WORKING PAPER SERIES

Upstream, Downstream & Common Firm Shocks

Everett Grant

Amazon.com

Julieta Yung

Federal Deposit Insurance Corporation

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Everett Grant [†] **Julieta Yung** [‡]

Amazon.com FDIC

April 2022

Abstract

We develop a quantitative approach to evaluate the roles of upstream (supplier-to-user), downstream (user-to-supplier) and common factor shock transmission across firms. Inter-firm networks are estimated from U.S. equities over 1989-2017 using machine learning techniques. We then employ a multi-sector DSGE model as a lens through which to interpret them and calculate sectoral exposures from input-output tables. We find that: (i) common factors drive an increasing variance share of returns; (ii) equity return based networks reflect real interconnections across firms, with supplier disruptions being more prominent than downstream exposures; (iii) removing the impact of common factors is increasingly important for revealing inter-firm connections.

Keywords: firm networks, upstream versus downstream, input-output linkages, shock propagation, aggregate shocks, equity returns.
JEL Codes: C32, C55, E23, E44, G01.

Broad transmission of shocks across firms has been endemic to crises over the past few decades, from the 2008 Global Financial Crisis, to the 2011 Tohoku earthquake's world-wide footprint, to supply chain disruptions during and following the COVID-19 pandemic. The frequency and severity of these events has led to a focus on discerning what channels shocks flow through as they spread across firms and industries, and how these mechanisms have evolved over time.[1] As a concrete natural experiment exemplifying upstream (supplier-to-user) and downstream (user-to-supplier) exposures and their linkage to firm equity prices, in 2018 the Commerce Department unexpectedly announced a prohibition on U.S. firms selling to the Chinese telecommunications firm ZTE Corp for its failing to comply with prior sanctions. This unexpected shock demonstrated the importance of firms' upstream connections with suppliers, as ZTE's equity price declined over 60% and it neared insolvency.[2] This event can also be viewed as a downstream shock for U.S. firms like Qualcomm, Intel, and AT&T that supplied ZTE and experienced significant contagion, with several suppliers having equity declines over 35% or appreciably deteriorated financial conditions.

This paper employs a novel approach to evaluate the importance of upstream versus downstream production network exposures, as well as the role of common factors in the propagation of shocks. We develop an empirical approach utilizing the information contained in equity prices to estimate common factors and inter-firm networks for U.S. companies over 1989-2017. The method yields two flavors of these networks: those for total returns inclusive of common factors and those isolating firms' idiosyncratic returns. We then use a multi-sector dynamic stochastic general equilibrium (DSGE) model with inter-temporal assets as a lens through which to examine the empirical results, specifically how upstream, downstream, and common factor shocks relate to equity returns. The DSGE model allows us to calculate theoretical consumer and supplier industry centralities from input-output tables, measuring exposures to upstream productivity and downstream consumer taste shocks through the production network, respectively. We compare these theoretical upstream and downstream exposures to the realized historical responses of firms' equity returns to one another captured in our empirical networks. Finally, we use simulations from the DSGE model to interpret our results and assess the relative importance of these shock propagation channels.

We find that, as illustrated in the ZTE example above, exposure to suppliers is economically important and statistically significant with a 0.62 average correlation between the upstream exposure networks derived from U.S. input-output tables and the firm level equity

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[1] The importance of input-output, financial, trade and common shock transmission at country and sector levels has been explored in many papers, including Brooks and Del Negro (2006), Burstein et al. (2008), di Giovanni and Levchenko (2010), Grant (2016), Imbs (2004), Johnson (2014) and Loayza et al. (2001).

[2] See https://www.wsj.com/articles/in-zte-battle-u-s-suppliers-are-collateral-damage-1524562201 and https://www.wsj.com/articles/ztes-operations-shutdown-stymies-major-phone-customers-1525957596.

return response networks. However, the downstream exposure appears more muted, with statistically insignificant average correlations of only 0.22 between the macroeconomic exposure in this direction and the equity return response networks. These results can be suggestive of a low short-term elasticity of substitution across inputs passing shocks downstream but greater flexibility with customers. For example, a manufacturer facing a disruption from a supplier going bankrupt might be unable to produce in the short term because of frictions in retooling its production process to substitute the use of parts from another source; however, if there were a bankruptcy of a customer then it could adjust its sales strategy to other clients, with less business impact.

Since financial networks often exhibit group structure, with some series exhibiting high inter-dependence or clustering, our empirical approach consists of several steps. First, we estimate common factors from equity return series with the use of principal component analysis. Second, we isolate the residual, idiosyncratic firm returns by subtracting the contributions of the common factors.[3] Third, we estimate inter-firm networks from these defactored returns, following the work of Bonaldi et al. (2015), Demirer et al. (2018), Diebold and Yilmaz (2009, 2014, 2016), and Grant and Yung (2021) in deriving them from vector-autoregressions (VARs), being the first to ground such analysis in a structural DSGE model to enhance interpretation.[4] We utilize the variable selection method from Chudik et al. (2018) to deal with the curse of dimensionality and avoid over-fitting the data in such a large VAR and apply our method to daily equity returns for samples of 500 to 1,600 U.S. firms from 1989 to 2017.

There are several salient features of our estimated factors and networks. On the common factor side, the econometric model yields three significant common factors in the US equity market. The first is the average daily return across firms in our sample, which is highly correlated with the market beta and economic growth, as measured by U.S. broad equity market returns, industrial production or GDP. We label the second factor the price one, as it is highly correlated with U.S. PPI, CPI, the value of the dollar, and 10-year breakeven inflation. The third factor has a strong correlation with commodities, particularly petroleum based energy commodities. These common factors — especially the broad market beta factor — become more important over the period we study, explaining 11.7% of the equity return variation over the first ten years of the sample and 35.0% over the final ten.

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[3]For alternative methods that disentangle systematic from idiosyncratic components in financial networks see Barigozzi et al. (2014); Guðmundsson and Brownlees (2021); Brownlees et al. (2021).

[4]Pertinent papers in the recent literature have also introduced factor structure in DSGE models for the purpose of understanding the transmission of shocks (Giannone et al., 2006), studying the relative contribution of idiosyncratic and aggregate components to the macroeconomy (Foerster et al., 2011), and relating macroeconomic shocks to the factor space (Onatski and Ruge-Murcia, 2013).

Once we obtain the common factors, their loadings, and the VAR of idiosyncratic equity returns, we estimate networks across firms using generalized forecast error variance contributions to infer the magnitude and direction of these relationships. These are similar to the standard generalized forecast error variance decompositions of Pesaran and Shin (1998), however, we do not include the equity variance adjustment in the denominator. We omit this because that adjustment varies by firm and over time, and we desire the network edge weights to be comparable across both. We estimate firm networks at both the total return level inclusive of the effects of the common factors in the individual firms' returns, and at the idiosyncratic return level. Using simulations, we show that the former networks that do not disentangle the effects of common factors are comparable to others in the literature. These networks are more reflective of similar loadings on common factors than of the bilateral relationships they are often purported to estimate, while the idiosyncratic return based networks reflect well the bilateral network connections that they are intended to capture.

The theoretical literature studying common shocks, and upstream and downstream inter-industry connections has grown significantly in the past several years.[5] To evaluate these three potential transmission channels, we expand the model of Baqaee (2018) from a one period setting to a multi-sector DSGE model with inter-temporal assets. In the model, heterogeneous monopolistically competitive firms may sell their goods to the household sector and to other firms as intermediate inputs, creating a production network.

The DSGE model indicates firm equity returns depend on three aggregates: the market beta, accounting for the stochastic discount factor and aggregate growth; the price level; and the supply of raw inputs. These align with our analysis of the common factors from the empirical model. Additionally, the DSGE model provides methods to calculate upstream and downstream exposure networks from U.S. input-output data and helps us understand the equity return network mechanisms.

We compare the equity return networks with input-output based upstream and downstream exposures to assess their relative importance for explaining firms' equity responses to one another. We do so by aggregating the equity return networks to the sector level and comparing them with several types of input-output table based exposures. These include the raw input-output tables, their Leontief inverses, and the theoretical upstream and downstream exposures from our DSGE model. We treat these tables as sectoral network adjacency matrices and calculate their correlations using a procedure to bootstrap the expected correlation distributions over networks of similar structure, calculating their statistical sig-

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[5]Recent papers on endogenous network formation and input-output linkages are Atalay et al. (2011), Gabaix (2011), Acemoglu et al. (2012), Acemoglu et al. (2016b), Acemoglu et al. (2017), Acemoglu and Azar (2020), Taschereau-Dumouchel (2017), Oberfield (2018), and Bernard et al. (2019). See Carvalho and Tahbaz-Salehi (2019) for further literature relating production networks to macroeconomic fluctuations.

nificance. We find that upstream exposures (shocks to a firm's suppliers) are more important than downstream exposures (shocks to customers), as the correlations are economically and statistically significant between the equity networks and the upstream networks; however, that is not the case for the downstream ones. Further, the upstream exposure networks have higher correlations with the equity response networks than do the Leontief inverses, suggesting an important role for market structure, the demand elasticities across goods, and the markups priced into the DSGE based networks.

The idiosyncratic equity response networks have 34% higher correlations with the upstream exposure networks on average than those that include the common factors — as past network estimation procedures implicitly do — suggesting that they more accurately reflect the underlying firm connections. That the variance contribution based idiosyncratic return networks have higher correlations than the total return ones accords with the DSGE model, where after the common factors are removed from equity returns one can derive a VAR(1) relationship between them, with exposures through the upstream and downstream centralities entering the innovations. These results support the use of the idiosyncratic equity network estimation method to study transmission across firms, especially towards the end of our sample when common factors become a greater driver of the equity returns, skewing the networks estimated inclusive of common factors that see declining correlations with the input-output derived networks over time.

In Section 4.4, we use simulations of our DSGE model to help understand these findings and discuss them in the context of the relevant theoretical and empirical literatures. Our results are consistent with the empirical literature finding significant exposures to suppliers, such as Menzly and Ozbas (2010) studying cross-predictability of equity returns based on lagged returns in direct supplier and customer industries, and Boehm et al. (2019) and Carvalho et al. (2016) analyzing the ramifications of the 2011 Tohoku earthquake. We find that the reverse direction does not appear to be as relevant to publicly traded firms, extending the results of the latter papers beyond the context of this natural experiment.

Our firm equity return based networks have several benefits: they are derived from publicly available data; they can be estimated in real-time; and they reveal firm heterogeneity and avoid the aggregation bias inherent in sectoral level input-output tables. Alternately, data on firm output, input usage, supplier connections, etc., is typically available at a lower frequency — making it difficult to estimate the propagation of transient shocks — and is either aggregated, available for a small sample of firms only, or comes from confidential microdata.

In addition, our empirical model can be applied to analyze the impact of aggregate shocks across firms. For example, given a hypothetical downward shock to commodity prices, we find

it would most adversely affect energy and base materials companies, while an airline in the network is expected to have the largest positive response. That an airline would be affected in this manner likely reflects the high fuel costs faced by the industry and exemplifies how understanding these networks can be used by firm managers, policy makers, and investors to identify and hedge such latent risks.

1 Econometric Model of Firm Exposures

We estimate the common factors of publicly traded U.S. equity returns, and derive networks capturing how shocks are propagated between these firms. Our approach combines two major tools designed to address big data estimation problems: principal component analysis (PCA) and variable selection methods. In this section, we describe our empirical approach and use simulated data to demonstrate it, introducing the network visualization algorithm that we later apply to U.S. equity return data.

1.1 Common Factor Estimation

Consider a panel dataset with $t = 1, 2, ..., T$ daily observations and N firms, where $R_t = (R_{1t}, R_{2t}, ..., R_{Nt})'$ represents the vector of observed daily log equity returns for each firm. Individual firms are assumed to be small enough relative to the whole economy that their idiosyncratic components do not influence the underlying K common factors F_t, which follow a VAR process of order L:

$$F_t = \Gamma(L)F_{L,t-1} + \eta_t; \quad \eta_t \overset{iid}{\sim} \mathcal{N}(0, \Upsilon), \tag{1}$$

where $\Gamma(L)$ represents the $K \times KL$ matrix of coefficients, $F_{L,t-1} = (F'_{t-1}, F'_{t-2}, \dots, F'_{t-L})'$ is a $KL \times 1$ vector of lagged factors, and η_t is a $K \times 1$ vector of shocks with variance-covariance Υ.[6] The F_t may reflect economy-wide macroeconomic shocks, or those for individual industries, regions, etc. as they and their loadings are directly recovered from the data with minimal econometric restrictions used for identification.

We use the panel BIC information criteria method from Bai and Ng (2002) to select the number of factors — the form they suggest to account for potential correlation in the idiosyncratic errors. We estimate the factors and loadings, (Λ), with the return series' covariance matrix based PCA inclusive of the means, and then fit the factors to Equation

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[6]We also estimated the factor VAR with potentially non-zero constants; however, they were generally estimated to be small and not statistically significant so we omit them from the model for simplicity of exposition.

(1) with $L = 1$ selected using standard BIC.[7] See the Online Appendix for further details on how the factors and loadings are calculated.

1.2 Estimating the Idiosyncratic Firm Return VAR

Once the common variation for each firm (ΛF_t) is estimated, the idiosyncratic firm returns, R_t^I, are obtained as the residual variation: $R_t^I = R_t - \Lambda F_t$. The relationships between the idiosyncratic returns are modeled using a VAR(1) process:[8]

$$R_t^I = \rho_0 + \rho R_{t-1}^I + \epsilon_t. \tag{2}$$

To deal with the curse of dimensionality and avoid over-fitting when estimating the large VAR of firm idiosyncratic returns in Equation (2), we utilize the One Covariate at a time Multiple Testing (OCMT) variable selection procedure of Chudik et al. (2018). The OCMT procedure is intuitive in that one need only run a series of OLS regressions of the dependent variable on the potential explanatory variables, testing whether they have a statistically significant relationship with the dependent variable. The key feature of this approach is that the critical values are adjusted for the fact that this test will be repeated for the potential explanatory variables. The prior literature on VAR based network estimation following Diebold and Yilmaz's work has generally used LASSO or adaptive elastic-net for variable shrinkage and selection; however, the OCMT procedure has several benefits over those algorithms: it is computationally faster and more efficient; it is statistically founded with clear individual variable inclusion rules; and there is not the randomness that can occur with the other methods due to cross-validation sampling selection and optimizer seeding.[9]

The OCMT procedure is based on evaluating the net impact of each of N potential explanatory variables, $R_{1,t-1}^I, R_{2,t-1}^I, \ldots, R_{N,t-1}^I$, on a dependent variable, $R_{i,t}^I$, in a linear

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[7]Foerster et al. (2011) pointed out that there may be estimation issues when trying to calculate inter-industry shock propagation due to the effects of common factors on all industries. In their work they used input-output and inter-sectoral capital network data to adjust for this. However, our goal is to estimate inter-firm networks without imposing any ex-ante network assumptions, so we make the trade-off to not apply their adjustments and estimate the common factors with PCA instead. They found that aggregate factors contributed less volatility over their sample period, while sectoral shocks did not change in importance and therefore became relatively more significant after the mid-1980s.

[8]We allowed for lags of the idiosyncratic returns of up to ten days and found that lags of more than one day were rarely selected by different information criteria and variable selection methods, supporting the use of one lag in our model specification.

[9]For example, OCMT is over twenty times faster than adaptive elastic-net when estimating the simulated networks in Section 1.4.

model of the form:

$$R_{i,t}^I = \rho_{i0} + \sum_{n=1}^{N} \rho_{in} R_{n,t-1}^I + \epsilon_{it}, \text{ for } t = 1, 2, \ldots, T, \tag{3}$$

where N is small relative to T and a subset of the ρ_{in} coefficients are non-zero. The intuition is that if an explanatory variable's coefficient is non-zero, then its mean net impact on $R_{i,t}^I$ should be significantly different from zero, where the mean net impact of variable $R_{n,t-1}^I$ is:

$$\theta_n = \sum_{l=1}^{N} \rho_{il} \vartheta_{nl}$$

and $\vartheta_{nl} = cov(R_{n,t-1}^I, R_{l,t-1}^I)$. Each variable is considered individually through a series of bivariate regressions of $R_{i,t}^I$ on each $R_{n,t-1}^I$ series and a constant estimated with OLS. The t-ratio of $\hat{\rho}_{in}$ from each regression is then compared to a critical value that takes into account the multiple testing aspect of this approach. The OCMT test of $\rho_{in} \neq 0$ is:

$$|t_{\hat{\rho}_{in}}| > \Phi^{-1}\left(1 - \frac{p}{2N^\delta}\right),$$

where p is the size of the test, and Φ^{-1} is the inverse of the cumulative standard normal distribution. The denominator of the second term can take a number of functional forms, but we choose this simple form and $\delta = 1$ for the first iteration. We then add the included variables to the test regressions along with a constant and repeat the test with $\delta = 2$ until no further variables are added.[10] The final step is to then estimate Equation (3) using OLS with only the selected variables and a constant included, setting the coefficients on all of the other variables to zero.

