing to the fact that the set of entities in our study all belong to the same region and the investment-grade group.

example in Duffie and Singleton, 2003). As a result, the equity tranche spreads tend to be higher and senior tranche spreads tend to be lower than in the one-period simulation. The second reason is due to an estimation error specific to the multi-period simulation. Because of the discrete-time approximation, the simulation ignores the probability of default during the small intervals between two sub-periods. As a result, the multi-period simulation delivers single-name default probabilities that are lower than the benchmark PDs by 2.5 basis points on average. This simulation error causes the predicted tranche spreads from the multi-period simulation to be lower for all tranches.

8.3 Alternative estimates of CDS-implied correlations

The asset return correlation implied from the single-name CDS market can in principle be affected by the mapping from CDS spreads to PDs and from PDs to the underlying asset values. As a robustness check, we implement the following two exercises.

(1) An alternative mapping from CDS spreads to PDs. We approximate the default intensity by $q = \frac{\rho}{1-RR}$, which is quite popular among market practitioners in the context of investment-grade entities.

(2) An alternative mapping from PDs to the asset return correlation. We allow defaults to occur at any point in time before maturity of the associated contracts and capture this setting via a Merton framework. The details of the mapping under this scenario are outlined in Appendix B.2.

The two alternative mappings change little our initial estimates of asset return correlations.²¹ As a result, the correlation coefficients implied by these mappings lead to CDS- and GCorr-implied tranche spreads that confirm the findings reported in Section 6.

9 Conclusion

This paper examined how alternative estimates of risk neutral PDs and asset return correlations affect prices of portfolio credit risk. Asset return correlations implied by the single-name CDS market turn out to be substantially lower than the correlations consistent with observed spreads of a popular CDS index. This discrepancy suggests inconsistency in the way single-name and index markets incorporate correlated default

²¹The results are available upon request.

risk into prices. By contrast, the observed CDS-index tranche spreads seem to be based on correlations that are close to M KMV's estimates of physical asset return correlations, raising the question of whether there is correlation risk premium in the CDS index market.

Our analysis also sheds light on the so-called implied-correlation smile and on characteristics of the model used by participants in the CDS index market. In particular, the dispersion in individual PDs and pairwise correlation coefficients, albeit with a smaller overall impact than the level of these parameters, can help one to reconcile an implied correlation smile with the close match between GCorr-implied and observed spreads across four index tranches. In addition, our numerical simulations show that a one-factor model of asset returns is sufficient to explain the bulk of observed prices of portfolio credit risk.

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Appendix

A The Kalman filter estimation

The state-space specification for the Kalman filter is as follows:

$$\Delta v_t = H\xi_t \quad (4)$$

$$\xi_t = F\xi_{t-1} + v_t \quad (5)$$

$$E(v_t v_t') = Q \quad (6)$$

where

$$\begin{aligned} \Delta v_t &\equiv [\Delta v_{1,t} \cdots \Delta v_{N,t}]', \quad \xi_t \equiv [M_{1,t} \cdots M_{F,t} \quad Z_{1,t} \cdots Z_{N,t}]' \\ v_t &\equiv [\eta_{1,t} \cdots \eta_{F,t} \quad Z_{1,t} \cdots Z_{N,t}]' \end{aligned}$$

is a vector of
standard normal variables

$$H \equiv \begin{bmatrix} \alpha_{1,1} & \cdots & \alpha_{1,F} & \sqrt{1 - A_1' A_1} & 0 & 0 \\ \cdots & \cdots & \cdots & 0 & \cdots & 0 \\ \alpha_{N,1} & \cdots & \alpha_{N,F} & 0 & 0 & \sqrt{1 - A_N' A_N} \end{bmatrix} \quad (7)$$

where the vector A_f is defined in Section 3.3.1

$$F \equiv \begin{bmatrix} \psi_1 & 0 & 0 & \mathbf{0}_{F \times N} \\ 0 & \cdots & 0 & \\ 0 & 0 & \psi_F & \\ \mathbf{0}_{N \times F} & & \mathbf{0}_{N \times N} & \end{bmatrix} \quad \text{and} \quad Q \equiv \begin{bmatrix} 1 - \psi_1^2 & 0 & 0 & \mathbf{0}_{F \times N} \\ 0 & \cdots & 0 & \\ 0 & 0 & 1 - \psi_F^2 & \\ \mathbf{0}_{N \times F} & & \mathbf{0}_{N \times F} & \mathbf{I}_N \end{bmatrix} \quad (8)$$

To estimate the unknown parameters in H , F and Q as well as the unobserved factors ξ_t , we first carry out two preliminary steps:

1. Standardize each time series of asset returns. In other words, we first de-mean and then divide by the sample standard deviation each time series $\{\Delta v_{i,t}\}_{t=1}^T$.
2. Ensure that the estimated loading coefficients belong to the interval $[-1, 1]$, ie

parameterize H as follow ($i = 1, \dots, N, j = 1, \dots, F$):

$$\begin{aligned}
\alpha_{i,1} &= 2\Phi(l_{i,1}) - 1 \\
\alpha_{i,2} &= \sqrt{1 - \alpha_{i,1}^2} \cdot [2\Phi(l_{i,2}) - 1] \\
&\dots \\
\alpha_{i,F} &= \sqrt{1 - \alpha_{i,1}^2 - \dots - \alpha_{i,F-1}^2} \cdot [2\Phi(l_{i,F}) - 1] \\
\psi_j &= 2\Phi(b_j) - 1
\end{aligned}$$

Then we follow Hamilton (1994) to derive the conditional distribution:

$$\Delta v_t | \Delta v_{t-1} \sim N(H\hat{\xi}_{t|t-1}, HP_{t|t-1}H')$$

where $P_{t|t-1} = (F - K_{t-1}H)P_{t-1|t-2}(F' - H'K'_{t-1}) + Q$, the gain matrix $K_{t-1} \equiv FP_{t-1|t-2}H'(HP_{t-1|t-2}H')^{-1}$ and $\hat{\xi}_{t|t-1}$ is a linear function of Δv_{t-1} . Thus, the log likelihood function to maximize is

$$\begin{aligned}
\max_{\{H,F,Q\}} & \sum_{i=1}^N \sum_{t=1}^T \log f(\Delta v_t | \Delta v_{t-1}) \\
f(\Delta v_t | \Delta v_{t-1}) & \equiv (2\pi)^{-n/2} |HP_{t|t-1}H'|^{-1/2} \\
& \times \exp\left\{-\frac{1}{2}(\Delta v_t - H\hat{\xi}_{i,t|t-1})'(HP_{t|t-1}H')^{-1}(\Delta v_t | \Delta v_{t-1} - H\hat{\xi}_{i,t|t-1})\right\}
\end{aligned}$$

B Estimating the probability distribution of joint defaults

This appendix outlines three methods for estimating the probability distribution of the number of defaults in a given portfolio. Two of the methods rely directly on asset return correlations and carry out Monte Carlo simulations. The first one of these methods assumes that a default can occur only at a particular point in time, whereas the second one allows for a default to occur at any point in time prior to the maturity of the corresponding debt contract. The third method relies on a common-factor model of asset returns and employs the Gaussian copula.

B.1 One-period Monte Carlo simulation²²

This method estimates the probability distribution of defaults in a portfolio of N exposures when a default is driven by a single draw of a random variable. The method relies on estimates of pairwise asset correlations and PDs. For asset correlations we

²²Strictly speaking, this simulation is also a copula method but without the common factor structure.

use ρ_{ij} , as defined by equation (2) and thus make implicitly the assumption that the autoregressive parameter $f_i = 1$.

