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Abstract

We evaluate the roles of upstream (supplier-to-user), downstream (user-to-supplier) and common factor shock transmission across firms by deriving inter-firm networks and common factors from U.S. equities over 1989-2017. We overcome the econometric challenges of estimating the large factor-augmented vector autoregressive (FAVAR) system by developing two alternative approaches: one prioritizing computational efficiency and the other providing the full posterior distribution of all model parameters and factors. We find that: (i) common factors drive an increasing variance share of returns; (ii) supplier shocks are more evident in equity price movements than downstream exposures; (iii) removing the impact of common factors is increasingly important for revealing inter-firm connections.

Keywords: large FAVAR; upstream versus downstream; shock propagation; common factors; Bayesian estimation. JEL Codes: C11; C33; C55; E44.

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1 Introduction

The broad transmission of economic shocks has been endemic to crises over the past few decades, from the 2008 Global Financial Crisis, to the 2011 Tohoku earthquake's worldwide footprint, to supply chain disruptions during and following the COVID-19 pandemic. The frequency and severity of these events has led to a focus on discerning what channels shocks flow through as they spread across firms and industries, and how these mechanisms have evolved over time.[1] To help understand such shock diffusion, we evaluate the importance of upstream versus downstream production network exposures, as well as the role of common factors in the propagation of shocks across publicly traded U.S. firms.

We extend the literature estimating inter-firm networks from vector-autoregressions (VARs) of their asset prices (e.g., Bonaldi et al., 2015; Demirer et al., 2018; Diebold and Yilmaz, 2009, 2014, 2016; Grant and Yung, 2021) by utilizing a factor-augmented VAR (FAVAR) approach. The FAVARs are estimated on firms' daily equity returns to derive their connections with one-another and the common factors, in a novel way that can separate out the impact of the factors and model them as explicit nodes within the network. We provide two alternative methods to estimate the large FAVAR models on our panel samples of 524 to 1,548 U.S. firms from 1989 to 2017, which deal with the curse of dimensionality and avoid over-fitting the data: one leveraging recent advances in big data analysis to optimize for computational efficiency; and a Bayesian approach with priors allowing for variable shrinkage and scalable posterior draws to estimate the full joint posterior distribution of the parameters and common factor dynamics.

Our results indicate that common factors play an important role in the transmission of shocks, with the variance share of the top three common factors being over a third in our most recent samples. We also find that exposure to suppliers is economically important and statistically significant, with a 0.62 average correlation between sectoral upstream exposure networks calibrated to U.S. input-output tables and the equity return derived networks. However, the downstream exposure appears more muted, with statistically insignificant average correlations of only 0.22 between the macroeconomic exposure in this direction and the equity return networks. This behavior is suggestive of a low short-term elasticity of substitution across inputs passing shocks on from upstream but greater flexibility with downstream customers. Our results are consistent with the empirical literature finding significant exposures to suppliers, such as Menzly and Ozbas (2010) studying cross-predictability of equity returns based on lagged returns in direct supplier and customer industries, and Boehm et al.

[1] The importance of input-output, financial, trade and common shock transmission at country and sector levels has been explored in many papers, including Brooks and Del Negro (2006), Burstein et al. (2008), di Giovanni and Levchenko (2010), Grant (2016), Imbs (2004), and Johnson (2014).

(2019) and Carvalho et al. (2016) analyzing the ramifications of the 2011 Tohoku earthquake. We find that the reverse direction does not appear to be as relevant to publicly traded firms, extending the results of the latter papers beyond the context of this natural experiment. Our analysis recommends the use of the idiosyncratic equity network estimation method that isolates the impact of common factors to study bilateral transmission across firms, especially towards the end of our sample when common factors become a greater driver of the equity returns and confound the inter-firm connections.

Our first large FAVAR estimation approach is a three-step method that obtains the common factors and their loadings using principal component analysis (PCA), estimates a VAR of the dynamic relationship between the factors, and utilizes the Chudik et al. (2018) variable selection method to estimate a VAR of the idiosyncratic firm returns (their returns minus the impact of the common factors). This method is computationally efficient taking seconds to run, deterministic (i.e., not subject to solver randomization), and parallelizable to accommodate very large panels. The second approach is to caste the large FAVAR as a Bayesian state-space model. The priors are adapted to a large FAVAR context to keep the estimation numerically tractable and apply variable shrinkage, implementing Minnesota-style priors with artificial dummy observations, using normal-Wishart priors that allow for draws from the efficient matrix normal distribution, and the implementation of the Durbin and Koopman (2002) simulation smoother provided in Jarocinski (2015), which we further optimize for Matlab data structures to store our large system in memory. The state-space method is more computationally intensive than the first approach (taking three days to several weeks to run); however, it avoids the potential bias and compounded uncertainty inherent in a multi-step estimation process and provides the joint distribution for the system of common factors and inter-firm networks. The inter-firm networks are derived using generalized forecast error variance contributions from the FAVARs to infer the magnitude and direction of the connections. The FAVAR structure of our model allows us to derive firm networks at both the total return level inclusive of the effects of the common factors in the individual firms' returns, and at the idiosyncratic return level, with the influence from factors attributed to separate nodes.

Our firm equity return based networks have several benefits: they are derived from publicly available data; they can be estimated in real-time; and they reveal firm heterogeneity and avoid the aggregation bias inherent in sectoral level input-output tables. Alternately, data on cross-firm input usage, supplier connections, etc. is typically available at a lower frequency — making it difficult to estimate the propagation of transient shocks — and is either aggregated, available for a small sample of firms, or comes from confidential microdata.

To evaluate the upstream versus downstream transmission channels, we develop an illustrative DSGE model by expanding that of Baqaee (2018) from a one period setting to a multi-sector model with inter-temporal assets.[2] To assess the relative historical importance of upstream versus downstream firm exposures we: calibrate the DSGE model's parameters to U.S. input-output tables; calculate the sectoral upstream and downstream exposures as defined by those parameters; then compare these to the propagation of realized firm shocks as embedded in the equity return based networks that the large FAVARs produce.

The econometric model yields three significant common factors for the U.S. equity market, the first of which is highly correlated with the stock market beta and economic growth, as measured by U.S. broad equity index returns, industrial production and real GDP. We label the second factor the price one, as it is highly correlated with U.S. PPI, CPI, and the value of the dollar. The third factor has a strong correlation with commodities, particularly petroleum-based energy ones. These common factors — especially the first, market beta factor — become more important over the period we study, explaining 11.7% of the equity return variation over the first ten years of the sample and 35.0% over the final ten. The DSGE model aligns with these findings, as in it firms' equity returns depend on three aggregates: the market beta, accounting for the stochastic discount factor and aggregate growth; the price level; and the supply of raw inputs.[3]

Evaluating the equity return networks against the DSGE model's input-output based upstream and downstream exposures, we find that upstream exposures (shocks to a firm's suppliers) are more relevant than downstream exposures (shocks to customers). The correlations are economically and statistically significant between the equity networks and the upstream networks; however, that is not the case for the downstream ones. In addition, the idiosyncratic equity response networks have about 30% higher correlations with the upstream exposure networks than those that include the common factors. That the idiosyncratic return networks have higher correlations than the total return ones accords with the DSGE model, where after the common factors are removed from equity returns one can derive a VAR(1) relationship between them, with exposures through the upstream and downstream centralities entering the innovations. These results are consistent across both FAVAR estimation approaches, with the Bayesian state-space method that accounts for model uncertainty having marginally lower correlations, which remain statistically significant in the same instances as the three-step approach. Using simulations, we show that the version of our

[2]Papers on endogenous network formation and input-output linkages include Atalay et al. (2011), Gabaix (2011), Acemoglu et al. (2012), Acemoglu et al. (2016), Acemoglu et al. (2017), Acemoglu and Azar (2020), Taschereau-Dumouchel (2017), Oberfield (2018), and Bernard et al. (2022). See Carvalho and Tahbaz-Salehi (2019) for further literature relating production networks to macroeconomic fluctuations.

[3]Factor structure has been previously introduced into DSGE models to understand shock transmission (Giannone et al., 2006), study contributions of idiosyncratic and aggregate components to the macroeconomy (Foerster et al., 2011) and relate macroeconomic shocks to the factor space (Onatski and Ruge-Murcia, 2013).

empirical networks that disentangles the effects of common factors is better able to reflect bilateral connections across firms, consistent with earlier work separating systematic from idiosyncratic components in financial networks (e.g., Barigozzi et al., 2014; Guðmundsson and Brownlees, 2021; Brownlees et al., 2021). Many prior papers in the network estimation literature do not do so, with their networks more reflective of similar loadings on common factors than of the bilateral relationships between network members they often purport to estimate.

The prominence of upstream versus downstream shock transmission is important in the context of the broader theoretical literature following Long and Plosser (1983) examining multi-sector economies. This literature differs on which of these two channels are operational. For example, Acemoglu et al. (2012) concluded that under Cobb-Douglas intermediate input aggregation, an industry's impact on the aggregate economy depends only on its role as a supplier of inputs through propagation from upstream, and not as a consumer.[4] Further, Section 4 of Baqaee (2018) showed these results hold under a more general set of models than those with Cobb-Douglas intermediate input aggregation. The models of Johnson (2014), Barrot and Sauvagnat (2016), and Baqaee and Farhi (2019) likewise have downstream but not upstream shock propagation. Our empirical findings support the many papers that rely on Cobb-Douglas input aggregation or other modeling simplifications that minimize exposure to downstream firms. Alternately, Baqaee (2018) and Luo (2020) achieved both downstream and upstream shock propagation by including firm entry-and-exit and a credit channel, respectively.

The relative significance of the upstream exposures could indicate a low short-term elasticity of substitution across inputs, which would match work at the country level. Examining trade across 30 countries, Ng (2010) found that bilateral trade in complements/upstream intermediate goods contributes to cross-country business-cycle comovement, while trade in substitutes/downstream final goods reduces it, with a net positive impact of trade on comovement. Additionally, Burstein et al. (2008) and Johnson (2014) identified that low elasticities of substitution between inputs is key to explaining the degree of synchronization in international business cycles, with Miranda-Pinto (2021) finding that the elasticity of substitution between intermediates and labor in particular can have a marked impact on GDP volatility. Relatedly, Barrot and Sauvagnat (2016) found that production elasticities near zero best match real world shock amplifications in a calibrated network model, and Atalay (2017) found that strong input complementarities play an important role in industry level shock transmission.

The next section introduces our econometric model of firm exposures from equity returns

[4] Under Cobb-Douglas, offsetting price and quantity effects remove propagation to suppliers (Shea, 2002).

and our two proposed large FAVAR estimation approaches. Section 3 describes the data, then the estimated common factors and equity based networks are outlined in Section 4. Section 5 summarizes the DSGE model's main results, including simulations that illuminate the upstream versus downstream channels for shock propagation. Section 6 compares the equity based networks with input-output derived upstream and downstream exposures to assess their relative importance in firms' equity responses to one another. Section 7 concludes.

2 Econometric Model of Firm Equity Returns

In this section, we first describe the structure of the large FAVAR framework that we use to model publicly traded U.S. equity returns, separating common factors from idiosyncratic firm returns. Second, we provide two alternative methods to estimate the system: one leveraging recent advances in big data analysis to optimize for computational efficiency, and the other, a Bayesian estimation approach to estimate the full joint posterior distribution of the parameters and common factor dynamics. Finally, we propose an econometric approach to derive inter-firm networks once the system is estimated with either method, capturing how common and idiosyncratic shocks propagate across the firms and factors.

Consider a panel dataset with T observations and N firms, where $R_t = (R_{1t}, R_{2t}, ..., R_{Nt})'$ represents the vector of observed log equity returns at time t. Firm returns are broken into three components, such that $R_t = R_t^F + R_t^I + \tau_t$. The first component, R_t^F, is driven by a set of K common factors with heterogeneous loadings across firms: $R_t^F \equiv \Lambda F_t$, where Λ is an $N \times K$ loading matrix on the factors and F_t is the vector of those factors. The second component, R_t^I, captures idiosyncratic firm returns unrelated to the common factors, but possibly related across firms or over time. The final term, $\tau_t \overset{iid}{\sim} \mathcal{N}(0, \mathcal{T})$, captures transient noise in the equity returns. The covariance matrix $\mathcal{T}$ is assumed to be diagonal and relatively small.

Individual firms are assumed to be small enough relative to the whole economy that their idiosyncratic components do not influence the common factors, which follow a VAR(1) process:

$$F_t = \Gamma F_{t-1} + \eta_t; \quad \eta_t \overset{iid}{\sim} \mathcal{N}(0, \Upsilon), \tag{1}$$

where Γ is a $K \times K$ matrix, and η_t is a $K \times 1$ vector of shocks with covariance Υ. The F_t may reflect economy-wide macroeconomic shocks, or those for individual industries, regions, etc., with the factors and firm-specific loadings being recovered from the data with minimal econometric restrictions used for identification.[5] The dynamics between the idiosyncratic

[5]Foerster et al. (2011) showed that the effects of common factors on all industries can create estimation

firm returns are also modeled as a VAR(1):

$$R_t^I = \rho_0 + \rho R_{t-1}^I + \epsilon_t, \tag{2}$$

where ρ_0 is $N \times 1$, ρ is $N \times N$, and $\epsilon_t \overset{iid}{\sim} \mathcal{N}(0, \Sigma)$. This structure implies that firms' innovations, ϵ_t, may be cross-sectionally correlated within a period, reflecting direct exposure to one-another or similar underlying economic conditions, but are serially uncorrelated, as equity prices quickly adjust to reflect new information.[6]

There are two primary challenges with estimating this system: the factors and idiosyncratic returns are not directly observed; and the curse of dimensionality and potential for over-fitting given the large parameter space. In the following subsections, we describe two different approaches to estimate the model: 1) a three-step method that estimates the factors with PCA, models Equation (1) with a VAR(1) on those factor series, then estimates the idiosyncratic VAR parameters with an efficient, parallelizable variable selection method; and 2) a Bayesian Gibbs sampling method to estimate the state-space formulation of the model. Further details for each method can be found in the Online Appendix. The key trade-offs are that the first method is highly efficient and scalable, while the second method is computationally more expensive but explicitly estimates the joint posterior distribution of the model's parameters, including the unobserved common factors' uncertainty and dynamics.

2.1 Three-Step Estimation Method

This first estimation approach combines two major tools designed to address big data problems: PCA and variable selection methods. This technique is in line with the standard approach in the FAVAR literature following Bernanke et al. (2005) to estimate the unobserved factors and loadings with PCA, then separately estimate the VAR model. PCA is a computationally efficient, deterministic, semi-parametric approach to uncover the space spanned by the common components, as set forth in the literature following Stock and Watson (2002) that uses PCA to summarize the information in large macroeconomic datasets.

issues when trying to calculate inter-industry shock propagation. They use input-output and inter-sectoral capital network data to adjust for this. Our goal is to estimate inter-firm networks without imposing any ex-ante network assumptions, so we make the trade-off to not apply their adjustments and estimate the common factors purely empirically. They found that aggregate factors contributed less volatility over their sample period, while sectoral shocks did not change in importance and therefore became relatively more significant after the mid-1980s.

[6] We indeed find no evidence of serial correlation in the errors. The choice of one-lag VARs for modeling factors and idiosyncratic returns in Equations (1) and (2), respectively, is supported in our data by the standard Bayesian Information Criterion (BIC). Further, when including additional lags in either the Γ or ρ matrices, parameter estimates for longer lags are generally near zero and not statistically significant.

Step one is to select the number of common factors, K, with the panel Bayesian Information Criterion (BIC) method from Bai and Ng (2002) — which accounts for potential correlation in the idiosyncratic errors — and apply covariance matrix-based PCA inclusive of the means on the firm return series to estimate the factors and their loadings. Once the common variation for each firm (ΛF_t) is estimated with these, the idiosyncratic firm returns, R_t^I, are obtained under the simplifying assumption that the τ are negligible: $R_t^I = R_t - \Lambda F_t$. The second method below relaxes this assumption. Step two is to estimate Equation (1) via OLS on the factor series.

Step three utilizes the One Covariate at a time Multiple Testing (OCMT) variable selection procedure of Chudik et al. (2018) to deal with the curse of dimensionality and avoid over-fitting to estimate the large VAR of firm idiosyncratic returns in Equation (2). The OCMT procedure is intuitive in that one need only run a series of OLS regressions of the dependent variable on the potential explanatory variables, testing whether they have a statistically significant relationship with the dependent variable based on their coefficients' t-statistics. The key feature of this approach is that the critical values for accepting a variable are adjusted for the fact that this test is repeated for the many potential explanatory variables. The prior literature on VAR-based network estimation following Diebold and Yilmaz's work has generally used LASSO, Ridge, or adaptive elastic-net for variable shrinkage and selection (e.g., Diebold and Yilmaz, 2009, 2014, 2016; Bonaldi et al., 2015; Demirer et al., 2018; Grant and Yung, 2021). The OCMT procedure has several benefits over those algorithms: it is computationally faster, more efficient and parallelizable; it is statistically founded with clear individual variable inclusion rules; and there is not the randomness that can occur with the other methods due to cross-validation sampling selection, optimizer seeding, etc. For example, OCMT is over twenty times faster than adaptive elastic-net when estimating our networks. To estimate Equation (2), we perform OCMT on each row of it, combine the coefficient vectors to obtain ρ_0 and ρ and estimate Σ from the residuals.

2.2 State-Space Estimation Method

In this section, we cast the common factors and idiosyncratic returns as unobserved state variables and reformulate the whole system as a state-space model. This system is estimated using a Gibbs sampling approach following Carter and Kohn (1994), which alternates between drawing from the conditional distributions of the VAR parameters for the observation and state equations given the unobserved series, and drawing from the conditional distribution of the F_t and R_t^I series.

The observation equation for the state-space system is:

$$R_t = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \dots & \lambda_{1K} & 1 & 0 & \dots & 0 \\ \lambda_{21} & \lambda_{22} & \dots & \lambda_{2K} & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \lambda_{N1} & \lambda_{N2} & \dots & \lambda_{NK} & 0 & 0 & \dots & 1 \end{bmatrix} \begin{bmatrix} F_t \\ R_t^I \end{bmatrix} + \tau_t = [\Lambda \; I_N] \begin{bmatrix} F_t \\ R_t^I \end{bmatrix} + \tau_t. \quad (3)$$

The state equation is:

$$\begin{bmatrix} F_t \\ R_t^I \end{bmatrix} = \begin{bmatrix} 0_{K\times 1} \\ \rho_0 \end{bmatrix} + \begin{bmatrix} \Gamma & 0_{K\times N} \\ 0_{N\times K} & \rho \end{bmatrix} \begin{bmatrix} F_{t-1} \\ R_{t-1}^I \end{bmatrix} + \begin{bmatrix} \eta_t \\ \epsilon_t \end{bmatrix}. \quad (4)$$

We employ Bayesian VAR estimation in the vein of Litterman (1986)'s Minnesota prior to address the large number of parameters in Equation (2), utilizing dummy observations following Bańbura et al. (2010) to keep the model estimation numerically tractable. The Λ coefficient priors are centered on the PCA factor loadings, and the Γ, ρ_0, and ρ coefficients are centered at zero, reflecting a belief that the underlying data series follow random-walks in log levels.

This approach departs from the variable shrinkage and selection methods commonly used in the literature following Diebold and Yilmaz on estimating networks from VARs in two ways. First, as with the previous estimation method, this approach incorporates common factors, yet this time the factors and their dynamics are jointly estimated along with the remainder of the model. Second, the Bayesian methodology extends the LASSO and Ridge estimation procedures generally used in the literature by adding the full posterior distributions to calculate the network distributions from, with the posterior mode for the coefficients being LASSO under Laplace priors on them, and Ridge the mode under the Gaussian priors that we employ.

2.3 Firm Network Estimation

After we estimate the system with either the three-step or state-space approach, we derive the inter-firm network from generalized forecast error variance contributions (GFEVc). Other papers, including several in the Diebold-Yilmaz network series and thereafter, have used the generalized forecast error variance decompositions (GFEVD) or generalized impulse response functions (GIRF) of Pesaran and Shin (1998) to derive network edges given an estimated VAR system, the formulas for which we provide in the Online Appendix. We instead adjust the GFEVD to create our GFEVc's. The difference is that in our GFEVc's we do not divide through by the equity variance adjustment in the denominator. We do this because those adjustments vary by firm and over time, and we would like the edge weights to be comparable across both dimensions.

We calculate the GFEVc's from the reduced form of the VAR created by stacking the equations in our system:

$$\begin{bmatrix} R_t \\ R_t^I \\ F_t \end{bmatrix} = \begin{bmatrix} \rho_0 \\ \rho_0 \\ 0_{K\times 1} \end{bmatrix} + \begin{bmatrix} 0_{N\times N} & \rho & \Lambda\Gamma \\ 0_{N\times N} & \rho & 0_{N\times K} \\ 0_{K\times N} & 0_{K\times N} & \Gamma \end{bmatrix} \begin{bmatrix} R_{t-1} \\ R_{t-1}^I \\ F_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_t + \Lambda\eta_t + \tau_t \\ \epsilon_t \\ \eta_t \end{bmatrix}. \tag{5}$$

For notational convenience, we label the coefficient matrix on the lag term of this reduced-form system A_1. The covariance of the errors is:

$$\Theta = \begin{bmatrix} \Sigma + \Lambda\Upsilon\Lambda' + \mathcal{T} & \Sigma & \Lambda\Upsilon \\ \Sigma & \Sigma & 0_{N\times K} \\ \Upsilon\Lambda' & 0_{K\times N} & \Upsilon \end{bmatrix},$$

where by construction, innovations in ϵ_t and τ_t are independent of one-another and of η_t, yielding the zero blocks in Θ. The formula for the one-period ahead GFEVc network is:

$$GFEVc = \left[\Theta^2 + (A_1\Theta)^2\right] Diag(\Theta)^{-1}, \tag{6}$$

where the $Diag$ operation yields a square matrix with the diagonal entries of the given input along the diagonal and zeroes elsewhere, and the exponents are all applied to the individual elements of the matrices. This formula provides a network adjacency matrix with the edge sources along the columns and the destinations along the rows. We then create two types of networks from this matrix.

2.3.1 Total-Return Networks (R to R)

In our first network type, the edges from each source firm, s, to each destination one, d, reflect the response of the destination's total returns, R_t, when there are shocks affecting the source's total returns. In Equation (6), this would mean the entry for column s and row d. This specification forms a weighted, directed network capturing the way that firms' returns respond to one another, regardless of whether the innovations are to the firms' idiosyncratic returns or the aggregate factors. These networks are similar to those in Demirer et al. (2018) and Grant and Yung (2021) in this way.

2.3.2 Idiosyncractic Return Networks (R^I & Factors to R & Factors)

For our second network type, we distinguish the effects of firms' idiosyncratic shocks from those to the common factors. When doing so, we take the novel approach of explicitly treating the factors as nodes in the network. In these networks there are $N + K$ nodes, with one for each of the firms and factors. The edge from firm s to firm d will be the expected variance contribution to d's returns when shocks affect firm s's idiosyncratic returns, and likewise for a shock affecting factor $k = 1, 2, ..., K$. In these cases, we take the entry for column $N + s$ and row d of Equation (6) as the inter-firm edge, and the entry for column $2N + k$ and row d for the edge from factor k to firm d. In the case where the destination is factor k, then the edge from firm s to it will be the entry in column $N + s$ and row $2N + k$ — giving a zero effect — and the edge from another factor, f, to it will be the entry in column $2N + f$ and row $2N + k$. Additionally, in our analysis below we use the subset of this network formed by the connections from firms' idiosyncratic returns to other firms and refer to these networks as the R^I to R ones.

To understand the dynamics captured by the two network types, we simulate data and run our procedure above for estimating inter-firm networks, both when including the effects on common factors between firms with "R to R" networks, and when separating them out with "R^I & Factors to R & Factors" networks (Online Appendix Section D.4). The results clearly show that the "R to R" networks identify the organization of the system around the factors very well, while the "R^I & Factors to R & Factors" networks are better able to effectively estimate the bilateral simulated firm relationships by separating out the factors' influence. We also compared several methods used in the prior literature (i.e., LASSO with GIRFs based networks, Ridge with GFEVD, AEN). The methods used in the prior literature produce networks with similar structure to the "R to R" networks, demonstrating the value of our method for identifying bilateral firm relationships. These simulation results are in line with the main arguments of Bailey et al. (2016) and Hale and Lopez (2019) that one needs to account for common factors before estimating a network in order to properly recover the bilateral connections between its members.

3 Data

To select our sample of U.S. firms, we take the union of all firms that are in the top 25% by market capitalization for any year in our sample. If a firm is in our sample at any point, then we obtain its equity pricing data for as long as recorded. We do this to filter out the smallest firms and ensure that those in our sample have actively traded, liquid equity securities that are highly researched and followed, providing them with accurate price discovery. As such, the efficient markets hypothesis indicates that the firms' equity prices should reflect all available information about them, including how they are connected through the channels we wish to study. Our dataset includes the daily Bloomberg closing prices for $5,454$ firms between December 30$^{\text{th}}$, 1988 and June 20$^{\text{th}}$, 2017. The closing equity values are total-return indices inclusive of returns from dividends to avoid spurious price jumps when dividends are paid that do not reflect a change in the valuation of the underlying firm. We also gather the Bureau of Economic Analysis (BEA) sector and Bloomberg industry for each firm and collect U.S. input-output use tables from the BEA for 1997 through 2015 for our macroeconomic exposure networks described below.

We analyze both a balanced panel of firms to study the long-term firm network, and unbalanced rolling panels to account for firm entrance and exit from the equity markets. Specifically, we examine the set of 524 firms continuously traded throughout our whole sample — both over the full period and in rolling 10-year periods — and broader rolling samples of firms continuously traded over each 10-year window, with up to 1,548 firms.

4 U.S. Inter-Firm Networks

This section provides empirical results from applying our methodology to estimate inter-firm networks from U.S. equity daily log returns. In Section 4.1, we examine the networks' estimated common factors, with a focus on their sample variance shares over time and comparisons with macroeconomic and financial indicators. In Section 4.2, we utilize spatial graphical algorithms to depict the networks and study the evolution of their dynamics.

4.1 Common Factors

We apply our three-step and state-space methods to our U.S. equity data and here focus on the common factors obtained by their PCA and Bayesian estimation approaches, respectively. In selecting the number of common factors, K, the Bai and Ng (2002) panel information criteria suggests a range between one and three common factors in U.S. daily log equity returns, with a mode of three. The top three PCA factors combined account for 27% of the cross-sectional variation in the entire dataset.[7] Subsequent PCA factors each contribute less than 2%, so using three factors as suggested by the statistical test is consistent with the

[7] Our finding that equities are roughly 70% driven by idiosyncratic firm shocks is in line with Campbell et al. (2001), who found that over the period from 1962 to 1997, U.S. equities were 17% market driven, 12% by industry developments, and 71% by idiosyncratic considerations.

scree plot jump selection approach typically used to select the number of factors in PCA analysis.

The cumulative variance share of the top three factors increased significantly the past several decades, from around 12% at the start of our sample period in the early 1990s to over 35% towards the end in the mid-2010s.[8] The variance share for the first factor, for example, increased from 8.6% over the 1989-1998 period to 31.2% over 2008-2017, with the largest increase occurring when 2008 entered the rolling windows. The second and third factors saw their variance shares increase by just over a fifth with 0.5% and 0.2% increases. These results suggest a significant change in the importance of the common factors following the Global Financial Crisis, with implications for portfolio diversification strategies.[9] Further, the 2010-2017 variance shares for the top three factors remained elevated — they explain 25.1%, 2.4%, and 1.2% of the variance, respectively — even when not including the 2008-2009 crisis period with particularly high equity correlations.[10]

The first factor loads positively on all firms in the sample. This implies that positive shocks to the first factor translate into higher equity returns for all firms. In fact, the first factor series is almost identical to the sample average log returns for each day, both in levels and in year-over-year changes. For this reason, we refer to this factor as the market beta, reflecting both time variation in discount factors or risk premia, and economic growth, akin to the Fama and French market risk factor (Fama and French, 1995).[11] The firms with the highest loadings on the first factor are predominantly from the technology, consumer cyclical and financial sectors, while those with the lowest loadings are royalty trusts, consumer non-cyclical and utility firms (refer to the Online Appendix for details). This makes intuitive sense as the former are pro-cyclical sectors, while the latter are generally considered passive, acyclical investment sectors.

To help interpret the dynamics of each factor, Figure 1 compares the year-over-year changes of each PCA factor in black to the year-over-year changes in the median of the corresponding Bayesian factor in blue, in addition to the shaded 68% and 95% credible intervals of the latter. We use year-over-year changes rather than levels to avoid spurious correlations in these time-series data. Each of the three rows corresponds to one of the

[8] The Online Appendix contains time series plots of the cumulative sample variance shares explained by the top three factors in rolling ten year firm samples, and for our long-run balanced panel.