To estimate Equation (2), we perform this analysis on each row of it, and combine the estimated coefficient vectors to arrive at our estimates of ρ_0 and ρ. We do not find evidence of serial correlation in the errors, so we assume that $\epsilon_t \overset{iid}{\sim} \mathcal{N}(0, \Sigma)$ and use the VAR residuals to derive our estimate of Σ. This assumption implies that firms' innovations, ϵ_t, may be cross-sectionally correlated within a period, reflecting exposure to similar underlying economic conditions, but are serially uncorrelated, as equity prices quickly adjust to reflect new information.

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[10]These values for δ are the lower bound to asymptotically select the proper variables, and in untabulated results we repeat our main analysis for a series of alternate values finding similar conclusions.

1.3 Firm Network Estimation

Once we estimate Equations (1) and (2), we then derive the inter-firm network from generalized forecast error variance contributions (GFEVc) across the system. Other papers, including several in the Diebold-Yilmaz network series and thereafter, have used generalized forecast error variance decompositions to derive network edges given an estimated VAR system, the formula for which we provide in the appendix.[11] We instead adjust these to create our GFEVc's. The difference is that in our GFEVc's we do not divide through by the equity variance adjustment in the denominator. We do this because those adjustments vary by firm and over time, and we would like the edge weights to be comparable across both.

We calculate the GFEVc's from the reduced form of the VAR created by stacking the equations in our system:

$$
\begin{bmatrix} R_t \\ R_t^I \\ F_t \end{bmatrix} = \begin{bmatrix} \rho_0 \\ \rho_0 \\ 0_{K \times 1} \end{bmatrix} + \begin{bmatrix} 0_{N \times N} & \rho & \Lambda\Gamma \\ 0_{N \times N} & \rho & 0_{N \times K} \\ 0_{K \times N} & 0_{K \times N} & \Gamma \end{bmatrix} \begin{bmatrix} R_{t-1} \\ R_{t-1}^I \\ F_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_t + \Lambda\eta_t \\ \epsilon_t \\ \eta_t \end{bmatrix}, \quad (4)
$$

where $0_{K \times N}$ is a K by N matrix of zeroes. The covariance of the errors is:

$$
\Theta = \begin{bmatrix} \Sigma + \Lambda\Upsilon\Lambda' & \Sigma & \Lambda\Upsilon \\ \Sigma & \Sigma & 0_{N \times K} \\ \Upsilon\Lambda' & 0_{K \times N} & \Upsilon \end{bmatrix},
$$

where by construction, innovations in ϵ_t are independent of those in η_t, yielding the zero blocks in Θ. For notational convenience, we label the coefficient matrix on the lag term of this reduced-form system A_1, such that the formula for the one period ahead GFEVc network is:

$$
GFEVc = \left[\Theta^2 + (A_1\Theta)^2\right] Diag(\Theta)^{-1}, \quad (5)
$$

where the $Diag$ operation yields a square matrix with the diagonal entries of the given input along the diagonal and zeroes elsewhere, and the exponents are all applied to the individual elements of the matrices. This formula provides a network adjacency matrix with the edge sources along the columns and the destinations along the rows. We then create two types of networks from this matrix.

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[11]See Appendix Section A for common factor estimation details, GIRF and GFEVD network edge equations, and VAR estimation bias when not accounting for common factors. In addition to the GFEVc based networks in the main text, we also estimate the generalized forecast error variance decomposition based networks (Appendix Figures D.3 - D.6) and generalized impulse response function (Appendix Figures D.7 - D.10) based networks with similar results.

1.3.1 R to R — Total Return Networks

In our first network type, the edges from each source firm, s, to each destination one, d, reflect the response of the destination's total returns, R_t, when there are shocks affecting the source's total returns. In Equation (5), this would mean the entry for column s and row d. This specification forms a weighted, directed network capturing the way that firms' returns respond to one another, regardless of whether the innovations are to the firms' idiosyncratic returns or the aggregate factors. These networks are similar to those in Demirer et al. (2018) and Grant and Yung (2021) in this way.

1.3.2 R^I & Factors to R & Factors — Idiosyncractic Return Networks

In our second approach, we distinguish the effects of firms' idiosyncratic shocks from those to the common factors. When doing so, we take the novel approach of explicitly treating the factors as nodes in the network. In these networks there are $N + K$ nodes, with one for each of the firms and factors. The edge from firm s to firm d will be the expected variance contribution to d's returns when shocks affect firm s's idiosyncratic returns, and likewise for a shock affecting factor $k = 1, 2, ..., K$. In these cases, we take the entry for column $N + s$ and row d of Equation (5) as the inter-firm edge, and the entry for column $2N + k$ and row d for the edge from factor k to firm d. In the case where the destination is factor k, then the edge from firm s to it will be the entry in column $N + s$ and row $2N + k$ — giving a zero effect — and the edge from another factor, f, to it will be the entry in column $2N + f$ and row $2N + k$. Additionally, in our analysis below we use the subset of this network formed by the connections from firms' idiosyncratic returns to other firms and refer to these networks as the R^I to R ones.

1.4 Network Estimation Simulation

To illustrate the above estimation procedure, we apply it to simulated data from the system defined by Equation (4).[12] The simulated data has nine distinct groups with bilateral connectedness where the observations within each group depend on one-another's lagged values plus idiosyncratic shocks, giving ρ a block diagonal form. These groups can be thought to model distinct industries. There are also three common factors, with the first portion of simulated series only loading on the first factor, the second loading on the first two factors, the third loading on the second factor, the fourth loading on the second and third factors, and the fifth loading only on the third factor. We have nine even groups in ρ to create

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[12]The details of how the data were simulated can be found in Appendix Section B, along with a comparison of the same networks estimated using different approaches (i.e., GIRF, GFEVD, AEN).

partitions within and overlapping each of our five Λ coefficient groups, so that there is a complex interaction between the two for our process to attempt to disentangle. Finally, the ρ_0 constant vector is drawn from a mean zero normal distribution.

Figure 1 shows the estimated networks when including or separating out the effects of common factors as described in the prior two sub-sections. These plots use the ForceAtlas2 method from Bastian et al. (2014). ForceAtlas2 is a force-directed layout algorithm to display network spatialization, transforming a network into a map where nodes with greater connectedness are closer together. At a high level, all of the nodes are repulsed from one another like charged particles, while edges attract their nodes like springs — yielding the name for this class of algorithm, spring plots. The final node positions provide a balanced state, helping to interpret the data without having to incorporate any other attributes of the network members. To read these plots, think of a map without a key showing the direction of true north or a scale. In that case, as with these plots, the precise orientation of the figures is not informative and rotations do not have a clear meaning, but the relative proximity of features on the plots to one another and the center of the figure do, as do any clusters that arise and inform the underlying topology. This technique is superior to other network visualization methods such as heat maps primarily because the number of network members makes many other methods hard to read, and the spring plots are able to capture third party or greater relationships (e.g., two nodes that are each closely connected to a third but not to each other will be close in the plot).

The first row of the figure provides the connections between the simulated R data series with an "R to R" total return network. The second row's network has edge weights between firms and factors based on the expected responses of firms' total returns from idiosyncratic shocks to other firms or factors in an "R^I & Factors to R & Factors" network. This allows us to simultaneously examine the roles of shocks to firms and aggregate factors.

The columns of Figure 1 differ only in the legends used for coloring the nodes. The first column has the nodes colored based on the factor(s) they load on. The second column is colored based on the nine blocks of non-zero ρ coefficient groups. To understand the results of the estimation process, we start by analyzing the "R to R" network in the first row. It is evident in the left plot that our procedure has grouped the nodes by the factors that each firm is directly affected by, with those loading on the first factor only at the bottom right in red, then those loading on the first two factors just to the left of that group in green, followed by those directly affected by only the second factor adjacent to that group, and so on. These total return networks are similar to those uncovered using the approach common in other papers in the literature of applying a version of adaptive elastic-net and then calculating network edges from either the raw or standardized variance decompositions

or impulse responses — as can be seen in Appendix Figure B.3 versus the first row of Figure 1 — with those also being driven by the common factor loadings.

The right plot in the first row shows the same network with each node in the identical position as the first plot. Here, the ρ coefficient groups can be seen in the clustering; however, it is their overlap with the factor loading groups that drives the patterns in them — all of the nodes in the first ρ coefficient group load only on the first factor so they are in the bottom right group in black, those in the second ρ coefficient group in gray are mixed in between those loading only on the first factor and those loading on both of the first two factors so they are split between the first two clusters, and so on.

The second row's plots demonstrate how our procedure is able to extract and separate the direct inter-firm connections captured in the ρ coefficient groups from the common factors. The factors have roughly even total influences on the network and are near the center of the figures, so the major dynamics of these figures are determined by the R^I sourced edges. When looking at the left column, it at first appears that the firm factor groups are central to the organization; however, it is clear from the right column and perfect clustering by the ρ coefficient groups that they in fact are the drivers. Clustering in this network is governed by the ρ coefficient group structure, unlike the "R to R" network.[13]

Combined, the results from Figure 1 are quite striking — the "R to R" networks identify the organization of the system around the factors very well, while the other plots that isolate the factors' influence are able to effectively estimate the bilateral node relationships.[14] Finally, we compare our estimated network matrices with ones calculated algebraically from the simulation model constants. The correlation for the total return network is 0.996, and that for the idiosyncratic network is 0.98 with both being statistically significant at the 1% level, indicating that the output of our estimation procedure is similar to the networks that we wished to uncover.

2 U.S. Inter-Firm Networks

We apply our methodology to daily U.S. log equity returns from 1989 through 2017 in order to estimate the U.S. equity common factors and inter-firm network.

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[13]Note that using our estimation approach but calculating the network edge weights with either generalized variance decompositions or impulse response functions instead of the GFEVcs yields similar results, with the total return networks being driven by the common factors and the idiosyncratic networks by the ρ groups. See Appendix Figures B.1 and B.2 for spring plots using these alternative approaches.

[14]These simulation results are in line with the main argument of Bailey et al. (2016) and Hale and Lopez (2019) that one needs to account for common factors before estimating a network in order to properly recover the bilateral connections between its members.

2.1 Data

To select our sample of U.S. firms, we take the union of all firms that are in the top 25% by market capitalization for any year in our sample. If a firm is in our sample at any point, then we obtain their equity pricing data for as long as recorded. We do this to filter out the smallest firms to ensure that those in our sample have actively traded, liquid equity securities that are highly researched and followed, providing them with accurate price discovery. As such, the efficient markets hypothesis indicates that the firms' equity prices should reflect all available information about them, including how they are connected through the channels we wish to study. Our data set includes the daily Bloomberg closing prices for $5,454$ firms between December 30th, 1988 and June 20th, 2017. The closing equity values are total return indices inclusive of returns from dividends to avoid spurious price jumps when dividends are paid that do not reflect a change in the valuation of the underlying firm. We also gather the Bureau of Economic Analysis (BEA) sector and Bloomberg industry for each firm and collect U.S. input-output use tables from the BEA for 1997 through 2015 for our macroeconomic exposure networks.

We analyze both a balanced panel of firms to study the long-term firm network, and others to account for changes to it from companies entering and exiting. Specifically, we examine the set of 524 firms continuously traded throughout our whole sample — both over the full period and in rolling 10-year periods — and broader rolling samples of firms continuously traded over each 10-year window.

2.2 U.S. Equity Common Factors

Applying our analysis to the 524 firms continuously traded throughout our sample produces three common factor series for their daily log equity returns using the Bai and Ng (2002) panel information criteria. The BIC suggests a range between one and three common factors in the data, with a mode of three. These three factors combined account for 27% of the cross-sectional variation in the entire dataset.[15] Subsequent factors each contribute less than 2%, so we favor a parsimonious approach that is also consistent with the scree plot test typically used to infer the number of factors in PCA.

The importance of the top three factors increased significantly over time. Figure 2 plots the cumulative shares of the sample variance explained by the top factors in rolling ten year windows for all firms traded continuously throughout each period.[16] The variance share

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[15] Our finding that equities are roughly 70% driven by idiosyncratic firm shocks is in line with Campbell et al. (2001), who found that over the period from 1962 to 1997, U.S. equities were 17% market driven, 12% by industry developments, and 71% by idiosyncratic considerations.

[16] Appendix Figure C.6 has this plot for the balanced 524 firm sample from 1989-2017, with similar results.

of the top three factors increased from around 12% at the start of our sample period in the early 1990s to over 35% towards the end in the mid-2010s.[17] Further, the 2010-2017 variance shares for these factors remained near the elevated levels in the figure — they explained 25.1%, 2.4%, and 1.2% of the variance, respectively — even when not including the 2008-2009 crisis period with particularly high equity correlations. The variance share for the first factor, for example, increased from 8.6% over the 1989-1998 period to 31.2% over 2008-2017, with the largest increase occurring when 2008 entered the rolling windows. The second and third factors saw their variance shares increase by just over a fifth with 0.5% and 0.2% increases. These results suggest a significant change in the importance of the common factors following the Global Financial Crisis, with implications for portfolio diversification strategies.[18]

The first factor loads positively on all firms in the sample. This implies that positive shocks to the first factor translate into higher equity returns for all firms. In fact, as can be observed in the top row of Figure 3, the first factor series is almost identical to the sample average log returns for each day, both in levels and in year-over-year changes. For this reason, we refer to this factor as the market beta, reflecting both time variation in discount factors or risk premia, and economic growth, akin to the Fama and French market risk factor (Fama and French, 1995).[19]

Appendix Section C shows the loadings of each industry on every factor (Figure C.1), as well as lists of firms with the highest and lowest loadings per factor. The firms with the highest loadings on the first factor are predominantly from the technology, consumer cyclical and financial sectors, while those with the lowest loadings are royalty trusts, consumer non-cyclical and utility firms. This makes intuitive sense as the former are pro-cyclical sectors, while the latter are generally considered passive, acyclical investment sectors.

The second row of Figure 3 shows how the first factor compares to another equity market average, the S&P 500 Index. The correlation between the series is 0.89 in levels and 0.83 in year-over-year changes, supporting the interpretation of the first factor as the market beta. The third and fourth rows in Figure 3 show that our first factor comoves with the U.S. industrial production index and GDP, with year-over-year change correlations above 0.50. Below we focus on the year-over-year changes in the other two factors and real series, as the

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[17]Bartram et al. (2018) also found an increasing share of firm level equity returns from common rather than idiosyncratic factors. Studying the period 1965-2017, they found that average idiosyncratic risk declined to an all-time low at the end of their sample.

[18]Note that the structures of the idiosyncratic networks before and after the Global Financial Crisis were highly correlated at around 95%; however, the inter-sector sums increased about 2.8× on average.

[19]The idea of time variation in risk premia (and hence investors' expected stochastic discount factor) is consistent with empirical evidence from the finance literature, documenting its importance in accounting for excess volatility in asset prices and predictability of returns (e.g., Cochrane 1991, 2011; Campbell 2014).

trends in levels may lead to spuriously high correlations.

Fluctuations in the second factor closely align with movements of the U.S. price level. In Figure 4 we compare the second factor with different measures of the price level, including the Producer Price Index (PPI), Consumer Price Index (CPI), the value of the dollar, and U.S. 10-Year Breakeven Inflation (BEI) calculated from TIPs and nominal U.S. Treasury bonds.[20] The correlation in absolute terms between year-over-year changes in the second factor and PPI is 0.60, that with CPI is 0.51, and those with the dollar range around 0.40, either when taking into account the trade-weighted value of the dollar or looking at several major currencies individually. Note that the PPI absolute correlation being higher than the CPI one fits with our results below that the firm level equity market networks are more correlated with the upstream rather than downstream exposures. In the last figure, it can be seen that breakeven inflation is positively correlated with our factor at 0.48. Interestingly, if the breakeven inflation rate is lagged six months then the absolute correlation increases to 0.71, suggesting that the forward looking market implied inflation has predictive power for the price level factor in our sample. The second factor loads negatively on the energy, financial, basic materials, and utilities sectors, and positively on the technology sector. All of the top ten firms loading on the second factor are technology companies, and the bottom ten consists of nine energy firms and a petroleum shipping company (Appendix Table C.2).