1. Let R denote the Cholesky factor of the correlation matrix, which has ρ_{ij} as its ij -th entry.
2. Generate N random draws x_0 from independent standard normal distributions.
3. Calculate $x = R'x_0$.
4. Denoting the i -th member of x by x_i ($i = 1, \dots, N$) and the associated PD by PD_i , entity i is said to default if and only if $x_i < \Phi^{-1}(PD_i)$.
5. Repeat steps 2 to 4 a large number of times to estimate the probability of $n \in \{0, \dots, N\}$ defaults.

B.2 Multi-period Monte Carlo simulation

This appendix outlines an alternative simulation procedure, which delivers an estimate of the probability distribution of defaults in a portfolio of borrowers. We consider N borrowers, each one of which can default in any one of multiple time periods and ask the following generic question: *What is the probability that n defaults occur over time horizon τ ?*

To answer this question, we start by assuming that the risk-neutral asset-value process is given by:

$$\frac{dV_{i,t}}{V_{i,t}} = \mu_i dt + \sigma_i dW_{i,t} \quad (9)$$

where μ_i is the (constant) risk-neutral drift, σ_i is the asset volatility and $W_{i,t}$ is a standard Wiener processes. Further, let entity i default as soon as the distance-to-default $DD_{i,t} \equiv \frac{\ln V_{i,t} - \ln D_i}{\sigma_i}$ crosses zero. The variable $dDD_{i,t}$ has a drift $\mu^* \equiv \frac{\mu - \sigma^2/2}{\sigma}$ and a unit variance, implying that the risk-neutral probability of default over the next τ years is given by:

$$PD_{i,t}(\tau) = 1 - \Phi\left(\frac{DD_{i,t} + s\mu_i^*}{\sqrt{\tau}}\right) + \exp(-2s\mu_i^*) \Phi\left(\frac{-DD_{i,t} + s\mu_i^*}{\sqrt{\tau}}\right) \quad (10)$$

The last equation allows for constructing time series of the distance-to-default variable. For any $DD_{i,t}$ and μ_i^* , $PD_{i,t}(s)$ is a concave function of τ , implying that the default intensity decreases over time. In the light of the maintained assumption for deriving default intensities (q) from CDS spreads, we set the values of $DD_{i,t}$ and μ_i^* to be such as to imply a 1-year PD and a 5-year average default intensity equal to the corresponding $q_{i,t}$.

To simulate defaults in a portfolio, we need to simulate paths of the distance-to-default variables for all the constituent entities. To carry out these simulations for a particular point in time t , we need the initial distance to default $DD_{i,t}$, and the drift parameters μ_i^* , which we calculate as described in the previous paragraph. In addition, we need the correlation matrix of the distance-to-default random variables, which we estimate by calculating $\text{corr}(\Delta DD_{i,t}, \Delta DD_{j,t})$ for all pairs $i - j$ in the sample.

Finally, we calculate the probability of n defaults over time horizon τ , allowing for a default to occur as soon as a distance-to-default variable falls below zero. Specifically, we record whether a particular simulation delivered n defaults, ie whether $\sum_{i=1}^N I_{i,\tau} = n$, where $I_{i,\tau} = 1$ if $DD_{i,t} \leq 0$ for some $t \in [0, \tau]$ and $I_{i,\tau} = 0$ otherwise. The ratio of the number of simulations for which the equality $\sum_{i=1}^N I_{i,\tau} = n$ holds to the total number of simulations is our estimate of the probability that n defaults occur over horizon τ when the portfolio consists of N names

B.3 Gaussian copula

This appendix outlines the copula method, which relies on a common-factor model of assets and has been developed by Li (2000), Laurent and Gregory (2005) and Andersen and Sidenius (2005). For illustrative purposes, we assume that assets are driven by a single common factor and use notation from Appendix A. Denoting the common factor, the loading coefficient on that factor and the PD of entity i by M , α_i and $PD_{i,t}$, respectively, the joint default probability can be calculated in three steps.

The first step is to calculate the conditional default probability for individual entity i on date t , $PD_i(t|M_t)$. When the asset value $V_{i,t} = \alpha_i M_t + \sqrt{1 - \alpha_{i,t}^2} Z_{i,t}$ and M_t and $Z_{i,t}$ are independent standard normal variables, it follows that

$$q_i(t|M) = \Phi \left(\frac{\Phi^{-1}(PD_{i,t}) - \alpha_i M}{\sqrt{1 - \alpha_i^2}} \right)$$

where $PD_{i,t}$ is the unconditional probability of default.

The second step is to calculate the conditional probability of an arbitrary number of defaults. Suppose we know the probability of $k \in \{0, 1, \dots, K\}$ defaults in a set of K entities: $p^K(k, t|M)$. Then, adding one more entity to the set leads to the following update of the default distribution:

$$\begin{aligned} p^{K+1}(0, t|M) &= p^K(0, t|M)(1 - PD_{K+1}(t|M)) \\ p^{K+1}(k, t|M) &= p^K(k, t|M)(1 - PD_{K+1}(t|M)) \\ &\quad + p^K(k-1, t|M)PD_{K+1}(t|M) \quad k = 1, \dots, K \\ p^{K+1}(K+1, t|M) &= p^K(K, t|M)PD_{K+1}(t|M) \end{aligned}$$

This recursion is started by setting the initial condition $p^0(0, t|M) = 1$.

The final step is to calculate the unconditional probability of k defaults:

$$p(k, t) = \int_{-\infty}^{\infty} p(k, t|M)\phi(M)dM$$

In approximating the integral, we vary M between -5 and 5 and set the grid size $dM = 0.02$.

The generalization to multiple factors is conceptually straightforward but increases the computation time. We choose the grid size to be 0.1 and 0.4 for the 2-factor and 3-factor models respectively.

Table 1: Summary statistics of PDs and recovery rates

	<i>mean</i>	<i>std dev</i>	<i>min</i>	<i>5%</i>	<i>25%</i>	<i>50%</i>	<i>75%</i>	<i>95%</i>	<i>max</i>
PDs (bps)									
<i>Daily averages</i>	85.3	12.6	63.7	65.4	74.6	87.9	97.5	102.1	105.4
<i>Averages over time</i>	85.3	50.0	23.5	34.9	58.4	71.2	91.1	216.9	281.1
Recovery rates (%)									
<i>Daily averages</i>	38.4	0.9	36.4	37.3	37.7	38.3	39.5	39.7	39.7
<i>Averages over time</i>	38.4	0.7	36.3	37.3	38.1	38.5	38.9	39.5	41.0

Notes: The summary statistics reflect all entities that belonged to any of the first three CDX.NA.IG.5Y releases. The underlying data start on Oct. 21 2003 and end on Mar. 20 2005. The first row reports summary statistics of the *daily* cross-sectional averages of PDs and recovery rates. The second row reports summary statistics of time averages of *individual* PDs and recovery rates.