[9] Note that the structures of the idiosyncratic networks before and after the Global Financial Crisis were highly correlated at around 95%; however, the inter-sector sums increased about 2.8× on average.

[10] Bartram et al. (2018) also found an increasing share of firm level equity returns from common rather than idiosyncratic factors. Studying the period 1965-2017, they found that average idiosyncratic risk declined to an all-time low at the end of their sample.

[11] The idea of time variation in risk premia (and hence investors' expected stochastic discount factor) is consistent with empirical evidence from the finance literature, documenting its importance in accounting for excess volatility in asset prices and predictability of returns (e.g., Cochrane 1991, 2011).

factors, and within each plot the factors are compared to similarly moving real economic or financial indicators in red.

Panel (a) shows how year-over-year changes in the first factor compare with those of the most commonly referenced U.S. equity market average, the S&P500 Index. The correlations between the S&P500 Index and both the first PCA and Bayesian factors are 0.82, supporting the interpretation of the first factor as the market beta. Panel (b) shows that our first factor broadly reflects the growth of the U.S. economy, as it co-moves with the U.S. industrial production index, with year-over-year change correlations around 0.56. Results in the Online Appendix show similar results when U.S. growth is measured using real GDP.

Fluctuations in the second factor closely align with movements of the U.S. price level.[12] In the second row of Figure 1, we compare the second factor with different measures of inflation, including the year-over-year changes in the inverted Consumer Price Index (CPI) in Panel (c) and the inverted Producer Price Index (PPI) in Panel (d). The correlation of year-over-year changes in the second Bayesian factor and inverted PPI is 0.60, and that with inverted CPI is 0.47. Note that the PPI correlation being higher than the CPI one fits with our results below that the firm level equity market networks are more correlated with the upstream rather than downstream exposures. Similarly high correlations are found when comparing the second factor with other U.S. price-level related indices, including the trade weighted value of the dollar, dollar-euro exchange rate, dollar-British pound exchange rate, and U.S. 10-Year breakeven inflation calculated from TIPs and nominal U.S. Treasury bonds.

The second factor loads negatively on the energy, financial, basic materials, and utilities sectors, and positively on the technology sector. All of the top ten firms loading on the second factor are technology companies, and the bottom ten consists of nine energy firms and a petroleum shipping company.

The third factor has large positive average loadings for the energy sector, followed by the technology and base materials sectors. Nine of the top ten firms by their loadings on the third factor are energy related. Figure 1 displays the movements of the third factor relative to the year-over-year changes in the Goldman Sachs Commodity Index (GSCI) in Panel (e) and the price of Brent crude oil in Panel (f). These observations indicate that the third factor is similar to a raw commodity input measure, with the correlation between price changes in oil and this factor especially high at 0.68. We also compare the third factor against various components of the GSCI index and find the correlations to be positive except for cocoa, and generally economically significant in magnitude. The highest correlations are

[12] A role for price pressures in the inter-firm network is supported by, for example, Smets et al. (2019), who found evidence of inflation being passed through production networks.

with the petroleum-based energy commodities, followed by industrials and then precious metals. Additionally, around 2009 commodities seem to go from the third to the second most important common factor in our sub-sample analyses — possibly because of a lower inflation environment making the price level less important, while there were large subsequent commodity price changes. If one looks on either side of this break then the pertinent factor has a greater correlation with commodity prices. For example, when looking at all firms in our sample traded continuously from 2008 through 2017, the correlation of the year-over-year changes in the second factor with oil is 0.77, and that for the GSCI is 0.79. Around this time the credible interval on the second factor also widens significantly, suggesting a change in the underlying data generating process.

4.2 Inter-Firm Equity Return Derived Networks

The inter-firm networks that we estimate are complex objects, with N^2 or $(N+K)^2$ elements, depending on whether the common factors are included or not. To help understand and visualize these networks, we employ the ForceAtlas2 method from Bastian et al. (2014) that is commonly used across disciplines to plot networks. ForceAtlas2 is a force-directed layout algorithm to display network spatialization, transforming a network into a map where nodes with greater connectedness are closer together. At a high level, all of the nodes are repulsed from one another like charged particles, while edges attract their nodes like springs — yielding the name for this class of algorithm, spring plots. The final node positions provide a balanced state, helping to interpret the data without having to incorporate any other attributes of the network members. To read these plots, think of a map without a key showing the direction of true north, nor a scale. In that case, as with these plots, the precise orientation of the figures is not informative and rotations do not have a clear meaning, but the relative proximity of features on the plots to one another and the center of the figure do, as do any clusters that arise and inform the underlying topology. This technique is superior to other network visualization methods such as heat maps because the number of network members makes many other methods hard to read, and the spring plots are able to capture third party or greater relationships. For example, if a rubber and a glass manufacturer both have strong ties to Ford, but weak ones with each other, then they would still be close in these figures because they would both be near Ford.

Figure 2 provides spring plots for "R to R" total-return networks, inclusive of the common factors, for different time periods. Panel (a) displays the long-run inter-firm network of the 524 firms continuously traded from 1989-2017, and Panels (b), (c) and (d) show the sets of all firms traded within different decades, with about $1,500$ firms in each of those plots.

Figure 3 includes similar plots, but for the "R^I & Factors to R & Factors" idiosyncratic-return networks, which split out the common factors and treat them as network nodes themselves. Note that the parameters entered into the ForceAtlas2 algorithm for these are the same, making them comparable. Each circular node in these figures represents a firm and is colored by its BEA sector. We further distinguish the Real Estate Investment Trust (REIT) sector (in gray) from the financial sector given its distinct return profile. In Figure 3, the factors are represented by purple stars with the factor number above them.

Figure 2 (a) reveals that sectoral clusters are an important feature of the network, with firms within the same sector grouped together, such as utilities (blue) and commodities (orange) on the top right. Additionally, the finance (black), REITS (gray) and consumer (yellow) sectors are near the center of the network. This observation coincides with seven of the top nine firms by the sum of their weights out to others being financial firms, and the analysis of Grant and Yung (2021) where financial firms were at the center of the global firm network. Further showing the importance of finance, the number two and three firms are industrial diversified firms, and that third firm — General Electric — was designated a non-bank systemically important financial institution by the Financial Stability Oversight Council due to its high level of financial dealings up until it significantly changed its businesses in June 2016.

The rolling samples in Panels (b)-(d) also show the finance sector typically at the center in all three decades and the clustering by sector is present in the different periods. Notably, the networks show firms becoming more tightly grouped over time regardless of sector, suggesting greater equity market integration and matching the increased common factor variance shares mentioned above. Due to this feature, these plots — and the other network estimation methods used in the past in this literature — become less informative about individual connections between firms, as all of the equities move more with the common factors and fall into one compact cluster at the center of the network.

Figure 3 shows the idiosyncratic-return networks that separate the effects of the common factors from the individual equities, and in so doing "unfold" the underlying firm-to-firm connections that we want to study. In Panel (a), one can see that the first factor — the market beta — is at the center of the network denoted by the purple star, and that finance firms are near it. What can be further seen is that REITs form a distinct gray cluster farther from the center of the network, unlike in Figure 2 where they are nearly dead center and indistinguishable in location from the other financial firms. Additionally, the commodity firms are on the northern periphery of the network, near the third commodity factor, and the consumer sector firms are close to the second, price-level factor. This implies that the energy, base material, and utility firms near the third factor do not comove as much with the broad market factor, and the commodity factor is more influential for them, while consumables are more linked with prices.

Panels (b)-(d) show that sectoral clusters become more pronounced over time in the idiosyncratic-return networks, possibly reflecting increased specialization and more integrated within-industry production processes, or the rise in sector specific investment funds.[13] Similar to Panel (a), the REITs are a distinct cluster from the remainder of the financial firms in the decade plots. They are close to the utility firm cluster, which is interesting as both are often seen as safe, acyclical, dividend-oriented investment sectors. The commodity sector can be seen to move between being near the second and third factors, in line with the factor discussion in the prior section.

To illustrate an application of these equity return networks to study shock propagation in real-time, in the Online Appendix we model the impact of market beta and commodity price shocks across firms using our estimated networks. This analysis shows that financial, consumer cyclical and commodity firms are most affected by a market beta/growth shock, with the financial sector at the center of the network in close proximity to this factor. A negative commodity price shock, on the other hand, most adversely affects energy and base materials companies. In fact, the top 10 declining equities following the shock are for firms in the oil and gas extraction sub-sector. On the other hand, United Continental Holdings — the parent company of United Airlines — would be expected to have the largest positive response. This result likely reflects the high fuel costs faced by airlines and exemplifies how these networks can inform firm managers, policy makers, and investors of their latent risks.

5 Theoretical Model of Firm Exposures

Having extended the empirical methods to construct inter-firm networks from log equity returns, in this section we theoretically ground that approach. Our theoretical model is an extension of the one-period, multi-sector DSGE model of Baqaee (2018) to a dynamic, stochastic setting with inter-temporal assets, the details of which we leave to the Online Appendix. There are four key results applicable to our analysis:

1. Firms' equity returns depend on three common factors, and their exposures to upstream and downstream productivity and demand shocks through the production network.

[13] Evidence of changing production processes — with increases in production fragmentation and specialization along the production chain — is especially strong in the trade literature given the high quality data on cross-border goods flows. Timmer et al. (2014) found that cross-border intermediate input trade measured as the foreign value-added content of production has rapidly increased since the early 1990s, and Bridgman (2012) pointed to a rapid expansion of manufactured parts traded over the past forty years.

2. The three common factors represent: the broad market beta, capturing changes in the stochastic discount factor and GDP; the real price level; and the supply of raw inputs.
3. Firms' upstream and downstream exposures can be calculated from their place in the real input-output network.
4. These upstream and downstream exposures are embedded in the innovations of a VAR(1) of firms returns on one-another, with the common factors removed.

These results have several implications for our analysis of upstream, downstream and common firm shocks. First and foremost is that they support the choice in the literature to estimate inter-firm networks from VARs of firms' equity returns. Second, they highlight the importance of our empirical extension to distinctly evaluate the role of common factors as additional network nodes, separate from firms' idiosyncratic returns. Third, they substantiate our use of three common factors and their interpretations above. Finally, they provide an approach to evaluate the relative historical importance of upstream versus downstream firm exposures: use U.S. input-output tables to calculate sectoral upstream and downstream exposures, then compare these to the propagation of realized firm shocks as embedded in equity return based networks aggregated to the sectoral level. We perform the latter analysis in Section 6 to assess the importance of upstream and downstream shocks.

5.1 Equity Returns, Common Factors & Network Centralities

We now examine firm equity returns in the model, finding that they depend on three common factors and each firm's proximity through the inter-firm network to the sources of productivity and demand shocks. To begin, there are two types of production network centralities that arise in the model, capturing how each firm acts as a consumer of raw inputs, and as a supplier of final goods to the households.

The *consumer centrality* measures the degree to which a firm consumes raw inputs itself and through others, and with that its exposure to shocks to its own and other upstream firms' productivity. The vector of consumer centralities, $\tilde{\alpha}_t$, is defined as:

$$\tilde{\alpha}_t \equiv \underbrace{\left[I_J - \mu^{1-\sigma}\Omega\right]^{-1} \mu^{1-\sigma}}_{\equiv \Psi_d} \upsilon_t. \tag{7}$$

Ψ_d is a function of the firms' positions within the production network and can be thought of as an elasticity adjusted Leontief inverse, where I_J is the $J \times J$ identity matrix (J is the number of industries), μ is a square matrix with adjustments for the industries' intra-industry elasticity of substitution ($\mu_j \equiv \frac{\varphi_j}{\varphi_j - 1}$) on the diagonal, σ is the inter-industry elasticity of substitution, and Ω summarizes the production technology, having the weights of each industry's goods as inputs in the others' production functions. Additionally, v_t is a vector of the sectoral productivity parameters, which follow a standard autoregressive data generating process:

$$\Delta v_t = c + \Xi \Delta v_{t-1} + \Delta \varepsilon_t, \tag{8}$$

with c a constant vector, Ξ an autoregressive coefficient matrix, and the $\Delta\varepsilon_t$ random shocks.

In the other direction, the *supplier centrality* measures how a firm is exposed to household demand shocks for its own and other downstream firms' goods. The supplier centrality vector, $\tilde{\beta}_t$, is defined as:

$$\tilde{\beta}_t' \equiv \beta_t' \underbrace{\left[I_J - \mu^{-\sigma}\Omega\right]^{-1}}_{\equiv \Psi_S}, \tag{9}$$

where β_t is a vector of β_{tj} consumer taste weights. $\tilde{\beta}_{tj}$ reflects the network adjusted final consumption share of firms in industry j. The households are assumed to have a base set of preference weights, $\bar{\beta}$, subject to random taste shocks, Z_t:

$$\beta_t = \bar{\beta} + Z_t. \tag{10}$$

We next derive the log steady-state equity price of firm i in industry j $\big(ln(q(j,i))\big)$ from the equity prices' Euler equation, which links firms' equity prices to macroeconomic fundamentals and the production network centralities:

$$ln(q(j,i)) = \underbrace{ln\left(\frac{1}{\varphi_j}\right)}_{\text{Markup}} + \underbrace{ln\Bigg(\underbrace{\frac{1}{1-\psi}}_{\substack{\text{Discount}\\ \text{Factor}}} \underbrace{P_c U}_{\substack{\text{GDP /}\\ \text{Aggregate}\\ \text{Demand}}}\Bigg)}_{\substack{\text{Broad Market}\\ \text{Beta}}} + \underbrace{ln\left(\frac{P_c}{\tilde{R}}\right)^{\sigma-1}}_{\substack{\text{Real Price}\\ \text{Level}}} + \underbrace{ln\tilde{z}^{\sigma-1}}_{\substack{\text{Raw}\\ \text{Input}\\ \text{Supply}}} + \underbrace{ln\tilde{\alpha}_j}_{\substack{\text{Consumer}\\ \text{Centrality}}} + \underbrace{ln\tilde{\beta}_j}_{\substack{\text{Supplier}\\ \text{Centrality}}}, \tag{11}$$

where the steady-state value of the stochastic discount factor is ψ, that of GDP is equal to the aggregate consumption price index (P_c) times the total consumption index (U), $\tilde{R}$ is the price for the composite of raw inputs, and $\tilde{z}$ is the labor-capital raw inputs aggregate. This equation shows that firm equity prices depend on the markups, three common factors, and the upstream and downstream network centralities. The three common factors represent: the broad market beta, capturing the discount factor and GDP; the real price level; and the supply of raw inputs. These three common factors align with those that we found using our empirical model, which is notable given that we based our empirical factors on standard PCA on the daily equity returns, without applying any identifying assumptions or rotations.

The log equity returns induced by changes in the common factors, the productivities and demand parameters can be approximated by differencing the first order Taylor expansion of

Equation (11) around the steady-state. Letting R_t be a vector of the firm log equity returns, ΛF_t the common factor loadings and their log changes, and R_t^I a vector of idiosyncratic firm returns, we have:

$$R_t = \Lambda F_t + R_t^I = \Lambda F_t + \underbrace{Diag\left(\frac{1}{\tilde{\alpha}}\right)\Psi_d}_{\mathcal{U}\equiv\text{Upstream Exposure}} \Delta v_t + \underbrace{Diag\left(\frac{1}{\tilde{\beta}}\right)\Psi_S'}_{\mathcal{D}\equiv\text{Downstream Exposure}} \Delta\beta_t = \Lambda F_t + \mathcal{U}\Delta v_t + \mathcal{D}\Delta\beta_t. \quad (12)$$

In the upstream exposure matrix, $\mathcal{U}$, each entry measures the exposure of the row sector to a productivity shock from the column sector, both directly and possibly indirectly through other sectors whose products are between theirs in a production chain. The idiosyncratic response of a firm in industry j to innovations in an upstream source industry s would be $\frac{\iota_j'\Psi_d\iota_s}{\tilde{\alpha}_j}\Delta v_s = \mathcal{U}_{js}\Delta v_s$, where ι_j is a selection vector with a one in the j^{th} position and zeroes elsewhere.

The downstream exposure matrix, $\mathcal{D}$, provides exposures to demand shocks through the network. The supplier centrality quantifies the intensity with which the household consumes from an industry, both directly and indirectly through its downstream users. The downstream exposure matrix captures the potential for propagation of taste shocks for downstream goods to each industry as the ratio of its centrality to downstream industries' relative to its total downstream exposure. The idiosyncratic return from a taste shock to a downstream industry s is $\frac{\iota_j'\Psi_S'\iota_s}{\tilde{\beta}_j}\Delta\beta_s = \mathcal{D}_{js}\Delta\beta_s$.

Further, rearranging the idiosyncratic return portion of Equation (12), and assuming $\Delta\varepsilon_t$ and ΔZ_t from Equations (8) & (10) are vectors of mean zero i.i.d. random shocks, yields:

$$\begin{aligned} R_t^I &= \rho_0 + \rho R_{t-1}^I + \mathcal{U}\Delta\varepsilon_t + \mathcal{D}\Delta Z_t + \zeta_t \\ \implies R_t^I &= \rho_0 + \rho R_{t-1}^I + \epsilon_t \end{aligned} \quad (13)$$

where ρ_0 is a constant vector of firm fixed effects, ρ is an $N \times N$ influence matrix, and ζ_t is a residual orthogonal to the two shocks. This formula matches Equation (2), where the R_t^I idiosyncratic returns follow a VAR(1) process. Further, this indicates that the upstream and downstream exposures are embedded in the ϵ_t residuals of this system and should be recoverable from our GFEVc's. We utilize the results of this section below to empirically evaluate upstream and downstream exposures in our estimated equity return networks.

5.2 Theoretical Upstream & Downstream Shock Propagation

To provide intuition on upstream versus downstream exposure propagation, we simulated productivity (v_j) and taste (β_j) shocks in our DSGE model for a network of five industries and a household sector, forming the classical "X-type" production network that is often used as an example in this literature.[14] Figure 4 Panel (a) displays the structure of the production network, with the central firm 2 (orange) consuming inputs from firms 1 (black) and 4 (green), while supplying its output to firms 3 (blue) and 5 (yellow). The latter two firms ultimately sell their goods to the household (gray).

Panels (b) through (e) show firms' idiosyncratic responses to different shocks in the network, with the common components removed to uncover input-output connections in firms' equity returns, leaving only the final two upstream and downstream components in Equation (12). In each case, the y-axis measures the change in equity prices from being in steady-state at the initial parameter levels to the new steady-state after the associated shock. The x-axis measures the relevant upstream ($\mathcal{U}$) or downstream ($\mathcal{D}$) exposure to the sector in which the shock originates, multiplied by the change in the specified parameter. For the productivity shocks, this is $\mathcal{U}_{js}\Delta v_s$, and for the taste shock it is $\mathcal{D}_{js}\Delta\beta_s$, where s is the source sector for the shock and j is the target one. These are the network adjusted use of raw inputs through sector s in the first term and the indirect sales through industry s in the second, scaled by the overall network adjusted raw input use and sales of a firm in industry j, respectively. The first feature of these plots that stands out is that the idiosyncratic responses to the shocks lay along the 45-degree lines, indicating that the dynamic responses of the idiosyncratic log returns match the expectations given the corresponding upstream and downstream exposures.

Panel (b) simulates a productivity shock for one of the most upstream firms (firm 1), and Panel (d) for one of the most downstream firms (firm 3) to compare supply network shock propagation in each extreme case. In each case, the firm experiencing a productivity shock, marked with an X, has the largest centrality to itself and hence the greatest idiosyncratic equity response. Since there are no firms downstream from firm 3, the other sectors have zero upstream centrality exposures to it, hence zero idiosyncratic returns to its productivity shock in Panel (d). On the other hand, firm 1's productivity shock affects the idiosyncratic returns of firms 2, 3, and 5, since they are all downstream from it. Further, firm 2 is more directly exposed to firm 1 so it has a greater response, while firms 3 & 4 have the same upstream exposures to firm 1 through firm 2, so they have the same equity responses. Bringing these two examples together, Panel (c) shows a productivity shock for the centrally located firm 2. The two upstream firms (1 & 4) have zero idiosyncratic returns, while the two downstream firms (3 & 5) have positive returns as they benefit from firm 2's productivity improvement.

Finally, Panel (e) shows that firms 1, 2 & 4 are similarly affected by a taste shock to downstream firm 3's production, reflecting that they have comparable downstream reliance

[14] We explore other network forms in the Online Appendix with qualitatively similar responses to shocks.

on firm 3. Firm 5's only relationship to firm 3 is as a direct competitor in the final goods market, therefore, since it is not upstream of firm 3, it has a zero idiosyncratic response to the taste shock. These simulations illustrate our prior insights: the importance of removing common factors to uncover the network connections between firms; and that once the common factors are removed, equity returns reflect the proximity of firms through the two exposure matrices.

6 Analysis of Upstream & Downstream Exposures

We now bring together our empirical and theoretical models to evaluate the historical importance of upstream and downstream exposures in the U.S. We do so by comparing the empirical equity return based networks that embed the realized propagation of shocks across firms aggregated to the sectoral level, with U.S. input-output table derived sectoral upstream and downstream exposures.[15] We consider both the "R to R" (total-return) and the inter-firm portion of our "R^I & Factors to R & Factors" equity networks to focus on the relationships between firms' idiosyncratic returns, removing the impact of common factors. Further, we examine both the three-step and state-space estimated networks.

For the input-output based networks, we follow Equation (12) to estimate the upstream and downstream exposure matrices, $\mathcal{U}$ and $\mathcal{D}$.[16] We utilize the BEA's U.S. input-output use table data from 1997 through 2015. The BEA refers to the use tables as a "recipe" matrix because they show the inputs necessary to produce the output of each sector. These tables provide the dollar expenditures on commodities from each sector by households, the government, and other firms as intermediate inputs.[17] The upstream and downstream exposure matrices calibrated from these data are arranged so that the supplier of an input is in the column and the user is in the row to align with the orientation of our equity-based networks.

[15] The correlation between the number of firms in our 1989-2017 sample in each BEA sector and sectors' output shares is 0.85-0.9 over time, indicating our sample has representative coverage of the sectors.

[16] We follow the literature in assuming a Cobb-Douglas form ($\sigma = 1$) when calibrating our model parameters from the input-output data. This assumption is standard in the literature because the industry-level prices cannot be cleanly separated from their output quantities in the sectoral data, while with a Cobb-Douglas structure only the expenditures matter, not the breakdown between prices and quantities. If we take the parameters as estimated assuming a Cobb-Douglas form and then vary σ, we find that our results are minimally affected by changes to this parameter, so beyond helping us take the model to the data, this assumption does not appear to affect our main results.

[17] We exclude the "Other services, except government" and "Government" sectors to focus on the private sector. The BEA input-output tables we use throughout are North American Industry Classification System based, with surveys at the establishment level. We also tried using the older Standard Industrial Classification based data that is at the firm level — in case the discrepancy between the level of our equity networks and the establishment level input-output networks skewed our results — but this change made little difference for the years where we have both use table types.

We compare the equity and input-output exposure networks using the element-by-element correlations between their adjacency matrices. To calculate the significance of these correlations, we borrow a network correlation distribution bootstrapping method from the machine learning literature: the Quadratic Assignment Procedure (QAP). The QAP allows us to estimate the distribution of the network correlations conditioned on the observed network structure. Doing so instead of using the typical procedure to calculate the significance of correlations between two data series is particularly important given the structure of the networks we study, where the weights from sectors to themselves are all expected to be substantial (i.e., the correlations are ex-ante expected to be sizable and positive due to the large diagonal entries in the adjacency matrices).

The test evaluates the expected share of random re-orderings of the first matrix that have a higher correlation with the second matrix than the first matrix itself does. The new re-ordering is the same for the rows and columns in the bootstrapped network, creating a hypothetical network with similar structure to that observed. In our case, letting $\varrho(\cdot,\cdot)$ be the pairwise correlation of the entries of two provided matrices, IO the upstream or downstream input-output network adjacency matrix under examination, and F the candidate firm equity network adjacency matrix, this means using random re-ordered draws of the IO matrix, IO_q, to estimate $\mathbb{E}\big[\mathbb{1}[\varrho(IO,F) < \varrho(IO_q,F)]\big] \approx \frac{1}{Q}\sum_{q=1}^{Q}\mathbb{1}[\varrho(IO,F) < \varrho(IO_q,F)]$ to derive the test's p-value for the three-step estimation based networks, where we take ten thousand draws. We extend this approach to account for estimation uncertainty using the state-space method's Bayesian draws. In addition to re-orderings of IO_q, we also draw $F^{(q)}$, an equity-based network from the retained Markov Chain Monte Carlo draws: $\mathbb{E}\big[\mathbb{1}[\varrho(IO,F) < \varrho(IO_q,F)]\big] \approx \frac{1}{Q}\sum_{q=1}^{Q}\mathbb{1}[\varrho(IO,F^{(q)}) < \varrho(IO_q,F^{(q)})]$ for one million draws.

Table 1 presents the correlations between the input-output based networks and the corresponding firm-equity based networks obtained with either the three-step estimation method in Panel (A) or the state-space estimation method in Panel (B). The rows indicate the estimation periods for the firm-equity based networks (Equity Network Period), and input-output year near their midpoint that they are compared with (IO Year). The top results are for rolling 10-year windows ending from 1989 through 2017, followed by the full 1989 to 2017 networks at the bottom. The equity-based networks include the same 524 firms from our long-run balanced sample to remove the potentially confounding impact of a changing sample on the correlations over time. We also provide the averages and standard deviations for the 10-year windows.

Within each panel, we provide four comparisons of the combinations of the two input-output network types (upstream $\mathcal{U}$ and downstream $\mathcal{D}$ exposure matrices), and the two equity network types (total-return and idiosyncratic-return networks). The stars indicate statistical significance as calculated using QAP.

There are two key takeaways from Table 1. First, as to our core question, the correlations indicate a greater role for upstream exposures than for downstream ones, as the correlations between the equity-response networks and the upstream exposures are consistently higher than those for the downstream exposures. Further, the correlations with the upstream exposures are all statistically significant, whereas none of those for the downstream exposures are so. For example, focusing on the full 1989-2017 sample in the final row, we see that the total-return network has correlations of 0.45 (three-step method) and 0.41 (state-space method) with the upstream exposure matrix, while the downstream correlations are 0.04 (three-step method) and 0.02 (state-space method). Similar results hold for the idiosyncratic-return networks.

These results indicate that shocks from upstream suppliers of which a firm is either directly or indirectly a customer, matter more for short-term equity responses than shocks from downstream in the production process. This is suggestive of low short-term elasticities of substitution across inputs with more flexibility on the downstream, customer side. It may also reflect greater average heterogeneity of firm's customer sets relative to the firm's individual supply chains, so that downstream sourced shocks are implicitly insured against.

This result is consistent with, for example, the significant supplier disruptions experienced in the aftermath of the 2011 Tohoku earthquake. While Carvalho et al. (2016) identified quantitatively large upstream and downstream spillovers after the quake, their empirical analysis found directed propagation from upstream to be robust to parametrization, with the positive/negative propagation of shocks from downstream firms dependent on the elasticity parameters instead. Importantly, they assessed that the transmission of shocks over input-output linkages accounted for a 1.2% decline in Japanese GDP in the year following the earthquake. Yet, these effects were not localized to the immediate area. Boehm et al. (2019) found that Japanese affiliates abroad, reliant on imports from the affected zones, saw output drop about one for one with imports of intermediate goods from Japan in the wake of the disaster, suggesting extremely low elasticities of substitution for inputs in the short-term. Similarly, Jones (2011) studied production linkages and intermediate input use, finding that problems along a production chain can sharply reduce output under input complementarity.

The second takeaway is that the correlations for the idiosyncratic-return networks are consistently higher than for the total-return networks for all of the different time periods, both sectoral exposure network types, and both estimation methods. For the upstream exposures, the average idiosyncratic-return network correlation is about 30% higher than the total-return one (0.46 versus 0.62 for the three-step method, and 0.42 versus 0.52 for the state-space one). For the downstream exposures, the idiosyncratic-return networks also yield higher results than the total-return networks; however, none of these are statistically significant. The total-return network correlations with the downstream exposures average only 0.06 (0.02 for the state-space estimation), while the average for the idiosyncratic networks is 0.22 (0.14). These results accord with Equation (13), where the common factors are first removed from the equity returns before deriving the VAR(1) connecting them with the upstream and downstream centralities, underscoring the importance of explicitly modeling the impact of the common factors to uncover inter-sectoral connections.

Additionally, note that the estimation method used is not crucial for delivering the results of this exercise, and that the key findings hold across the three-step and state-space equity network estimation methods. This observation is important with regard to robustness of our results, as well as recommending our three-step method as a more efficient estimation approach that can still deliver the key results.