The third factor has a large positive average loading on the energy sector, followed by the technology and base materials sectors. Nine of the top ten firms by their loadings on the third factor are energy related (Appendix Table C.3). Figure 5 displays the movements of the third factor relative to the price of Brent crude oil and the Goldman Sachs Commodity Index (GSCI). These series indicate that the third factor is similar to a raw input factor, with the correlation between price changes in oil and this factor especially high at 0.64. See Appendix Figures C.2 to C.5 for similar plots of the third factor against various components of the GSCI index. The correlations are positive except for cocoa and generally economically significant in magnitude. The highest correlations are with the petroleum based energy commodities, followed by industrials and then precious metals. Additionally, around 2009 commodities seem to go from the third to the second most important common factor in our sub-sample analyses — possibly because of a lower inflation environment making the price level less important, while there were large subsequent commodity price changes.[21] If one looks on

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[20]A role for price pressures in the inter-firm network is supported by Smets et al. (2019), who found evidence of inflation being passed through production networks.

[21]See Appendix Figures C.8, C.9 and C.10 for the sum of the edge weights of the factors to each sector and one-another over time. These make the transition of commodities between the second and third factors clear, as well as some other interesting behavior like the greatly increased influence of the first two factors on the finance sector beginning in 2009. Appendix Figures C.11 and C.12 expand this to include all bilateral weight sums between sectors and factors over time.

either side of this break then the pertinent factor has a greater correlation with commodity prices. For example, when looking at all firms in our sample traded continuously from 2008 through 2017, the correlation of the year-over-year changes in the second factor with oil is 0.77, and that for the Goldman Sachs Commodity Index is 0.79.

The first factor loads relatively evenly across sectors while the second and third have the largest loadings in absolute value on the energy and technology sectors, indicating that they are key sources of industry specific variance in our sample. Although we focus on the first three factors for our main network specification, in untabulated analysis we explore variations in which we include one to five common factors yielding similar results. Refer to Appendix Table A.1 for details of the PCA results.

2.3   U.S. Equity Return Based Firm Networks

The inter-firm network of the 524 firms continuously traded from 1989-2017 is shown in spring plots in Figures 6 (without removing the common factors) and 7 (after separating out the common factors). These plots also use the ForceAtlas2 algorithm to determine the node locations so that both direct relationships between firms, and those through third parties are reflected. For example, if two tire manufacturers both have strong ties to Ford but weak ones with each other, then they would still be close in these figures because they would both be near Ford. The parameters entered into the ForceAtlas2 algorithm for these and the subsequent firm network figures are the same, and therefore they are comparable. Each node is colored by the BEA sector to which the firm belongs. Note that we distinguish the Real Estate Investment Trust (REIT) sector in these plots from the financial sector given its distinct return profile and separate cluster.

The "R to R" network for the full sample is in Figure 6 panel (a), with those for different subperiods in panels (b), (c) and (d). Focusing on panel (a), it is clear that sector clusters are an important feature of the network, with firms within the same sector grouped together, such as utilities (purple) and commodities (orange), with many firms belonging to the man- ufacturing sector (light blue). Additionally, the finance (black), REITS (gray) and consumer (yellow) sectors are near the center of the network. This observation coincides with seven of the top nine firms by the sum of their weights out to others being financial firms, as in Grant and Yung (2021) examining global firm networks derived from equity prices. Additionally, the number two and three firms are industrial diversified firms, and that third firm — Gen- eral Electric — was designated a non-bank systemically important financial institution by the Financial Stability Oversight Council due to its high level of financial dealings up until it significantly changed its businesses in June 2016. See Appendix Table C.4 for the list of

top firms by their sum of weights out to others.

Panels (b), (c) and (d) show the networks of all firms continuously traded within each of the past three decades, with about $1,500$ firms in each plot. When estimating networks over rolling samples, the factors are calculated using only data from the estimation sample period and do not include future or out of sample information. These networks show the finance sector typically at the center in all three decades and the clustering by sector is present in the different periods. Notably, the networks show firms becoming more tightly grouped over time regardless of sector, suggesting greater equity market integration and matching the increased common factor variance shares mentioned above. Along with this feature, these plots — and the other network estimation methods used in the past in this literature — become less informative as all of the equities move more with the common factors and fall into one compact cluster at the center of the network.

Figure 7 shows the "R^I & Factors to R & Factors" networks that separate the effects of the common factors from the individual equities, and in so doing "unfold" the underlying firm to firm connections that we want to study. In panel (a) one can see that the first factor — the market beta — is at the center of the network denoted by the purple star, and that finance firms are near it. What can be further seen is that REITs form a distinct gray cluster farther from the center of the network, unlike in Figure 6 where they are nearly dead center and indistinguishable in location from the other financial firms. Additionally, the commodity firms are on the periphery of the network, near the third commodity factor at the top of the plot, and the consumer sector firms are close to the second, price-level factor. This implies that the energy, base material, and utility firms near the third factor do not comove as much with the broad market factor and the commodity factor is more influential for them.

Panels (b), (c) and (d) show that sector clusters become more pronounced over time in the defactored networks, possibly reflecting increased specialization and more integrated within-industry production processes, or the rise in sector specific investment funds.[22] Similar to panel (a), the REITs are a distinct cluster from the remainder of the financial firms. They are close to the utility firm cluster, which is interesting as both are often seen as safe, acyclical dividend oriented investment sectors. The commodity sector can be seen to move between being near the second and third factors.

To illustrate how these equity return networks can be used to study shock propagation

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[22]Evidence of changing production processes — with increases in production fragmentation and specialization along the production chain — is especially strong in the trade literature given the high quality data on cross-border goods flows. Timmer et al. (2014) found that cross-border intermediate input trade — measured as the foreign value-added content of production — has rapidly increased since the early 1990s, and Bridgman (2012) pointed to a rapid expansion of manufactured parts traded over the past forty years. For other recent contributions to this long literature see for example Hummels et al. (2001) and Bems et al. (2011).

in real-time, in Appendix Section D.4 we model the impact of market beta and commodity price shocks across firms using networks estimated with our methodology. This analysis shows that financial, consumer cyclical and commodity firms are most affected by a market beta/growth shock, with the financial sector at the center of the network in close proximity to this factor.

A negative commodity price shock, on the other hand, most adversely affects energy and base materials companies. In fact, the top 10 declining equities following the shock are for firms in the oil and gas extraction sub-sector. On the other hand, United Continental Holdings — the parent company of United Airlines — would be expected to have the largest positive response. This result likely reflects the high fuel costs faced by airlines. Modeling these expected dynamics can help managers and policymakers analyze potential exposures to common shocks, and inform investors' diversification choices to confront the systemic risk they face, as in the case of a fund that is long airlines recognizing their latent commodity risk.

3 Multi-Sector DSGE Model of Firm Exposures

In this section, we provide a theoretical model as a lens through which to examine exposures to upstream, downstream and common factor shocks. Our model is an extension of the one-period multi-sector DSGE model of Baqaee (2018) to a dynamic, stochastic setting with inter-temporal assets. This extension allows us to derive equity prices from a standard Euler equation, which is shown to link firms' equity returns to macroeconomic fundamentals and production network centralities. The model has two sets of agents acting in discrete time: a unit continuum of identical households; and heterogeneous firms divided across J industries, each producing a single differentiated product in a monopolistically competitive environment. Every period, firms choose how much of their goods to sell to the household sector and to other firms as intermediate inputs.[23]

Each period proceeds in three stages. In the first stage, the households — who are the sole owners of inter-period capital — determine how to allocate their labor, and their capital holdings across renting to the firms and a technology to produce further capital for tomorrow. Each firm determines how much labor and capital to employ, the amount of other firms' output to use as intermediate inputs in its production process, and the price it will charge. In the second stage, production of goods and next period capital occur, and the productivity (v_{t+1}) and taste (β_{t+1}) shocks for $t + 1$ are realized. In the final stage, the

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[23]We keep the model here parsimonious and outline an extended version with industry total factor productivity, credit, varied market size and commodity price shocks in Online Appendix Section E.

firms pay the households their wages, return on capital, and profits as equity dividends. The households also make their consumption purchases and trade firm equities then.

3.1 Household's Problem

The representative household maximizes expected discounted utility:

$$E_0 \sum_{t=0}^{\infty} \psi_t U_t$$

where

$$U_t = \left(\sum_{j=1}^{J} \beta_{tj}^{\frac{1}{\sigma}} c_{tj}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \quad ; \quad c_{tj} = \left(\int_{M_j} c_t(j,i)^{\frac{\varphi_j-1}{\varphi_j}} di \right)^{\frac{\varphi_j}{\varphi_j-1}}.$$

U_t is the total consumption index discounted by the stochastic discount factor ψ_t, c_{tj} is the composite consumption index for industry $j = 1, 2, ..., J$, σ is the inter-industry elasticity of substitution, $c_t(j,i)$ is consumption of the output from firm i in industry j, $M_j = 1$ is the mass of firms in industry j, and φ_j is the intra-industry elasticity of substitution across varieties.[24] The households are assumed to have a base set of preferences, which are subject to random taste shocks:

$$\beta_t = \bar{\beta} + Z_t \tag{6}$$

where β_t is a vector of the β_{tj} terms.

At the beginning of each period, the household must choose how to allocate its capital holdings across investing in further capital for tomorrow and renting it to firms, K_t^r, at a market rate of r_t. The household inelastically supplies one unit of labor each period, and both the labor and capital markets are perfectly competitive, with all participants taking prices as given. Post-production, the household makes its consumption and firm equity purchases, constrained by the following budget:

$$\sum_{j=1}^{J} \int p_t(j,i) c_t(j,i) di = w_t + K_t^r r_t + \sum_{j=1}^{J} \int s_t(j,i) q_t(j,i) di - \sum_{j=1}^{J} \int s_{t+1}(j,i) [q_t(j,i) - \pi_t(j,i)] di.$$

w_t is the wage per unit of labor and $p_t(j,i)$ is the price of the good from firm i in industry j. The wage is the numeraire in the economy, with $w_t = 1$ for all periods. The $q_t(j,i)$ are the cum-dividend firm equity prices, $s_t(j,i)$ are the holdings of those equities and $\pi_t(j,i)$ are the profits repaid as dividends to the equity holders. There is a unit supply of each firm's

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[24]The standard simplifying assumption from the literature that households' and firms' elasticities are the same is chosen for mathematical tractability.

equity, with initial equal holdings across the households.

Iterating the household's first order condition for $s_{t+1}(j,i)$ over periods, along with the equity transversality condition, yields the cum-dividend equity price equation as a function of expected discounted real dividends:

$$q_t(j,i) = E_t \sum_{\tau=0}^{\infty} \frac{\psi_{t+\tau}}{\psi_t} \frac{P_{ct}}{P_{c,t+\tau}} \pi_{t+\tau}(j,i),$$ (7)

where P_{ct} is the aggregate consumption price index of the industry price indices, P_{tj}:

$$P_{ct} \equiv \left(\sum_{j=1}^{J} \beta_{tj} P_{tj}^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \quad ; \quad P_{tj} \equiv \left(\int p_t(j,i)^{1-\varphi_j} di \right)^{\frac{1}{1-\varphi_j}}.$$

3.2 Firms' Problem

Within each industry j there is a unit continuum of firms, and the firms use labor, capital, and other firms' goods as inputs to their production processes. Since the firms do not have any inter-period choice variables, they solve a series of independent problems each period seeking to maximize profits:

$$\pi_t(j,i) = p_t(j,i) \left[c_t(j,i) + D_t(j,i) \right] - \left[w_t L_t(j,i) + r_t K_t(j,i) + \sum_{l=1}^{J} \int p_t(l,n) x_t(j,i,l,n) dn \right].$$

Firm i in industry j makes its intermediate input decision for purchases of good n from industry l, $x_t(j,i,l,n)$, and agrees to pay prices for those, $p_t(l,n)$, before production occurs. $D_t(j,i) \equiv \sum_{l=1}^{J} \int x_t(l,n,j,i) dn$ is the total demand for good i from firms to use as an intermediate input. The amount of labor, $L_t(j,i)$, and capital, $K_t(j,i)$, employed by firm i are also decided upon in the first stage.

The firm's output, $y_t(j,i)$, is given by the following production function:

$$y_t(j,i) = \left[v_{tj}^{\frac{1}{\sigma}} \left(K_t(j,i)^{\gamma} L_t(j,i)^{1-\gamma} \right)^{\frac{\sigma-1}{\sigma}} + \sum_{l=1}^{J} \omega_{jl}^{\frac{1}{\sigma}} \left(\int x_t(j,i,l,n)^{\frac{\varphi_l-1}{\varphi_l}} dn \right)^{\frac{\varphi_l}{\varphi_l-1} \frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$ (8)

where γ is the production process' capital share. The v_{tj} industry productivity parameters are realized during production in the prior period, which can be thought of as firms learning or improving fabrication techniques during production that they implement the following period. Let the firm productivity parameters follow a standard AR(1) data generating

process:

$$\Delta v_t = \chi + \Xi \Delta v_{t-1} + \Delta \varepsilon_t. \tag{9}$$

χ is a vector of constants, Ξ is a matrix of autoregressive coefficients, and the $\Delta \varepsilon_t$ are random shocks. The amount that firm i sells to the households will be its final output minus the quantity it already sold to other firms, $c_t(j,i) = y_t(j,i) - D_t(j,i)$. Finally, the ω_{jl} are share parameters for the goods of industry l in production for industry j, and the $J \times J$ matrix of these entries, Ω, characterizes the real firm network of the economy — that is, the input-output structure of the firms' production processes.

3.3 Industry Centralities

Each firm in our model makes three choices each period about where to place itself in the production network: how much to consume of others' goods as intermediate inputs; how much to sell to other firms as a supplier of intermediate inputs; and how much to sell as final goods directly to the households. From these choices each firm will act both as a "consumer" of raw inputs (i.e., labor and capital) and a "supplier" of final goods, though they may do so either directly or indirectly through other firms in one or more production chains. The degrees to which firms act as consumers of inputs and suppliers of outputs through the full input-output network are captured by the two centralities that we introduce in this section. *Consumer centrality* measures the degree to which a firm consumes raw inputs itself and through others, and with that its exposure to shocks to its own and other upstream firms' productivity parameters. Likewise, *supplier centrality* measures how a firm is exposed to household demand for its own and other downstream firms' goods. This therefore represents exposure to shocks to the demand parameters for a firm and those downstream from it.

3.3.1 Consumer Centrality

Using within industry symmetry, and the ratio between firm prices and marginal costs, the industry price indices (P_{tj}) can be related to the quantity and prices of the raw capital and labor inputs through upstream input-output connections:

$$P_t^{1-\sigma} = \underbrace{\left[I_J - \mu^{1-\sigma}\Omega\right]^{-1}\mu^{1-\sigma}}_{\equiv \Psi_d} v_t \tilde{z}_t^{\sigma-1} \tilde{R}_t^{1-\sigma} \equiv \tilde{\alpha}_t \tilde{z}_t^{\sigma-1} \tilde{R}_t^{1-\sigma} \tag{10}$$

where P_t is a vector of the P_{tj} industry price indices, $\tilde{z}_t$ is the labor-capital aggregate, and $\tilde{R}_t$ is the price for this composite of raw inputs:

$$\tilde{z}_t \equiv K_t^{r\gamma} L_t^{1-\gamma} \; ; \; \tilde{R}_t \equiv r_t K_t^r + w_t L_t = \frac{1}{1-\gamma}.$$

Additionally, v_t is a vector of the productivity parameters, I_J is the $J \times J$ identity matrix, and μ is a square matrix with the industries' $\mu_j \equiv \frac{\varphi_j}{\varphi_j - 1}$ values on the diagonal. Ψ_d is a function of the firms' positions within the production network from Equation (8) and can be thought of as a markup adjusted Leontief inverse. The vector of consumer centralities for the labor-capital aggregate is defined as $\tilde{\alpha}_t \equiv \Psi_d v_t$, suggesting that a firm's direct and indirect demand for raw inputs depends on the economy's production capabilities (v_t), technology (Ω), and the elasticities of substitution. The $\tilde{\alpha}_{tj}$ consumer centrality term captures the importance of industry j as a user of raw inputs and measures its network adjusted factor use.

3.3.2 Supplier Centrality

The supplier centrality can be calculated by examining the total downstream demand for a firm's goods from other industries and consumers. The supplier centrality is determined from the following system of the stacked total demand equations:

$$(P_t^\sigma y_t)' = \beta_t' \underbrace{\left[I_J - \mu^{-\sigma}\Omega\right]^{-1}}_{\equiv \Psi_S} P_{ct}^\sigma U_t \equiv \tilde{\beta}_t' P_{ct}^\sigma U_t. \tag{11}$$

β_t is a vector of β_{tj} consumer taste weights, and y_t is a vector of industry aggregate outputs:

$$y_{tj} \equiv \left(\int y_t(j,i)^{\frac{\varphi_j-1}{\varphi_j}} di\right)^{\frac{\varphi_j}{\varphi_j-1}}.$$

The supplier centrality vector is defined as $\tilde{\beta}_t \equiv \Psi_S' \beta_t$, relating a firm's role as a supplier in the network to the consumer preferences for goods and services of itself and downstream industries. $\tilde{\beta}_{tj}$ therefore reflects the network adjusted final consumption share of firms in industry j, with all of these stacked in the $\tilde{\beta}_t$ vector.