Table 2: Comparing three prices of index tranche spreads

A. Average tranche spreads				
	<i>index market</i>	<i>CDS-implied</i>	<i>GCorr-implied</i>	
<i>0-3 %</i>	1705.4	1856.3	1572.8	
<i>3-7 %</i>	303.9	313.1	330.4	
<i>7-10 %</i>	111.1	69.9	112.4	
<i>10-15 %</i>	45.5	15.9	42.2	
<i>15-30 %</i>	12.5	0.8	5.9	
B. Pricing differences				
	<i>average</i>	<i>(%)</i>	<i>MAE</i>	<i>(%)</i>
Index vs. CDS-implied				
<i>0-3 %</i>	137.4	8.1	140.5	8.2
<i>3-7 %</i>	4.8	1.6	39.6	12.7
<i>7-10 %</i>	-42.6	-38.3	42.6	38.3
<i>10-15 %</i>	-29.9	-65.7	29.9	65.7
<i>15-30 %</i>	-11.7	-93.6	11.7	93.6
Index vs. GCorr-implied				
<i>0-3 %</i>	-143.8	-8.4	156.5	9.2
<i>3-7 %</i>	22.5	7.4	45.8	15.1
<i>7-10 %</i>	-0.3	-0.3	10.9	9.8
<i>10-15 %</i>	-4.0	-8.8	8.4	18.5
<i>15-30 %</i>	-6.8	-53.9	6.8	53.9

Notes: The statistics in panel B cannot be calculated directly from panel A because there are 46 days with missing observations in the index market.

Table 3: Common-factor approximation of asset return correlations

A. CDX.NA.IG.5Y release 1								
<i>Tranche</i>	<i>mean</i>	<i>std dev</i>	<i>skew</i>	<i>kurt</i>	<i>min</i>	<i>max</i>	<i>MAE</i>	<i>MSE</i>
CDS-implied	0.1333	0.0994	0.2516	5.0457	-0.5692	0.7962	–	–
1-factor	0.1331	0.0570	0.6648	3.2081	0.0199	0.3614	0.0590	0.0823
2-factor	0.1332	0.0721	-0.3302	5.6162	-0.4620	0.4712	0.0515	0.0696
3-factor	0.1329	0.0809	0.1004	5.0364	-0.4455	0.5185	0.0437	0.0597
B. CDX.NA.IG.5Y release 2								
<i>Tranche</i>	<i>mean</i>	<i>std dev</i>	<i>skew</i>	<i>kurt</i>	<i>min</i>	<i>max</i>	<i>MAE</i>	<i>MSE</i>
CDS-implied	0.1298	0.1006	0.3092	5.0887	-0.5692	0.7962	–	–
1-factor	0.1298	0.0568	0.6213	3.2008	0.0117	0.3644	0.0604	0.0837
2-factor	0.1297	0.0728	-0.0730	5.7524	-0.4321	0.4912	0.0526	0.0707
3-factor	0.1294	0.0824	0.2880	5.3939	-0.4470	0.5490	0.0441	0.0599
C. CDX.NA.IG.5Y release 3								
<i>Tranche</i>	<i>mean</i>	<i>std dev</i>	<i>skew</i>	<i>kurt</i>	<i>min</i>	<i>max</i>	<i>MAE</i>	<i>MSE</i>
CDS-implied	0.1283	0.0995	0.3095	5.0887	-0.5692	0.7962	–	–
1-factor	0.1284	0.0556	0.6846	3.2589	0.0188	0.3636	0.0599	0.0830
2-factor	0.1283	0.0711	-0.1049	5.9161	-0.4542	0.4855	0.0526	0.0706
3-factor	0.1280	0.0802	0.2706	5.6569	-0.4569	0.5552	0.0446	0.0605
D. All 136 entities included in the three releases								
<i>Tranche</i>	<i>mean</i>	<i>std dev</i>	<i>skew</i>	<i>kurt</i>	<i>min</i>	<i>max</i>	<i>MAE</i>	<i>MSE</i>
GCorr	0.2380	0.0764	0.8589	4.6444	0.0464	0.6500		
1-factor	0.2378	0.0750	0.6295	3.7691	0.0469	0.5518	0.0122	0.0177
2-factor	0.2380	0.0756	0.8235	4.4433	0.0458	0.5782	0.0076	0.0115

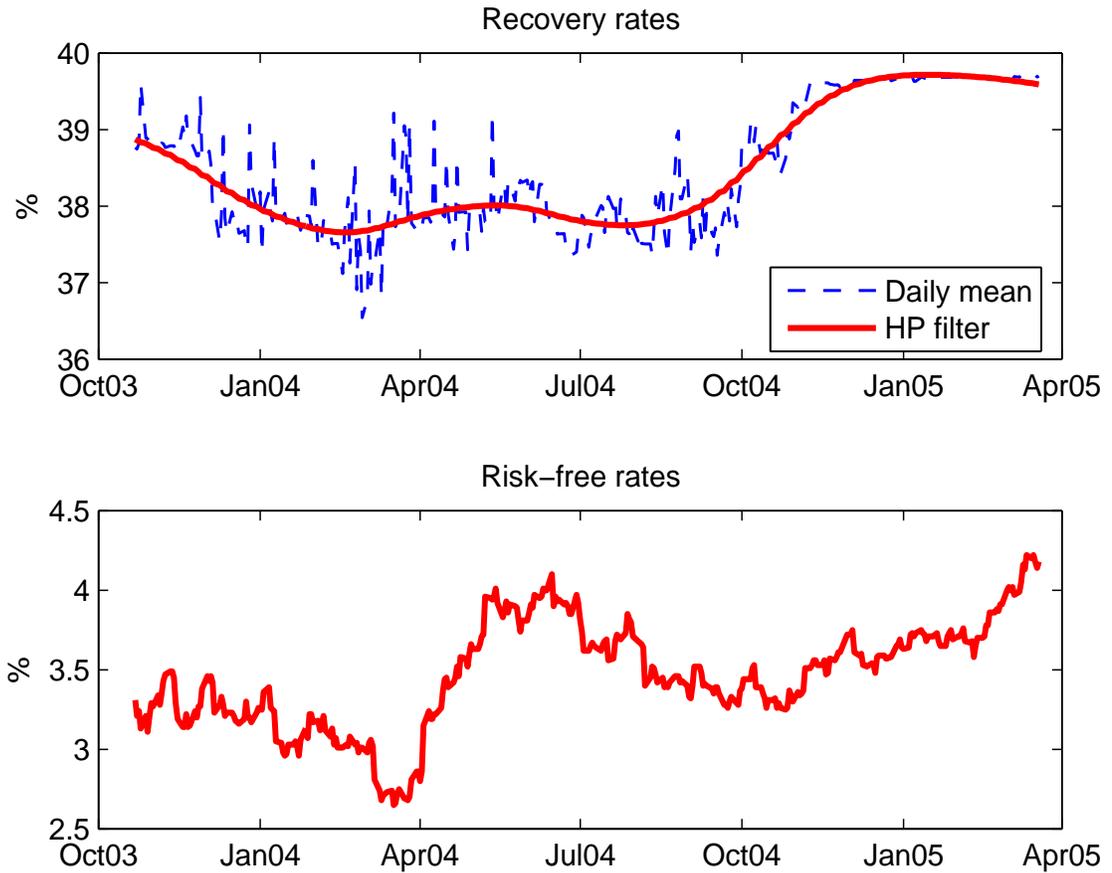
Notes: “MAE” stands for mean absolute errors and “MSE” for mean squared errors. Panels A to C summarise the match between CDS-implied asset return correlations and correlations implied by common factor models (under the Kalman-filter specification). Panel D summarises the match between GCorr correlations and correlations implied by the stripped-down specification.