Our main takeaways are robust to different assumptions about network specifications and sampling choices. In Table 2, Panel (A) shows that using a balanced panel of firms over time is not a critical assumption. When instead including all firms within each 10-year window, such that firms enter and exit the sample over time, the average correlation across these samples for the idiosyncratic-return networks is 0.61 for upstream exposure (versus 0.62 above) and 0.19 for downstream exposure (versus 0.22 above). We also construct model-free networks where the edge weights are the bilateral daily equity return correlations between each firm pair, with results in the final two columns of the table. The correlations of these model-free networks with the input-output exposures for the rolling firm sets in Panel (A) ($\mathcal{U}$ correlation of 0.41 and $\mathcal{D}$ correlation of 0.01) and constant balanced firm panel in Panel (B) ($\mathcal{U}$ correlation of 0.40 and $\mathcal{D}$ correlation of 0.02) are similar to those obtained when comparing the total-return networks. In fact, these simple networks are highly correlated with the total-return networks (0.95-0.99 correlations when aggregated at the BEA sector level); however, these two approaches have lower correlations with the input-output exposures than the idiosyncratic-return networks, reinforcing the need to account for common factors when using VARs to evaluate their empirical connections to properly uncover inter-firm relationships. The table also shows that our results hold for the lower frequency changes in market expectations captured using monthly returns, and that the choice of daily data frequency used for the empirical analysis does not drive our results.

Finally, in untabulated analysis, we also compare the equity networks against the standard Leontief inverse and find that the equity-based networks — particularly the idiosyncratic ones — strongly reflect the underlying macroeconomic relationships between sectors captured therein. The similarity in results for the upstream exposure and Leontief inverse is not surprising, as they are closely related. In fact, the two are the same under Cobb-Douglas aggregation ($\sigma = 1$) or perfect competition ($\lim_{\varphi_j \to \infty} \forall j$, so that $\mu_j = 1 \, \forall j$). The correlations for the upstream exposures are consistently greater than those for the Leontief inverses for both equity-return network types, implying a meaningful role for market competition, the demand elasticities across goods, and markups in the structure of inter-firm networks.

7 Conclusion

In this paper, we propose an approach to derive inter-firm networks from equity returns, which can isolate the effects of common factors and explicitly model them as nodes within the network. We present two alternative methods to estimate the large FAVAR that provides the foundation for these networks, each of which utilizes big data econometric techniques, with their own trade-offs. We ground this approach in a theoretical model, then utilize that model to evaluate the historical propagation of upstream versus downstream exposures. We find upstream exposure (shocks to a firm's direct and indirect suppliers) to be more evident than downstream exposure (shocks to its direct and indirect customers) in the equity returns based networks. Our findings have meaningful implications for understanding the reactions of firms relative to one another. Further, these networks potentially allow for the real-time monitoring of economic developments at a frequency and disaggregated level that would otherwise be difficult to study in macroeconomic data. Additionally, our results support the use of the idiosyncratic equity networks to study connections across firms, removing the confounding influence of common factors.

Our work suggests several areas that warrant further investigation. First, understanding why the common factors that drive equity returns have increased in influence. Delving into the precise channels through which firms are connected — e.g., intermediate goods, services or credit — to understand the specific mechanisms of contagion would also be fruitful. Finally, a greater importance of upstream firms relative to downstream ones could be applied to businesses hedging risks, or designing trade and international economic policies.

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TABLES AND FIGURES

Table 1: Comparisons of Firm Equity Return Networks & Input-Output Exposures

		(A) Three-Step Estimation Method				**(B) State-Space Estimation Method**			
Equity Network Type:		Total-Return		Idiosyncratic-Return		Total-Return		Idiosyncratic-Return	
Equity Network Period	IO Year	Upstream Supplier Exposure ($\mathcal{U}$)	Downstream Customer Exposure ($\mathcal{D}$)	Upstream Supplier Exposure ($\mathcal{U}$)	Downstream Customer Exposure ($\mathcal{D}$)	Upstream Supplier Exposure ($\mathcal{U}$)	Downstream Customer Exposure ($\mathcal{D}$)	Upstream Supplier Exposure ($\mathcal{U}$)	Downstream Customer Exposure ($\mathcal{D}$)
1989-1998	1997	0.50***	0.08	0.61***	0.20	0.42**	0.02	0.49***	0.10
1990-1999	1997	0.51***	0.10	0.60***	0.19	0.42**	0.03	0.48**	0.08
1991-2000	1997	0.52***	0.11	0.60***	0.19	0.43**	0.03	0.49**	0.09
1992-2001	1997	0.51***	0.11	0.60***	0.20	0.43**	0.04	0.51**	0.10
1993-2002	1998	0.49***	0.09	0.61***	0.20	0.42**	0.03	0.52**	0.12
1994-2003	1999	0.47***	0.07	0.61***	0.19	0.42**	0.03	0.51**	0.11
1995-2004	2000	0.47***	0.07	0.60***	0.19	0.42***	0.03	0.51**	0.11
1996-2005	2001	0.45***	0.07	0.60***	0.19	0.40***	0.03	0.50***	0.11
1997-2006	2002	0.45***	0.07	0.60***	0.20	0.41***	0.03	0.50***	0.12
1998-2007	2003	0.45***	0.07	0.60***	0.21	0.40***	0.03	0.52***	0.14
1999-2008	2004	0.45***	0.05	0.62***	0.22	0.41***	0.03	0.53**	0.15
2000-2009	2005	0.46***	0.03	0.63***	0.21	0.42**	0.02	0.53**	0.14
2001-2010	2006	0.45***	0.03	0.63***	0.21	0.43**	0.01	0.53**	0.13
2002-2011	2007	0.45***	0.02	0.63***	0.22	0.43**	0.01	0.53**	0.13
2003-2012	2008	0.44***	0.03	0.63***	0.22	0.42**	0.02	0.53**	0.15
2004-2013	2009	0.41***	0.03	0.62***	0.23	0.39***	0.02	0.52**	0.17
2005-2014	2010	0.42***	0.02	0.63***	0.25	0.41**	0.02	0.53**	0.19
2006-2015	2011	0.43***	0.03	0.63**	0.29	0.42**	0.02	0.54**	0.24
2007-2016	2012	0.44***	0.03	0.63**	0.30	0.42**	0.02	0.53**	0.20
2008-2017	2013	0.44***	0.03	0.63**	0.30	0.42**	0.02	0.53**	0.20
Average		0.46	0.06	0.62	0.22	0.42	0.02	0.52	0.14
Std. Dev.		(0.03)	(0.03)	(0.01)	(0.04)	(0.01)	(0.01)	(0.02)	(0.04)
1989-2017	2001	0.45***	0.04	0.62***	0.21	0.41***	0.02	0.54**	0.14

Note: The table displays correlations between U.S. input-output table based upstream ($\mathcal{U}$) and downstream ($\mathcal{D}$) exposures from Section 5 and firm equity return based networks from Section 4, aggregated at the BEA sector level. Equity networks in Panel (A) are constructed with the three-step estimation method, and those in Panel (B) with the state-space estimation method. The total-return equity networks include the impact of similar loadings on common factors, whereas the idiosyncratic-return networks separate out common factors to focus on firms' direct bilateral connections. The equity return based networks use the same balanced panel of 524 firms continuously traded over 1989-2017, but for the different time frames listed in the "Equity Network Period" column, with "IO Year" the year of the input-output tables used for comparison. Statistical significance calculated using QAP with ten thousand draws for the three-step approach and one million for the state-space method, where *** p<0.01, ** p<0.05, and * p<0.1.

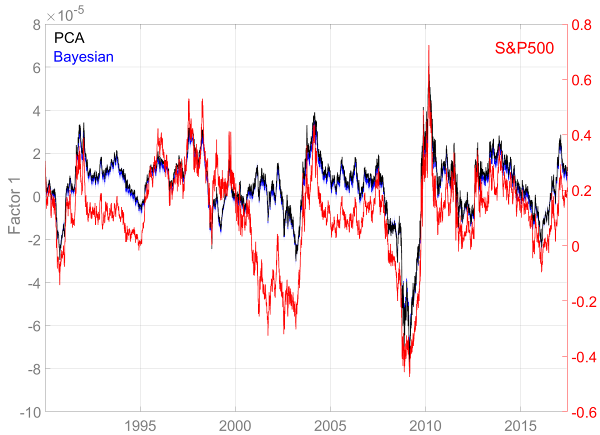
Table 2: Firm Equity Return Networks & Input-Output Exposures: Robustness

	Equity Network Type:	Total-Return		Idiosyncratic-Return		Bilateral Return Correlations	
		Upstream Supplier Exposure ($\mathcal{U}$)	Downstream Customer Exposure ($\mathcal{D}$)	Upstream Supplier Exposure ($\mathcal{U}$)	Downstream Customer Exposure ($\mathcal{D}$)	Upstream Supplier Exposure ($\mathcal{U}$)	Downstream Customer Exposure ($\mathcal{D}$)
(A) Average of Unbalanced Panels (10-year rolling sub-samples ending 1998 to 2017)	Daily Data	0.44	0.04	0.61	0.19	0.41	0.01
(B) Balanced Panel	Monthly Data	0.43***	0.04	0.55***	0.14		
(Equity Period: 1989-2017, IO Year: 2001)	Daily Data	0.45***	0.04	0.62***	0.21	0.40***	0.02

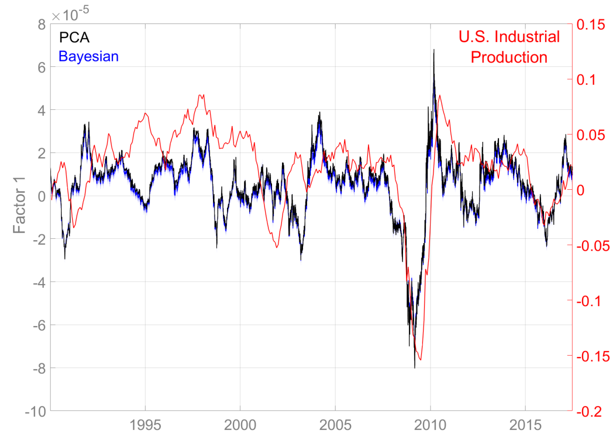
Note: The table displays correlations between U.S. input-output table based upstream ($\mathcal{U}$) and downstream ($\mathcal{D}$) exposures from Section 5 and firm equity return based networks from Section 4 in the first four data columns, aggregated at the BEA sector level with the three-step estimation method. The total-return equity networks include the impact of similar loadings on common factors, whereas the idiosyncratic-return networks separate out common factors to focus on firms' direct bilateral connections. The final two columns instead use equity networks where firms' bilateral equity return correlations are the edge weights. Panel (A) averages across unbalanced 10-year rolling sub-samples ending from 1998 to 2017, with each containing the maximum number of firms continuously traded over its period and compared to the input-output year nearest its midpoint. Panel (B) displays correlations for networks from a balanced panel of 524 firms continuously traded over 1989-2017, relative to the 2001 input-output based network, either from daily or monthly data. Statistical significance for Panel (B) is calculated using QAP with ten thousand draws, where *** $p<0.01$, ** $p<0.05$, and * $p<0.1$.

Figure 1: Equity Return Common Factors and U.S. Economic Indicators

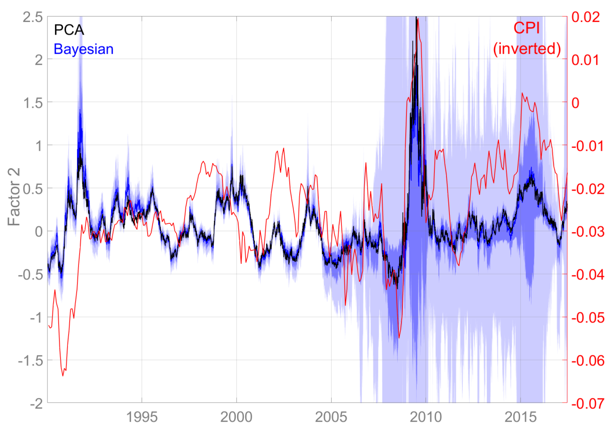
(a) First Factor vs. S&P500



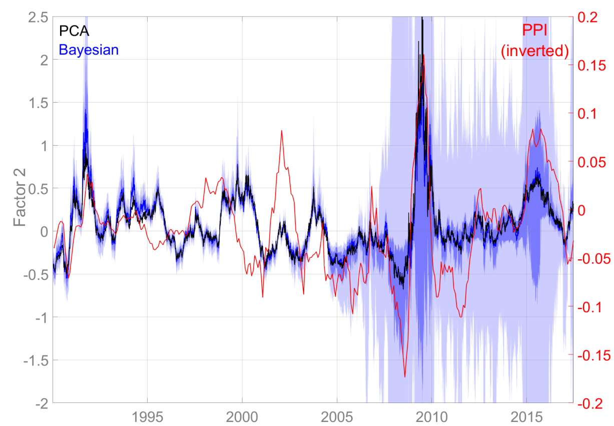
(b) First Factor vs. Industrial Production



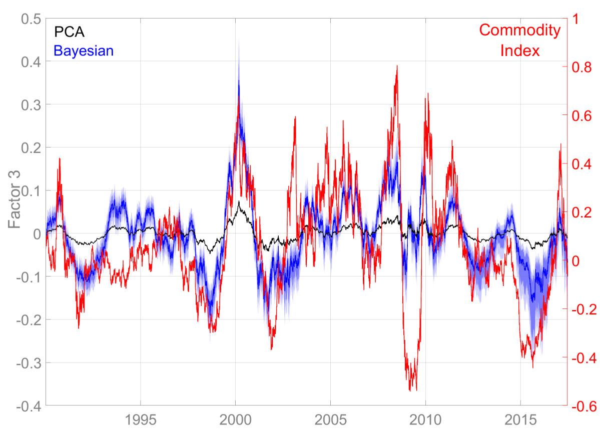
(c) Second Factor vs. CPI



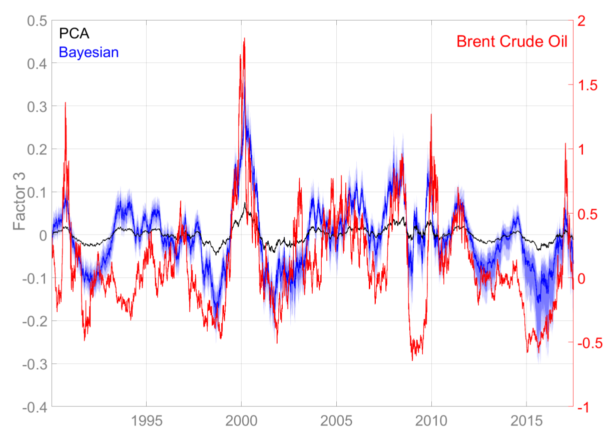
(d) Second Factor vs. PPI



(e) Third Factor vs. GSCI Commodity Index

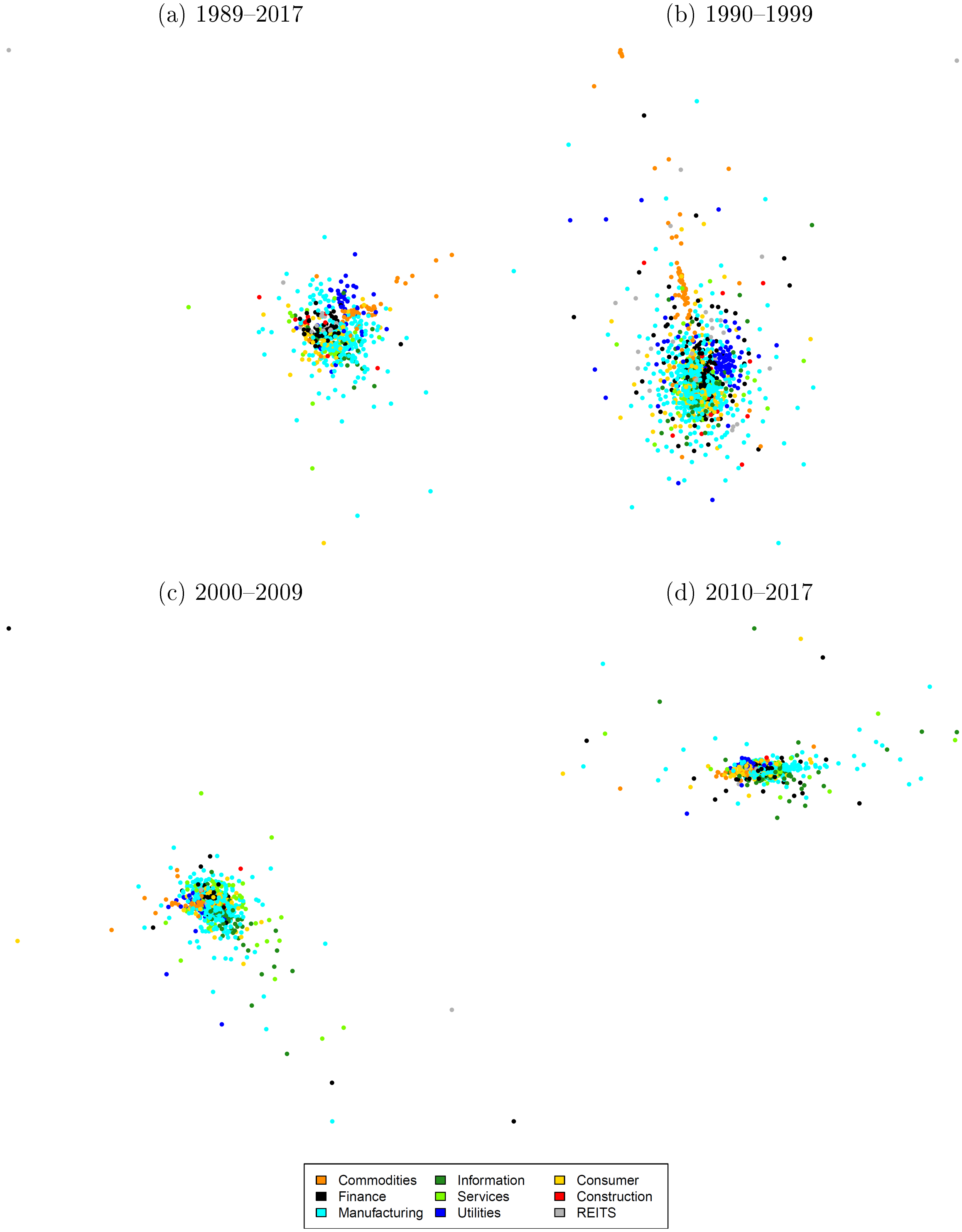


(f) Third Factor vs. Brent Crude Oil



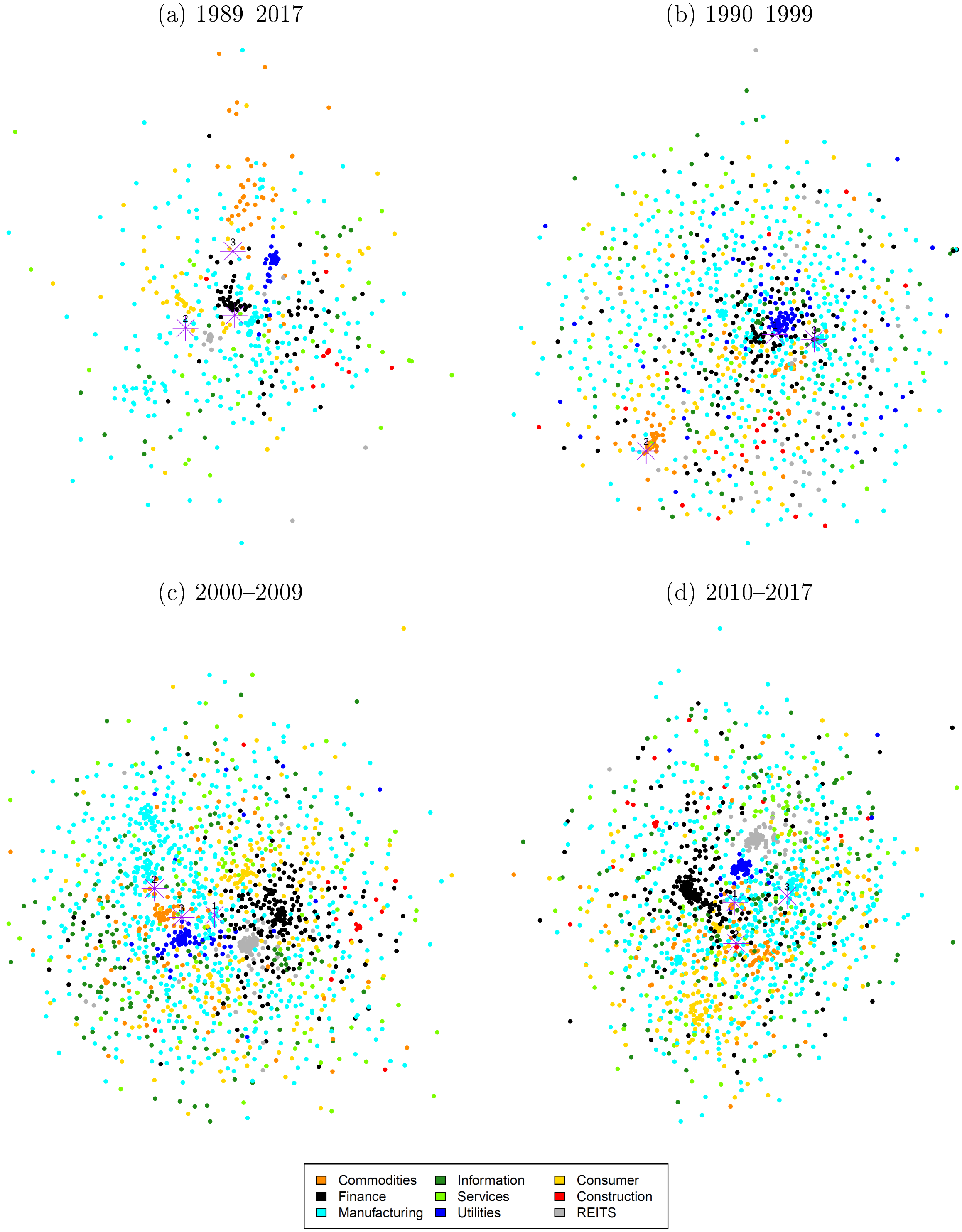
Note: The three common factors are estimated from the sample of U.S. daily log equity returns continuously traded from 1989 through 2017, with either PCA (black) or the median Bayesian draws (blue), with dark and light shaded areas denoting the latter's corresponding 68% and 95% intervals, on the left axis. The right axis is for a comparable macroeconomic or financial indicator (red). All series are expressed in year-over-year changes.

Figure 2: Total-Return U.S. Firm Equity Return Based Networks



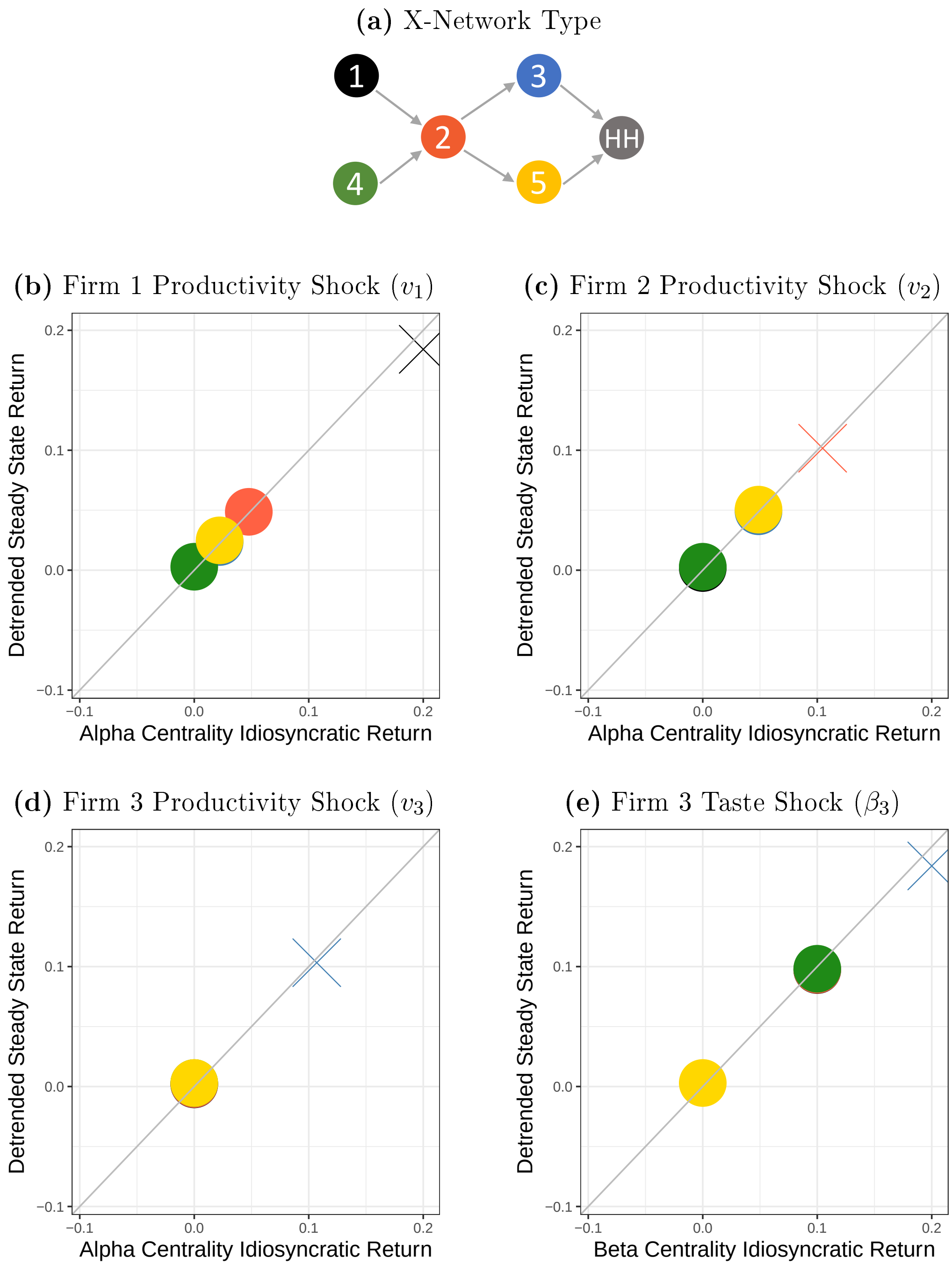
Note: The figure displays firm equity return based networks estimated over different sample periods with the three-step method. Each dot represents a firm colored by its BEA sector, network edges are calculated using GFEVc's inclusive of the common factors, and the proximity of dots to one another depends on how connected firms are using the ForceAtlas2 algorithm.

Figure 3: Idiosyncratic-Return U.S. Firm Equity Return Based Networks



Note: The figure displays firm equity return based networks estimated over different sample periods with the three-step method. Each dot represents a firm colored by its BEA sector, network edges are calculated using GFEVc's after distinguishing the effects of common factors, and the proximity of dots to one another depends on how connected nodes are using the ForceAtlas2 algorithm. Factors are depicted as purple stars with the corresponding number in black above.

Figure 4: Simulated Idiosyncratic Equity Responses to Shocks in DSGE Model



Note: Panel (a) shows the X-type network, where every node represents either a different firm (1 to 5) or the household sector (HH). Panels (b) through (e) plot the simulated idiosyncratic returns from moving between steady-states at the initial and new parameters after the shock on the y-axis, against the upstream or downstream exposure to the source node multiplied by the change in its specified parameter on the x-axis. The 45-degree line equating these two is included for reference. The source firm for each shock is denoted with an X-marker. The idiosyncratic returns are the latter two terms of Equation (12) after removing the common factors, $R_t^I = \mathcal{U}\Delta v_t + \mathcal{D}\Delta\beta_t$.

Online Appendix

Analysis of Upstream, Downstream & Common Firm Shocks Using a Large Factor-Augmented Vector Autoregressive Approach

Everett Grant — Amazon.com

Julieta Yung — FDIC

A Common Factor Estimation Using PCA

There are different methods that can be used with PCA to calculate the common factor and idiosyncratic return series. In Section 2 of the paper we use the "Covariance PCA with Full Factor Series" approach.

PCA solves the following optimization problem:

$$\min_{\{\Lambda, F_t\}} \frac{1}{NT} \sum_{i=1}^{N} \sum_{t=1}^{T} \left(R_{it} - \lambda_i F_t\right)^2 ,$$

where $\Lambda' \equiv \{\lambda_1', \lambda_2', ..., \lambda_N'\}$ is the combination of the vectors of firm specific factor weights. Once the common variation in the data, ΛF_t, is estimated, R_t^I is obtained as the residual firm returns.