3.4 Equity Returns, Common Factors & Network Centralities

In this section, we examine firm equity returns in the model, finding that they depend on three common factors and each firm's proximity through the inter-firm network to the source

of a productivity or demand shock. To begin, we derive the log steady-state equity price of firm i in industry j by first noting that the standard constant elasticity of substitution and monopolistic competition result holds, with firms' profits being a fixed markup $\left(\frac{1}{\varphi_j}\right)$ of their sales. Second, multiplying Equations (10) and (11) gives $P_{tj}y_{tj}$ on the left hand side and allows us to derive sales in terms of economy wide aggregates and the industry centralities. Finally, using these facts with equity pricing Equation (7) at the steady-state and taking logs gives:

$$ln(q(j,i)) = \underbrace{ln\left(\frac{1}{\varphi_j}\right)}_{\text{Markup}} + \underbrace{ln\left(\underbrace{\frac{1}{1-\psi}}_{\substack{\text{Discount} \\ \text{Factor}}} \underbrace{P_c U}_{\substack{\text{GDP /} \\ \text{Aggregate} \\ \text{Demand}}}\right)}_{\substack{\text{Broad Market} \\ \text{Beta}}} + \underbrace{ln\left(\frac{P_c}{\tilde{R}}\right)^{\sigma-1}}_{\substack{\text{Real Price} \\ \text{Level}}} + \underbrace{ln\tilde{z}^{\sigma-1}}_{\substack{\text{Raw} \\ \text{Input} \\ \text{Supply}}} + \underbrace{ln\tilde{\alpha}_j}_{\substack{\text{Consumer} \\ \text{Centrality}}} + \underbrace{ln\tilde{\beta}_j}_{\substack{\text{Supplier} \\ \text{Centrality}}}.$$

$$(12)$$

Firm equity prices depend on the markups, three common factors, and the upstream and downstream network centralities. The three common factors represent: the broad market beta, capturing the discount factor and GDP; the real price level; and the supply of raw inputs. These three common factors align with those that we found using our empirical model, which is notable given that we ran standard PCA on the daily equity returns, without applying any identifying assumptions or rotations.

The log equity returns induced by changes in these common factors, the productivities and demand parameters can be approximated by differencing the first order Taylor expansion of Equation (12) around the steady-state. Letting R_t be a vector of the firm log equity returns, ΛF_t the common factor loadings and their log changes, and R_t^I a vector of idiosyncratic firm returns, we have:

$$R_t = \Lambda F_t + R_t^I = \Lambda F_t + \underbrace{Diag\left(\frac{1}{\tilde{\alpha}}\right)\Psi_d}_{\mathcal{U}\equiv\text{Upstream Exposure}}\Delta v_t + \underbrace{Diag\left(\frac{1}{\tilde{\beta}}\right)\Psi_S'}_{\mathcal{D}\equiv\text{Downstream Exposure}}\Delta\beta_t = \Lambda F_t + \mathcal{U}\Delta v_t + \mathcal{D}\Delta\beta_t.$$

$$(13)$$

In the upstream exposure matrix, $\mathcal{U}$, each entry measures the exposure of the row sector to a productivity shock from the column sector, both directly and possibly indirectly through other sectors whose products are between theirs in a production chain. The idiosyncratic response of a firm in industry j to innovations in an upstream source industry s would be $\frac{\iota_j'\Psi_d\iota_s}{\tilde{\alpha}_j}\Delta v_s = \mathcal{U}_{js}\Delta v_s$, where ι_j is a selection vector with a one in the jth position and zeroes elsewhere.

The downstream exposure matrix, $\mathcal{D}$, provides exposures to demand shocks through the network. The supplier centrality quantifies the intensity with which the household consumes

from an industry, both directly and indirectly through its downstream sales. The downstream exposure matrix captures the potential for propagation of taste shocks for downstream goods to each industry as the ratio of its centrality to downstream industries' relative to its total downstream exposure. The idiosyncratic return from a taste shock to a downstream industry s is $\frac{\iota_j' \Psi_S' \iota_s}{\bar{\beta}_j} \Delta\beta_s = \mathcal{D}_{js} \Delta\beta_s$.

Further, rearranging the idiosyncratic return portion of Equation (13), and assuming ΔZ_t and $\Delta\varepsilon_t$ from Equations (6) and (9), respectively, are vectors of mean zero i.i.d. random shocks, yields:

$$R_t^I = \rho_0 + \rho R_{t-1}^I + \mathcal{U}\Delta\varepsilon_t + \mathcal{D}\Delta Z_t + \zeta_t$$
$$\implies R_t^I = \rho_0 + \rho R_{t-1}^I + \epsilon_t \tag{14}$$

where ρ_0 is a constant vector of firm fixed effects, ρ is an $N \times N$ influence matrix, and ζ_t is a residual orthogonal to the two shocks. This formula matches Equation (2), where the R_t^I idiosyncratic returns follow a VAR(1) process. Further, this indicates that the upstream and downstream exposures are embedded in the ϵ_t residuals of this system. We utilize the results of this section in the next one to empirically evaluate upstream and downstream exposures in our estimated U.S. equity return based networks.

4 U.S. Upstream & Downstream Exposures

Our equity based networks quantify how firms' equity returns comove as a consequence of shocks to the economy. Using these networks we next evaluate the significance of upstream/supplier and downstream/demand side macroeconomic exposures for a large number of firms over the past three decades. To this end, we aggregate the equity response networks at the sector level and compare them with U.S. input-output table based networks outlined in Section 4.1, including the upstream and downstream exposure matrices from our DSGE model.

In this analysis, we study both our R to R (total return) and the inter-firm portion of our R^I & Factors to R & Factors (idiosyncratic return) equity networks, estimated over various time frames. We will refer to the latter as the R^I to R networks for short. The edge weights are summed at the BEA sector level to create response matrices at the level of the U.S. input-output tables.[25] In these matrices, the source of an edge is in the column and the

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destination in the row. Additionally, as an alternate aggregation we scale the networks so that the edges into each sector sum to 100%, excluding self-loops from firms to themselves. This alternative accounting is meant to produce edge weight shares to parallel the sectoral output normalization that we apply in some of our input-output network comparisons below.

We compare networks using the element-by-element correlations between their adjacency matrices. To calculate the significance of these correlations we utilize a network correlation distribution bootstrapping method from the machine learning literature, the Quadratic Assignment Procedure (QAP). This algorithm creates a distribution of network correlations by randomly reassigning the order of the nodes in one of the networks and moving the adjacency matrix entries accordingly. The new order will be the same for the rows and columns in the bootstrapped network, creating a hypothetical network with similar structure to that observed. We repeat this procedure ten thousand times for each network comparison that we do, saving the set of correlations and then calculating the p-value for the actual network correlation based on the simulated distribution. Doing this rather than using the standard procedure to calculate the significance of correlations between two data series is particularly important given the structure of the networks we study, where the weights from sectors to themselves are all expected to be substantial (i.e., the correlations are ex-ante expected to be sizable and positive due to the large diagonal entries). The QAP bootstrapping procedure will retain this property of the networks, and provide correlations for distributions of networks with this trait. The QAP procedure also implicitly accounts for the sparsity of the network, the scale (i.e., range of edge weight magnitudes), and whether there are star nodes with outsized weights in or out of them from many other agents.

4.1 Measures of Sectoral Upstream & Downstream Exposures

We estimate the upstream and downstream exposure matrices, $\mathcal{U}$ and $\mathcal{D}$ from Equation (13), from our theoretical model using U.S. input-output data from the BEA for 1997 through 2015. The BEA refers to the use tables as a "recipe" matrix because they show the inputs necessary to produce the output of each sector. These tables provide the expenditures on commodities from each sector by households and firms as intermediate inputs, valued in dollars. The commodities or products used are in the rows, and the consumer is in the column. The entries in each row sum to the output of that commodity. The columns also contain the components of value added — employee compensation, taxes, and profits. Therefore, including these values, the sum of the entries in a column equal that sector's

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networks and the input-output networks skewed our results. This change made little difference for the years where we have both use table types.

output.[26]

We generate six types of networks from the U.S. input-output use tables at the BEA sector level. The matrices are arranged so that the supplier of an input is in the column and the user is in the row, as this best aligns with the orientation of our equity based networks.[27] First, we take the raw input-output use tables, which we call the "Raw IO" network. Second, we use those tables with the input expenditures divided by the total output of the using sector to get a measure of the share of value derived from the other sectors as intermediate inputs. We label these networks as "IO Output Normalized." The third network is the standard Leontief inverse calculated using the IO Output Normalized matrices. The fourth and fifth networks are the upstream and downstream exposure matrices, $\mathcal{U}$ and $\mathcal{D}$. We match the input-output data to our model parameters by assuming a Cobb-Douglas form ($\sigma = 1$). This assumption is standard in the literature because otherwise we run into the issue that the industry-level prices cannot be cleanly separated from their output quantities in the data, while with a Cobb-Douglas structure, only the expenditures matter, not the breakdown between prices and quantities. If we take the parameters as estimated assuming a Cobb-Douglas form and then vary σ, we find that our results are minimally affected by changes to this parameter, so beyond helping us take the model to the data, this assumption does not appear to be a crucial one. Finally, to match Equation (13) we add the two exposures to see how the combined centralities compare in our "Upstream + Downstream Exposure" networks.

4.2   Long-Run Upstream & Downstream Exposures

The results of comparing these networks with the equity based ones over our full sample period can be seen in two ways in the panels of Table 1. The top panel shows the results of comparing our 1989-2017 balanced panel network with the input-output data from the middle of that period in 2001. The bottom panel has the averages for broader rolling member 10-year networks ending in 1998 through 2017 against the input-output year with data available nearest their midpoints. The stars in Panel A indicate statistical significance as calculated using the QAP procedure. The results are similar for the two methods of looking across the sample period, so we focus on Panel A.

The first column contains the correlations of the equity networks with the Raw IO tables. We provide these correlations to demonstrate that the underlying input-output tables are positively related to our equity networks; however, we do not focus on these as the correla-

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[26]We exclude the "Other services, except government" and "Government" sectors as the former does not cleanly match the sectors of the firms we analyze, and we wish to focus on the private sector.

[27]This is not a critical assumption due to the large degree of symmetry in both the input-output and equity return based networks we examine. Our correlation results are changed by less than 0.02 if we transpose the equity based networks.

tions are potentially spurious being driven by a handful of diagonal entries due to the skewed nature of the Raw IO tables. For example, the R^I to R network for 1989-2017 has a correlation of 0.89 with the 2001 Raw IO network; however, if the manufacturing self usage entry on the diagonal is dropped then this declines to 0.55. If that entry and the finance self usage entry are dropped, the correlation declines to 0.19. If no diagonal entries are included, the correlation increases to 0.32.[28] Nevertheless, we provide these as a baseline to demonstrate that the equity based networks are reflective of the unadulterated input-output data.

The next column contains the IO Output Normalized networks against our equity networks. These values are economically and statistically significant, indicating a strong correlation between both types of equity networks and the normalized input-output tables. In this case there is no clear frontrunner between the two equity network types; however, when taking into account the full, network-adjusted relationships inclusive of pass-through via the Leontief inverse in the next column, the idiosyncratic networks come out ahead. The correlation of the R to R network with the Leontief inverse is 0.39, while that for the idiosyncratic network is 56% higher at 0.61. The correlations for the upstream exposures exhibit a similar pattern: the R to R network has a correlation of 0.45, while that for the idiosyncratic network is 38% higher at 0.62. These correlations are statistically significant, and demonstrate that the equity based networks — particularly the idiosyncratic ones — strongly reflect the underlying macroeconomic relationships between sectors.[29] That the idiosyncratic return networks have higher correlations than the total return ones accords with Equation (14) from the DSGE model, where the common factors are first removed from equity returns before deriving the VAR(1) connecting them with the upstream and downstream centralities.

For the downstream exposures, the idiosyncratic networks also yield higher results than the R to R networks; however, none of these are statistically significant. The total return network correlation with the downstream exposures is only 0.04 (0.06 for the normalized network), while that for the idiosyncratic network is 0.21 (0.30 for the normalized network).

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[28]Note that this issue is far greater for the Raw IO matrices than the other networks given the large concentration of values along the diagonals, and we do not believe that it materially affects our main results. If we remove the diagonal elements from the analysis for the unnormalized equity networks then the correlations decline to about a third of their values for the Raw IO and IO Output Normalized networks. However, the upstream exposure and Leontief inverse measures decline by only about half, with the upstream exposure correlations remaining greater than those for the Leontief inverse. The downstream exposure correlations continue to be positive but minimal. Excluding the diagonal elements for the normalized equity networks sees the Raw IO correlations marginally decrease, and the IO Output Normalized correlations slightly increase. The upstream exposure and Leontief inverse correlations increase to the upper 0.6's, while downstream exposure correlations double, remaining positive but low. All of these results agree with our main findings.

[29]This similarity in results for the upstream exposure and Leontief inverse is not surprising as they are strongly related. In fact, the two are the same under our Cobb-Douglas assumption if the μ_j values are all one, which occurs in the case of perfect competition ($\lim_{\varphi_j \to \infty} \forall j$).

In the final column, we study the Upstream + Downstream Exposures to examine whether including both terms unearths a further association with the equity responses. In these cases, it appears that the poor fit of the downstream exposures dominates the better fit of the upstream exposures, with correlations very similar to those for the downstream exposures and not statistically significant.

Together, these results indicate that shocks from upstream suppliers of which a firm is either directly or indirectly a customer, matter more for short-term equity responses as measured by our networks than shocks from downstream in the production process. This is suggestive of low short-term elasticities of substitution across inputs with more flexibility on the downstream, customer side. It may also reflect greater heterogeneity of customers than suppliers, so that downstream sourced shocks are implicitly insured against and netted out. Additionally, publicly traded equities might not reflect macroeconomic demand shocks well.

These relationships not only hold for the changes in market expectations captured in the one-day equity return based networks, but also for networks calculated using longer return periods, suggesting that the choice of data frequency used for the empirical analysis does not drive the key results in the paper. Appendix Table D.2 shows the same network correlations for monthly equity return based networks against the input-output networks in order to capture lower frequency comovements in equity returns. The results are similar, reflecting persistence in the importance of shocks to upstream firms over those to downstream firms.

Finally, removing the common factors is an important step to uncover the inter-sectoral connections. As further robustness, we repeat our analysis for networks where the edge weights are the bilateral daily equity return correlations between each firm pair, with the results in Table D.1. These simple networks are highly correlated with the R to R ones from our econometric model (0.95-0.99 correlations when aggregated at the BEA sector level); hence, they have similar correlations with the IO based networks, reinforcing the need to account for common factors to properly identify the network structure.

4.3 Evolution of Upstream & Downstream Exposures

In this section, we examine changes in how our equity based networks compare to the input-output derived networks over time. Table 2 lists the correlations between rolling ten year equity return networks ending in 1998 through 2017 and those derived from the input-ouput tables closest to their midpoints. These networks include the same 524 firms over time to remove the impact of a changing sample on the correlations. The top panel contains these correlations for the total return and idiosyncratic return networks. The bottom panel contains the amount by which the idiosyncratic return correlations exceed those of the R to

R networks on the left, and the percentage improvement in fit for the model based Leontief inverse and upstream exposure networks over the basic IO Output Normalized networks on the right.

There are a few key takeaways from Panel A worth emphasizing. First, the correlations for the idiosyncratic return networks are consistently higher than for the total return networks, across all of the input-output table based network transformations. This again suggests improved fit for the idiosyncratic return networks with the underlying real economic relationships, and the need to account for common factors. Second, the idiosyncratic return network correlations are relatively static over time, while those for the total return networks decline. This is likely a result of the common factors becoming more influential over our period as shown in Figure 2. For example, the improvement of the idiosyncratic network over the total return one in terms of correlation hovered near 0.1 for upstream exposure at the start of the sample, doubling to around 0.2 in the last five years. These two facts support the use of our procedure to measure inter-firm networks over those previously proposed in the literature that produce results similar to the R to R networks, and they indicate that these network relationships have been remarkably consistent over time.

Finally, the improvement of the correlations for the idiosyncratic network with the model based Leontief inverse and upstream exposure networks over the simple IO Output Normalized networks emphasizes the importance of considering propagation through the full network and not just to immediate neighbors. The right section of Panel B shows the percentage increase in the correlations for these two model-based networks over the IO Output Normalized one. These are especially large towards the end of sample, possibly reflecting production processes involving more specialization and intermediate input use. The correlations for the upstream exposures are consistently greater than those for the Leontief inverses for both equity return network types, implying a meaningful role for market competition, the demand elasticities across goods and markups in the structure of inter-firm networks.