Table 4: **Sensitivity analysis of CDX tranche spreads**

A. Benchmark spreads					
<i>Tranche</i>	<i>0-3%</i>	<i>3-7%</i>	<i>7-10%</i>	<i>10-15%</i>	<i>15-30%</i>
CDS-implied	1856.3	313.1	69.9	15.9	0.8
GCorr-implied	1572.8	330.4	112.4	42.2	5.9
index market	1705.4	303.9	111.1	45.5	12.5
B. Remove dispersion in PDs					
<i>Tranche</i>	<i>0-3%</i>	<i>3-7%</i>	<i>7-10%</i>	<i>10-15%</i>	<i>15-30%</i>
CDS-implied	1860.2	322.4	75.7	18.3	1.1
GCorr-implied	1499.1	343.2	128.4	52.6	8.4
C. Remove dispersion in correlation coefficients					
<i>Tranche</i>	<i>0-3%</i>	<i>3-7%</i>	<i>7-10%</i>	<i>10-15%</i>	<i>15-30%</i>
CDS-implied	1948.4	299.7	56.6	11.0	0.5
GCorr-implied	1557.9	341.1	113.6	40.8	5.3
D. Remove dispersion in PDs and correlations					
<i>Tranche</i>	<i>0-3%</i>	<i>3-7%</i>	<i>7-10%</i>	<i>10-15%</i>	<i>15-30%</i>
CDS-implied	1915.5	316.8	67.0	14.6	0.8
GCorr-implied	1501.9	353.4	127.2	49.2	7.2
E. Sensitivity to the factor structure					
<i>Tranche</i>	<i>0-3%</i>	<i>3-7%</i>	<i>7-10%</i>	<i>10-15%</i>	<i>15-30%</i>
CDS-implied					
1-factor	1898.1	302.9	65.8	14.9	0.8
2-factor	1889.7	305.0	66.8	15.2	0.8
3-factor	1886.0	310.6	69.5	16.1	0.8
GCorr-implied					
1-factor	1577.9	328.8	111.9	42.1	5.9
F. Sensitivity to numerical methods					
<i>Tranche</i>	<i>0-3%</i>	<i>3-7%</i>	<i>7-10%</i>	<i>10-15%</i>	<i>15-30%</i>
CDS-implied					
one-period MC	1850.0	310.5	68.6	15.6	0.8
multi-period MC	1836.4	244.6	42.7	7.9	0.3

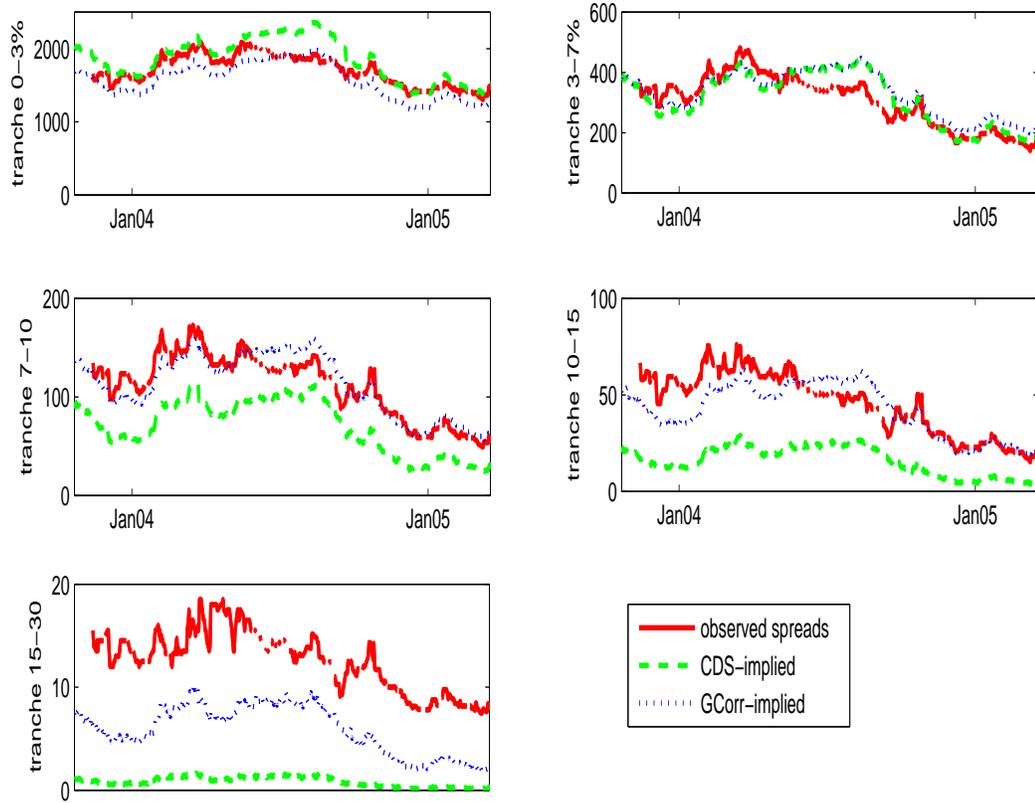
Notes: As implied by average daily tranche spreads between November 21, 2003 and March 18, 2005 (369 business days in total). The only exception is panel F, in which the results are based on average tranche spreads calculated every 20 days during the same sample period (18 observations in total).

Figure 1: Default recovery rates and risk-free rates of return



Note: (1) Recovery rates are provided by Markit. The recovery rate on each day refers to the cross-sectional average of the 125 entities that are included in the “on-the-run” CDX.NA.IG.5Y release. The HP filter adopts $\lambda = 64000$. (2) The risk-free rate of return is proxied for by 5-year Treasury rates.

Figure 2: Observed and implied spreads of CDS index tranches



Note: The observed tranche spreads in the CDS index market are provided by JP Morgan. The two sets of implied tranche spreads are based on the one-period Monte Carlo simulation method (Appendix B.1).