There are four alternative methods we could use to calculate the common factors and decompose the data. The alternatives are based on whether the covariance or correlation matrix is used to calculate the major axes of the common factors, and whether the means are included in the common factors or they are detrended. For these formulas, R represents the $T \times N$ matrix of combined R_t vectors. Let μ be the $T \times N$ matrix of the firm means of the R return series repeated along each column, ϑ be an $N \times N$ matrix of the R series' standard deviations along the diagonal and zeroes elsewhere, R_0 be the demeaned return series $(R - \mu)$, and R_S be the standardized R series $((R - \mu)\vartheta^{-1})$. Also, V is the $N \times K$ matrix of the first K eigenvectors from the covariance matrix (in descending order of sample variance explained), and V_c is the matrix of the first K eigenvectors from the correlation matrix.

(1) Standard Covariance PCA: Covariance Eigenvectors & Detrended Factors

The formula to recover the original data using standard PCA involves projecting the data series without their means into the reduced dimension space. The means are excluded so that the directions of maximal variation are captured, rather than the means of the return series driving the newly created factors. The means are then added to these series after they are projected back to the original space:

$$\text{Recovery Formula: } \hat{R} = \mu + R_0 V V'.$$

The recovery error — or the variation explained by the excluded eigenvectors — is:

$$R - \hat{R} = R - (\mu + R_0 V V') = R_0 - R_0 V V' = R_0 (I_N - V V').$$

As more eigenvectors are included in V, the VV' term approaches the identity matrix and the recovery error goes to zero.

Using these factors in our method ($F = R_0V; \Lambda' = V'$) we can then define what is captured by the R^I term:

$$R = F\Lambda' + R^I = R_0VV' + R^I$$

$$\implies R^I = R - R_0VV' = \mu + R_0(I_N - VV').$$

The idiosyncratic returns are then the average returns for each firm plus the recovery error, or the variation not explained along the included eigenvectors.

(2) Covariance PCA with Full Factor Series

There may be broad common trends that one might wish to capture, rather than allocating them to the individual firm return series. In that case, we would instead calculate the factor series with the full — rather than the demeaned — data.

This would mean that the factor series are calculated as $F = RV$ but Λ' would still be equal to V'. In that case the R^I term is:

$$R = F\Lambda' + R^I = RVV' + R^I$$

$$\implies R^I = R - RVV' = (R_0 + \mu)(I_N - VV') = \mu + R_0(I_N - VV') - \mu VV'.$$

These idiosyncratic returns include the average returns and recovery error as in the first case, but the shared trend already accounted for in the factors is removed in the last term. The last term is equal to the means of these full factor series projected back to the full firm space.

(3) Standard Correlation Matrix PCA: Correlation Eigenvectors & Detrended Factors

The formula to recover the original data using standard correlation matrix based PCA involves using the eigenvectors of the correlation matrix (V_c) to project the standardized data series, $R_S = (R - \mu)\vartheta^{-1}$, into the reduced dimension space. The means are excluded so that the dimension of maximal variation is captured, rather than the means driving the newly created factors, and the series are standardized to have the same variation so that they are accounted for equally when deriving the eigenvectors defining the directions of maximal variation for the common factor series. To recover the data, the derived common factor series must be adjusted for both the series means and standard deviations:

$$\text{Recovery Formula: } \hat{R} = \mu + R_S V_c V_c' \vartheta = \mu + R_0 \vartheta^{-1} V_c V_c' \vartheta.$$

The recovery error is:

$$R - \hat{R} = R - (\mu + R_0 \vartheta^{-1} V_c V_c' \vartheta) = R_0 (I_N - \vartheta^{-1} V_c V_c' \vartheta).$$

Using these factors in our method ($F = R_S V_c ; \Lambda' = V_c' \vartheta$) we can then define what is captured by the R^I term:

$$R = F\Lambda' + R^I = R_S V_c V_c' \vartheta + R^I$$

$$\implies R^I = R - R_S V_c V_c' \vartheta = R - R_0 \vartheta^{-1} V_c V_c' \vartheta = \mu + R_0 (I_N - \vartheta^{-1} V_c V_c' \vartheta).$$

The idiosyncratic returns are then the average returns for each firm plus the recovery error.

(4) Correlation PCA with Full Factor Series

We can also use factor series with the common trends included using the correlation matrix eigenvectors for the data projections, with $F = R\vartheta^{-1} V_c$ and $\Lambda' = V_c' \vartheta$. In that case the R^I term is:

$$R = F\Lambda' + R^I = R\vartheta^{-1} V_c V_c' \vartheta + R^I$$

$$\implies R^I = R - R\vartheta^{-1} V_c V_c' \vartheta = R(I_N - \vartheta^{-1} V_c V_c' \vartheta) = \mu + R_0 (I_N - \vartheta^{-1} V_c V_c' \vartheta) - \mu \vartheta^{-1} V_c V_c' \vartheta.$$

These idiosyncratic returns include the average returns and recovery error as in the first case, but the shared trend already accounted for in the factors is removed in the last term as in the covariance based PCA.

A.1 PCA Common Factors from U.S. Equity Returns (1989-2017)

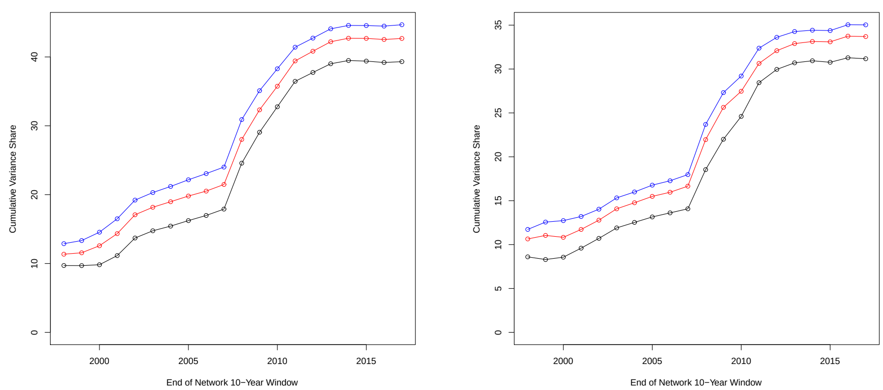
Table A.1: 20 Largest Eigenvalues by PCA for U.S. Equity Returns (1989-2017)

Top	Value	%	Cum. %	Top	Value	%	Cum. %
1	692.99	22.63	22.63	*11*	23.00	0.75	34.68
2	70.03	2.29	24.91	*12*	22.07	0.72	35.40
3	66.17	2.16	27.07	*13*	21.18	0.69	36.09
4	39.82	1.30	28.37	*14*	20.65	0.67	36.76
5	34.68	1.13	29.51	*15*	19.92	0.65	37.41
6	32.10	1.05	30.55	*16*	18.90	0.62	38.03
7	28.68	0.94	31.49	*17*	18.12	0.59	38.62
8	27.52	0.90	32.39	*18*	17.71	0.58	39.20
9	24.10	0.79	33.17	*19*	17.20	0.56	39.76
10	23.03	0.75	33.93	*20*	16.82	0.55	40.31

Note: Top 20 eigenvalues of the covariance matrix of log daily equity returns for the continuously-traded sample from 1989 through 2017 (T = 7,424 and N = 524).

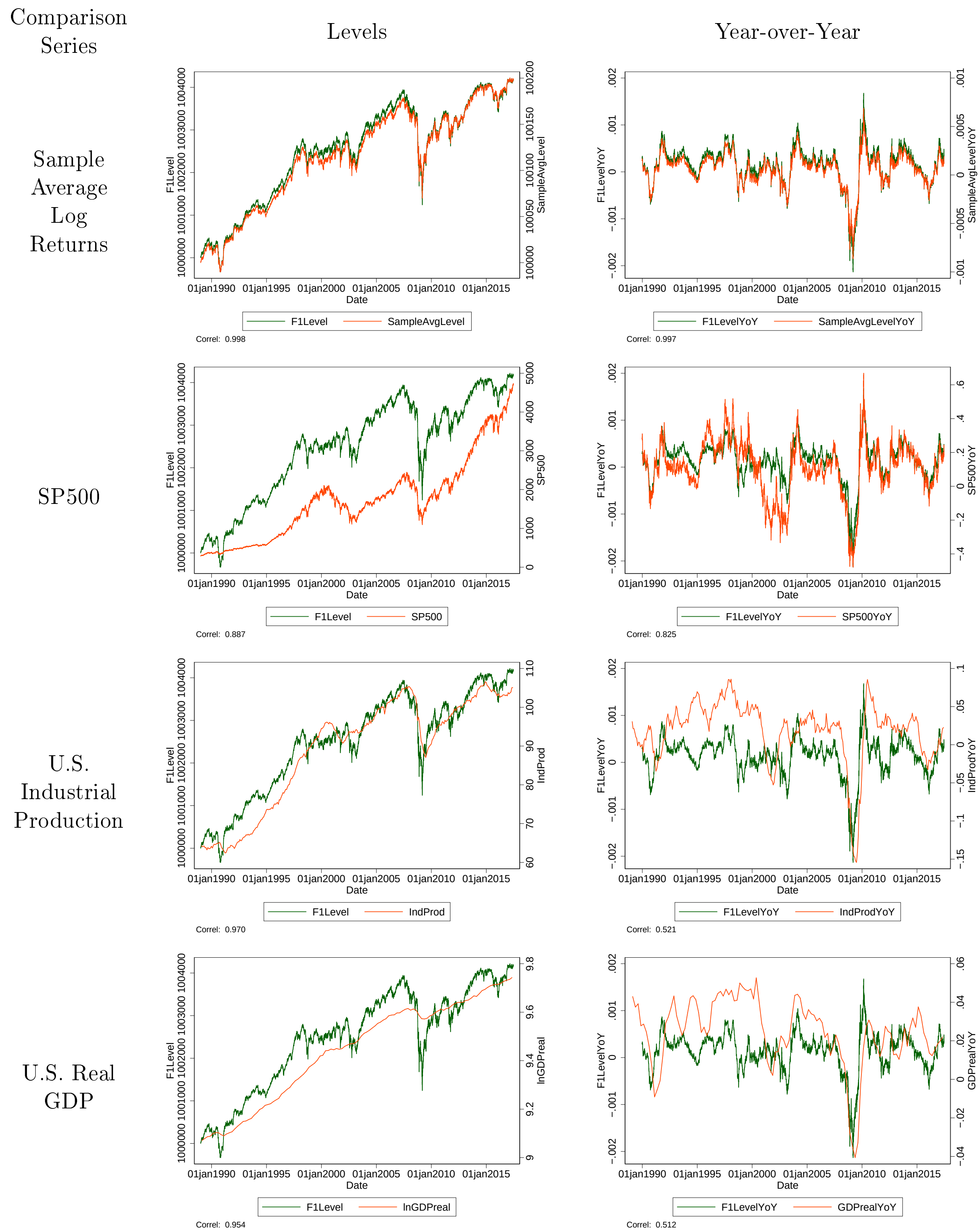
Figure A.1: Variance Share of Top 3 PCA Factors

(a) Balanced Firm Sample (b) Rolling 10-Year Samples



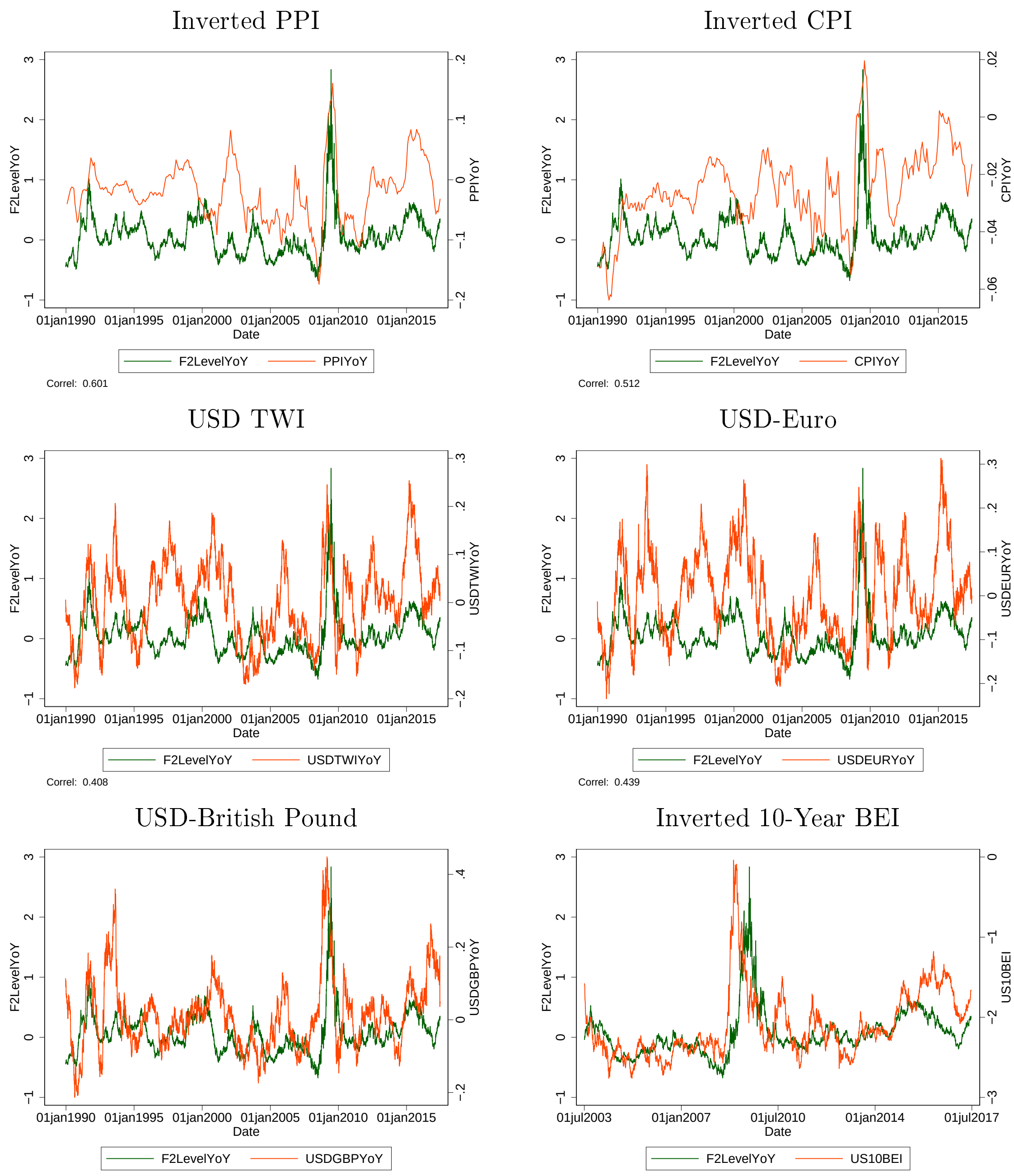
Note: The first factor is shown in black, the second one in red, and the third one in blue. Factors extracted by principal component analysis on the covariance matrix of U.S. daily log equity returns.

Figure A.2: First PCA Factor, Equity Markets, & Growth of the U.S. Economy (1989-2017)



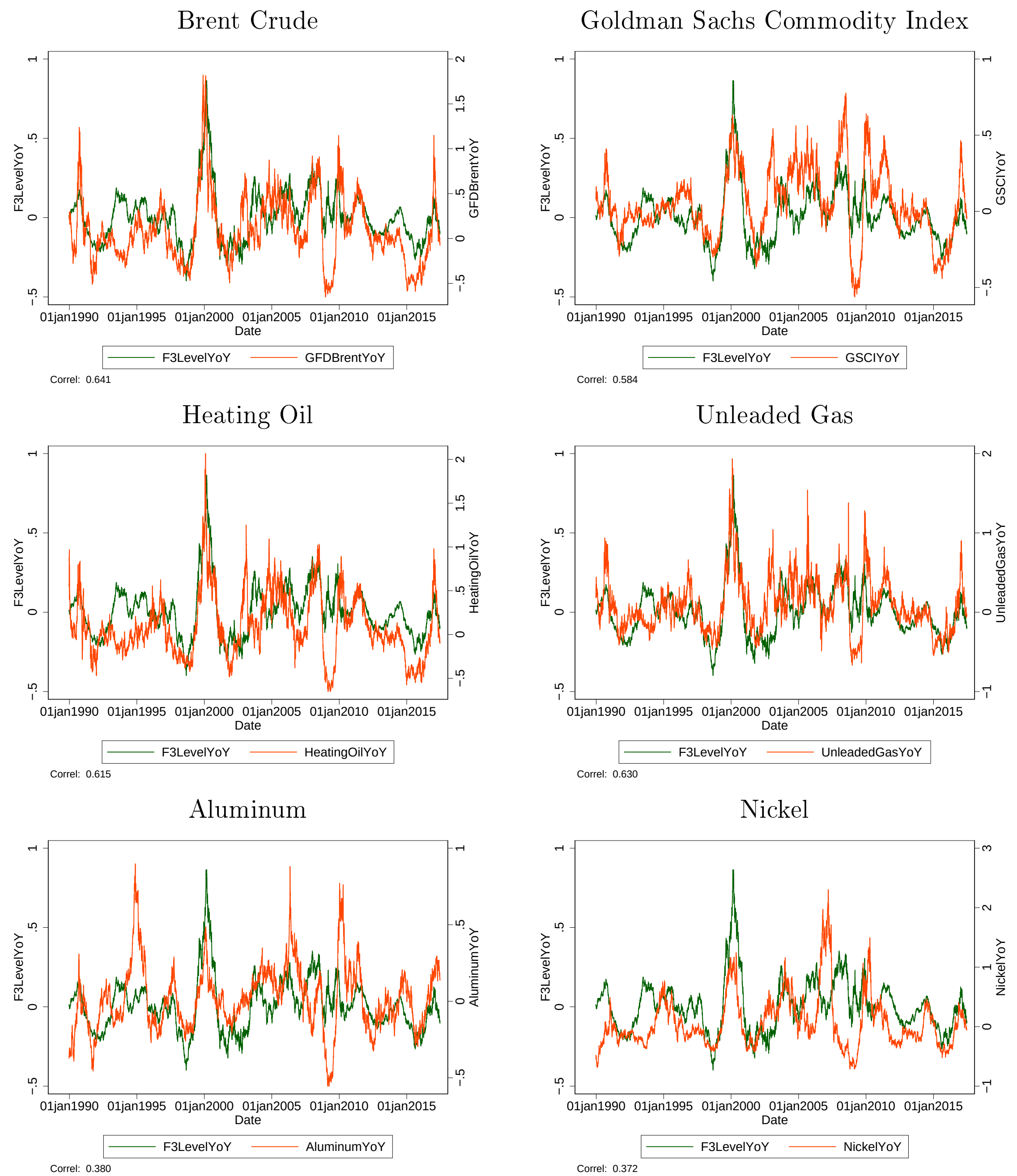
Note: "F1Level" is the cumulative sum of the first factor extracted by principal component analysis on the covariance matrix of U.S. daily log equity returns for the portion of our sample continuously traded from 1989 through 2017 (T = 7,424 and N = 524).

Figure A.3: Second PCA Factor and U.S. Prices, Year-over-Year Plots (1989-2017)



Note: "F2Level" is the cumulative sum of the second factor extracted by principal component analysis on the covariance matrix of U.S. daily log equity returns for the portion of our sample continuously traded from 1989 through 2017 (T = 7,424 and N = 524). The breakeven inflation, PPI and CPI return series are negated to match the direction of the factor series.

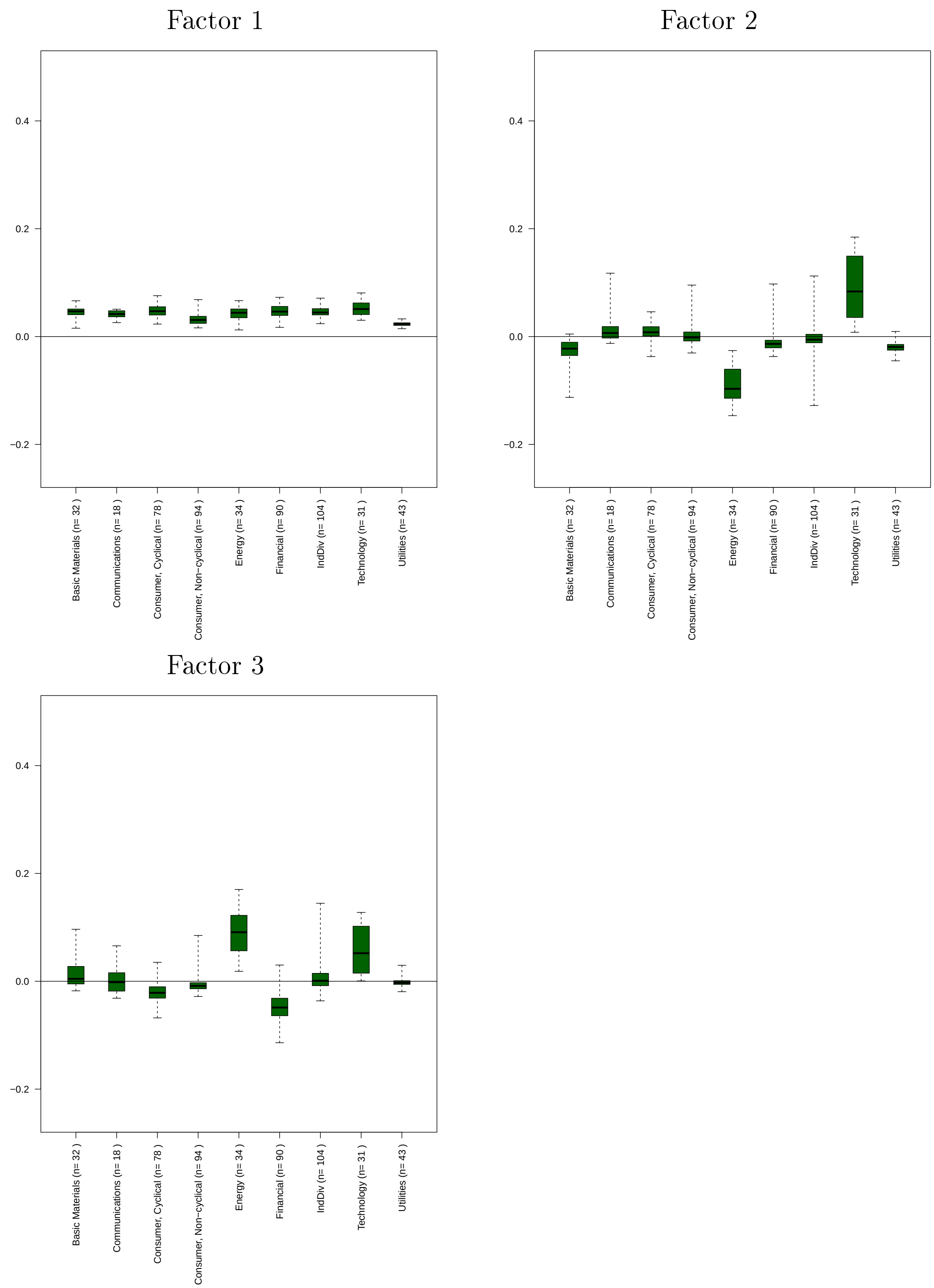
Figure A.4: Third PCA Factor and Commodities, Year-over-Year Plots (1989-2017)



Note: "F3Level" is the cumulative sum of the third factor extracted by principal component analysis on the covariance matrix of U.S. daily log equity returns for the portion of our sample continuously traded from 1989 through 2017 (T = 7,424 and N = 524).

A.2 PCA Common Factor Loadings

Figure A.5: U.S. Factor Λ Coefficient Distributions by Industry (1989-2017)



Note: Loadings on the first three factors extracted by principal component analysis on the covariance matrix of U.S. daily log equity returns for the portion of our sample continuously traded from 1989 through 2017 (T = 7,424 and N = 524).

Table A.2: Top Firm Loadings on First Factor

Panel A: Highest Factor Loadings

	Λ	Name	Industry	Industry Subgroup	BEA Sector	BEA Subgroup	Country
1	0.08	XCERRA CORP	Technology	Semiconductor Equipment	Manufacturing	Computer and electronic products	US
2	0.08	KULICKE & SOFFA	Technology	Semiconductor Equipment	Manufacturing	Computer and electronic products	SGP
3	0.08	HOVNANIAN ENT-A	Consumer, Cyclical	Bldg-Residential/Commer	Construction	Construction	US
4	0.07	SAFEGUARD SCIENT	Financial	Venture Capital	Finance	Securities, commodity contracts, and investments	US
5	0.07	TEREX CORP	Industrial	Machinery-Constr&Mining	Manufacturing	Machinery	US
6	0.07	LINCOLN NATL CRP	Financial	Life/Health Insurance	Finance	Securities, commodity contracts, and investments	US
7	0.07	OFFICE DEPOT INC	Consumer, Cyclical	Retail-Office Supplies	Retail	General merchandise stores	US
8	0.07	ENZO BIOCHEM INC	Consumer, Non-cyclical	Medical-Biomedical/Gene	Manufacturing	Chemical products	US
9	0.07	MICRON TECH	Technology	Electronic Compo-Semicon	Manufacturing	Computer and electronic products	US
10	0.07	MBIA INC	Financial	Financial Guarantee Ins	Finance	Securities, commodity contracts, and investments	US

Panel B: Lowest Factor Loadings

	Λ	Name	Industry	Industry Subgroup	BEA Sector	BEA Subgroup	Country
1	0.01	NORTH EURO OIL	Energy	Oil-US Royalty Trusts	Mining	Oil and gas extraction	US
2	0.01	SOUTHERN CO	Utilities	Electric-Integrated	Utilities	Utilities	US
3	0.02	NEWMONT MINING	Basic Materials	Gold Mining	Mining	Mining, except oil and gas	US
4	0.02	SABINE ROYALTY	Energy	Oil-US Royalty Trusts	Mining	Oil and gas extraction	US
5	0.02	GENERAL MILLS IN	Consumer, Non-cyclical	Food-Misc/Diversified	Manufacturing	Food and beverage and tobacco products	US
6	0.02	CONS EDISON INC	Utilities	Electric-Integrated	Utilities	Utilities	US
7	0.02	DYNEX CAPITAL	Financial	REITS-Mortgage	Finance	Real estate	US
8	0.02	WEC ENERGY GROUP	Utilities	Electric-Integrated	Utilities	Utilities	US
9	0.02	HORMEL FOODS CRP	Consumer, Non-cyclical	Food-Meat Products	Manufacturing	Food and beverage and tobacco products	US
10	0.02	KELLOGG CO	Consumer, Non-cyclical	Food-Misc/Diversified	Manufacturing	Food and beverage and tobacco products	US

Note: Sample includes the 524 firms continuously traded over 1989-2017.

Table A.3: Top Firm Loadings on Second Factor

Panel A: Highest Factor Loadings

	Λ	Name	Industry	Industry Subgroup	BEA Sector	BEA Subgroup	Country
1	0.18	LAM RESEARCH	Technology	Semiconductor Equipment	Manufacturing	Computer and electronic products	US
2	0.18	XCERRA CORP	Technology	Semiconductor Equipment	Manufacturing	Computer and electronic products	US
3	0.17	INTEGRAT DEVICE	Technology	Semicon Compo-Intg Circu	Manufacturing	Computer and electronic products	US
4	0.17	KULICKE & SOFFA	Technology	Semiconductor Equipment	Manufacturing	Computer and electronic products	SGP
5	0.17	KLA-TENCOR CORP	Technology	Semiconductor Equipment	Manufacturing	Computer and electronic products	US
6	0.16	SKYWORKS SOLUTIO	Technology	Electronic Compo-Semicon	Manufacturing	Computer and electronic products	US
7	0.16	TERADYNE INC	Technology	Semiconductor Equipment	Manufacturing	Computer and electronic products	US
8	0.15	ANALOG DEVICES	Technology	Semicon Compo-Intg Circu	Manufacturing	Computer and electronic products	US
9	0.15	CYPRESS SEMICON	Technology	Semicon Compo-Intg Circu	Manufacturing	Computer and electronic products	US
10	0.15	MICRON TECH	Technology	Electronic Compo-Semicon	Manufacturing	Computer and electronic products	US

Panel B: Lowest Factor Loadings

	Λ	Name	Industry	Industry Subgroup	BEA Sector	BEA Subgroup	Country
1	-0.15	ENSCO PLC-CL A	Energy	Oil&Gas Drilling	Mining	Oil and gas extraction	UK
2	-0.14	UNIT CORP	Energy	Oil Comp-Explor&Prodtn	Mining	Oil and gas extraction	US
3	-0.14	NOBLE CORP PLC	Energy	Oil&Gas Drilling	Mining	Oil and gas extraction	UK
4	-0.14	ROWAN COMPANIE-A	Energy	Oil&Gas Drilling	Mining	Oil and gas extraction	US
5	-0.14	NABORS INDS LTD	Energy	Oil&Gas Drilling	Mining	Oil and gas extraction	US
6	-0.13	TIDEWATER INC	Industrial	Transport-Marine	Transport	Air transportation	US
7	-0.12	PARKER DRILLING	Energy	Oil&Gas Drilling	Mining	Oil and gas extraction	US
8	-0.12	HELMERICH & PAYN	Energy	Oil&Gas Drilling	Mining	Oil and gas extraction	US
9	-0.12	BAKER HUGHES INC	Energy	Oil-Field Services	Mining	Oil and gas extraction	US
10	-0.11	APACHE CORP	Energy	Oil Comp-Explor&Prodtn	Mining	Oil and gas extraction	US

Note: Sample includes the 524 firms continuously traded over 1989-2017.