4.4 Upstream-vs-Downstream Exposures: Intuition & Discussion

We next discuss simulations of our DSGE model to help interpret these findings and consider them in the context of the broader literature on inter-firm shock transmission. We performed simulations to a set of productivity (v_j) and taste (β_j) shocks for a network of five industries and a household sector, forming the classical "X-type" production network that is often used as an example in this literature.[30] Figure 8 Panel (a) shows Firm 2 (orange) consuming

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[30]Appendix Section F contains the results for numerous other canonical firm networks from the literature — including the examples from the Baqaee (2018) and Acemoglu et al. series of network papers — with qualitatively similar relationships.

inputs from firms 1 (black) and 4 (green), while supplying its output to firms 3 (blue) and 5 (yellow). The latter two firms ultimately sell their goods to the household (gray).

Panels (b) through (e) show firms' idiosyncratic responses to different shocks in the network, with the common components removed to uncover input-output connections in firms' equity returns, as we learn from Equation (13). In each case, the y-axis measures the change in equity prices from being in steady-state at the initial parameter levels to the new steady-state after the associated shock. The x-axis measures the relevant upstream ($\mathcal{U}$) or downstream ($\mathcal{D}$) exposure to the sector in which the shock originates, multiplied by the change in the specified parameter. For the productivity shocks, this is $\mathcal{U}_{js}\Delta v_s$, and for the taste shock it is $\mathcal{D}_{js}\Delta\beta_s$, where s is the source sector for the shock and j is the target one. These are the network adjusted use of raw inputs through sector s in the first term and the indirect sales through industry s in the second, scaled by the overall network adjusted raw input use and sales of a firm in industry j, respectively. Note that the elasticities of substitution and Ω matrix enter into the centralities, and different parameterizations would alter the quantitative changes but the qualitative features would persist. The first feature of these plots that stands out is that the idiosyncratic responses to the shocks lay along the 45-degree lines, indicating that the dynamic responses of the idiosyncratic log returns match the expectations given the corresponding consumer and supplier centralities.

The two bottom left panels simulate productivity shocks for one of the most upstream and one of the most downstream firms to compare supply network shock propagation in each extreme case. The impact of a productivity shock to Firm 1 is provided in Panel (b) (Most upstream), and one to Firm 3 is in Panel (d) (Least upstream). In each case, the firm experiencing a productivity shock, marked with an X, has the largest centrality to itself and hence the greatest idiosyncratic equity response. Since there are no firms downstream from Firm 3, the other sectors have zero upstream centrality exposures to it, hence zero idiosyncratic returns to its productivity shock in Panel (d). On the other hand, Firm 1's productivity shock affects firms 2, 3, and 5, since they are all downstream from it. Further, Firm 2 is more directly exposed to Firm 1 so it has a greater response, while firms 3 & 4 have the same upstream exposures to Firm 1 through Firm 2, so have the same equity responses. Bringing these two examples together, Panel (c) shows a productivity shock for the central Firm 2. The two upstream firms (1 & 4) have zero idiosyncratic returns, while the two downstream firms (3 & 5) have positive returns as they benefit from Firm 2's productivity improvement.

Finally, Panel (e) shows how firms 1, 2 & 4 are similarly affected by a taste shock to downstream Firm 3's good, reflecting that they have comparable downstream reliance on Firm 3. Firm 5's only relationship to Firm 3 is as a direct competitor in the final goods

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market, therefore, since it is not upstream of Firm 3, it has a zero idiosyncratic response to the taste shock.

These simulations illustrate our prior insights: the importance of removing common factors to uncover the connections between firms; and that once the common factors are removed, equity returns reflect the proximity of firms through the two exposure matrices. In addition, they support our empirical finding that upstream exposure is more important than downstream exposure when it comes to productivity shocks or supply chain disruptions.

Our findings are consistent with the significant supplier disruptions experienced in the aftermath of the 2011 Tohoku earthquake. While Carvalho et al. (2016) identified quantitatively large upstream and downstream spillovers after the quake, their empirical analysis found directed propagation from upstream to be robust to parametrization, with the positive/negative propagation of shocks from downstream firms dependent on the elasticity parameters instead. Importantly, they assessed that the transmission of shocks over input-output linkages accounted for a 1.2% decline in Japanese GDP in the year following the earthquake. Yet, these effects were not localized to the immediate area. Boehm et al. (2019) found that Japanese affiliates abroad, reliant on imports from the affected zones, saw output drop about one for one with imports of intermediate goods from Japan in the wake of the disaster, suggesting extremely low elasticities of substitution for inputs in the short term. Similarly, Jones (2011) studied production linkages and intermediate input use, finding that problems along a production chain can sharply reduce output under input complementarity.

The prominence of upstream versus downstream shock transmission is important in the context of the broader theoretical literature following Long and Plosser (1983) examining multi-sector economies. This literature differs on which of these two channels are operational. For example, Acemoglu et al. (2012) concluded that under Cobb-Douglas intermediate input aggregation, an industry's impact on the aggregate economy depends only on its role as a supplier of inputs through propagation from upstream, and not as a consumer. Further, Section 4 of Baqaee (2018) showed these results hold under a more general set of models than those with Cobb-Douglas intermediate input aggregation. The models of Johnson (2014), Barrot and Sauvagnat (2016), and Baqaee and Farhi (2019) likewise have downstream but not upstream shock propagation. Our empirical findings and intuition from the DSGE simulations provide support for the many papers that rely on Cobb-Douglas input aggregation or other modeling simplifications that minimize the downstream exposure channel.[31] Alternately, Baqaee (2018) and Luo (2020) achieved both downstream and upstream shock propagation by including firm entry-and-exit and a credit channel, respectively.

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[31]There is no upstream propagation of shocks under Cobb-Douglas because the price and quantity effects cancel out (Shea, 2002).

Our results on upstream versus downstream exposures and the importance of common factors can be placed in several other literatures. The relative significance of the upstream exposures could indicate a low short-term elasticity of substitution across inputs, which would match work at the country level. Examining trade across 30 countries, Ng (2010) found that bilateral trade in complements/upstream intermediate goods contributes to cross-country business-cycle comovement, while trade in substitutes/downstream final goods reduces it, with a net positive impact of trade on comovement. Additionally, Burstein et al. (2008) and Johnson (2014) identified that low elasticities of substitution between inputs is key to explaining the degree of synchronization in international business cycles, with Miranda-Pinto (2021) finding that the elasticity of substitution between intermediates and labor in particular can have a marked impact on GDP volatility. Intermediate input complementarity is shown to synchronize cross-country business cycles in Backus et al. (1994) and Heathcote and Perri (2002). Relatedly, Loayza et al. (2001) analyzed output fluctuations in emerging economies, finding a central role for sectoral interdependence. Barrot and Sauvagnat (2016) found that production elasticities near zero best match real world shock amplifications in a calibrated network model, and Atalay (2017) found that strong input complementarities play an important role in industry level shock transmission.

Acemoglu et al. (2016a) used input-output linkages along with geographic connections of industries to empirically investigate the propagation of four different types of shocks through the U.S. input-output network. They found that in the case of demand-side shocks — China import shocks and federal government spending shocks — upstream propagation is substantially stronger than downstream effects; whereas in the case of supply-side shocks — TFP and foreign patenting shocks — downstream propagation is stronger.

5   Conclusion

There has been a documented increase in the comovement of firm equity returns over the past few decades, particularly during crisis events. This is true both within and across industries. Given the increased risks associated with this environment, we evaluate the relative importance of upstream, downstream and common factor exposures that may lead to these comovements. Specifically, we examine how shocks are transmitted between publicly traded firms relative to the predictions of input-output based networks.

We apply a machine learning VAR estimation method to study the inter-firm response network accounting for common factors using daily U.S. equity returns from 1989 to 2017. Our empirical work reveals that these common factors — especially the market beta — are important stock market drivers and have become more important over time, explaining

11.7% of equity return variation during the 1990s versus 35.0% post-2007. As the movements in equity prices attributable to common factors can be sizable, their influence may mask the underlying connections between firms, with many prior methods in the network estimation literature conflating the two.

The empirical literature has yet to reconcile theoretical models' discrepancies as to the importance of upstream and downstream propagation, so we compare these firm level equity based networks with macroeconomic upstream and downstream exposures through the lens of a DSGE model. We find upstream exposure (shocks to a firm's direct and indirect suppliers) to be more significant than downstream exposure (shocks to its direct and indirect customers). These results persist if we extend the analysis beyond the short-term daily returns and analyze monthly equity returns instead. Our findings have meaningful implications for understanding the reactions of firms relative to one another and that it is important to consider microeconomic linkages not captured in input-output tables. Further, these networks potentially allow for the real-time monitoring of economic developments at a frequency and disaggregated level that would otherwise be difficult to study.

Our work yields several insights that warrant further investigation. First, the common factors that drive equity returns have increased in influence and finding their root causes would have implications for portfolio diversification, firm management and public policy strategies. Our results can also indicate that publicly traded equities do not do a good job of capturing demand side macroeconomic shocks, suggesting a possible contributor to the divergences observed during periods such as the COVID-19 downturn. Delving into the precise channels through which firms are connected—e.g., intermediate goods, services or credit—to understand contagion would also be fruitful. Finally, a greater importance of upstream firms relative to downstream ones could be applied to businesses hedging risks and the design of trade and international economic policies.

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Tables and Figures

Table 1: Firm Equity vs. Input-Output Based Networks

Panel A: 1989-2017 Network

Equity Network Type	Raw IO	IO Output Normalized	Leontief Inverse	Upstream Exposure	Downstream Exposure	Upstream + Downstream Exposure
R to R	0.83***	0.49**	0.39**	0.45***	0.04	0.07
R^I to R	0.89***	0.54**	0.61**	0.62***	0.21	0.24
R to R, No Self-Loops and Scaled by Destination	0.39***	0.55***	0.27***	0.38***	0.06	0.08
R^I to R, No Self-Loops and Scaled by Destination	0.40*	0.51**	0.53**	0.59**	0.30	0.32

Panel B: Average Across 10-Year Networks with Maximum Number of Firms Ending 1998-2017

| Equity Network Type	Raw IO	IO Output Normalized	Leontief Inverse	Upstream Exposure	Downstream Exposure	Upstream + Downstream Exposure
R to R	0.78	0.47	0.38	0.44	0.04	0.06
R^I to R	0.88	0.56	0.59	0.61	0.19	0.22
R to R, No Self-Loops and Scaled by Destination	0.38	0.56	0.28	0.40	0.07	0.09
R^I to R, No Self-Loops and Scaled by Destination	0.41	0.54	0.51	0.58	0.31	0.34

Note: The firms in each equity network type are those that are continuously traded throughout that period, with edges estimated from GFEVCs, where "R to R" does not remove the three common factors while "R^I to R" does. Each column is for a network type estimated from U.S. input-output tables closest to the mid-point of the period: I/O networks without normalization (Raw IO); input expenditures divided by total output (IO Output Normalized); standard Leontief inverse using the IO Output Normalized (Leontief Inverse); $\mathcal{U}$ (Upstream) and $\mathcal{D}$ (Downstream) Exposure matrices from the theoretical model with $\sigma = 1$; and the two exposures together (Upstream + Downstream Exposures). In the top panel, statistical significance is estimated using the QAP procedure with 10,000 iterations *** p<0.01, ** p<0.05, * p<0.1.

Table 2: Firm Equity vs. Input-Output Based Networks Over Time

Panel A: Firm Equity Network Correlations with BEA Sector Level Input-Output Based Networks

EQ Network Period	IO Year	R to R Network Correlations					R^I to R Network Correlations				
		IO Output Normalized	Leontief Inverse	Upstream Exposure	Downstream Exposure	Upstream + Downstream Exposure	IO Output Normalized	Leontief Inverse	Upstream Exposure	Downstream Exposure	Upstream + Downstream Exposure
1989-1998	1997	0.55**	0.46**	0.50***	0.08	0.11	0.58***	0.60**	0.61***	0.20	0.23
1990-1999	1997	0.55**	0.47**	0.51***	0.10	0.13	0.57***	0.59**	0.60***	0.19	0.22
1991-2000	1997	0.55**	0.48**	0.52***	0.11	0.14	0.57***	0.59**	0.60***	0.19	0.22
1992-2001	1997	0.55**	0.47**	0.51***	0.11	0.14	0.57***	0.59**	0.60***	0.20	0.23
1993-2002	1998	0.54**	0.45**	0.49***	0.09	0.11	0.57***	0.59**	0.61***	0.20	0.22
1994-2003	1999	0.53**	0.43**	0.47***	0.07	0.10	0.56***	0.60**	0.61***	0.19	0.22
1995-2004	2000	0.52**	0.43**	0.47***	0.07	0.09	0.55***	0.59**	0.60***	0.19	0.22
1996-2005	2001	0.50**	0.42**	0.45***	0.07	0.09	0.54***	0.59**	0.60***	0.19	0.22
1997-2006	2002	0.51**	0.41**	0.45***	0.07	0.09	0.53***	0.59**	0.60***	0.20	0.22
1998-2007	2003	0.50**	0.41**	0.45***	0.07	0.09	0.54***	0.60**	0.62***	0.21	0.24
1999-2008	2004	0.49**	0.40**	0.45***	0.05	0.08	0.53***	0.61**	0.63***	0.21	0.24
2000-2009	2005	0.48**	0.39**	0.46***	0.03	0.06	0.53***	0.61**	0.63***	0.21	0.24
2001-2010	2006	0.48**	0.38**	0.45***	0.03	0.05	0.51***	0.61**	0.63***	0.22	0.25
2002-2011	2007	0.47**	0.38**	0.45***	0.02	0.05	0.51***	0.61**	0.63***	0.22	0.26
2003-2012	2008	0.45**	0.37**	0.44***	0.03	0.06	0.51***	0.60**	0.63***	0.22	0.26
2004-2013	2009	0.44**	0.35**	0.41***	0.03	0.05	0.50***	0.60**	0.62***	0.23	0.28
2005-2014	2010	0.45**	0.35**	0.42***	0.02	0.05	0.50***	0.61**	0.63***	0.25	0.32
2006-2015	2011	0.46**	0.36**	0.43***	0.03	0.06	0.50***	0.61**	0.63**	0.29	0.32
2007-2016	2012	0.45**	0.36**	0.44***	0.03	0.05	0.50***	0.61**	0.63**	0.30	0.32
2008-2017	2013	0.46**	0.36**	0.44***	0.03	0.06	0.50***	0.61**	0.63**	0.30	0.33
Average		0.50	0.41	0.46	0.06	0.08	0.54	0.60	0.62	0.22	0.25
Std. Dev.		0.04	0.04	0.03	0.03	0.03	0.03	0.01	0.01	0.04	0.04

Panel B: Correlation Comparisons

| EQ Network Period	IO Year	R^I Minus R Correlations					% Improvement Over IO Output Normalized for R^I	
		IO Output Normalized	Leontief Inverse	Upstream Exposure	Downstream Exposure	Upstream + Downstream Exposure	Leontief Inverse	Upstream Exposure
1989-1998	1997	0.03	0.14	0.11	0.12	0.12	3%	5%
1990-1999	1997	0.02	0.12	0.09	0.09	0.09	4%	5%
1991-2000	1997	0.02	0.11	0.08	0.08	0.08	4%	5%
1992-2001	1997	0.02	0.12	0.09	0.09	0.09	4%	5%
1993-2002	1998	0.03	0.14	0.12	0.11	0.11	4%	7%
1994-2003	1999	0.03	0.17	0.14	0.12	0.12	7%	9%
1995-2004	2000	0.03	0.16	0.13	0.12	0.13	7%	9%
1996-2005	2001	0.04	0.17	0.15	0.12	0.13	9%	11%
1997-2006	2002	0.03	0.18	0.15	0.13	0.13	9%	11%
1998-2007	2003	0.03	0.18	0.15	0.14	0.15	11%	13%
1999-2008	2004	0.05	0.20	0.17	0.17	0.17	11%	15%
2000-2009	2005	0.05	0.22	0.17	0.18	0.18	15%	19%
2001-2010	2006	0.05	0.23	0.18	0.18	0.19	15%	19%
2002-2011	2007	0.06	0.23	0.18	0.20	0.20	15%	19%
2003-2012	2008	0.06	0.24	0.19	0.19	0.20	19%	19%
2004-2013	2009	0.07	0.25	0.21	0.20	0.21	20%	24%
2005-2014	2010	0.05	0.25	0.21	0.23	0.23	18%	22%
2006-2015	2011	0.04	0.25	0.20	0.26	0.26	20%	22%
2007-2016	2012	0.05	0.25	0.19	0.27	0.27	22%	26%
2008-2017	2013	0.04	0.25	0.19	0.27	0.27	22%	26%
Average		0.04	0.19	0.16	0.16	0.17	12%	15%
Std. Dev.		0.01	0.05	0.04	0.06	0.06	7%	8%

Note: All networks are estimated as outlined in Table 1 notes, but with 10-year rolling networks of the same 524 firms continuously traded over 1989-2017 instead. Panel A displays correlations and statistical significance across different network types over time. Panel B displays the amount by which the "R^I to R" correlations exceed those of the "R to R" networks on the left, and the percentage improvement in fit for the model based "Leontief Inverse" and "Upstream Exposure" networks over the basic "IO Output Normalized" networks on the right.