Figure 3: Daily cross-sectional average PDs

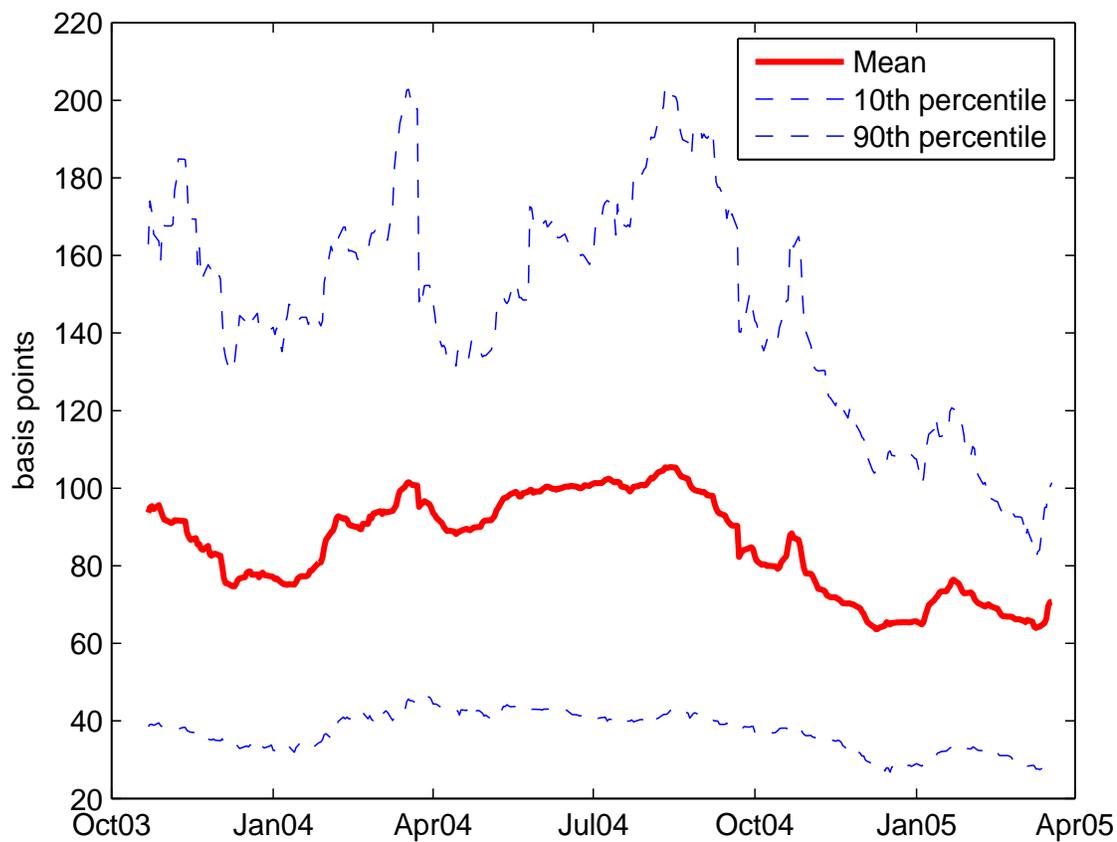
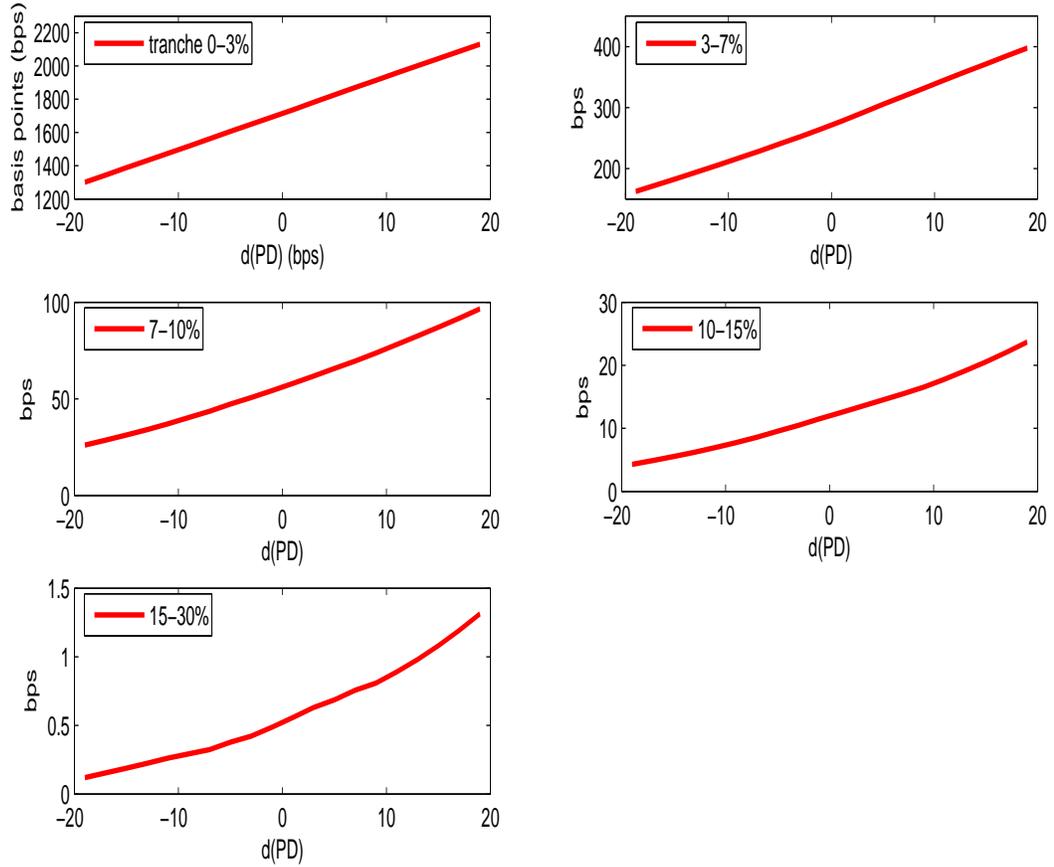
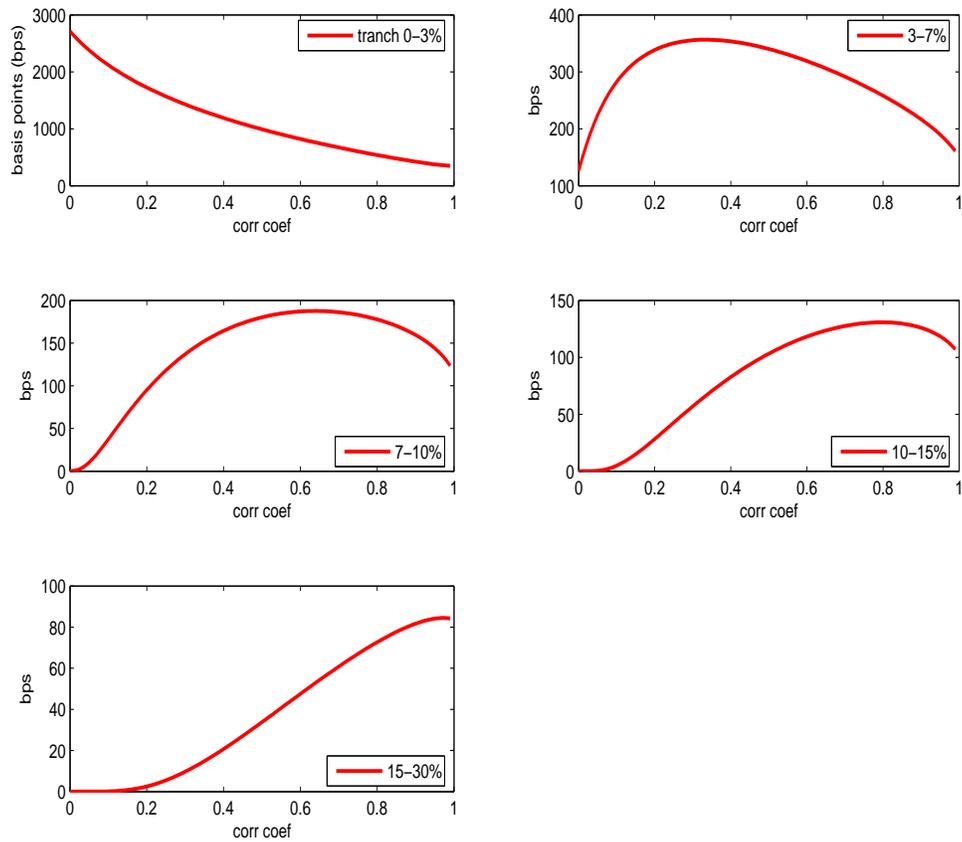