Table A.4: Top Firm Loadings on Third Factor

Panel A: Highest Factor Loadings

	Λ	Name	Industry	Industry Subgroup	BEA Sector	BEA Subgroup	Country
1	0.17	ENSCO PLC-CL A	Energy	Oil&Gas Drilling	Mining	Oil and gas extraction	UK
2	0.17	NOBLE CORP PLC	Energy	Oil&Gas Drilling	Mining	Oil and gas extraction	UK
3	0.16	UNIT CORP	Energy	Oil Comp-Explor&Prodtn	Mining	Oil and gas extraction	US
4	0.16	NABORS INDS LTD	Energy	Oil&Gas Drilling	Mining	Oil and gas extraction	US
5	0.16	ROWAN COMPANIE-A	Energy	Oil&Gas Drilling	Mining	Oil and gas extraction	US
6	0.16	PARKER DRILLING	Energy	Oil&Gas Drilling	Mining	Oil and gas extraction	US
7	0.14	TIDEWATER INC	Industrial	Transport-Marine	Transport	Air transportation	US
8	0.13	HELMERICH & PAYN	Energy	Oil&Gas Drilling	Mining	Oil and gas extraction	US
9	0.13	HALLIBURTON CO	Energy	Oil-Field Services	Mining	Oil and gas extraction	US
10	0.13	INTEGRAT DEVICE	Technology	Semicon Compo-Intg Circu	Manufactur-ing	Computer and electronic products	US

Panel B: Lowest Factor Loadings

	Λ	Name	Industry	Industry Subgroup	BEA Sector	BEA Subgroup	Country
1	-0.11	HUNTINGTON BANC	Financial	Super-Regional Banks-US	Finance	SCCI	US
2	-0.11	FIFTH THIRD BANC	Financial	Super-Regional Banks-US	Finance	SCCI	US
3	-0.10	REGIONS FINANCIA	Financial	Commer Banks-Southern US	Finance	SCCI	US
4	-0.10	SUNTRUST BANKS	Financial	Super-Regional Banks-US	Finance	SCCI	US
5	-0.09	KEYCORP	Financial	Super-Regional Banks-US	Finance	SCCI	US
6	-0.09	SYNOVUS FINL	Financial	Commer Banks-Southern US	Finance	SCCI	US
7	-0.09	ZIONS BANCORP	Financial	Commer Banks-Western US	Finance	SCCI	US
8	-0.09	WELLS FARGO & CO	Financial	Super-Regional Banks-US	Finance	SCCI	US
9	-0.09	FIRST HORIZON NA	Financial	Commer Banks-Southern US	Finance	SCCI	US
10	-0.08	MBIA INC	Financial	Financial Guarantee Ins	Finance	SCCI	US

Note: Sample includes the 524 firms continuously traded over 1989-2017. SCCI: Securities, commodity contracts, and investments.

B Three-Step Estimation Method Details

The first step of the procedure is to select the number of common factors, K, with the panel BIC method from Bai and Ng (2002) and use the "Covariance PCA with Full Factor Series" method described in Section A to obtain the common factors and the firms' loadings on them. Once the common variation for each firm (ΛF_t) is estimated with these, the idiosyncratic firm returns, R_t^I, are obtained under the simplifying assumption that the τ are minimal: $R_t^I = R_t - \Lambda F_t$. Step two is to estimate Equation (1) via OLS on the factor series. The third step utilizes the OCMT variable selection procedure, which we next describe in detail.

B.1 OCMT Variable Selection Procedure

The OCMT procedure in our use case involves evaluating the net impact of each of N potential explanatory variables, $R_{1,t-1}^I, R_{2,t-1}^I, \ldots, R_{N,t-1}^I$, on a dependent variable, $R_{i,t}^I$, in a linear model of the form:

$$R_{i,t}^I = \rho_{i0} + \sum_{n=1}^{N} \rho_{in} R_{n,t-1}^I + \epsilon_{it}, \text{ for } t = 1, 2, \ldots, T, \tag{B.1}$$

where N is small relative to T and a subset of the ρ_{in} coefficients are non-zero. The intuition is that if an explanatory variable's coefficient is non-zero, then its mean net impact on $R_{i,t}^I$ should be significantly different from zero, where the mean net impact of variable $R_{n,t-1}^I$ is:

$$\theta_{in} = \sum_{l=1}^{N} \rho_{il} cov(R_{n,t-1}^I, R_{l,t-1}^I).$$

Each variable is considered individually through a series of bivariate regressions of $R_{i,t}^I$ on each $R_{n,t-1}^I$ series and a constant estimated with OLS. The t-ratio of $\hat{\rho}_{in}$ from each regression is then compared to a critical value that takes into account the multiple testing aspect of this approach. The OCMT test of $\rho_{in} \neq 0$ is:

$$|t_{\hat{\rho}_{in}}| > \Phi^{-1}\left(1 - \frac{p}{2N^{\delta}}\right),$$

where p is the size of the test, and Φ^{-1} is the inverse of the cumulative standard normal distribution. The denominator of the second term can take a number of functional forms, but we choose this simple form and $\delta = 1$ for the first iteration. We then add the included variables to the test regressions along with a constant and repeat the test with $\delta = 2$ until no further variables are added. These values for δ are the lower bound to asymptotically select the proper variables, and in untabulated results we repeat our main analysis for a series of alternate values finding similar conclusions. The final step is to then estimate Equation (B.1) using OLS with only the selected variables and a constant included, setting the coefficients on all of the other variables to zero. To estimate Equation (2), we perform this analysis on each row of it, and combine the estimated coefficient vectors to arrive at our estimates of ρ_0 and ρ.

C State-Space Model Estimation Details

This section provides details of the Gibbs sampling priors, posteriors and estimation algorithm. To reiterate, the observation and state equations for our system are:

$$R_t = [\Lambda \; I_N] \begin{bmatrix} F_t \\ R_t^I \end{bmatrix} + \tau_t$$

and

$$\begin{bmatrix} F_t \\ R_t^I \end{bmatrix} = \begin{bmatrix} 0_{K\times 1} \\ \rho_0 \end{bmatrix} + \begin{bmatrix} \Gamma & 0_{K\times N} \\ 0_{N\times K} & \rho \end{bmatrix} \begin{bmatrix} F_{t-1} \\ R_{t-1}^I \end{bmatrix} + \begin{bmatrix} \eta_t \\ \epsilon_t \end{bmatrix}.$$

The first consideration for estimating the state-space system is the lack of unique identification for the F_t factors, which affects their data time series, their loadings, and the covariance matrix of their innovations. There are three issues: their scale is not identified (i.e., multiplying the loadings by two and dividing the factors by two would equally fit the system); their sign is not determined (i.e., multiplying the loadings and factors by negative one would equally fit); and their order is not unique (i.e., switching the first and third factors and their loadings would fit the same). We use three approaches to address these.

First, for the scale indeterminacy, we assume that the factors have variances of one, and that their innovations are uncorrelated. In other words, the covariance matrix of their innovations, Υ, is the I_K identity matrix. This is in the spirit of PCA, which we use to determine the priors for these factors. As an extension one could apply economic insights on top of the factors to identify them, for example by setting certain loadings to one or zero based on firms' industries or geographic regions, or other rotations based on external information (e.g., match impulse responses to findings from other research or a theoretical model); however, we do not apply such restrictions or rotations in order to let the data inform us about them with minimal assumptions or structure implied. We go into further details below on how we use PCA estimates and variance adjust them for this to create priors for the unobserved states and their loadings in the estimation algorithm.

Second, for the sign indeterminacy, for one of the firms available throughout our whole sample and in all of our networks (IBM), we require, without loss of generality, that its loadings on all factors are positive. If not, we multiply the factor and respective loadings on it for all firms by negative one. Third, for the factor order, as with PCA, our estimation orders the factors in descending order of the cumulative variance they explain for the R_t equity returns.

The above assumptions mean that we do not estimate Υ, leaving the following as the parameters we must estimate:

- Λ
- ρ_0
- $\mathcal{T}$
- F_t
- Γ
- ρ
- Σ
- R_t^I .

We next provide an overview of the priors and their respective posterior distributions where applicable, and then the estimation algorithm details.

C.1 Model Prior & Posterior Distributions

In this section, we discuss the model priors and the resulting posterior distributions, working from the top of the above state-space equations to the bottom. For expositional purposes, we do not list the full information set that each conditional distribution is contingent upon, but in general, it is the observed data and the most recently drawn parameter values and unobserved state series not being drawn in the current posterior distribution.

When setting the priors, our underlying assumption is that the common factors and idiosyncratic return series are random walks in levels. Hence, the mean of our prior for Γ is $0_{K\times K}$, for ρ_0 is $0_{N\times 1}$, and for ρ is $0_{N\times N}$. The initial values for the common factor series, $F_t^{(0)}$, are the first K PCA components of the equity returns using the "Covariance PCA with Full Factor Series" method, then scaled and sign adjusted as described above. The mean of the prior for the Λ matrix, $\bar{L}$, is centered at the PCA loadings after they are likewise adjusted for the common factors' scaling and signs. The initial values for the idiosyncratic return series are $R_t^{I(0)} = R_t - \bar{L}F_t^{(0)}$.

When constructing the variances for the priors of the coefficient matrices, we follow the idea of Litterman's Minnesota prior in scaling by the variables' relative variances. As customary in the literature, the residual variances from univariate models of the data series are used in determining these. Our assumptions about the data generating process imply a random walk without drift for each of the i idiosyncratic return series, which makes the variance of the i^{th} $R^{I(0)}$ return series itself the $\hat{\sigma}_i^2$ variance estimate.

C.1.1 Observation Equation Prior & Posterior Distributions

The prior for the common factor loading coefficients of the i^{th} equation of the observation system is:

$$\Lambda'_{Ri} \sim \mathcal{N}(\bar{L}'_{Ri}, \breve{L}_i) \ ; \ \breve{L}_i \equiv \kappa_\Lambda^2 \hat{\sigma}_i^2 I_K,$$

where κ_Λ controls the tightness of the prior, and Λ_{Ri} and $\bar{L}_{Ri}$ are the i^{th} rows of those matrices. We follow the BVAR literature in setting $\kappa_\Lambda = 0.2$. The $\mathcal{T}$ covariance matrix is assumed to be diagonal, and a conjugate prior for the $\mathcal{T}_{ii}$ terms on its diagonal fitting this is an independent inverse gamma distribution for each one. Specifically, our prior for each $\mathcal{T}_{ii}$ is:

$$\mathcal{T}_{ii} \sim \Gamma^{-1}\left(\frac{T_i^0}{2}, \frac{\kappa_\sigma \hat{\sigma}_i^2}{2}\right)$$

with the scale parameter and degrees of freedom set such that the mean of this distribution is $\kappa_\sigma \hat{\sigma}_i^2$ (i.e., $T_i^0 = 3 \ \forall \ i$). The κ_σ term is a scalar controlling the prior on the share of the variance explained by the idiosyncratic shocks. This is assumed to be small and we set this parameter to 1%.

The loadings on the idiosyncratic terms are known, hence to simplify the estimation we define $\boldsymbol{Y} \equiv \boldsymbol{R} - \hat{\boldsymbol{R}}^I$ with the observations stacked such that each of these is a $T \times N$ matrix. This implies that:

$$\boldsymbol{Y} = \boldsymbol{F}\Lambda' + \boldsymbol{\tau},$$

where $\boldsymbol{F}$ and $\boldsymbol{\tau}$ likewise have the observations over time stacked. Under these assumptions, Kadiyala and Karlsson (1997) show that the posterior distribution for the coefficients are normally distributed and can be drawn independently for each equation as:

$$\Lambda'_{Ri} \sim \mathcal{N}\left((\breve{L}_i^{-1} + \mathcal{T}_{ii}^{-1}\boldsymbol{F}'\boldsymbol{F})^{-1}(\breve{L}_i^{-1}\bar{L}'_{Ri} + \mathcal{T}_{ii}^{-1}\boldsymbol{F}'\boldsymbol{F}L_i^{OLS}), (\breve{L}_i^{-1} + \mathcal{T}_{ii}^{-1}\boldsymbol{F}'\boldsymbol{F})^{-1}\right),$$

where L_i^{OLS} is the vectorized OLS estimate of the coefficients, $L_i^{OLS} \equiv (\boldsymbol{F}'\boldsymbol{F})^{-1}\boldsymbol{F}'\boldsymbol{Y}_{Ci}$, with $\boldsymbol{Y}_{Ci}$ the i^{th} column of $\boldsymbol{Y}$. The posterior for $\mathcal{T}_{ii}$ is distributed inverse gamma:

$$\mathcal{T}_{ii} \sim \Gamma^{-1}\left(\frac{T_i^0 + T}{2}, \frac{\kappa_\sigma \hat{\sigma}_i^2 + (\boldsymbol{Y}_{Ci} - \boldsymbol{F}\Lambda'_{Ri})'(\boldsymbol{Y}_{Ci} - \boldsymbol{F}\Lambda'_{Ri})}{2}\right).$$

Note that these distributions can be drawn from in parallel across the i observation equations, which can be taken advantage of to speed up the Gibbs sampling algorithm.

C.1.2 State Transition Equation Prior & Posterior Distributions

The prior for Γ is:

$$vec(\Gamma') \sim \mathcal{N}(0_{K^2\times 1}, G) \ ; \ G \equiv \kappa_\Gamma^2 I_{K^2},$$

where κ_Γ controls the tightness of the prior, and we follow the BVAR literature in setting $\kappa_\Gamma = 0.2$. Its posterior is:

$$vec(\Gamma') \sim \mathcal{N}\Bigg(\big(G^{-1} + I_K \otimes \boldsymbol{F}'_{t-1}\boldsymbol{F}_{t-1}\big)^{-1}\big((I_K \otimes \boldsymbol{F}'_{t-1}\boldsymbol{F}_{t-1})\Gamma'_{OLS}\big), \big(G^{-1} + I_K \otimes \boldsymbol{F}'_{t-1}\boldsymbol{F}_{t-1}\big)^{-1}\Bigg),$$

where $\Gamma'_{OLS} \equiv vec\big((\boldsymbol{F}'_{t-1}\boldsymbol{F}_{t-1})^{-1}\boldsymbol{F}'_{t-1}\boldsymbol{F}\big)$, the vectorized OLS estimate of the coefficients.

For the VAR(1) of the idiosyncratic returns' transition equations, we utilize dummy variables to implement the priors in a more computationally tractable way, given the system's large size. To simplify the exposition, we restate these equations with stacked variable matrices of T rows each, and the lagged variables and constants combined into a single $T \times (1+N)$ matrix with the ones in the first column and lagged variables after it, denoted $\boldsymbol{X} \equiv [1_{T\times1}\ \boldsymbol{R}^I_{t-1}]$. The idiosyncratic returns transition equation is then:

$$\boldsymbol{R}^I = \boldsymbol{X}\boldsymbol{\rho} + \boldsymbol{\epsilon},$$

where $\boldsymbol{\rho} \equiv [\rho_0\ \rho]'$. The expected value for each element of $\boldsymbol{\rho}$ is zero. The variances of the coefficients on the constant terms will reflect diffuse priors, and the variance of the coefficient for dependent variable i on explanatory variable j will be of the Minnesota form:

$$VAR(\boldsymbol{\rho}_{1+j,i}) = \frac{\kappa_\rho^2 \hat{\sigma}_i^2}{\hat{\sigma}_j^2}. \tag{C.1}$$

We follow the BVAR literature in setting $\kappa_\rho = 0.2$. We specify an Inverse-Wishart prior for Σ, centered on the diagonal matrix with the $\hat{\sigma}_i^2$ terms on the diagonal. To efficiently implement these priors, we utilize artificial dummy observations:

$$\underset{(2N+1)\times N}{\boldsymbol{Y}_D} = \left[\begin{array}{c} 0_{N\times N} \\ \hdashline 0_{1\times N} \\ \hdashline diag(\hat{\sigma}_1, \hat{\sigma}_2, \ldots, \hat{\sigma}_N) \end{array}\right], \quad \underset{(2N+1)\times(N+1)}{\boldsymbol{X}_D} = \left[\begin{array}{cc} 0_{N\times1} & diag(\hat{\sigma}_1, \hat{\sigma}_2, \ldots, \hat{\sigma}_N)/\kappa_\rho \\ \hdashline \epsilon & 0_{1\times N} \\ \hdashline \multicolumn{2}{c}{0_{N\times(N+1)}} \end{array}\right].$$

The first block of dummy observations implements the priors for the coefficients in ρ, the second block the uninformative priors for the constants (ϵ is a small number), and the third block for the covariance matrix. Specifically, we employ a standard normal Inverse-Wishart prior of the form:

$$vec(\boldsymbol{\rho})|\Sigma \sim \mathcal{N}\Big(vec(\boldsymbol{\rho}_D), \Sigma \otimes (\boldsymbol{X}'_D\boldsymbol{X}_D)^{-1}\Big) \quad \text{and} \quad \Sigma \sim IW\big(\Sigma_D, N+2\big)\ .$$

The prior parameters are $\boldsymbol{\rho}_D = (\boldsymbol{X}'_D\boldsymbol{X}_D)^{-1}\boldsymbol{X}'_D\boldsymbol{Y}_D = 0_{(N+1)\times N}$, $\Sigma_D = (\boldsymbol{Y}_D - \boldsymbol{X}_D\boldsymbol{\rho}_D)'(\boldsymbol{Y}_D - \boldsymbol{X}_D\boldsymbol{\rho}_D) = diag(\hat{\sigma}_1^2, \hat{\sigma}_2^2, \ldots, \hat{\sigma}_N^2)$, and it is straightforward to show that the prior variance of $\boldsymbol{\rho}$ has the Minnesota form of Equation (C.1). The degrees of freedom for the inverse-Wishart prior is set to the minimum that guarantees the existence of the mean of Σ, which is Σ_D. It can be shown through algebraic substitution that the posterior has a similar form as the prior, but with the actual data appended to the dummy observation matrices. Let $\boldsymbol{Y}_* = [\boldsymbol{R}^{I\prime}\ \boldsymbol{Y}'_D]'$, $\boldsymbol{X}_* = [\boldsymbol{X}'\ \boldsymbol{X}'_D]'$, $\boldsymbol{\rho}_* = (\boldsymbol{X}'_*\boldsymbol{X}_*)^{-1}\boldsymbol{X}'_*\boldsymbol{Y}_*$ and $\Sigma_* = (\boldsymbol{Y}_* - \boldsymbol{X}_*\boldsymbol{\rho}_*)'(\boldsymbol{Y}_* - \boldsymbol{X}_*\boldsymbol{\rho}_*)$. We can then write the posterior as:

$$vec(\boldsymbol{\rho})|\Sigma, Y \sim \mathcal{N}\Big(vec(\boldsymbol{\rho}_*), \Sigma \otimes (\boldsymbol{X}'_*\boldsymbol{X}_*)^{-1}\Big) \quad \text{and} \quad \Sigma|Y \sim IW\big(\Sigma_*, T+N+2\big) \ .$$

The dummy observation approach helps to regularize the coefficients' variance matrix such that only $(\boldsymbol{X}'_*\boldsymbol{X}_*)$ must be inverted, improving the computational tractability of the posterior draws from their normal distribution.

C.1.3 Unobserved Factor Series Priors

Finally, we need the S_1 and P_1 priors for the simulation smoother algorithm used to draw the posterior values of the unobserved F_t and R_t^I states: $[F_1'\ R_1^{I\prime}]' \sim \mathcal{N}\big(S_1, P_1\big)$. Our unconditional expectation for the initial period states is that they are zero (i.e., the underlying series are random walks in levels), so S_1 is a $(K+N) \times 1$ zero vector. We assume they are uncorrelated, making P_1 a diagonal matrix with ones for the first K terms relating to the common factors and the N $\hat{\sigma}_i^2$ variances thereafter.

C.2 Gibbs Sampling Algorithm

The general approach of the estimation algorithm follows Carter and Kohn (1994), which alternates between drawing from the conditional distribution of the VAR parameters for the observation and state equations given the unobserved series, and then drawing from the conditional distribution of the F_t and R_t^I series given those parameters. The steps for the estimation algorithm are described below.

1. Set the priors for the factors and their loadings.

    (a) Determine the number of common factors, K. We use the panel BIC information criteria method from Bai and Ng (2002) to select the number of factors — the form they suggest to account for potential correlation in the idiosyncratic errors.

    (b) Run covariance matrix based PCA inclusive of the means on the firms' daily returns to get the prior means for the top K common factors and their loadings.

    (c) Divide the factors by their standard deviations and multiply their loadings by them.

(d) Confirm that IBM has a positive loading on each factor. If not, multiply that factor's values and its loadings for all firms by negative one.

(e) Set the initial draws of the $T \times K$ common factors matrix, $F^{(0)}$, to equal the adjusted PCA series.

(f) Set the mean of the prior for the Λ matrix, $\bar{L}$, as the PCA loadings adjusted for the common factors' scaling and signs.

(g) Set the initial draws of the $T \times N$ matrix of idiosyncratic returns by subtracting the adjusted PCA factor contributions from the observed returns: $R^{I(0)} = R - F^{(0)}\bar{L}'$.

(h) Get the idiosyncratic return series variances: $\hat{\sigma}_i^2 = VAR(R_i^{I(0)}) \; \forall \; i \in \{1, 2, \ldots, N\}$.

(i) Set S_1 to a $(K+N) \times 1$ zero vector.

(j) Set P_1 to a diagonal matrix with ones for the first K terms relating to the common factors and the $\hat{\sigma}_i^2$ variances for the idiosyncratic returns' entries.

2. Set the prior parameters for the observation equation.

   (a) The prior mean for the Λ coefficients is $\bar{L}$, as set in the prior step.

   (b) The prior covariance matrix for the Λ coefficients of each of the i equations of the observation system is $\breve{L}_i \equiv \kappa_\Lambda^2 \hat{\sigma}_i^2 I_K$ where $\kappa_\Lambda = 0.2$.

   (c) The degrees of freedom for each $\mathcal{T}_{ii}$ prior is 3.

   (d) The scale for each $\mathcal{T}_{ii}$ prior is $S_i \equiv \kappa_\sigma \hat{\sigma}_i^2$ where $\kappa_\sigma = 1\%$.

   (e) Set $\mathcal{T}_{ii}^{(0)} = S_i \; \forall \; i \in \{1, 2, \ldots, N\}$.

3. Set the prior parameters for the state equation.

   (a) The common factor state system:

      i. The prior mean for $vec(\Gamma')$ is $0_{K^2 \times 1}$.

      ii. The prior covariance matrix for $vec(\Gamma')$ is $G \equiv \kappa_\Gamma^2 I_{K^2}$ where $\kappa_\Gamma = 0.2$.

      iii. Υ is the I_K identity matrix.

   (b) The idiosyncratic return factor state system:

      i. Create the dummy observations with $\kappa_\rho = 0.2$ and $\epsilon = 1e^{-8}$:

$$\underset{(2N+1) \times N}{\boldsymbol{Y}_D} = \left[\begin{array}{c} 0_{N \times N} \\ \hdashline 0_{1 \times N} \\ \hdashline diag(\hat{\sigma}_1, \hat{\sigma}_2, \ldots, \hat{\sigma}_N) \end{array}\right], \quad \underset{(2N+1) \times (N+1)}{\boldsymbol{X}_D} = \left[\begin{array}{cc} 0_{N \times 1} & diag(\hat{\sigma}_1, \hat{\sigma}_2, \ldots, \hat{\sigma}_N)/\kappa_\rho \\ \hdashline \epsilon & 0_{1 \times N} \\ \hdashline \multicolumn{2}{c}{0_{N \times (N+1)}} \end{array}\right]$$

4. Cycle through iterations of Gibbs sampling, indexed by g, drawing from the conditional posterior distributions.

   (a) Run a parallel loop over the N observation equations.

      i. Subtract the idiosyncratic returns from the observed ones: $\boldsymbol{Y}_{Ci}^{(g)} \equiv R_i - R_i^{I(g-1)}$, each of these being $T \times 1$ vectors.

      ii. Draw the $\Lambda^{(g)}$ coefficients for observation equation i from:

$$\Lambda_{Ri}^{(g)'} \sim \mathcal{N}\Bigg((\breve{L}_i^{-1}+\frac{1}{\mathcal{T}_{ii}^{(g-1)}}\boldsymbol{F^{(g-1)\prime}}\boldsymbol{F^{(g-1)}})^{-1}(\breve{L}_i^{-1}\bar{L}_{Ri}'+\frac{1}{\mathcal{T}_{ii}^{(g-1)}}\boldsymbol{F^{(g-1)\prime}}\boldsymbol{Y}_{Ci}^{(g)}),(\breve{L}_i^{-1}+\frac{1}{\mathcal{T}_{ii}^{(g-1)}}\boldsymbol{F^{(g-1)\prime}}\boldsymbol{F^{(g-1)}})^{-1}\Bigg).$$

      iii. Draw $\mathcal{T}_{ii}^{(g)}$ from:

$$\mathcal{T}_{ii}^{(g)} \sim \Gamma^{-1}\Bigg(\frac{3+T}{2}, \frac{S_i + (\boldsymbol{Y}_{Ci}^{(g)} - \boldsymbol{F}^{(g-1)}\Lambda_{Ri}^{(g)'})'(\boldsymbol{Y}_{Ci}^{(g)} - \boldsymbol{F}^{(g-1)}\Lambda_{Ri}^{(g)'})}{2}\Bigg).$$

      iv. Collect the $\Lambda^{(g)}$ coefficients, ensuring that the factors are ordered in descending order of the cumulative variance their innovations explain for the R_t equity returns. If not, redraw the prior two steps.

   (b) Draw $\Gamma^{(g)'}$, checking for stability, from:

$$V^* \equiv \left(G^{-1} + I_K \otimes (\boldsymbol{F}_{t-1}^{(g-1)'}\boldsymbol{F}_{t-1}^{(g-1)})\right)^{-1}$$

$$\Gamma_{OLS}' \equiv vec\Bigg(\left(\boldsymbol{F}_{t-1}^{(g-1)'}\boldsymbol{F}_{t-1}^{(g-1)}\right)^{-1}\boldsymbol{F}_{t-1}^{(g-1)'}\boldsymbol{F}^{(g-1)}\Bigg)$$

$$vec(\Gamma^{(g)'}) \sim \mathcal{N}\Bigg(V^*\left((I_K \otimes \boldsymbol{F}_{t-1}^{(g-1)'}\boldsymbol{F}_{t-1}^{(g-1)})\Gamma_{OLS}'\right), V^*\Bigg).$$

   (c) Use direct Monte Carlo sampling from posterior of Σ and then $\boldsymbol{\rho}$ for the idiosyncratic VAR parameters, checking for stability of the coefficient vector.

      i. First calculate the necessary parameters:

$$\boldsymbol{X}^{(g)} \equiv [1_{T\times 1}\,\boldsymbol{R}_{t-1}^{I(g-1)}],$$

$$\boldsymbol{Y}_*^{(g)} = [\boldsymbol{R}^{I(g-1)'}\,\boldsymbol{Y}_D']',$$

$$\boldsymbol{X}_*^{(g)} = [\boldsymbol{X}^{(g)'}\,\boldsymbol{X}_D']',$$

$$\boldsymbol{\rho}_*^{(g)} = (\boldsymbol{X}_*^{(g)'}\boldsymbol{X}_*^{(g)})^{-1}\boldsymbol{X}_*^{(g)'}\boldsymbol{Y}_*^{(g)}$$

and

$$\Sigma_*^{(g)} = (\boldsymbol{Y}_*^{(g)} - \boldsymbol{X}_*^{(g)}\boldsymbol{\rho}_*^{(g)})'(\boldsymbol{Y}_*^{(g)} - \boldsymbol{X}_*^{(g)}\boldsymbol{\rho}_*^{(g)}).$$

ii. Draw $\Sigma^{(g)}$ from its posterior distribution:

$$\Sigma^{(g)}|Y \sim IW\big(\Sigma_*^{(g)}, T + N + 2\big).$$

iii. Draw $\boldsymbol{\rho}^{(g)}$ from its posterior distribution:. For large samples, one should draw from the equivalent but more efficient matrix normal distribution.

$$vec(\boldsymbol{\rho}^{(g)})|\Sigma^{(g)}, Y \sim \mathcal{N}\Bigg(vec(\boldsymbol{\rho}_*^{(g)}), \Sigma^{(g)} \otimes (\boldsymbol{X}_*^{(g)\prime}\boldsymbol{X}_*^{(g)})^{-1}\Bigg).$$

(d) Draw the $F^{(g)}$ and $R^{I(g)}$ states. We utilize the implementation of the Durbin and Koopman (2002) simulation smoother provided in Jarociński (2015), which we further optimize for Matlab data structures to store our large system in memory. Then draw the period-zero states conditional on these using backwards iteration (i.e., one step of the Carter and Kohn algorithm).