Figure 1: Spring Plots of Simulated Networks
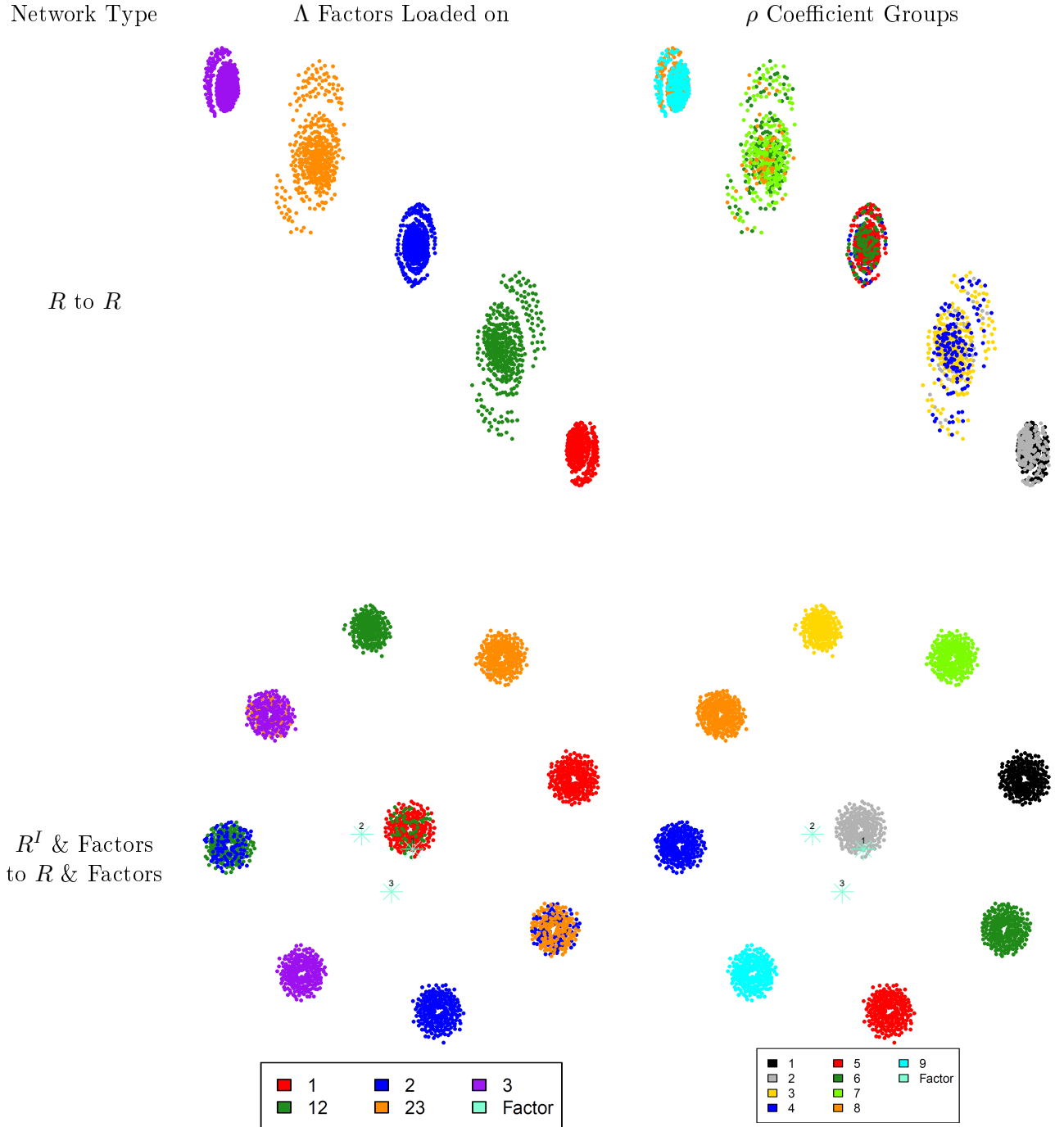
Note: Each row shows one of our network estimation methods for the simulated data. "R to R" does not remove common factors; and "R^I & Factors to R & Factors" separates common factors but keeps them in the network as nodes themselves. In both cases, each dot is a panel member (colored by factor loadings on the left and coefficient groups on the right), and the proximity of nodes to one another depends on how connected observations are. See Section 1.4 and Appendix Section B for details.

Figure 2: Variance Share Explained by Top 3 Factors, Rolling 10-Year Samples
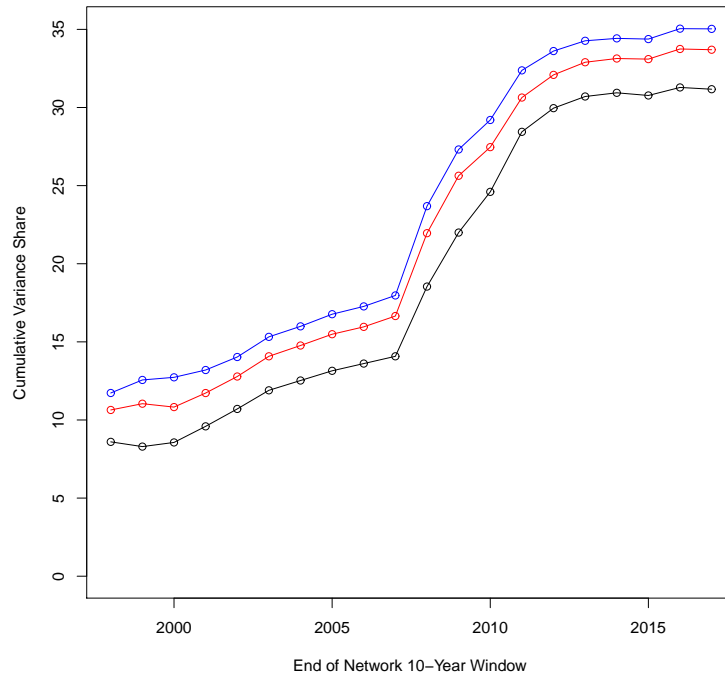
Note: Factor variance shares for rolling 10-year samples with all firms continuously traded within each time period, with factors extracted by principal component analysis on the variance-covariance matrix of the daily log equity returns. The first factor is shown in black, the second one in red, and the third one in blue.

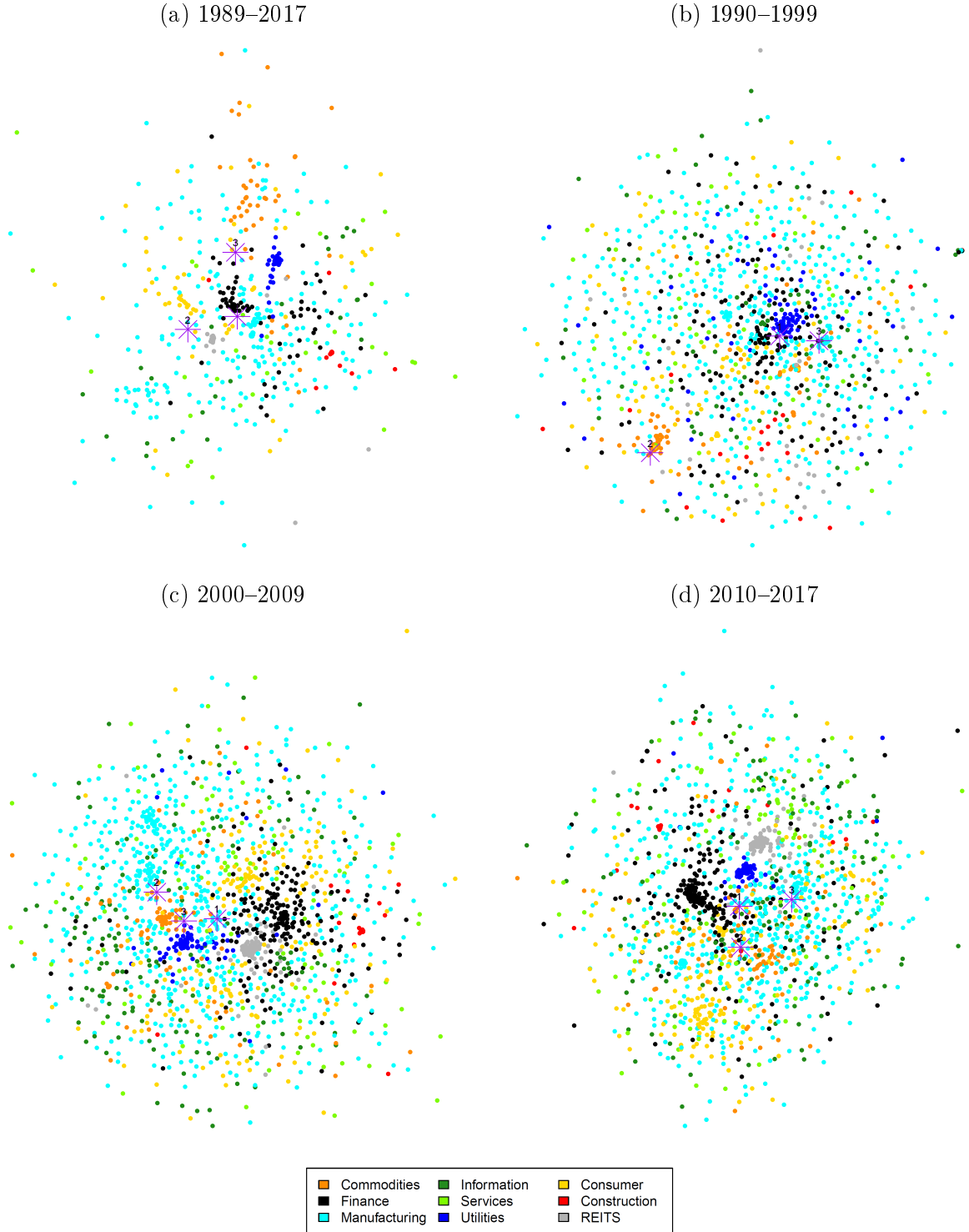Figure 3: First Factor, Equity Markets, & Growth of the U.S. Economy (1989-2017)
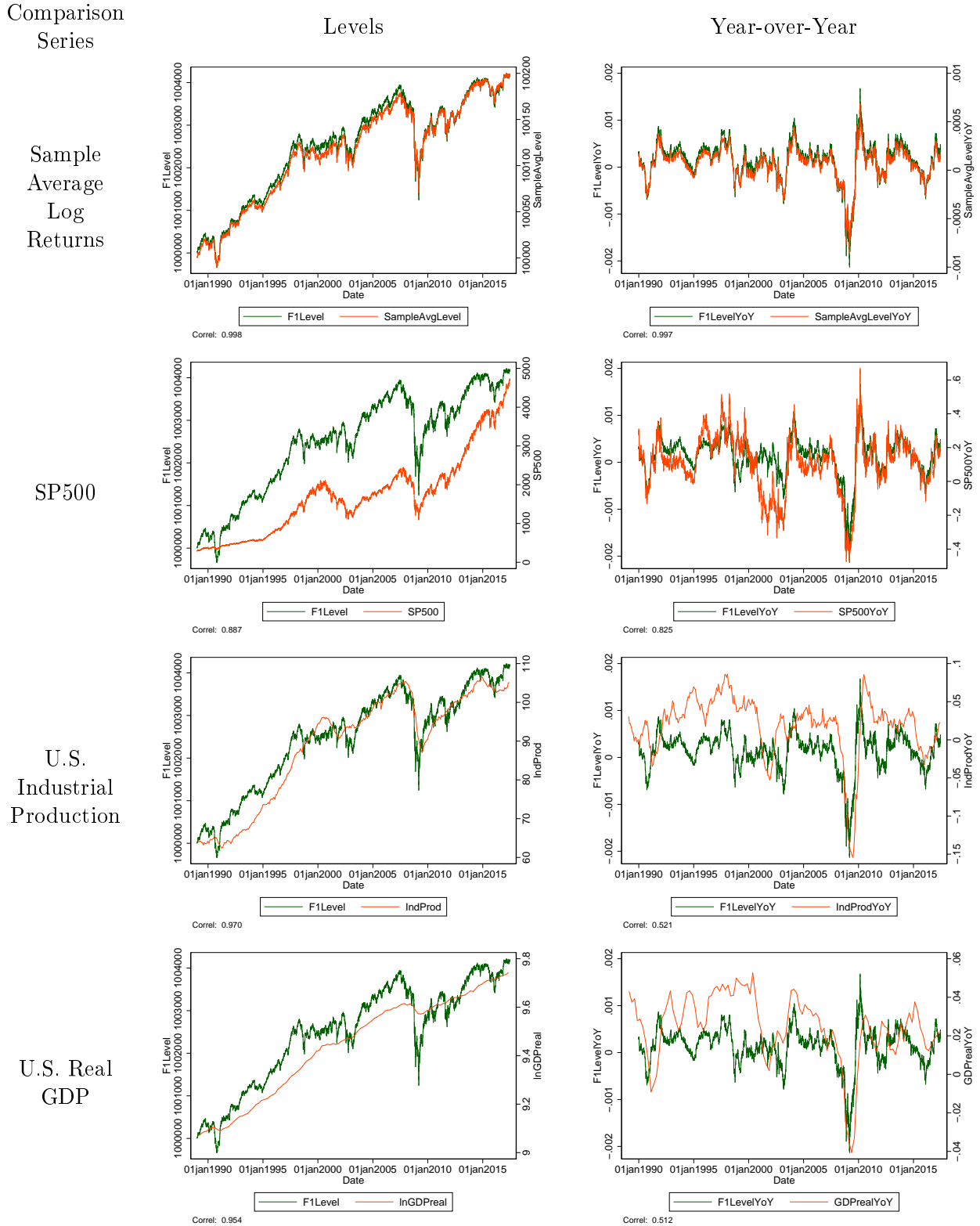
Note: "F1Level" is the cumulative sum of the first factor extracted by principal component analysis on the covariance matrix of U.S. daily log equity returns for the portion of our sample continuously traded from 1989 through 2017 (T = 7,424 and N = 524).

Figure 4: Second Factor and U.S. Prices, Year-over-Year Plots (1989-2017)
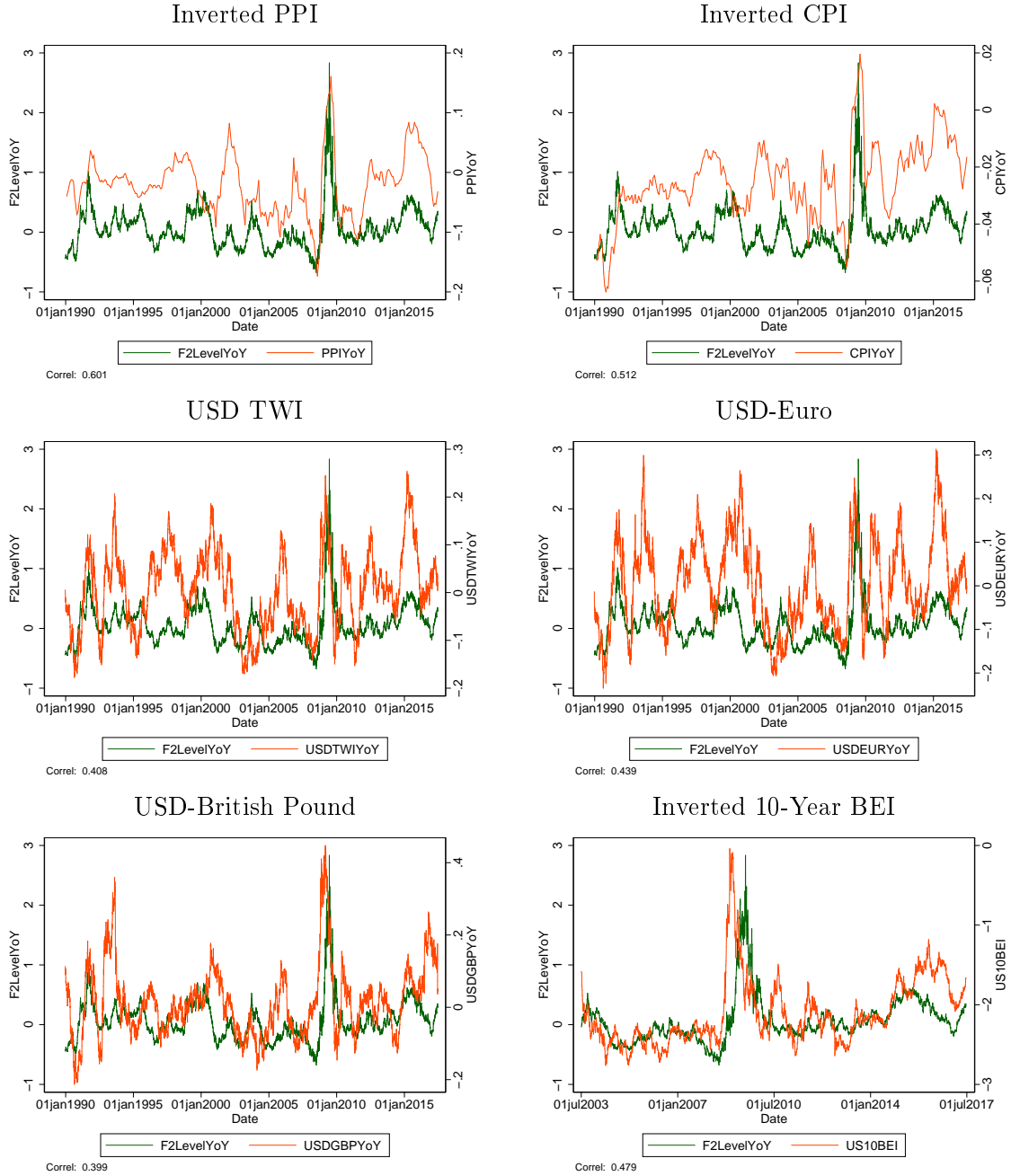
Note: "F2Level" is the cumulative sum of the second factor extracted by principal component analysis on the covariance matrix of U.S. daily log equity returns for the portion of our sample continuously traded from 1989 through 2017 (T = 7,424 and N = 524). The breakeven inflation, PPI and CPI return series are negated to match the direction of the factor series.