Figure 4: The sensitivity of tranche spreads to PDs: an example



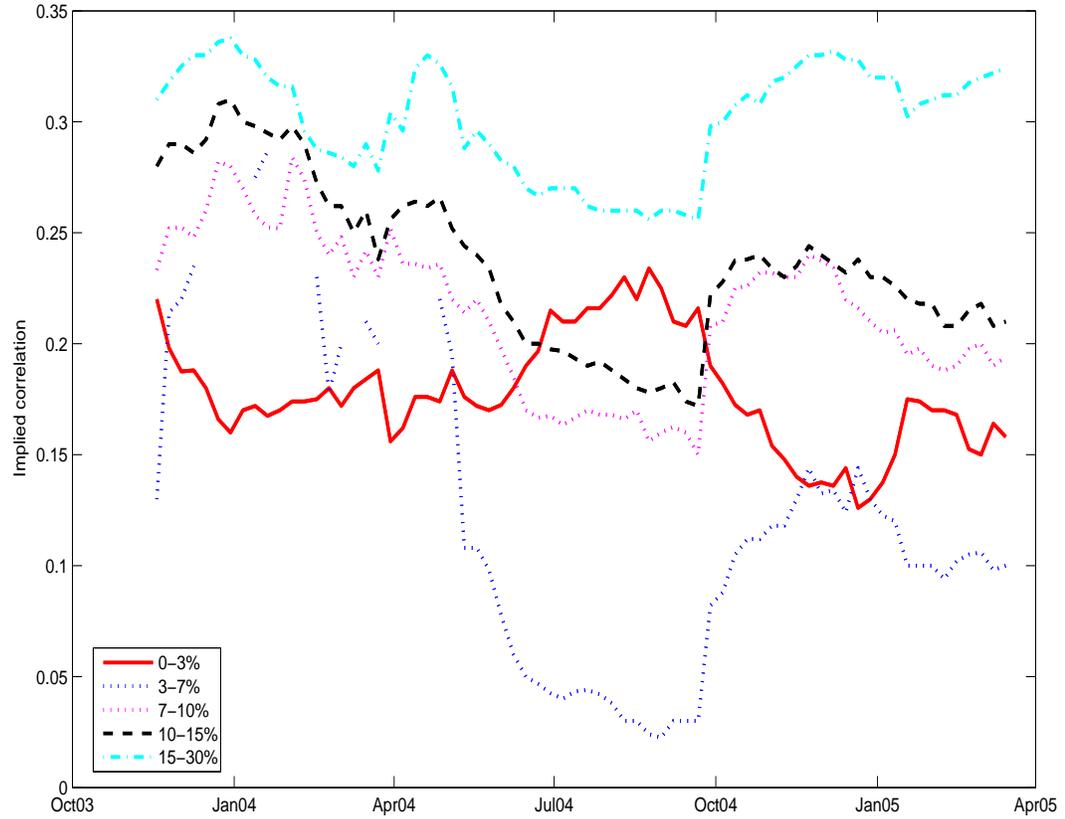
Note: The sample set includes the 125 entities in CDX.NA.IG.5Y release 3. The pricing of tranche spreads is based on the CDS-implied asset return correlation and the average recovery rate and average risk-free rate during the sample period. In the baseline case ($dPD=0$), individual PDs are set to the average PDs of each firm over the sample period, with a mean of 79 basis points and a standard deviation of 53 basis points across the 125 entities. We then change all individual PDs by the same amount and re-price the tranche spreads.

Figure 5: The sensitivity of tranche spreads to correlation coefficients: an example



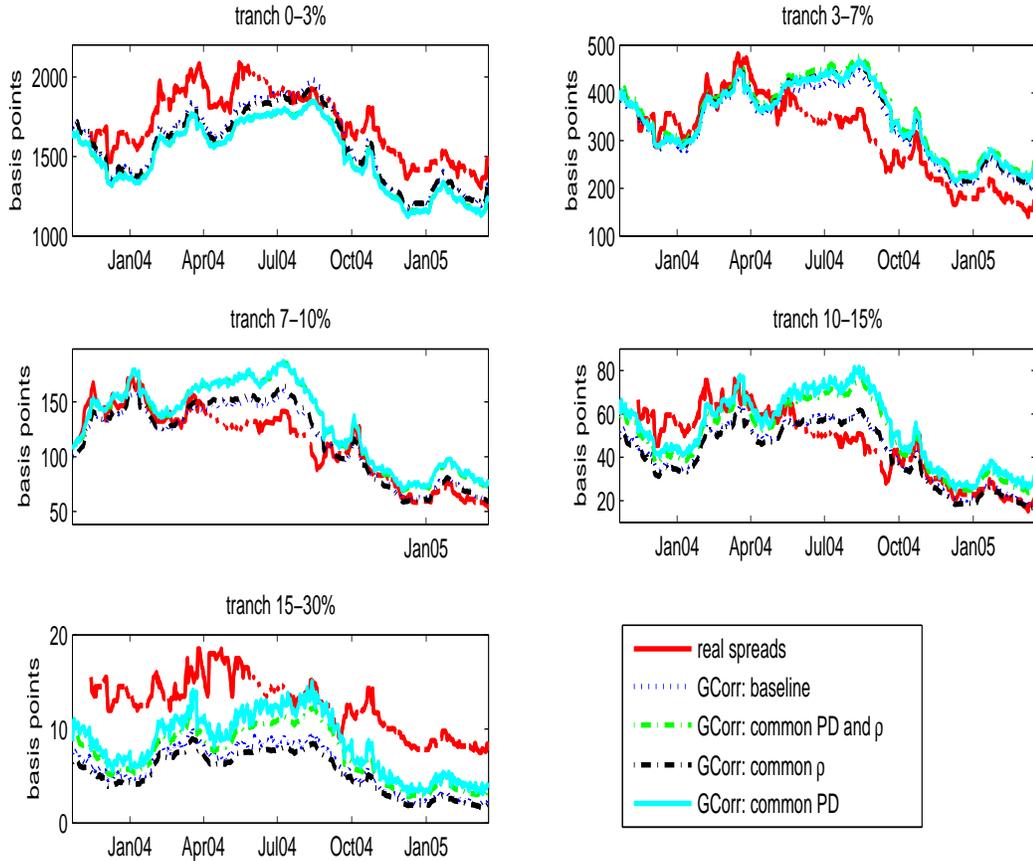
Note: In this example: the recovery rate is 40%, and the risk-free rate is 3.5%. The sample set includes the 125 entities in CDX.NA.IG.5Y release 3. Individual PDs equal the average PD of each firm over time, and all pairwise correlation coefficients are assumed to be equal. Tranche spreads are calculated by varying the correlation coefficient from 0 to 1.

Figure 6: Implied correlations in the index market, by tranche



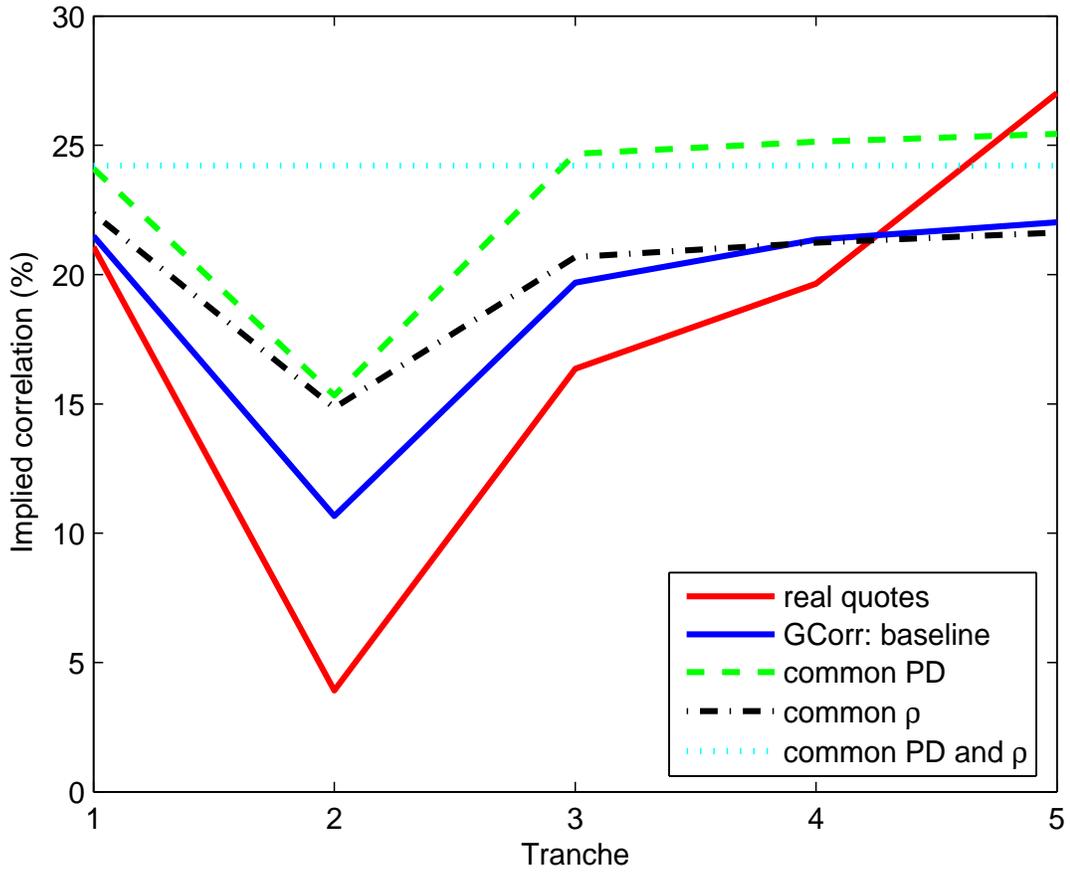
Note: The implied correlation is calculated from the observed tranche spreads on each day, on the assumption that the PDs and pairwise correlation coefficients are the same across entities.