5. Repeat the prior step the desired number of times, retaining the final draws after the burn-in period.

D Firm Network Estimation Details

D.1 One Period Ahead GIRF Network Edge Equation

The formula for the one period ahead GIRF (Generalized Impulse Response Function) of Pesaran and Shin (1998) is:

$$GIRF_{j\to i}(1) = \frac{e_i' A_1 \Theta e_j}{\sqrt{e_j'\Theta e_j}} = \frac{e_i' \begin{bmatrix} \rho\Sigma + \Lambda\Gamma(L)\Upsilon\Lambda' & \rho\Sigma & \Lambda\Gamma(L)\Upsilon \\ \rho\Sigma & \rho\Sigma & 0_{N\times S} \\ \Gamma(L)\Upsilon\Lambda' & 0_{S\times N} & \Gamma(L)\Upsilon \end{bmatrix} e_j}{\sqrt{e_j'\Theta e_j}}$$

D.2 One Period Ahead GFEVD Network Edge Equation

The formula for the one period ahead GFEVD (Generalized Forecast Error Variance Decomposition) of Pesaran and Shin (1998) is:

$$GFEVD_{j\to i}(1) = \frac{\Theta_{jj}^{-1}[(e_i'\Theta e_j)^2 + (e_i' A_1 \Theta e_j)^2]}{e_i'\Theta e_i + e_i' A_1 \Theta A_1' e_i}$$

where the e_i are appropriately sized selection vectors with zeroes in all cells except for the i^{th}, which is one.

D.3 VAR Estimation Bias from Omitting Common Factors

This equation provides the bias that would result from estimating $\tilde{\rho} = [\rho_0\ \rho]$ from a standard VAR with the total-returns, but not accounting for the common factors. The second term is the bias, and the $\boldsymbol{R}^1_{t-1}$ terms include a column vector of ones before the firm returns to account for the constant term:

$$E[\boldsymbol{R}^1_t \boldsymbol{R}^1_{t-1}{}'(\boldsymbol{R}^1_{t-1}\boldsymbol{R}^1_{t-1}{}')^{-1}] = \tilde{\rho} + (\Lambda\Gamma(L) - \tilde{\rho}\Lambda)\boldsymbol{F}_{L,t-1}\boldsymbol{R}^1_{t-1}{}'(\boldsymbol{R}^1_{t-1}\boldsymbol{R}^1_{t-1}{}')^{-1}$$

Note that the bias is zero if either Λ is zero or all of the $\boldsymbol{F}_{L,t-1}$ are zero. In either case, the bias is zero when the factors have no effect on the firms' total-returns.

D.4 Empirical Method Simulations

To illustrate the network estimation procedure, we apply it to simulated data from the system defined by Equation (5). The simulated system has three independent common factor series following vector-autoregressive processes with zero constants and coefficients on their first lags of 0.9. The innovations to these series are independent normally distributed, and have variances of 16. The values of Υ and Γ are:

$$\Upsilon = \begin{bmatrix} 16 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 16 \end{bmatrix} \qquad \text{and} \qquad \Gamma(1) = \begin{bmatrix} 0.9 & 0 & 0 \\ 0 & 0.9 & 0 \\ 0 & 0 & 0.9 \end{bmatrix}.$$

The coefficients of the simulated firms on the factors are broken into five sets by their factor loadings. The first fifth of the dataset has a Λ coefficient of one on the first factor only, the next fifth has 0.5 coefficients on the first two factors, the next fifth has a coefficient of one on the second factor only, the fourth partition has 0.5 coefficients on the second and third factors, and the final fifth has a coefficient of one on only the third factor. This setup enables us to analyze the results of our estimation procedure when there are complex interactions among the nodes' factor dependence.

To further create heterogeneity across the simulated markets — and to examine the effects of the idiosyncratic shocks — the innovations of the idiosyncratic return series for the markets are the first five powers of two, repeated in order along the diagonal of Σ, with the symmetric off-diagonal terms in the remainder of Σ coming from normally distributed random draws. If needed, the Σ matrix is adjusted to be diagonally dominant to ensure that it is positive definite.

To create an easily discernible pattern of varied connections across the firms for us to study, the ρ matrix is given a block diagonal form. The simulated markets are split into nine groups that have Uniform$[-0.9999, 0.9999]$ randomly distributed coefficients for markets within the same block, with zeroes elsewhere. If needed, the ρ matrix is multiplied by 0.9 until the modulus of its largest eigenvalue is less than one. These groups can be thought to model distinct industries. This yields the following form for the ρ matrix:

$$\rho = \begin{bmatrix} \rho_1 & 0_{250} & 0_{250} & & 0_{250} \\ 0_{250} & \rho_2 & 0_{250} & \cdots & 0_{250} \\ 0_{250} & 0_{250} & \rho_3 & & 0_{250} \\ & \vdots & & \ddots & 0_{250} \\ 0_{250} & 0_{250} & 0_{250} & 0_{250} & \rho_9 \end{bmatrix}$$

where the ρ_g are the randomly generated coefficients and 0_{250} represents a 250×250 matrix of zeroes. The ρ_0 constant vector is drawn from a mean zero normal distribution. We use nine even groups so that we have partitions within and overlapping each of our five adjacent Λ coefficient groups, so that there is a complex interaction between the ρ and Λ coefficient groups for our process to attempt to disentangle. Finally, we have fifty repetitions of each idiosyncratic innovation volatility and ρ_g coefficient group number combination. This makes for $50 \times 5 \times 9 = 2,250$ simulated markets.

Given these model parameters, to obtain the simulated data we first generate $12,500$ multivariate normal draws of ϵ_t and η_t. We then initialize R_1^I and F_1 to be zero vectors and calculate their data series from the generated innovations and above parameter matrices. Finally, we calculate the R values from these two series. The first $5,000$ values of the R series are treated as burn-in values and are dropped, with the remaining $7,500$ saved as the values to apply our estimation procedure to, close to our full sample's $T = 7424$.

Figure D.1 shows the estimated networks when including or separating out the effects of common factors. The first row of the figure provides the connections between the simulated R data series with an "R to R" total-return network. The second row's network has edge weights between firms and factors based on the expected responses of firms' total-returns from idiosyncratic shocks to other firms or factors in an "R^I & Factors to R & Factors" network. This allows us to simultaneously examine the roles of shocks to firms and aggregate factors.

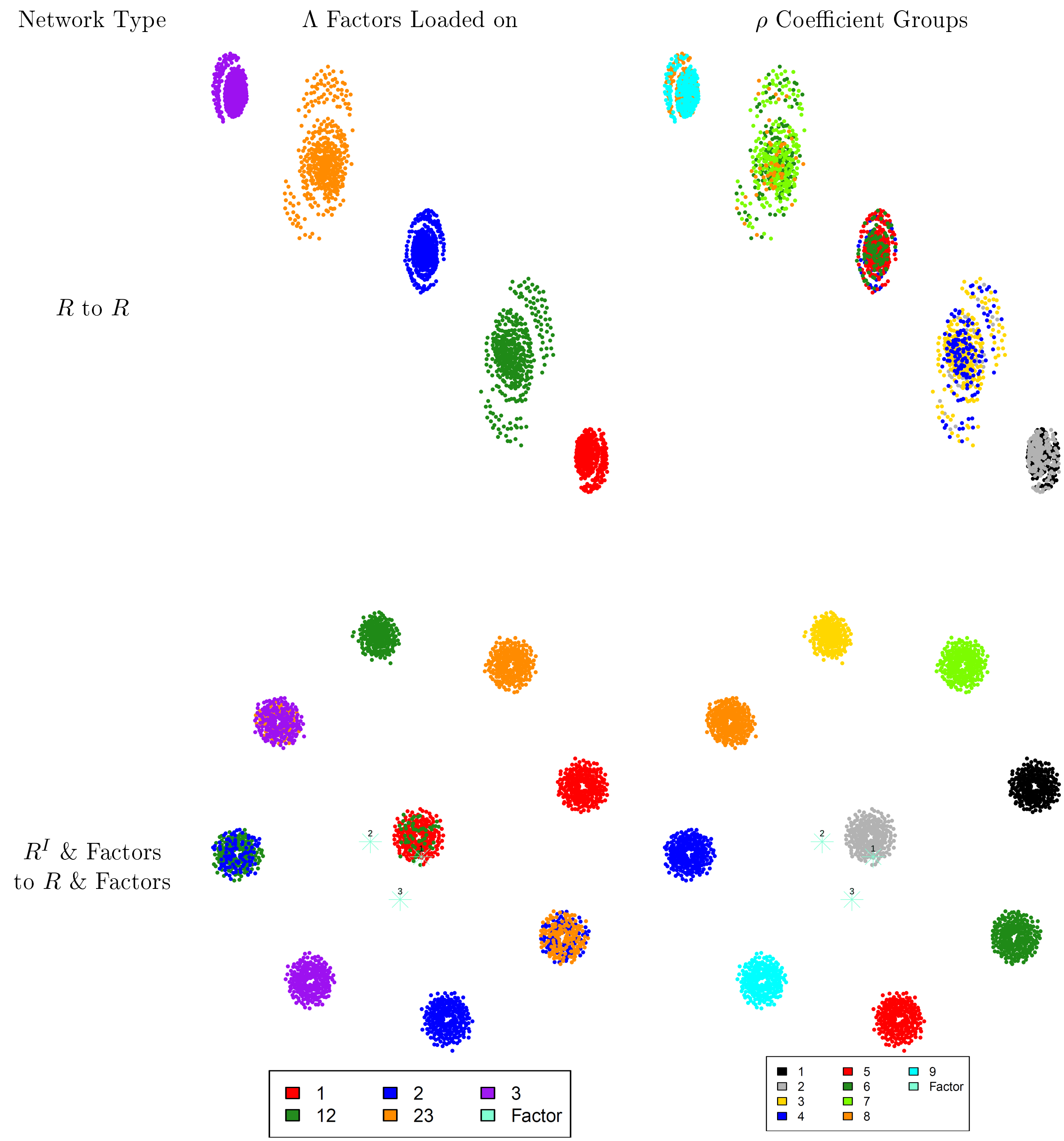
The columns of Figure D.1 differ only in the legends used for coloring the nodes. The first column has the nodes colored based on the factor(s) they load on. The second column is colored based on the nine blocks of non-zero ρ coefficient groups. To understand the results of the estimation process, we start by analyzing the "R to R" network in the first row. It is evident in the left plot that our procedure has grouped the nodes by the factors that each firm is directly affected by, with those loading on the first factor only at the bottom right in red, then those loading on the first two factors just to the left of that group in green, followed by those directly affected by only the second factor adjacent to that group, and so on. These total-return networks are similar to those uncovered using the approach common in other papers in the literature of applying a version of adaptive elastic-net and then calculating network edges from either the raw or standardized variance decompositions or impulse responses — as can be seen in Figure D.4 versus the first row of Figure D.1 — with those also being driven by the common factor loadings.

The right plot in the first row shows the same network with each node in the identical position as the first plot. Here, the ρ coefficient groups can be seen in the clustering; however, it is their overlap with the factor loading groups that drives the patterns in them — all of the nodes in the first ρ coefficient group load only on the first factor so they are in the bottom right group in black, those in the second ρ coefficient group in gray are mixed in between those loading only on the first factor and those loading on both of the first two factors so they are split between the first two clusters, and so on.

The second row's plots demonstrate how our procedure is able to extract and separate the direct inter-firm connections captured in the ρ coefficient groups from the common factors. The factors have roughly even total influences on the network and are near the center of the figures, so the major dynamics of these figures are determined by the R^I sourced edges. When looking at the left column, it at first appears that the firm factor groups are central to the organization; however, it is clear from the right column and perfect clustering by the ρ coefficient groups that they in fact are the drivers. Clustering in this network is governed by the ρ coefficient group structure, unlike the "R to R" network.

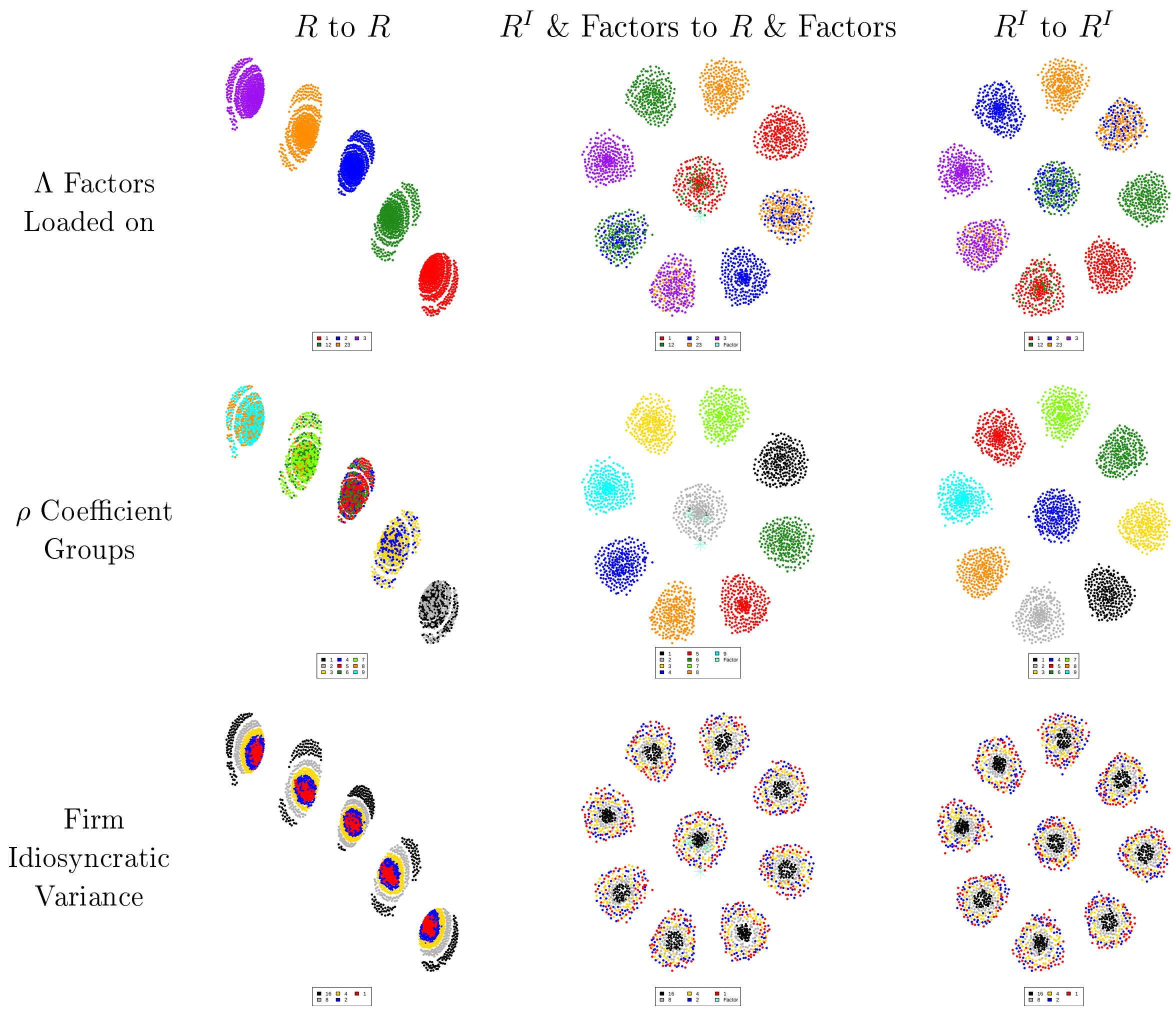
We compare our estimated network matrices with ones calculated algebraically from the simulation model constants. The correlation for the total-return network is 0.996, and that for the idiosyncratic network is 0.98 with both being statistically significant at the 1% level, indicating that the output of our estimation procedure is similar to the networks that we wished to uncover. Finally, we obtain network edges with alternative methods, such as GIRFs (Figure D.2), GFEVD (Figure D.3), and adaptive-elastic net or AEN (Figure D.4), which have also been commonly used in the literature. These alternative methods to calculate network edges yield similar results, with the total-return networks being driven by the common factors and the idiosyncratic networks by the ρ groups.

Figure D.1: Spring Plots of Simulated Networks with GFEVc



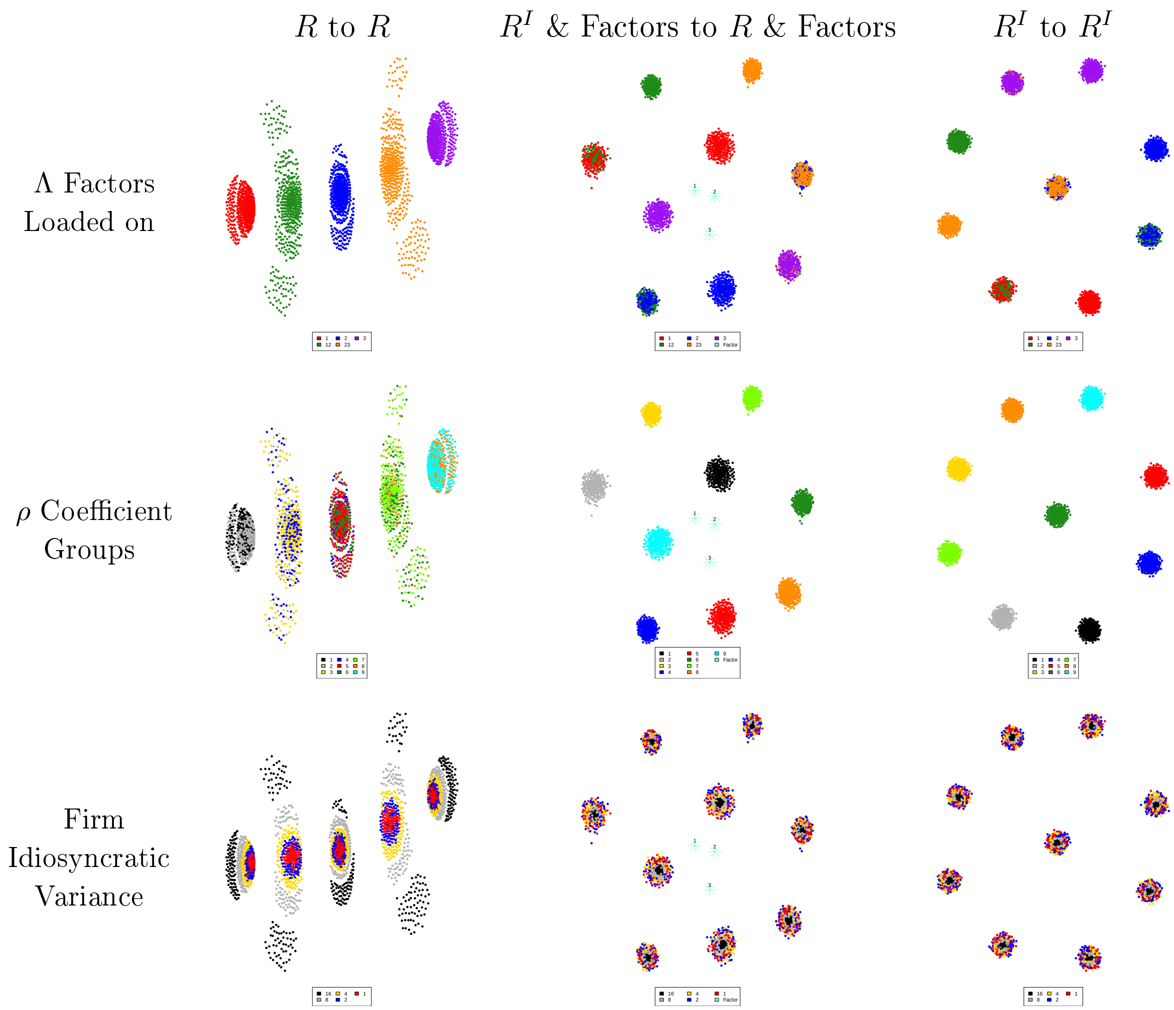
Note: Each row shows one of our network estimation methods for the simulated data. "R to R" does not remove common factors; and "R^I & Factors to R & Factors" separates common factors but keeps them in the network as nodes themselves. In both cases, each dot is a panel member (colored by factor loadings on the left and coefficient groups on the right), and the proximity of nodes to one another depends on how connected observations are.

Figure D.2: Spring Plots of Simulated Networks with GIRF



Note: Estimated networks from data simulations with 3 common factors when using the absolute value of generalized impulse response functions (GIRF) instead of GFEVc.

Figure D.3: Spring Plots of Simulated Networks with GFEVD



Note: Estimated networks from data simulations with 3 common factors when using generalized forecast error variance decompositions (GFEVD) instead of contributions.

Figure D.4: Spring Plots of Simulated Networks with Adaptive Elastic Net

Estimation Method	Λ Factors Loaded on	ρ Coefficient Groups	Firm Idiosyncratic Variance
GIRF, Standardized			
GFEVD, Standardized			
GIRF			
GFEVD			

Note: Generalized forecast error variance decomposition (GFEVD) and generalized impulse response function (GIRF) networks calculated from VAR estimated using adaptive elastic net instead of OCMT for model selection.

D.5 Top Firms by Weights Out to Others

Table D.1: U.S. Top Firms by Network Out Weights (1989-2017): "R to R"

Rank	Ticker	Name	Industry	Industry Group	Sum Weights Out
1	BEN	FRANKLIN RES INC	Financial	Diversified Finan Serv	337.25
2	DOV	DOVER CORP	IndDiv	Miscellaneous Manufactur	324.07
3	GE	GENERAL ELECTRIC	IndDiv	Miscellaneous Manufactur	323.75
4	JPM	JPMORGAN CHASE	Financial	Banks	323.54
5	NTRS	NORTHERN TRUST	Financial	Banks	320.45
6	CMA	COMERICA INC	Financial	Banks	319.47
7	PCH	POTLATCH CORP	Financial	REITS	319.02
8	LNC	LINCOLN NATL CRP	Financial	Insurance	318.1
9	AXP	AMERICAN EXPRESS	Financial	Diversified Finan Serv	317.81
10	PPG	PPG INDS INC	BasMater	Chemicals	316.39
11	EMR	EMERSON ELEC CO	IndDiv	Electrical Compo and Equip	315.61
12	LM	LEGG MASON INC	Financial	Diversified Finan Serv	314.18
13	STI	SUNTRUST BANKS	Financial	Banks	310.72
14	L	LOEWS CORP	Financial	Insurance	310.47
15	BK	BANK NY MELLON	Financial	Banks	308.07
16	TMK	TORCHMARK CORP	Financial	Insurance	306.14
17	ETN	EATON CORP PLC	IndDiv	Miscellaneous Manufactur	305.44
18	WFC	WELLS FARGO AND CO	Financial	Banks	302.69
19	TROW	T ROWE PRICE GRP	Financial	Diversified Finan Serv	302.25
20	IR	INGERSOLL-RAND	IndDiv	Miscellaneous Manufactur	299.68
21	KEY	KEYCORP	Financial	Banks	298.88
22	WRI	WEINGARTEN RLTY	Financial	REITS	298.32
23	PCAR	PACCAR INC	ConsCycl	Auto Manufacturers	297.32
24	TRN	TRINITY INDUSTRI	IndDiv	Miscellaneous Manufactur	295.99
25	ITW	ILLINOIS TOOL WO	IndDiv	Miscellaneous Manufactur	293.81

Note: Sample includes the 524 firms continuously traded over 1989-2017. GFEVc networks with 3 common factors (T = 7,424 and N = 524). Self-loops not included.

D.6 U.S. Inter-Firm Network Application: Market Beta & Commodity Factor Shocks

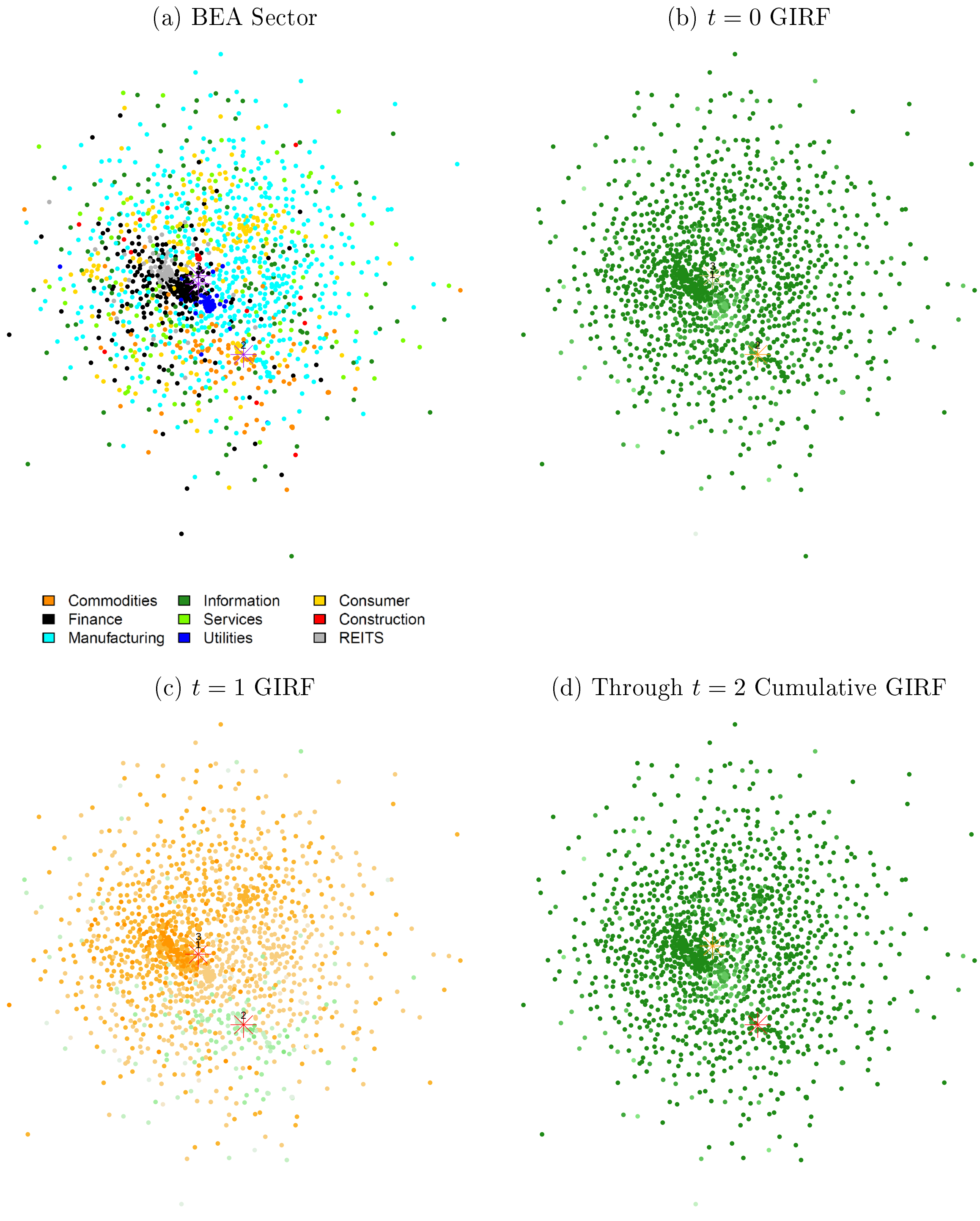
For these exercises, we combine our U.S. inter-firm network estimation with visualization algorithms to analyze responses to common shocks, with the novel addition of including common factors as separate nodes in the networks. We focus on the network of $1,416$ firms continuously traded over 2008-2017 obtained with the three-step estimation method. In the spring plots in Figures D.5 and D.6, the location of each firm is the same, though the colors are set based on their expected log returns over the given period following a shock to one of the common factors, with the darkest green for returns over 1% and the darkest red for those below -1%. Most of the price movement comes on impact, with some moderate returns for a few firms the day after, and virtually zero impact in the following days, as equity markets are quick to incorporate new information. The spring plots with BEA sector legends are also included for reference.

Figure D.5 shows that financial, consumer cyclical and commodity companies would be most affected by a one standard deviation positive market beta shock at $t = 0$. Visually, this is reflected in the fact that financial firms are at the center of the network in close proximity to the first factor. Additionally, a positive move in the first factor is correlated with a negative move in the second factor, which over this period reflects a commodity price increase in that factor. The second factor here has a year-over-year growth correlation of -0.79 with Brent crude oil and -0.74 with the Goldman Sachs Commodity Index. These plots show which firms are most driven by the market beta factor — both directly and indirectly through other firms and factors — and that the central finance firms have a high degree of influence on the network, in agreement with our findings that the majority of the top 25 firms by total-return network weights out are financial firms. Further, all of the firms' cumulative returns through $t = 2$ are positive, consistent with the findings in Figure A.5 that firms load positively on the first factor.