Figure 5: Third Factor and Commodities, Year-over-Year Plots (1989-2017)

Brent Crude

Goldman Sachs Commodity Index
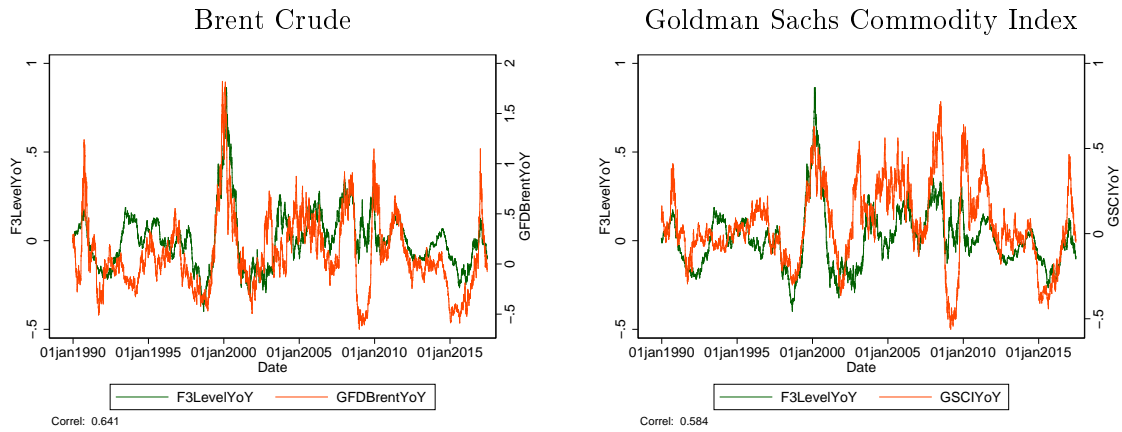
Note: "F3Level" is the cumulative sum of the third factor extracted by principal component analysis on the covariance matrix of U.S. daily log equity returns for the portion of our sample continuously traded from 1989 through 2017 (T = 7,424 and N = 524).

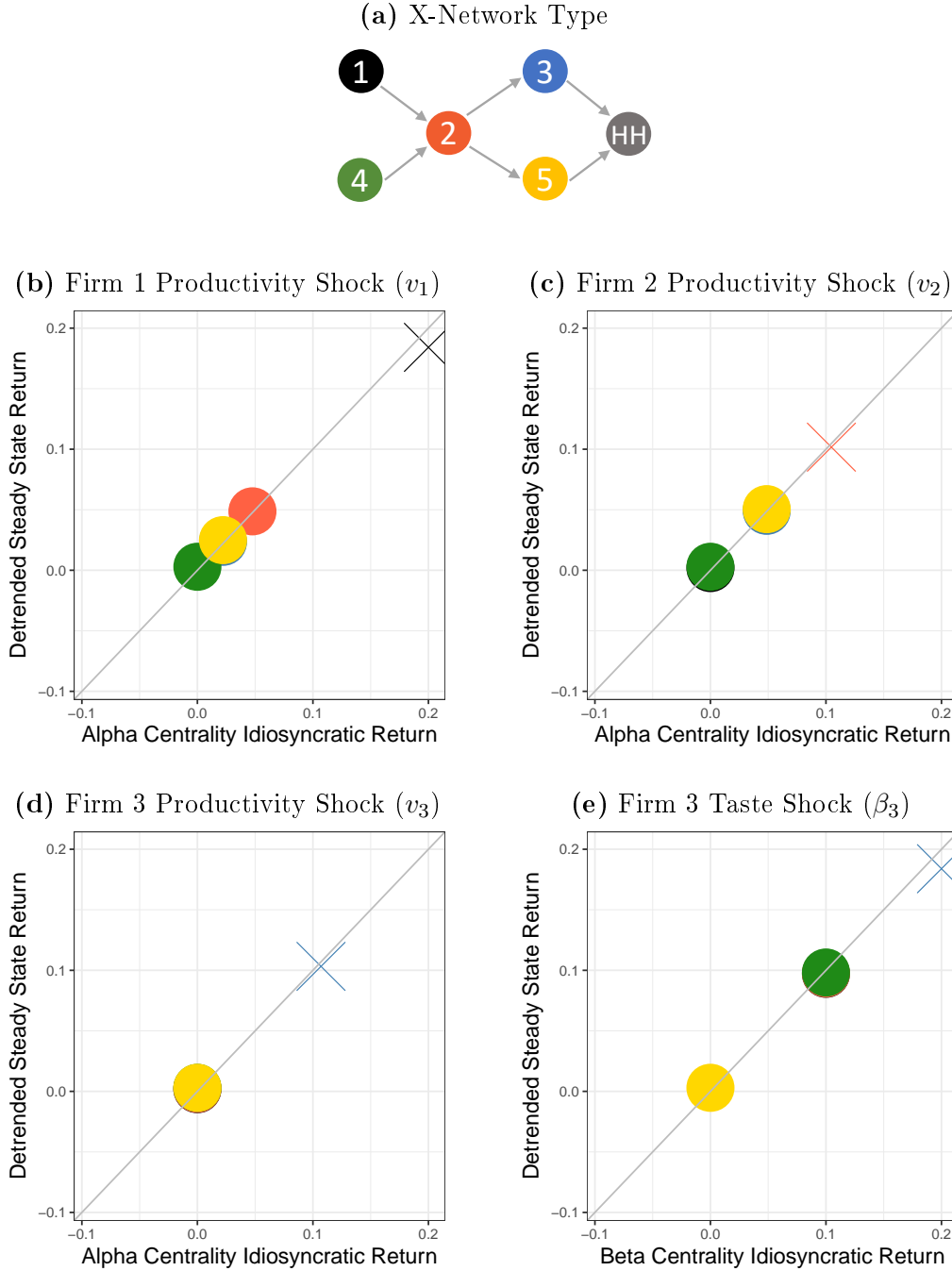Figure 6: R to R U.S. Firm Network Spring Plots
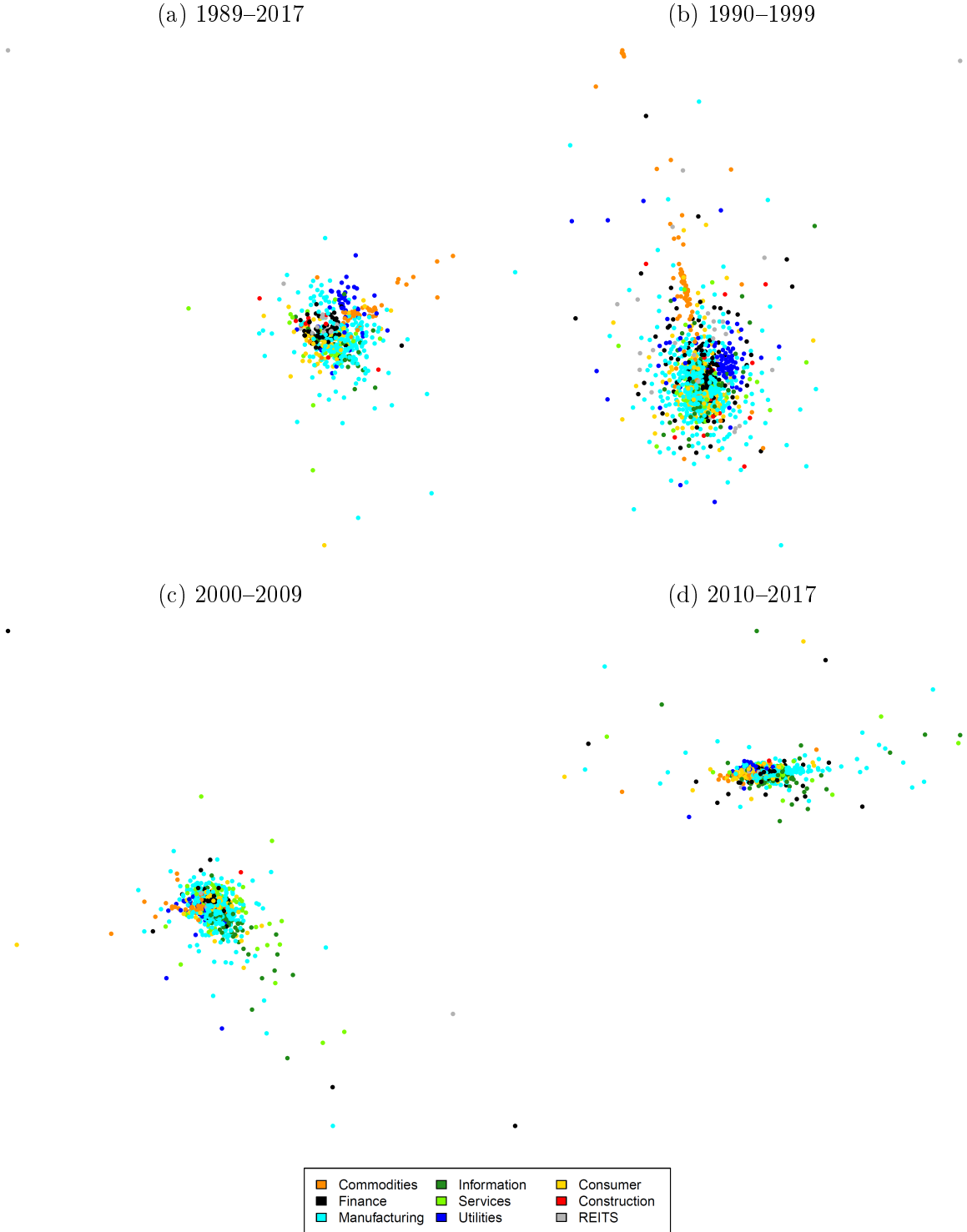
(a) 1989–2017

(b) 1990–1999

(c) 2000–2009

(d) 2010–2017

■ Commodities	■ Information	■ Consumer
■ Finance	■ Services	■ Construction
■ Manufacturing	■ Utilities	■ REITS

Note: Networks of U.S. daily log equity returns for samples of firms whose equities are continuously traded within each period. Each dot represents a firm colored by its BEA sector, network edges are calculated using GFEVcs without removing the common factors, and the proximity of dots to one another depends on how connected firms are.

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Figure 7: R^I & Factors to R & Factors U.S. Firm Network Spring Plots

(a) 1989–2017

(b) 1990–1999

(c) 2000–2009

(d) 2010–2017

■ Commodities		■ Information		■ Consumer	
■ Finance		■ Services		■ Construction	
■ Manufacturing		■ Utilities		■ REITS	

Note: Networks of U.S. daily log equity returns for samples of firms whose equities are continuously traded within each period. Each dot represents a firm colored by its BEA sector, network edges are calculated using GFEVcs after removing three common factors, and the proximity of dots to one another depends on how connected firms are. Factors are denoted by a purple star with the corresponding number in black.

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Figure 8: Simulated Idiosyncratic Equity Responses to Shocks in DSGE Model

(a) X-Network Type

(b) Firm 1 Productivity Shock (v_1)

(c) Firm 2 Productivity Shock (v_2)

(d) Firm 3 Productivity Shock (v_3)

(e) Firm 3 Taste Shock (β_3)

Note: Panel (a) shows the X-type network, where every node represents either a different firm (1 to 5) or the household sector (HH). Panels (b) through (e) plot the simulated idiosyncratic returns from moving between steady-states at the initial and new parameters after the shock on the y-axis, against the upstream or downstream exposure to the source node multiplied by the change in its specified parameter on the x-axis. The 45-degree line equating these two is included for reference. The source firm for each shock is denoted with an X-marker. The idiosyncratic returns are the latter two terms of Equation (13), $R_t^I = \mathcal{U}\Delta v_t + \mathcal{D}\Delta\beta_t$.

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Online Appendix

Upstream, Downstream & Common Firm Shocks

Everett Grant   **Julieta Yung**
Amazon.com   FDIC

A Identifying Idiosyncratic Variation & Common Factors

A.1 Estimation Details

There are four main methods that can potentially be used with PCA to calculate the common factor and idiosyncratic return series. In the main body of the paper we use the "Covariance PCA with Full Factor Series" approach.

PCA solves the following optimization problem:

$$\min_{\{\Lambda, F_t\}} \frac{1}{NT} \sum_{i=1}^{N} \sum_{t=1}^{T} \left(R_{it} - \lambda_i F_t\right)^2,$$

where $\Lambda' \equiv \{\lambda'_1, \lambda'_2, ..., \lambda'_N\}$, the combination of the vectors of firm specific factor weights. Once the common variation in the data, ΛF_t, is estimated, R_t^I is obtained as the residual firm returns.

There are four alternative methods we could use to calculate the common factors and decompose the data. The alternatives are based on whether the covariance or correlation matrix is used to calculate the major axes of the common factors, and whether the means are included in the common factors or they are detrended. For these formulas, R will represent the $T \times N$ matrix of combined R_t vectors. Let μ be the $T \times N$ matrix of the firm means of the R return series repeated along each column, ϑ be an $N \times N$ matrix of the R series' standard deviations along the diagonal and zeroes elsewhere, R_0 be the demeaned return series $(R - \mu)$, and R_S be the standardized R series $((R - \mu)\vartheta^{-1})$. Also, V is the $N \times K$ matrix of the first K eigenvectors from the covariance matrix (In descending order of sample variance explained), and V_c is the matrix of the first K eigenvectors from the correlation matrix.

Standard Covariance PCA: Covariance Eigenvectors & Detrended Factors

The formula to recover the original data using standard PCA involves projecting the data series without their means into the reduced dimension space. The means are excluded so that the directions of maximal variation are captured, rather than the means of the return series driving the newly created factors. The means are then added to these series after they are projected back to the original space:

$$\text{Recovery Formula: } \hat{R} = \mu + R_0 V V'$$

The recovery error — or the variation explained by the excluded eigenvectors — is:

$$R - \hat{R} = R - (\mu + R_0 V V') = R_0 - R_0 V V' = R_0(I_N - V V')$$

As more eigenvectors are included in V the VV' term will approach the identity matrix and the recovery error will go to zero.

Using these factors in our method $(F = R_0 V; \Lambda' = V')$ we can then define what is

captured by the R^I term:
$$R = F\Lambda' + R^I = R_0 VV' + R^I$$
$$\implies R^I = R - R_0 VV' = \mu + R_0(I_N - VV').$$

The idiosyncratic returns are then the average returns for each firm plus the recovery error, or the variation not explained along the included eigenvectors.

Covariance PCA with Full Factor Series

There may be broad common trends that we wish to capture, rather than allocating them to the individual firm return series. In that case, we would instead calculate the factor series with the full — rather than the demeaned — data.