Figure 7: Sensitivity of tranche spreads to dispersion in correlations and PDs



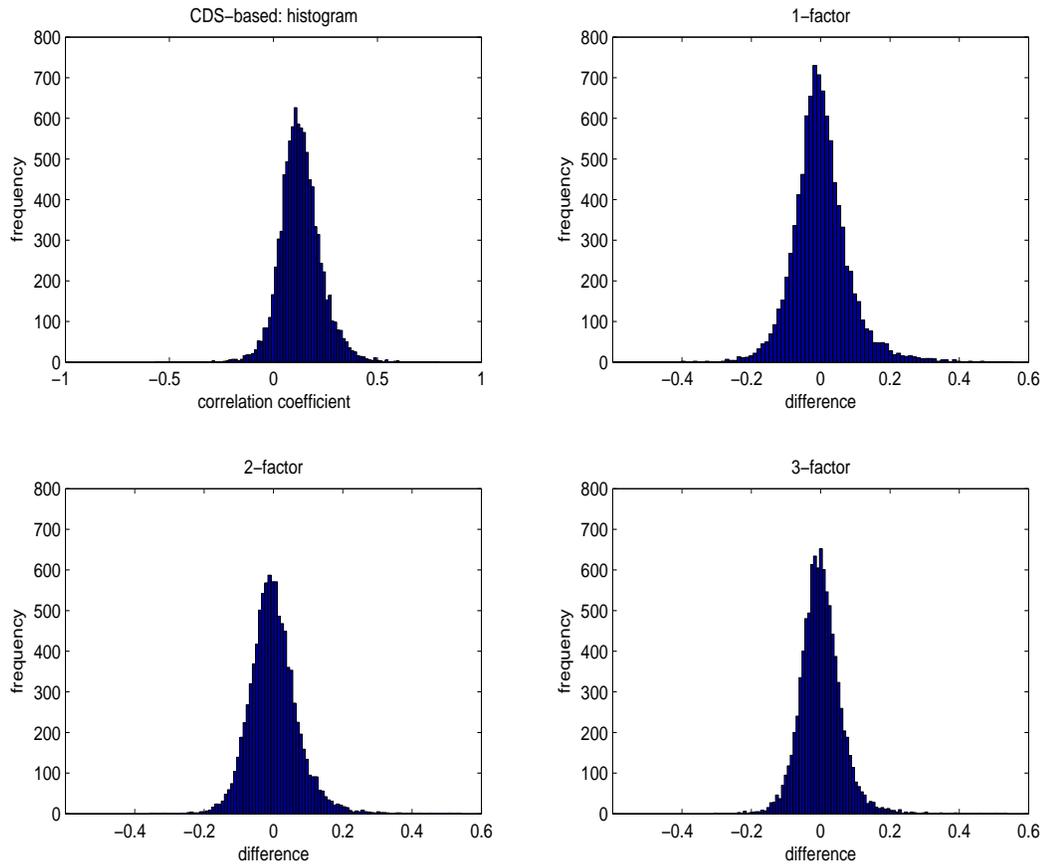
Note: The pricing is based on the GCorr correlation. The dark dash-dotted lines assume the same correlation coefficients but allows PDs to vary across entities. The cyan lines assume instead the same PD across entities (on each day), but do not alter the original GCorr correlations. The green dash-dotted lines assume that both correlation coefficients and PDs are the same across entities. The results of the latter two exercises are hardly distinguishable for the 0-3 and 7-10 tranches.

Figure 8: Implied correlation smile, an example



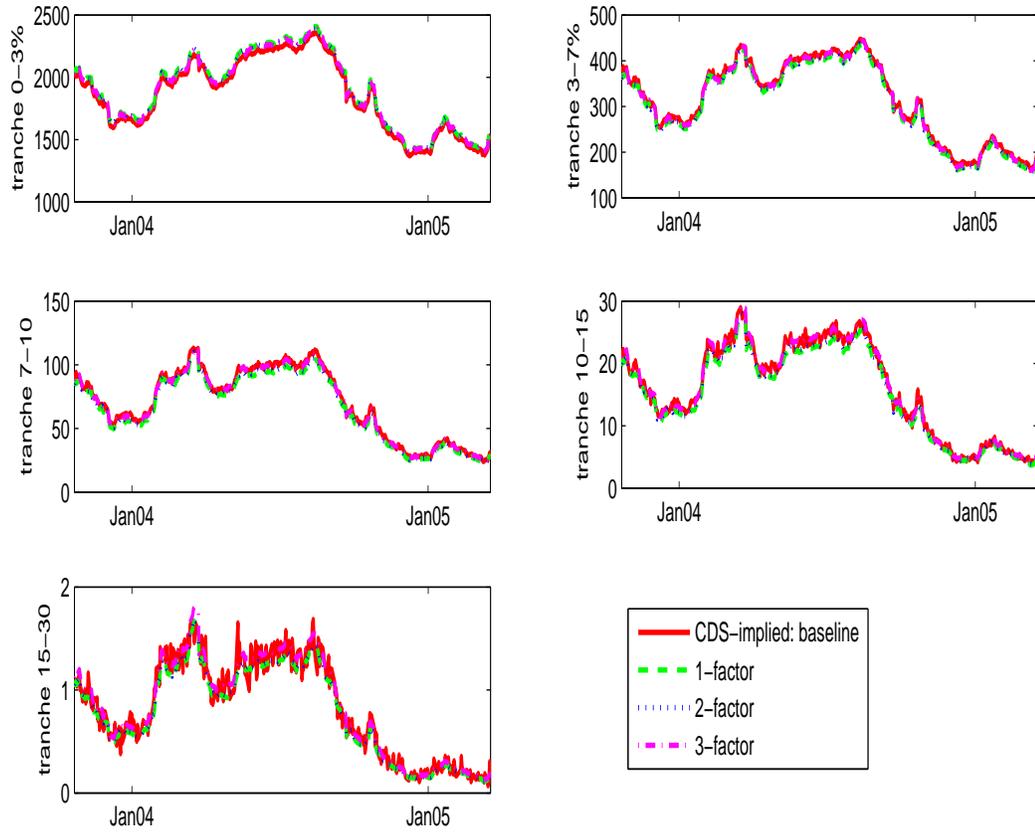
Note: The example pertains only to 30.06.2004. Tranches change across the horizontal axis: from 1 to 5, the tranches increase in seniority from the equity to the super-senior tranche, which are defined in the text. The calculation of implied correlation is divided into two steps. In the first step, we record observed spreads and baseline GCorr-implied spreads, as well as GCorr-implied spreads under certain restrictions on PDs and pairwise correlations. In the second step, implied tranche correlations are derived using the standard method, which assumes the same PD and correlation coefficients across entities.