Figure D.6 shows the effect of a commodity price drop as measured by a one standard deviation shock to the second factor. Lower commodity prices would unsurprisingly most adversely affect energy and base materials companies. In fact, the top 10 declining equities following the shock are for firms in the oil and gas extraction sub-sector. On the other hand, United Continental Holdings — the parent company of United Airlines — would be expected to have the largest positive response. This result likely reflects the high fuel costs faced by airlines, and recognizing this could be used by an airline's managers as an indicator that it should hedge its commodity exposure. The other firms in the top ten highest expected equity returns are banks and REITs, possibly because lower commodity prices benefit firms in other sectors of the economy that would be passed onto them. It is not only these central sectors mentioned that increase, but the consumer cyclical firms also have positive returns, supporting this interpretation. Overall, the responses to a shock to commodity prices have more variation in firm returns than those to the first factor, with some cumulative returns positive and some negative after two days.

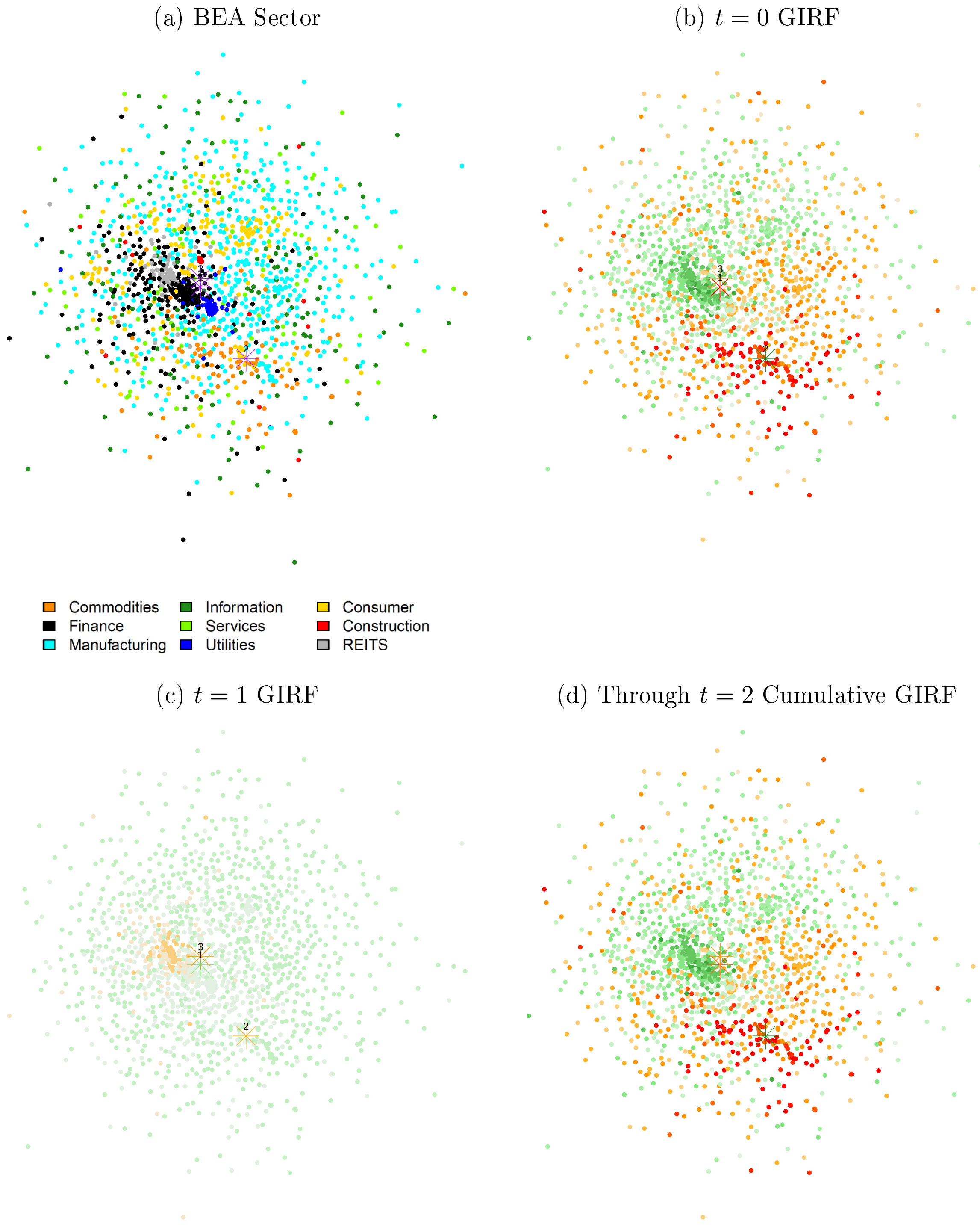
These observations of the expected inter-firm dynamics can help managers and policymakers analyze potential exposures to common shocks, and inform investors' diversification choices to confront the systemic risk they face, as in the case of a fund that is long airlines recognizing the latent commodity risk factor in that investment by also going long oil and gas extraction firms.

Figure D.5: Positive Market Beta Shock for the U.S. Firm Network (2008-2017)



Note: Networks are for the portion of our sample continuously traded from 2008-2017 with 3 factors. Factors extracted by covariance PCA on the matrix of daily log equity returns. The shock is a positive one to the first factor, correlating with a positive market beta shock. The node colors represent the return impact on each firm from $> 1\%$ in dark green to $< -1\%$ in red.

Figure D.6: Commodity Price Decline Shock for the U.S. Firm Network (2008-2017)



Note: Networks are for the portion of our sample continuously traded from 2008-2017 with 3 factors. Factors extracted by covariance PCA on the matrix of daily log equity returns. The shock is a positive one to the second factor, which over this period correlates with a commodity price decline. The node colors represent the return impact on each firm from $> 1\%$ in dark green to $< -1\%$ in red.

E Full Specification of Theoretical Model of Firm Exposures

The model has two sets of agents acting in discrete time: a unit continuum of identical households; and heterogeneous firms divided across J industries, each producing a single differentiated product in a monopolistically competitive environment. Every period, firms choose how much of their goods to sell to the household sector and to other firms as intermediate inputs. Each period proceeds in three stages. In the first stage, the households — who are the sole owners of inter-period capital — determine how to allocate their labor, and their capital holdings across renting to the firms and a technology to produce further capital for tomorrow. Each firm determines how much labor and capital to employ, the amount of other firms' output to use as intermediate inputs in its production process, and the price it will charge. In the second stage, production of goods and next period capital occur, and the productivity (v_{t+1}) and taste (β_{t+1}) shocks for $t+1$ are realized. In the final stage, the firms pay the households their wages, return on capital, and profits as equity dividends. The households also make their consumption purchases and trade firm equities.

E.1 Household's Problem

The representative household maximizes expected discounted utility:

$$E_0 \sum_{t=0}^{\infty} \psi_t U_t \text{ where } U_t = \left(\sum_{j=1}^{J} \beta_{tj}^{\frac{1}{\sigma}} c_{tj}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \text{ and } c_{tj} = \left(\int_{M_j} c_t(j,i)^{\frac{\varphi_j-1}{\varphi_j}} di \right)^{\frac{\varphi_j}{\varphi_j-1}}.$$

U_t is the total consumption index discounted by the stochastic discount factor ψ_t, c_{tj} is the composite consumption index for industry $j = 1, 2, ..., J$, σ is the inter-industry elasticity of substitution, $c_t(j,i)$ is consumption of the output from firm i in industry j, $M_j = 1$ is the mass of firms in industry j, and φ_j is the intra-industry elasticity of substitution across varieties. For mathematical tractability, we use the standard simplifying assumption in the literature that households' and firms' elasticities are the same. The households have a base set of preferences, subject to random taste shocks:

$$\beta_t = \bar{\beta} + Z_t \tag{E.1}$$

where β_t is a vector of the β_{tj} terms.

At the beginning of each period, households choose how to allocate capital holdings across investing in further capital for tomorrow and renting it to firms, K_t^r, at a market rate of r_t. Households inelastically supply one unit of labor each period, and both the labor and capital markets are perfectly competitive, with all participants taking prices as given. Post-production, the representative household makes its consumption and firm equity purchases, constrained by the following budget:

$$\sum_{j=1}^{J}\int p_t(j,i)c_t(j,i)di = w_t + K_t^r r_t + \sum_{j=1}^{J}\int s_t(j,i)q_t(j,i)di - \sum_{j=1}^{J}\int s_{t+1}(j,i)[q_t(j,i)-\pi_t(j,i)]di.$$

w_t is the wage per unit of labor and $p_t(j,i)$ is the price of the good from firm i in industry j. The wage is the numeraire in the economy, with $w_t = 1$ for all periods. The $q_t(j,i)$ are the cum-dividend firm equity prices, $s_t(j,i)$ are the holdings of those equities and $\pi_t(j,i)$ are the profits repaid as dividends to the equity holders. There is a unit supply of each firm's equity, with initial equal holdings across the households.

Iterating the household's first order condition for $s_{t+1}(j,i)$ over periods, along with the equity transversality condition, yields the cum-dividend equity price equation as a function of expected discounted real dividends:

$$q_t(j,i) = E_t \sum_{\tau=0}^{\infty} \frac{\psi_{t+\tau}}{\psi_t} \frac{P_{ct}}{P_{c,t+\tau}} \pi_{t+\tau}(j,i), \tag{E.2}$$

where P_{ct} is the aggregate consumption price index of the industry price indices, P_{tj}:

$$P_{ct} \equiv \left(\sum_{j=1}^{J}\beta_{tj}P_{tj}^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \quad ; \quad P_{tj} \equiv \left(\int p_t(j,i)^{1-\varphi_j}di\right)^{\frac{1}{1-\varphi_j}}.$$

E.2 Firms' Problem

Within each industry j there is a unit continuum of firms, and the firms use labor, capital, and other firms' goods as inputs to their production processes. Since the firms do not have any inter-period choice variables, they solve a series of independent problems each period seeking to maximize profits:

$$\pi_t(j,i) = p_t(j,i)\left[c_t(j,i) + D_t(j,i)\right] - \left[w_t L_t(j,i) + r_t K_t(j,i) + \sum_{l=1}^{J}\int p_t(l,n)x_t(j,i,l,n)dn\right].$$

Firm i in industry j makes its intermediate input decision for purchases of good n from industry l, $x_t(j,i,l,n)$, and agrees to pay prices for those, $p_t(l,n)$, before production occurs. $D_t(j,i) \equiv \sum_{l=1}^{J}\int x_t(l,n,j,i)dn$ is the total demand for good i from firms to use as an intermediate input. The amount of labor, $L_t(j,i)$, and capital, $K_t(j,i)$, employed by firm i are also decided upon in the first stage.

The firm's output, $y_t(j,i)$, is given by the following production function:

$$y_t(j,i) = \left[v_{tj}^{\frac{1}{\sigma}} \left(K_t(j,i)^{\gamma} L_t(j,i)^{1-\gamma} \right)^{\frac{\sigma-1}{\sigma}} + \sum_{l=1}^{J} \omega_{jl}^{\frac{1}{\sigma}} \left(\int x_t(j,i,l,n)^{\frac{\varphi_l - 1}{\varphi_l}} dn \right)^{\frac{\varphi_l}{\varphi_l - 1}\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{E.3}$$

where γ is the production process' capital share. The v_{tj} industry productivity parameters are realized during production in the prior period, which can be thought of as firms learning or improving fabrication techniques during production that they implement the following period. Let the firm productivity parameters follow a standard AR(1) data generating process:

$$\Delta v_t = c + \Xi \Delta v_{t-1} + \Delta \varepsilon_t. \tag{E.4}$$

c is a vector of constants, Ξ is a matrix of autoregressive coefficients, and the $\Delta\varepsilon_t$ are random shocks. The amount that firm i sells to the households will be its final output minus the quantity it already sold to other firms, $c_t(j,i) = y_t(j,i) - D_t(j,i)$. Finally, the ω_{jl} are share parameters for the goods of industry l in production for industry j, and the $J \times J$ matrix of these entries, Ω, characterizes the real firm network of the economy — that is, the input-output structure of the firms' production processes.

E.3 Industry Centralities

Each firm in our model makes three choices each period about where to place itself in the production network: how much to consume of others' goods as intermediate inputs; how much to sell to other firms as a supplier of intermediate inputs; and how much to sell as final goods directly to the households. From these choices each firm will act both as a "consumer" of raw inputs (i.e., labor and capital) and a "supplier" of final goods, though they may do so either directly or indirectly through other firms in one or more production chains. The degrees to which firms act as consumers of inputs and suppliers of outputs through the full input-output network are captured by the two centralities that we introduce in this section. *Consumer centrality* measures the degree to which a firm consumes raw inputs itself and through others, and with that its exposure to shocks to its own and other upstream firms' productivity parameters. Likewise, *supplier centrality* measures how a firm is exposed to household demand for its own and other downstream firms' goods. This therefore represents exposure to shocks to the demand parameters for a firm and those downstream from it.

E.3.1 Consumer Centrality

Using within industry symmetry, and the ratio between firm prices and marginal costs, the industry price indices (P_{tj}) can be related to the quantity and prices of the raw capital and labor inputs through upstream input-output connections:

$$P_t^{1-\sigma} = \underbrace{\left[I_J - \mu^{1-\sigma}\Omega\right]^{-1} \mu^{1-\sigma}}_{\equiv \Psi_d} v_t \tilde{z}_t^{\sigma-1} \tilde{R}_t^{1-\sigma} \equiv \tilde{\alpha}_t \tilde{z}_t^{\sigma-1} \tilde{R}_t^{1-\sigma} \tag{E.5}$$

where P_t is a vector of the P_{tj} industry price indices, $\tilde{z}_t$ is the labor-capital aggregate, and $\tilde{R}_t$ is the price for this composite of raw inputs:

$$\tilde{z}_t \equiv K_t^{r\gamma} L_t^{1-\gamma} \;\; ; \;\; \tilde{R}_t \equiv r_t K_t^r + w_t L_t = \frac{1}{1-\gamma}.$$

Additionally, v_t is a vector of the productivity parameters, I_J is the $J \times J$ identity matrix, and μ is a square matrix with the industries' $\mu_j \equiv \frac{\varphi_j}{\varphi_j - 1}$ values on the diagonal. Ψ_d is a function of the firms' positions within the production network from Equation (E.3) and can be thought of as a markup adjusted Leontief inverse. The vector of consumer centralities for the labor-capital aggregate is defined as $\tilde{\alpha}_t \equiv \Psi_d v_t$, suggesting that a firm's direct and indirect demand for raw inputs depends on the economy's production capabilities (v_t), technology (Ω), and the elasticities of substitution. The $\tilde{\alpha}_{tj}$ consumer centrality term captures the importance of industry j as a user of raw inputs and measures its network adjusted factor use.

E.3.2 Supplier Centrality

The supplier centrality can be calculated by examining the total downstream demand for a firm's goods from other industries and consumers. The supplier centrality is determined from the following system of the stacked total demand equations:

$$(P_t^\sigma y_t)' = \beta_t' \underbrace{\left[I_J - \mu^{-\sigma}\Omega\right]^{-1}}_{\equiv \Psi_S} P_{ct}^\sigma U_t \equiv \tilde{\beta}_t' P_{ct}^\sigma U_t. \tag{E.6}$$

β_t is a vector of β_{tj} consumer taste weights, and y_t is a vector of industry aggregate outputs:

$$y_{tj} \equiv \left(\int y_t(j,i)^{\frac{\varphi_j - 1}{\varphi_j}} di \right)^{\frac{\varphi_j}{\varphi_j - 1}}.$$

The supplier centrality vector is defined as $\tilde{\beta}_t \equiv \Psi_S' \beta_t$, relating a firm's role as a supplier in the network to the consumer preferences for goods and services of itself and downstream industries. $\tilde{\beta}_{tj}$ therefore reflects the network adjusted final consumption share of firms in industry j, with all of these stacked in the $\tilde{\beta}_t$ vector.

E.4 Equity Returns, Common Factors & Network Centralities

In this section, we examine firm equity returns in the model, finding that they depend on three common factors and each firm's proximity through the inter-firm network to the source of a productivity or demand shock. To begin, we derive the log steady-state equity price of firm i in industry j by first noting that the standard constant elasticity of substitution and monopolistic competition result holds, with firms' profits being a fixed markup $(\frac{1}{\varphi_j})$ of their sales. Second, multiplying Equations (E.5) and (E.6) gives $P_{tj}y_{tj}$ on the left hand side and allows us to derive sales in terms of economy wide aggregates and the industry centralities. Finally, using these facts with equity pricing Equation (E.2) at the steady-state and taking logs gives:

$$ln(q(j,i)) = \underbrace{ln\left(\frac{1}{\varphi_j}\right)}_{\text{Markup}} + \underbrace{ln\left(\underbrace{\frac{1}{1-\psi}}_{\substack{\text{Discount}\\\text{Factor}}} \underbrace{P_c U}_{\substack{\text{GDP /}\\\text{Aggregate}\\\text{Demand}}}\right)}_{\substack{\text{Broad Market}\\\text{Beta}}} + \underbrace{ln\left(\frac{P_c}{\tilde{R}}\right)^{\sigma-1}}_{\substack{\text{Real Price}\\\text{Level}}} + \underbrace{ln\tilde{z}^{\sigma-1}}_{\substack{\text{Raw}\\\text{Input}\\\text{Supply}}} + \underbrace{ln\tilde{\alpha}_j}_{\substack{\text{Consumer}\\\text{Centrality}}} + \underbrace{ln\tilde{\beta}_j}_{\substack{\text{Supplier}\\\text{Centrality}}}. \tag{E.7}$$

Firm equity prices depend on the markups, three common factors, and the upstream and downstream network centralities. The three common factors represent: the broad market beta, capturing the discount factor and GDP; the real price level; and the supply of raw inputs. These three common factors align with those that we found using our empirical model, which is notable given that we ran standard PCA on the daily equity returns, without applying any identifying assumptions or rotations.

The log equity returns induced by changes in these common factors, the productivities and demand parameters can be approximated by differencing the first order Taylor expansion of Equation (E.7) around the steady-state. Letting R_t be a vector of the firm log equity returns, ΛF_t the common factor loadings and their log changes, and R_t^I a vector of idiosyncratic firm returns, we have:

$$R_t = \Lambda F_t + R_t^I = \Lambda F_t + \underbrace{Diag\left(\frac{1}{\tilde{\alpha}}\right)\Psi_d}_{\mathcal{U}\equiv\text{Upstream Exposure}} \Delta v_t + \underbrace{Diag\left(\frac{1}{\tilde{\beta}}\right)\Psi_S'}_{\mathcal{D}\equiv\text{Downstream Exposure}} \Delta\beta_t = \Lambda F_t + \mathcal{U}\Delta v_t + \mathcal{D}\Delta\beta_t. \tag{E.8}$$

In the upstream exposure matrix, $\mathcal{U}$, each entry measures the exposure of the row sector to a productivity shock from the column sector, both directly and possibly indirectly through other sectors whose products are between theirs in a production chain. The idiosyncratic response of a firm in industry j to innovations in an upstream source industry s would be $\frac{\iota_j'\Psi_d\iota_s}{\tilde{\alpha}_j}\Delta v_s = \mathcal{U}_{js}\Delta v_s$, where ι_j is a selection vector with a one in the j^{th} position and zeroes elsewhere.

The downstream exposure matrix, $\mathcal{D}$, provides exposures to demand shocks through the network. The supplier centrality quantifies the intensity with which the household consumes from an industry, both directly and indirectly through its downstream sales. The downstream exposure matrix captures the potential for propagation of taste shocks for downstream goods to each industry as the ratio of its centrality to downstream industries' relative to its total downstream exposure. The idiosyncratic return from a taste shock to a downstream industry s is $\frac{\iota_j' \Psi_S' \iota_s}{\tilde{\beta}_j} \Delta\beta_s = \mathcal{D}_{js} \Delta\beta_s$.

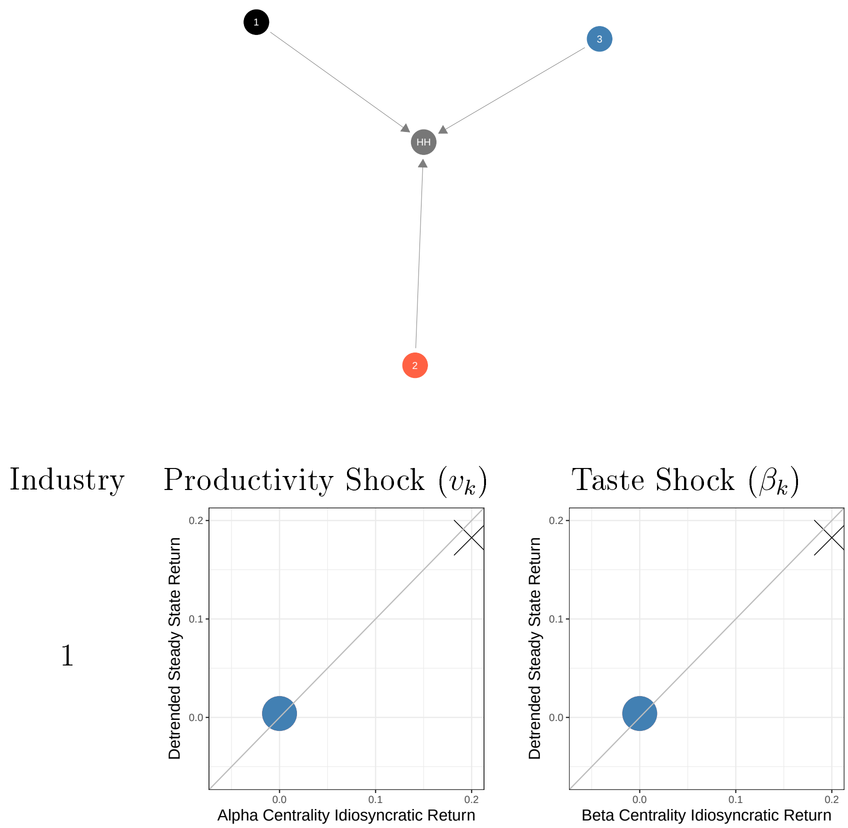
Further, rearranging the idiosyncratic return portion of Equation (E.8), and assuming ΔZ_t and $\Delta\varepsilon_t$ from Equations (E.1) and (E.4), respectively, are vectors of mean zero i.i.d. random shocks, yields:

$$\begin{aligned} R_t^I &= \rho_0 + \rho R_{t-1}^I + \mathcal{U}\Delta\varepsilon_t + \mathcal{D}\Delta Z_t + \zeta_t \\ \implies R_t^I &= \rho_0 + \rho R_{t-1}^I + \epsilon_t \end{aligned} \tag{E.9}$$

where ρ_0 is a constant vector of firm fixed effects, ρ is an $N \times N$ influence matrix, and ζ_t is a residual orthogonal to the two shocks. This formula matches Equation (2), where the R_t^I idiosyncratic returns follow a VAR(1) process. Further, this indicates that the upstream and downstream exposures are embedded in the ϵ_t residuals of this system.

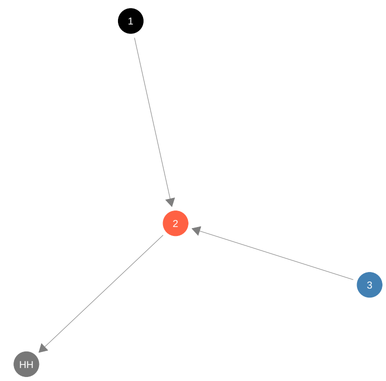
E.5 Theoretical Model Simulations to Upstream & Downstream Shocks

Figure E.1: Simulated Idiosyncratic Equity Responses to Shocks by Exposures: Star Network



Note: The y-axis measures the idiosyncratic return from being in steady-state at the initial parameter levels to moving to a steady-state after the shock. The x-axis measures the upstream or downstream exposure to the source sector multiplied by the change in that sector's specified parameter. The lines in the lower plots are the 45-degree lines equating these two. The source firm is denoted with an x-marker. The industry legend is 1-black, 2-red, 3-blue, 4-green, and 5-yellow. Gray is the household.

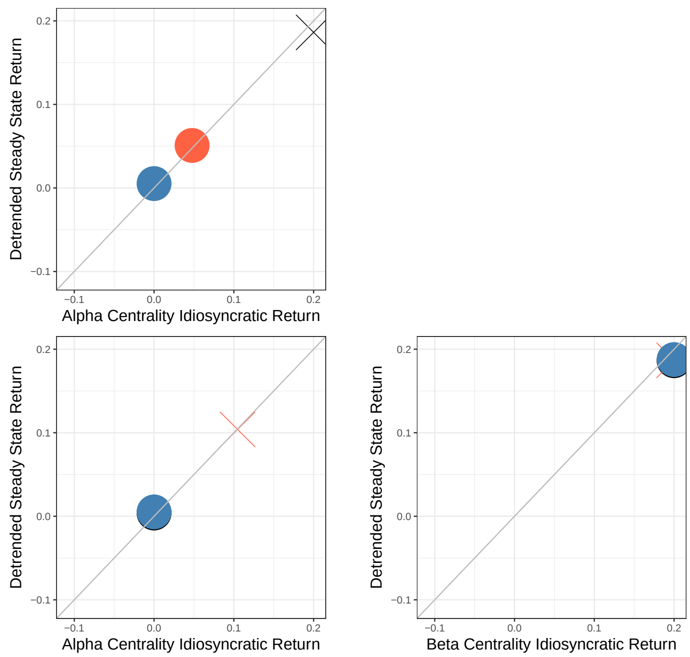
Figure E.2: Simulated Idiosyncratic Equity Responses to Shocks by Exposures: Y-Network



Industry | Productivity Shock (v_k) | Taste Shock (β_k)

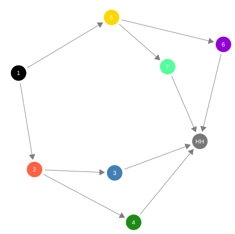
1

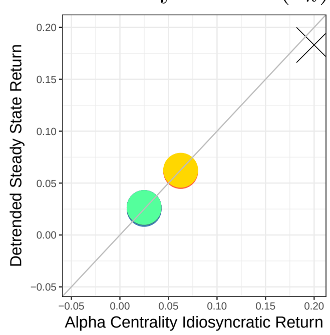
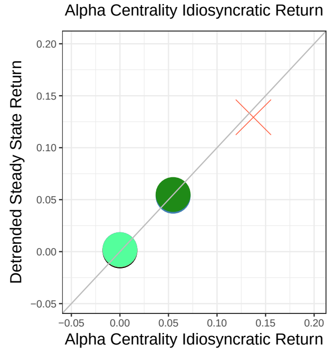
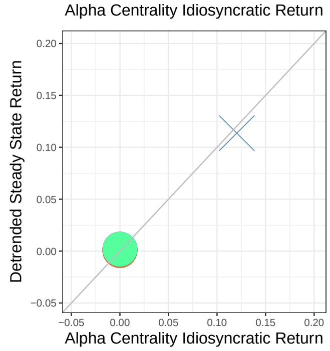
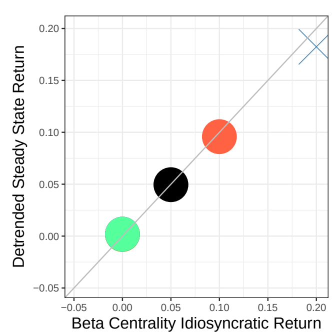
2



Note: The y-axis measures the idiosyncratic return from being in steady-state at the initial parameter levels to moving to a steady-state after the shock. The x-axis measures the upstream or downstream exposure to the source sector multiplied by the change in that sector's specified parameter. The lines in the lower plots are the 45-degree lines equating these two. The source firm is denoted with an x-marker. The industry legend is 1-black, 2-red, 3-blue, 4-green, and 5-yellow. Gray is the household.