Figure 9: Common factor models of asset returns



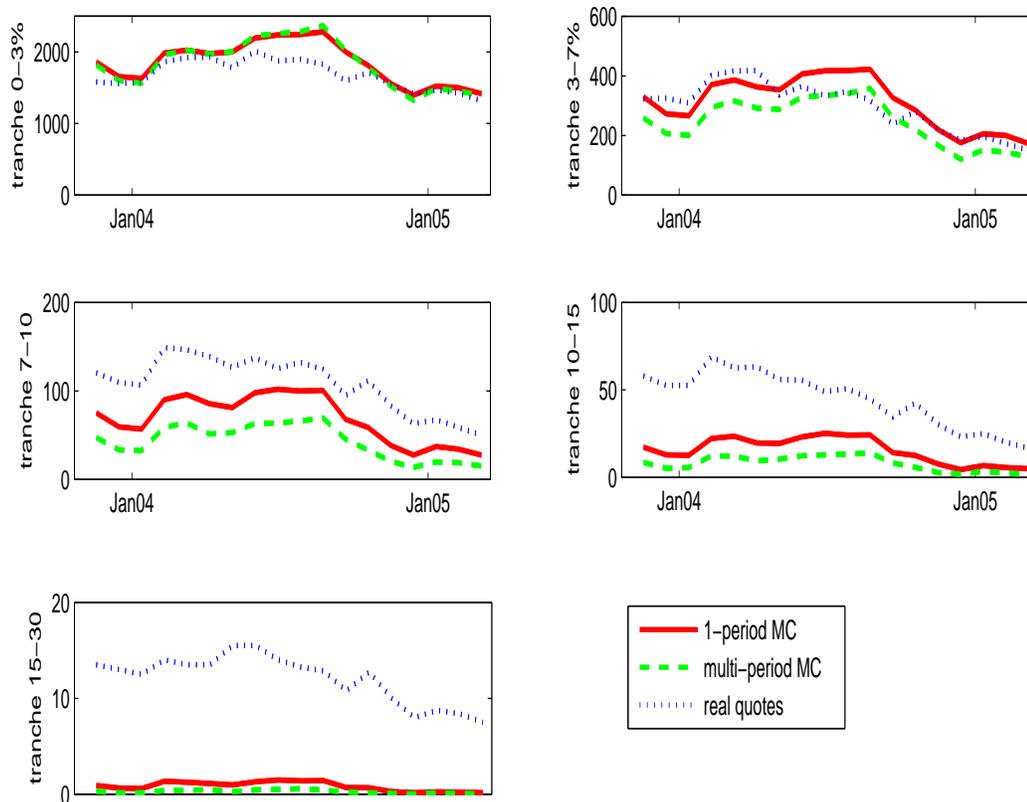
Note: The upper-left panel shows the distribution of the CDS-implied correlation coefficients. The 1-, 2- and 3- common factor models are estimated via a Kalman filter (see Appendix B). The distributions of correlation coefficient discrepancies are shown in the other three panels.

Figure 10: Sensitivity of tranche spreads to the assumed correlation structure



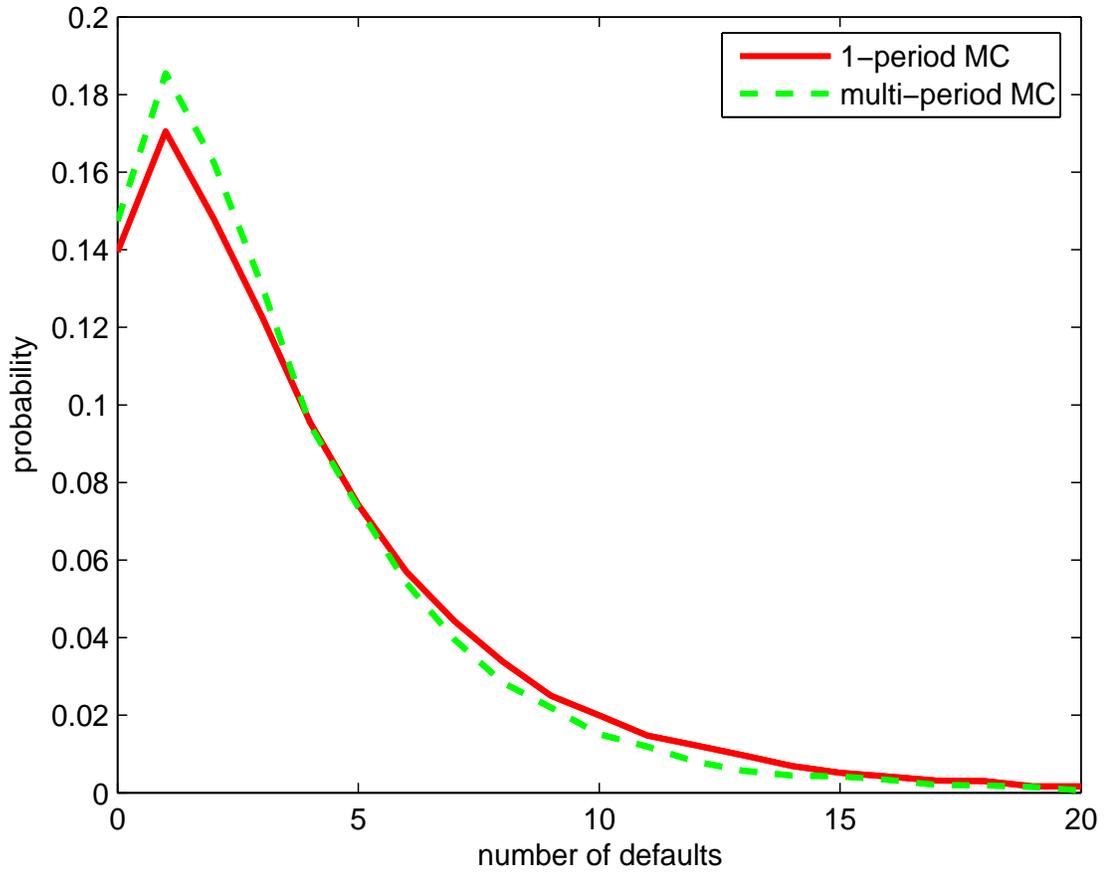
Note: Solid lines represent prices based on the CDS-implied asset return correlation and the one-period Monte Carlo simulation method. The other results are obtained using a Gaussian copula.

Figure 11: Pricing implications of different mappings from asset return to default correlations



Note: Both simulations use the CDS-implied asset return correlation, but they differ in that one relies on the one-period Monte Carlo simulation method (Appendix B.1) whereas the other one relies on the multi-period Monte Carlo simulation specification (Appendix B.2).

Figure 12: Joint default distributions, 5-year horizon



Note: The results pertain to March 7, 2005. The tranche spreads are 1417.1, 172.9, 27.5, 5.0 and 0.2 basis points using the one-period Monte Carlo simulation and 1386.2, 128.0, 15.0, 1.6 and 0.025 basis points using the multi-period simulation.