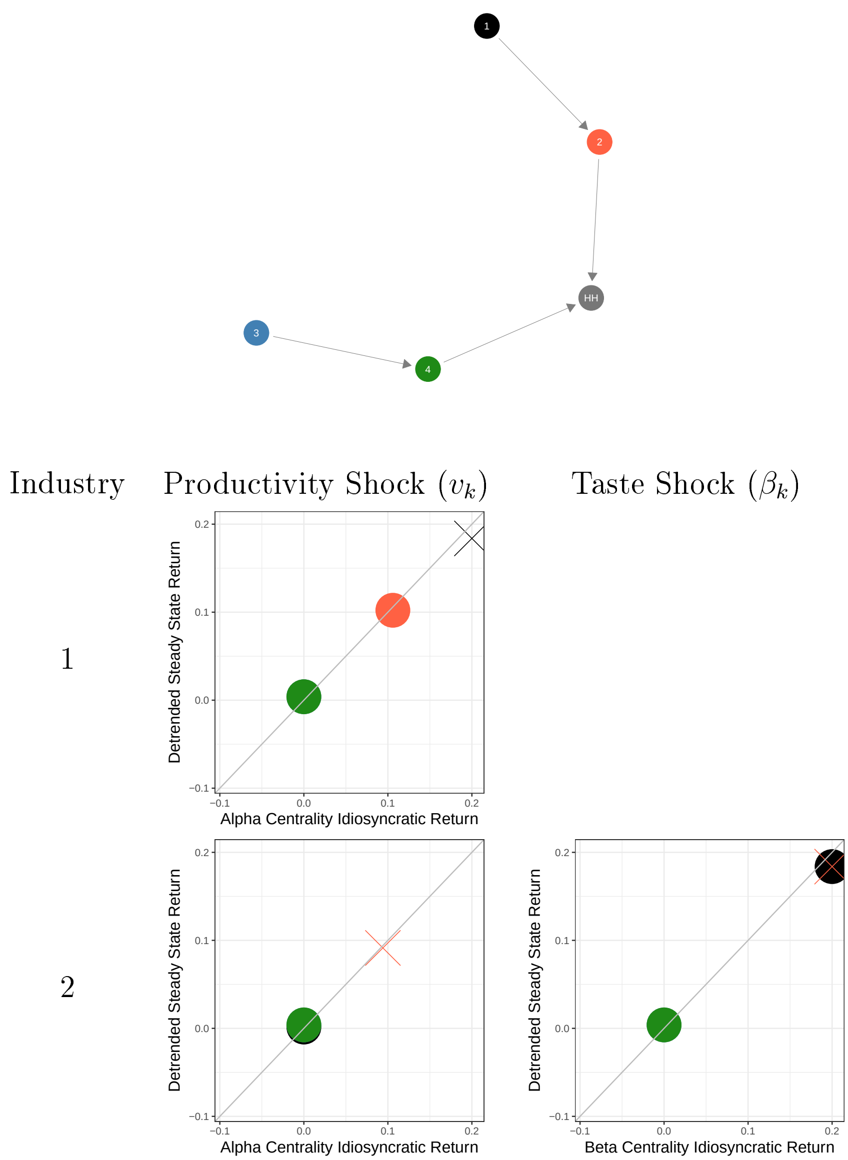
Figure E.3: Simulated Idiosyncratic Equity Responses to Shocks by Exposures: Nested Network



Industry	Productivity Shock (v_k)	Taste Shock (β_k)
1		
2		
3		

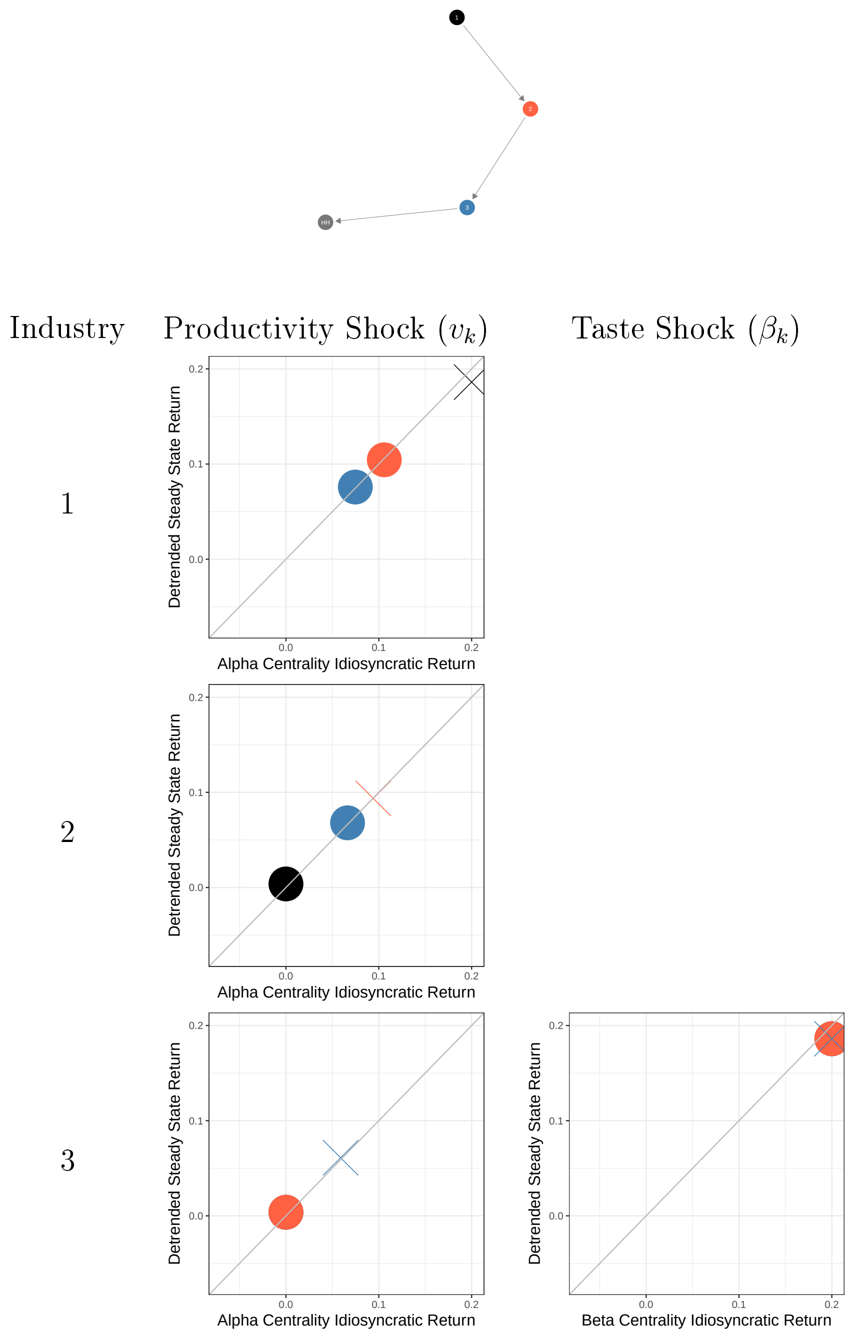
Note: The y-axis measures the idiosyncratic return from being in steady-state at the initial parameter levels to moving to a steady-state after the shock. The x-axis measures the upstream or downstream exposure to the source sector multiplied by the change in that sector's specified parameter. The lines in the lower plots are the 45-degree lines equating these two. The source firm is denoted with an x-marker. The industry legend is 1-black, 2-red, 3-blue, 4-green, and 5-yellow. Gray is the household.

Figure E.4: Simulated Idiosyncratic Equity Responses to Shocks by Exposures: Parallel Network



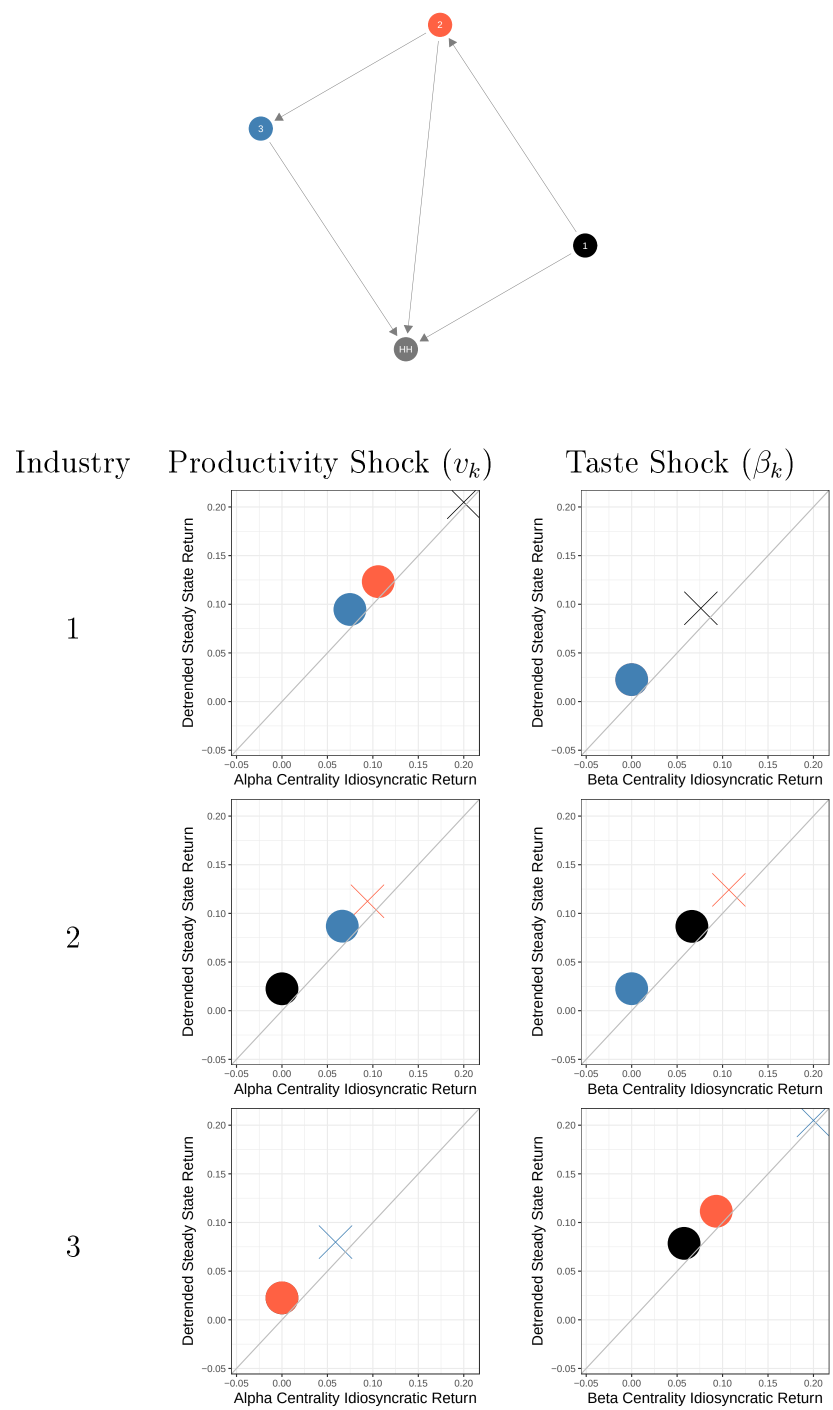
Note: The y-axis measures the idiosyncratic return from being in steady-state at the initial parameter levels to moving to a steady-state after the shock. The x-axis measures the upstream or downstream exposure to the source sector multiplied by the change in that sector's specified parameter. The lines in the lower plots are the 45-degree lines equating these two. The source firm is denoted with an x-marker. The industry legend is 1-black, 2-red, 3-blue, 4-green, and 5-yellow. Gray is the household.

Figure E.5: Simulated Idiosyncratic Equity Responses to Shocks by Exposures: Linear Network



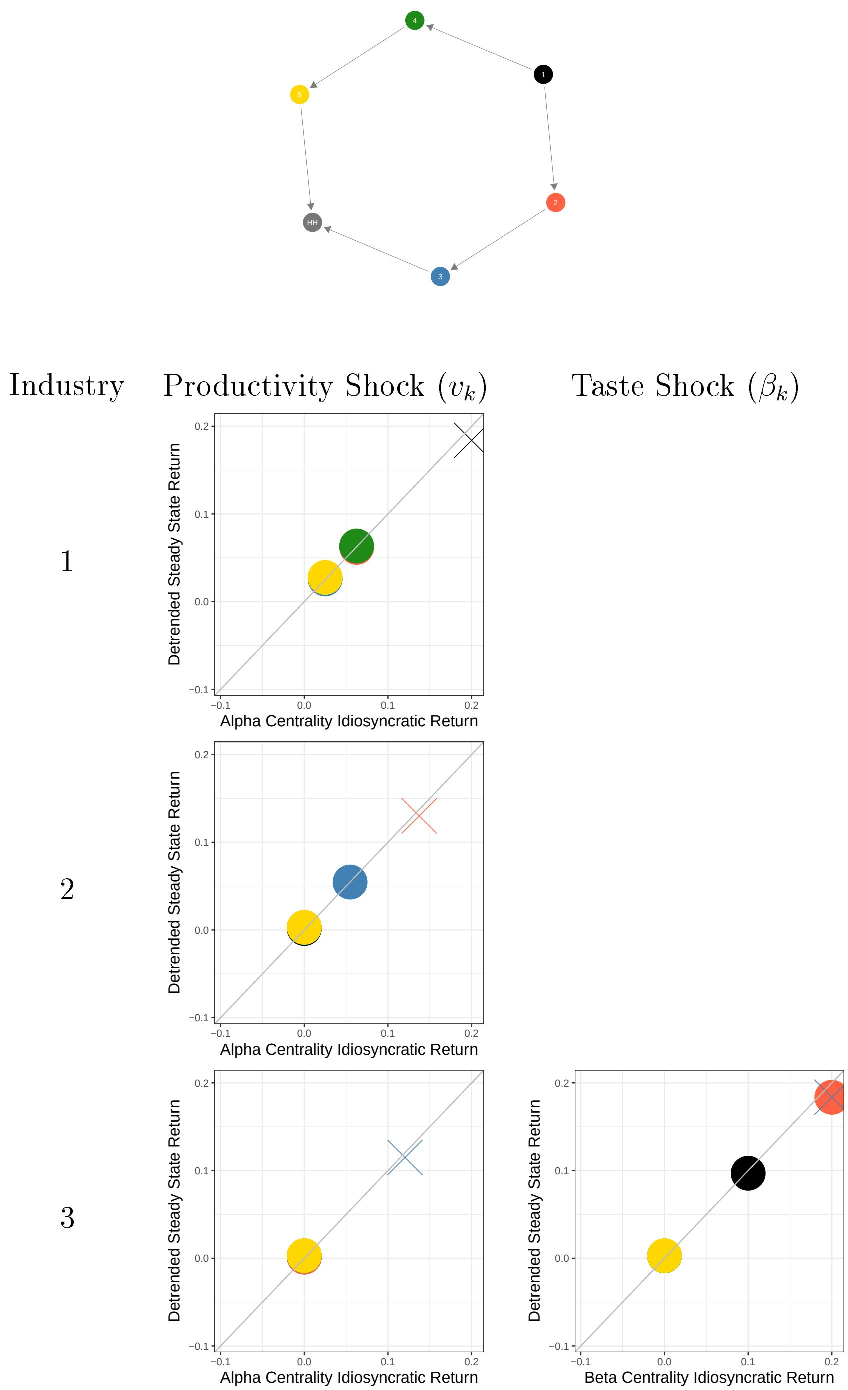
Note: The y-axis measures the idiosyncratic return from being in steady-state at the initial parameter levels to moving to a steady-state after the shock. The x-axis measures the upstream or downstream exposure to the source sector multiplied by the change in that sector's specified parameter. The lines in the lower plots are the 45-degree lines equating these two. The source firm is denoted with an x-marker. The industry legend is 1-black, 2-red, 3-blue, 4-green, and 5-yellow. Gray is the household.

Figure E.6: Simulated Idiosyncratic Equity Responses to Shocks by Exposures: Dense Linear Network



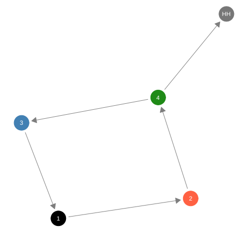
Note: The y-axis measures the idiosyncratic return from being in steady-state at the initial parameter levels to moving to a steady-state after the shock. The x-axis measures the upstream or downstream exposure to the source sector multiplied by the change in that sector's specified parameter. The lines in the lower plots are the 45-degree lines equating these two. The source firm is denoted with an x-marker. The industry legend is 1-black, 2-red, 3-blue, 4-green, and 5-yellow. Gray is the household.

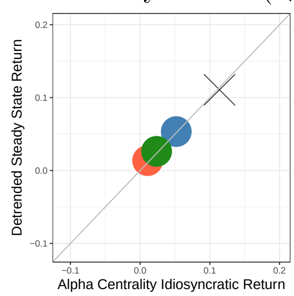
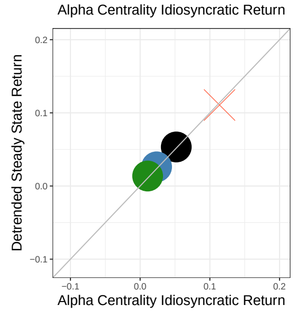
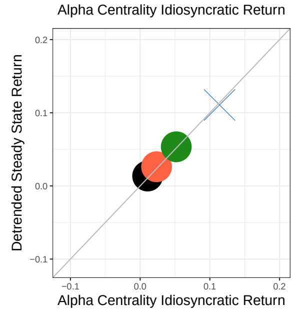
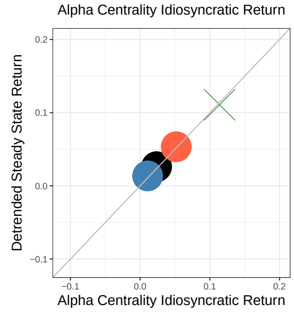
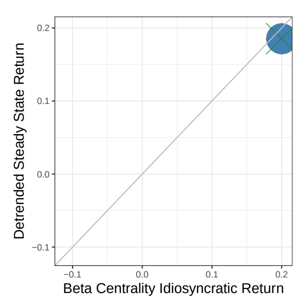
Figure E.7: Simulated Idiosyncratic Equity Responses to Shocks by Exposures: Diamond Network



Note: The y-axis measures the idiosyncratic return from being in steady-state at the initial parameter levels to moving to a steady-state after the shock. The x-axis measures the upstream or downstream exposure to the source sector multiplied by the change in that sector's specified parameter. The lines in the lower plots are the 45-degree lines equating these two. The source firm is denoted with an x-marker. The industry legend is 1-black, 2-red, 3-blue, 4-green, and 5-yellow. Gray is the household.

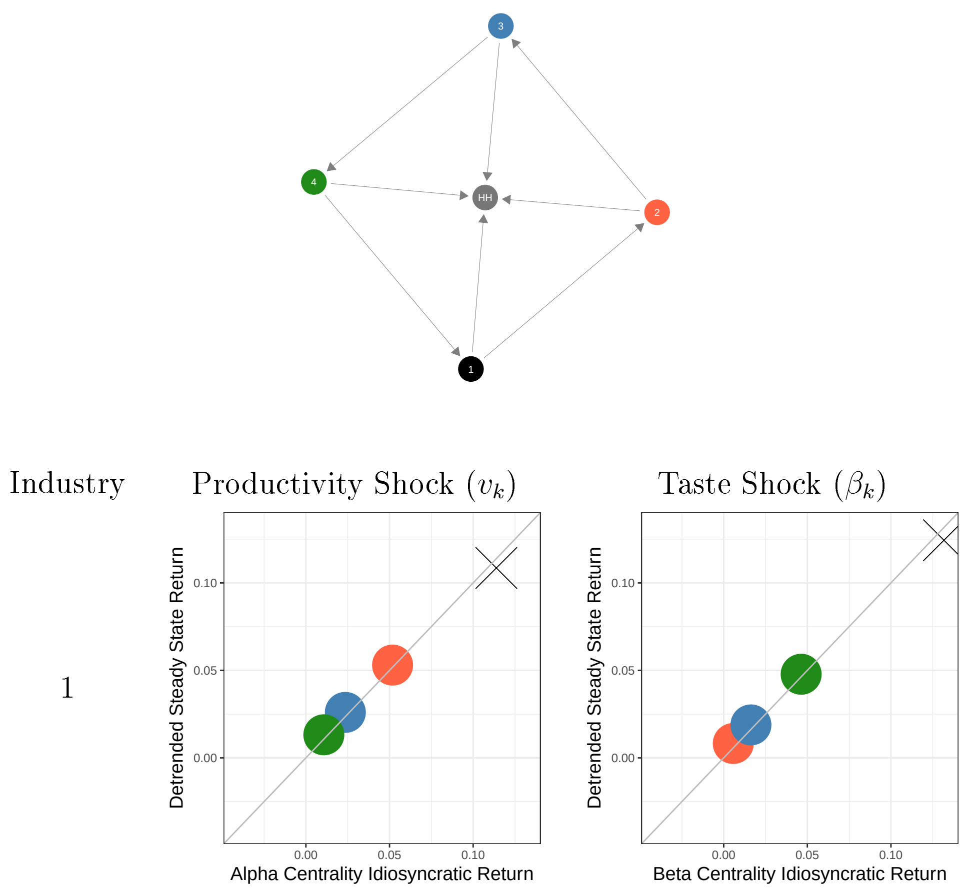
Figure E.8: Simulated Idiosyncratic Equity Responses to Shocks by Exposures: Circle



Industry	Productivity Shock (v_k)	Taste Shock (β_k)
1		
2		
3		
4		

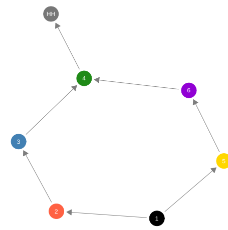
Note: The y-axis measures the idiosyncratic return from being in steady-state at the initial parameter levels to moving to a steady-state after the shock. The x-axis measures the upstream or downstream exposure to the source sector multiplied by the change in that sector's specified parameter. The lines in the lower plots are the 45-degree lines equating these two. The source firm is denoted with an x-marker. The industry legend is 1-black, 2-red, 3-blue, 4-green, and 5-yellow. Gray is the household.

Figure E.9: Simulated Idiosyncratic Equity Responses to Shocks by Exposures: Dense Circle Network

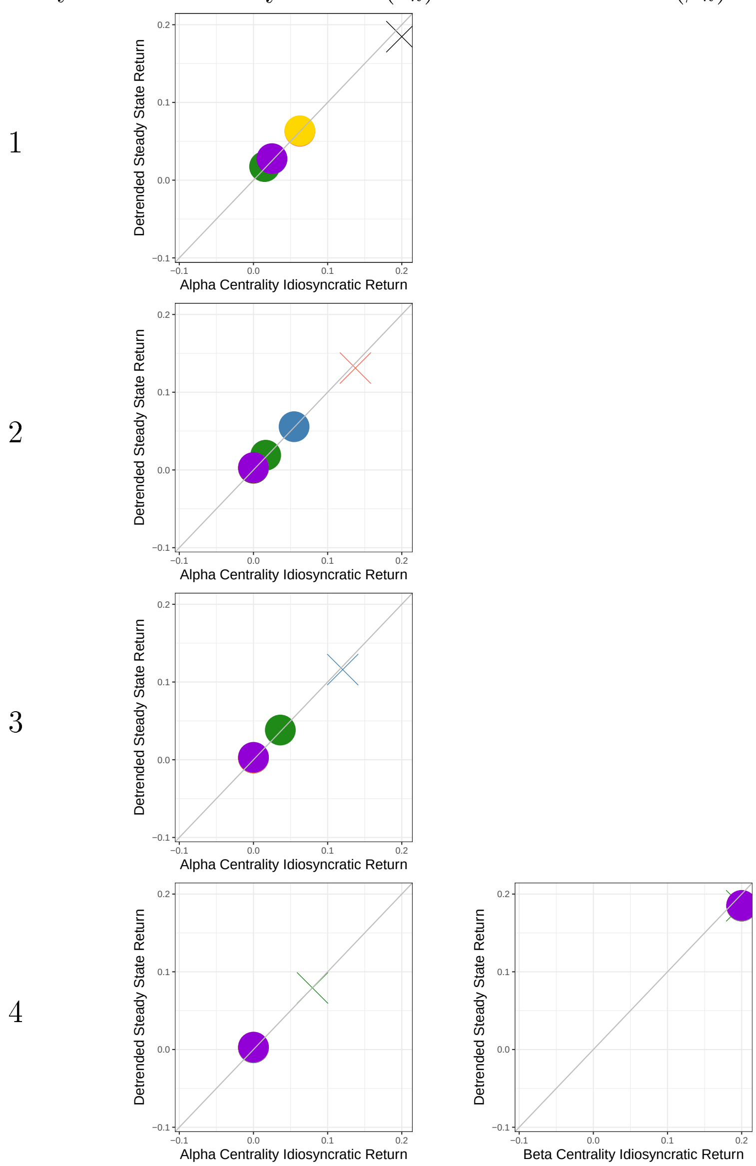


Note: The y-axis measures the idiosyncratic return from being in steady-state at the initial parameter levels to moving to a steady-state after the shock. The x-axis measures the upstream or downstream exposure to the source sector multiplied by the change in that sector's specified parameter. The lines in the lower plots are the 45-degree lines equating these two. The source firm is denoted with an x-marker. The industry legend is 1-black, 2-red, 3-blue, 4-green, and 5-yellow. Gray is the household.

Figure E.10: Simulated Idiosyncratic Equity Responses to Shocks by Exposures: 1-2-2-1

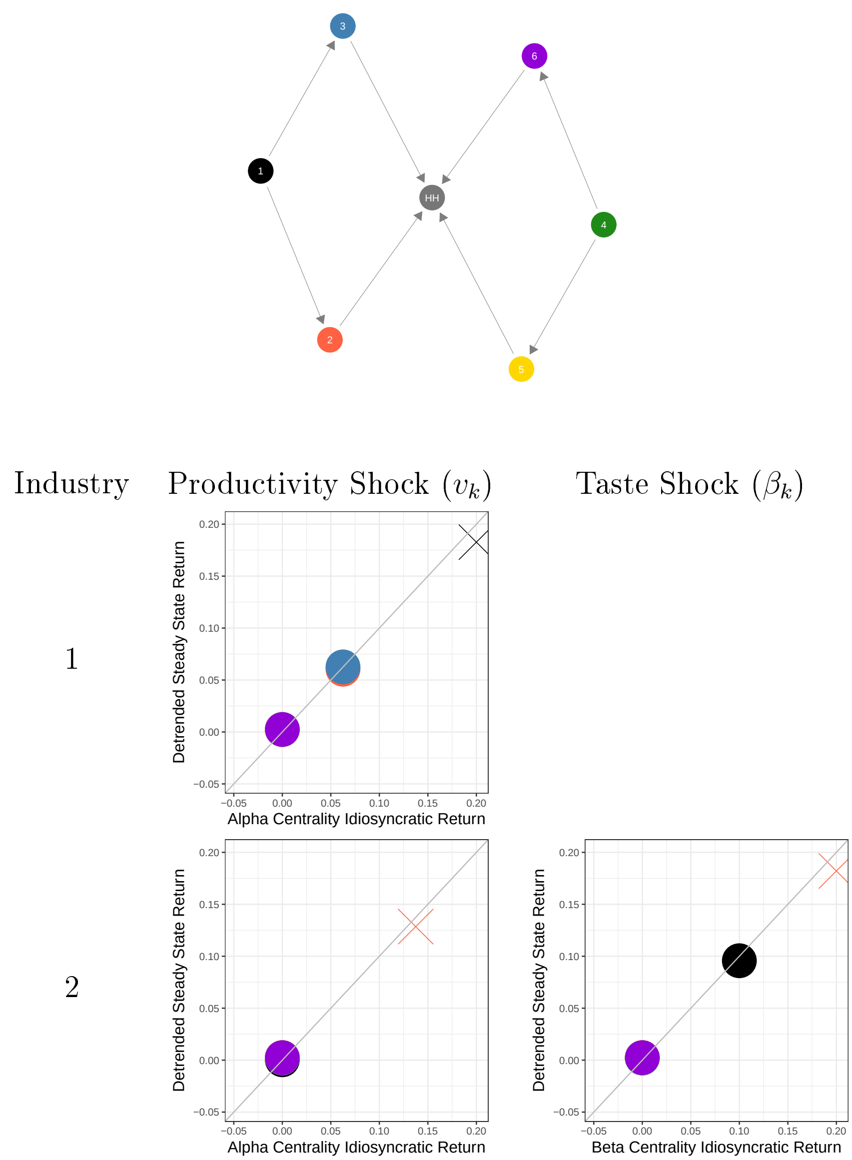


Industry Productivity Shock (v_k) Taste Shock (β_k)



Note: The y-axis measures the idiosyncratic return from being in steady-state at the initial parameter levels to moving to a steady-state after the shock. The x-axis measures the upstream or downstream exposure to the source sector multiplied by the change in that sector's specified parameter. The lines in the lower plots are the 45-degree lines equating these two. The source firm is denoted with an x-marker. The industry legend is 1-black, 2-red, 3-blue, 4-green, and 5-yellow. Gray is the household.

Figure E.11: Simulated Idiosyncratic Equity Responses to Shocks by Exposures: 2 Nests Network



Note: The y-axis measures the idiosyncratic return from being in steady-state at the initial parameter levels to moving to a steady-state after the shock. The x-axis measures the upstream or downstream exposure to the source sector multiplied by the change in that sector's specified parameter. The lines in the lower plots are the 45-degree lines equating these two. The source firm is denoted with an x-marker. The industry legend is 1-black, 2-red, 3-blue, 4-green, and 5-yellow. Gray is the household.

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