This would mean that the factor series are calculated as $F = RV$ but Λ' would still be equal to V'. In that case the R^I term is:

$$R = F\Lambda' + R^I = RVV' + R^I$$

$$\implies R^I = R - RVV' = (R_0 + \mu)(I_N - VV') = \mu + R_0(I_N - VV') - \mu VV'.$$

These idiosyncratic returns include the average returns and recovery error as in the first case, but the shared trend already accounted for in the factors is removed in the last term. The last term is equal to the means of these full factor series projected back to the full firm space.

Standard Correlation Matrix PCA: Correlation Eigenvectors & Detrended Factors

The formula to recover the original data using standard correlation matrix based PCA involves using the eigenvectors of the correlation matrix (V_c) to project the standardized data series, $R_S = (R - \mu)\vartheta^{-1}$, into the reduced dimension space. The means are excluded so that the dimension of maximal variation is captured, rather than the means driving the newly created factors, and the series are standardized to have the same variation so that they are accounted for equally when deriving the eigenvectors defining the directions of maximal variation for the common factor series. To recover the data the derived common factor series must be adjusted for both the series means and standard deviations:

$$\text{Recovery Formula: } \hat{R} = \mu + R_S V_c V_c' \vartheta = \mu + R_0 \vartheta^{-1} V_c V_c' \vartheta$$

The recovery error is:

$$R - \hat{R} = R - (\mu + R_0 \vartheta^{-1} V_c V_c' \vartheta) = R_0(I_N - \vartheta^{-1} V_c V_c' \vartheta)$$

Using these factors in our method ($F = R_S V_c; \Lambda' = V_c' \vartheta$) we can then define what is captured by the R^I term:
$$R = F\Lambda' + R^I = R_S V_c V_c' \vartheta + R^I$$
$$\implies R^I = R - R_S V_c V_c' \vartheta = R - R_0 \vartheta^{-1} V_c V_c' \vartheta = \mu + R_0(I_N - \vartheta^{-1} V_c V_c' \vartheta).$$

The idiosyncratic returns are then the average returns for each firm plus the recovery error.

Correlation PCA with Full Factor Series

We can also use factor series with the common trends included using the correlation matrix eigenvectors for the data projections, with $F = R\vartheta^{-1}V_c$ and $\Lambda' = V_c'\vartheta$. In that case the R^I term is:

$$R = F\Lambda' + R^I = R\vartheta^{-1}V_cV_c'\vartheta + R^I$$

$$\implies R^I = R - R\vartheta^{-1}V_cV_c'\vartheta = R(I_N - \vartheta^{-1}V_cV_c'\vartheta) = \mu + R_0(I_N - \vartheta^{-1}V_cV_c'\vartheta) - \mu\vartheta^{-1}V_cV_c'\vartheta.$$

These idiosyncratic returns include the average returns and recovery error as in the first case, but the shared trend already accounted for in the factors is removed in the last term as in the covariance based PCA.

Table A.1: 20 Largest Eigenvalues by PCA for U.S. Equity Returns (1989-2017)

Top	Value	%	Cum. %	Top	Value	%	Cum. %
1	692.99	22.63	22.63	*11*	23.00	0.75	34.68
2	70.03	2.29	24.91	*12*	22.07	0.72	35.40
3	66.17	2.16	27.07	*13*	21.18	0.69	36.09
4	39.82	1.30	28.37	*14*	20.65	0.67	36.76
5	34.68	1.13	29.51	*15*	19.92	0.65	37.41
6	32.10	1.05	30.55	*16*	18.90	0.62	38.03
7	28.68	0.94	31.49	*17*	18.12	0.59	38.62
8	27.52	0.90	32.39	*18*	17.71	0.58	39.20
9	24.10	0.79	33.17	*19*	17.20	0.56	39.76
10	23.03	0.75	33.93	*20*	16.82	0.55	40.31

Note: Top 20 eigenvalues of the covariance matrix of log daily equity returns for the portion of our sample continuously traded from 1989 through 2017 (T = 7,424 and N = 524).

A.2 One Period Ahead GIRF Network Edge Equation

$$GIRF_{j\to i}(1) = \frac{e_i'A_1\Theta e_j}{\sqrt{e_j'\Theta e_j}} = \frac{e_i'\begin{bmatrix} \rho\Sigma + \Lambda\Gamma(L)\Upsilon\Lambda' & \rho\Sigma & \Lambda\Gamma(L)\Upsilon \\ \rho\Sigma & \rho\Sigma & 0_{N\times S} \\ \Gamma(L)\Upsilon\Lambda' & 0_{S\times N} & \Gamma(L)\Upsilon \end{bmatrix} e_j}{\sqrt{e_j'\Theta e_j}}$$

A.3 One Period Ahead GFEVD Network Edge Equation

The formula for the one period ahead GFEVD of Pesaran and Shin (1998) is:

$$GFEVD_{j\to i}(1) = \frac{\Theta_{jj}^{-1}[(e_i'\Theta e_j)^2 + (e_i'A_1\Theta e_j)^2]}{e_i'\Theta e_i + e_i'A_1\Theta A_1'e_i}$$

where the e_i are appropriately sized selection vectors with zeroes in all cells except for the ith, which is one.

A.4　VAR Estimation Bias

This equation provides the bias that would result from estimating $\tilde{\rho} = [\rho_0 \ \rho]$ from a standard VAR with the total returns, but not accounting for the common factors. The second term is the bias, and the $\boldsymbol{R}_{t-1}^1$ terms include a column vector of ones before the firm returns to account for the constant term:

$$E[\boldsymbol{R}_t^1 \boldsymbol{R}_{t-1}^1{}'(\boldsymbol{R}_{t-1}^1 \boldsymbol{R}_{t-1}^1{}')^{-1}] = \tilde{\rho} + (\Lambda\Gamma(L) - \tilde{\rho}\Lambda)\boldsymbol{F}_{L,t-1}\boldsymbol{R}_{t-1}^1{}'(\boldsymbol{R}_{t-1}^1 \boldsymbol{R}_{t-1}^1{}')^{-1}$$

Note that the bias is zero if either Λ is zero or all of the $\boldsymbol{F}_{L,t-1}$ are zero. In either case, the bias is zero when the factors have no effect on the firms' total returns.

B Empirical Method Simulations

The simulated system has three independent common factor series following vector-autoregressive processes with zero constants and coefficients on their first lags of 0.9. The innovations to these series are independent normally distributed, and have variances of 16. The values of Υ and Γ are:

$$\Upsilon = \begin{bmatrix} 16 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 16 \end{bmatrix} \quad \text{and} \quad \Gamma(1) = \begin{bmatrix} 0.9 & 0 & 0 \\ 0 & 0.9 & 0 \\ 0 & 0 & 0.9 \end{bmatrix}.$$

The coefficients of the simulated firms on the factors are broken into five sets by their factor loadings. The first fifth of the dataset has a Λ coefficient of one on the first factor only, the next fifth has 0.5 coefficients on the first two factors, the next fifth has a coefficient of one on the second factor only, the fourth partition has 0.5 coefficients on the second and third factors, and the final fifth has a coefficient of one on only the third factor. This setup enables us to analyze the results of our estimation procedure when there are complex interactions among the nodes' factor dependence.

To further create heterogeneity across the simulated markets — and to examine the effects of the idiosyncratic shocks — the innovations of the idiosyncratic return series for the markets are the first five powers of two, repeated in order along the diagonal of Σ, with the symmetric off-diagonal terms in the remainder of Σ coming from normally distributed random draws.[1]

To create an easily discernible pattern of varied connections across the firms for us to study, the ρ matrix is given a block diagonal form. The simulated markets are split into nine groups that have Uniform$[-0.9999, 0.9999]$ randomly distributed coefficients for markets within the same block, with zeroes elsewhere.[2] This yields the following form for the ρ matrix:

$$\rho = \begin{bmatrix} \rho_1 & 0_{250} & 0_{250} & & 0_{250} \\ 0_{250} & \rho_2 & 0_{250} & \cdots & 0_{250} \\ 0_{250} & 0_{250} & \rho_3 & & 0_{250} \\ & \vdots & & \ddots & 0_{250} \\ 0_{250} & 0_{250} & 0_{250} & 0_{250} & \rho_9 \end{bmatrix}$$

where the ρ_g are the randomly generated coefficients and 0_{250} represents a 250×250 matrix of zeroes. The ρ_0 constant vector is drawn from a mean zero normal distribution. We use nine even groups so that we have partitions within and overlapping each of our five adjacent Λ coefficient groups, so that there is a complex interaction between the ρ and Λ coefficient groups for our process to attempt to disentangle. Finally, we have fifty repetitions of each idiosyncratic innovation volatility and ρ_g coefficient group number combination. This makes for $50 \times 5 \times 9 = 2,250$ simulated markets.

Given these model parameters, to obtain the simulated data we first generate $12,500$ multivariate normal draws of ϵ_t and η_t. We then initialize R_1^I and F_1 to be zero vectors and calculate their data series from the generated innovations and above parameter matrices. Finally, we calculate the R values from these two series. The first $5,000$ values of the R series are treated as burn-in values and are dropped, with the remaining $7,500$ saved as the values to apply our estimation procedure to, close to our full sample's $T = 7424$.

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[1]If needed, the Σ matrix is adjusted to be diagonally dominant to ensure that it is positive definite.
[2]If needed, the ρ matrix is multiplied by 0.9 until the modulus of its largest eigenvalue is less than one.
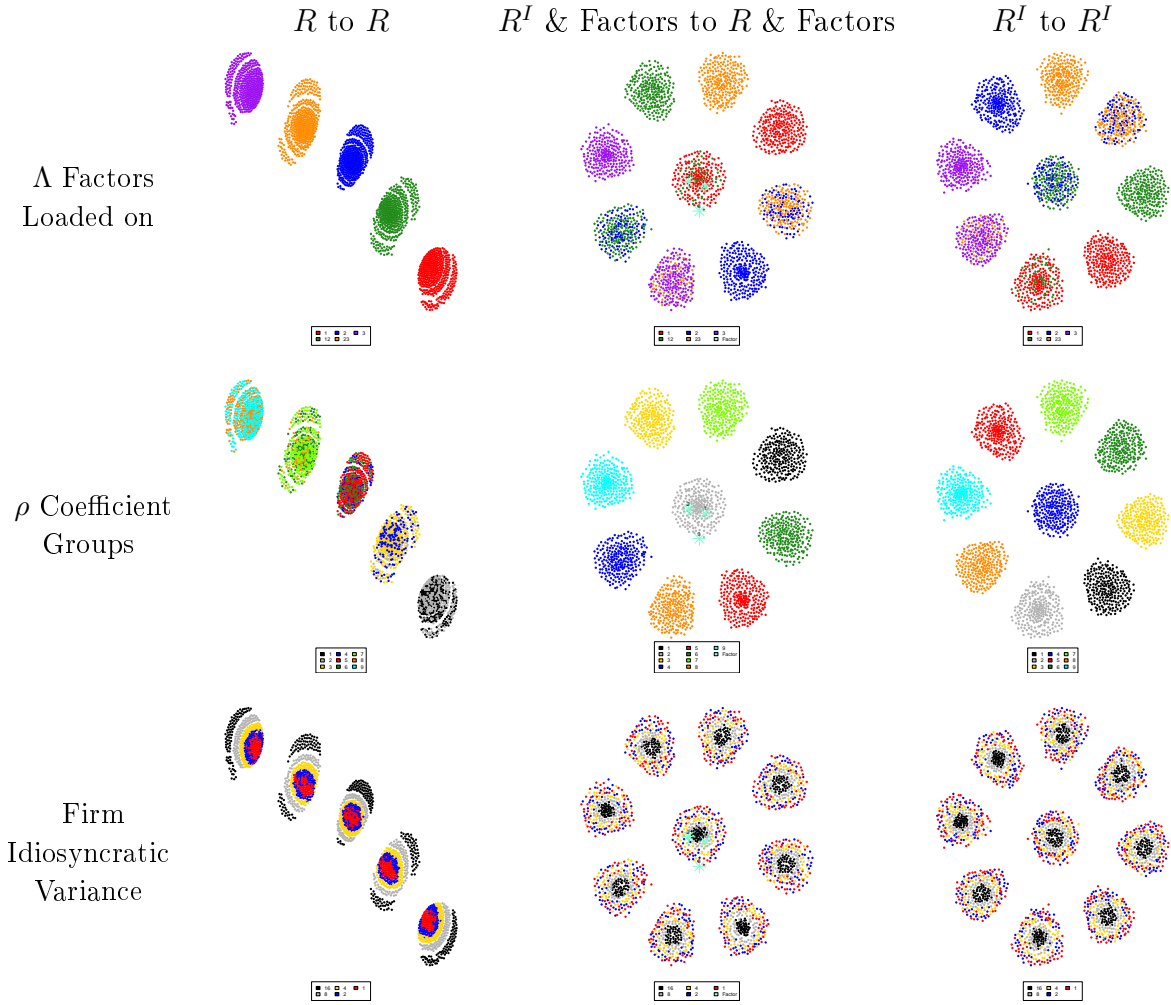
B.1 Simulated Networks with Different Methods: GFEVD, GIRF, AEN

Figure B.1: Spring Plots of Simulated Networks with GFEVD
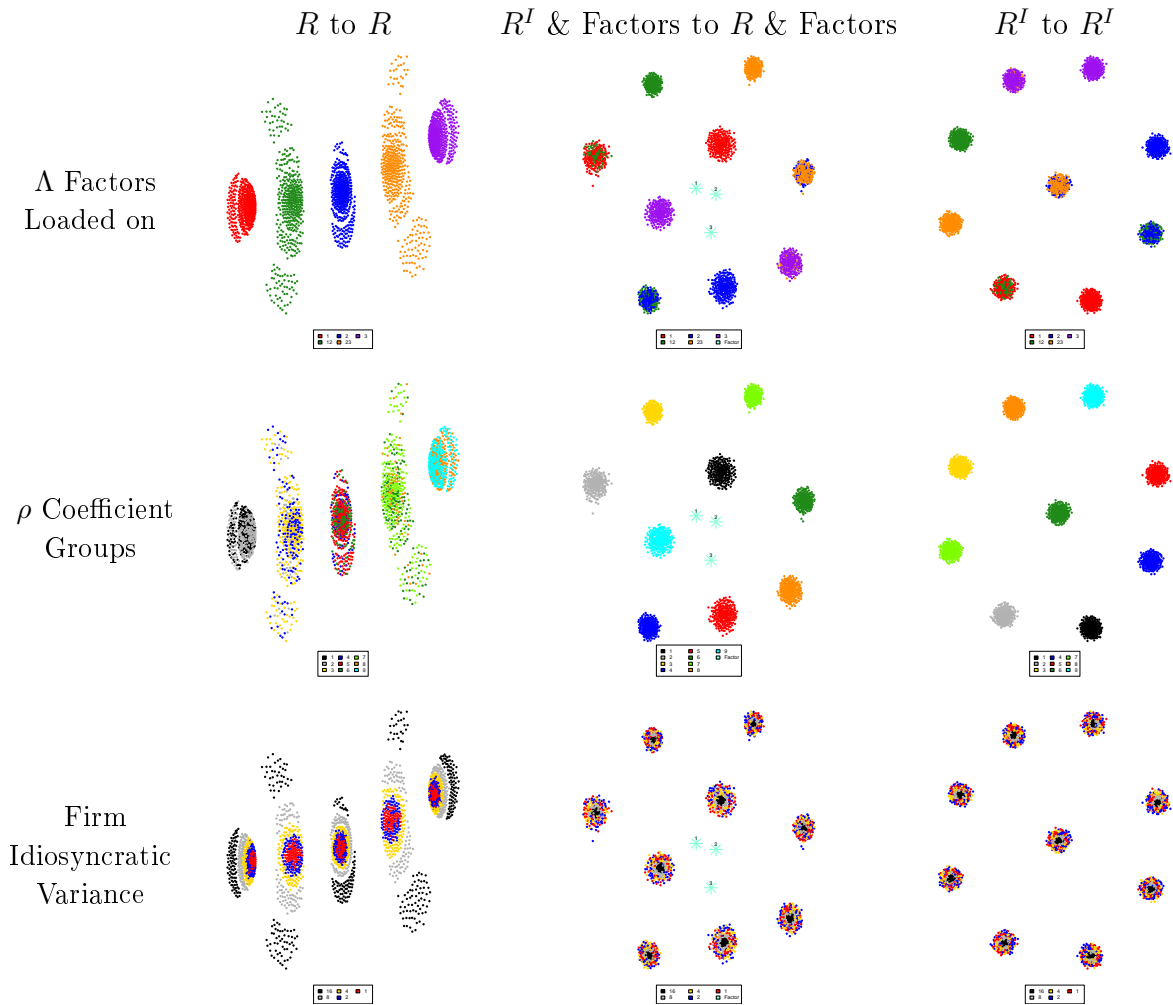
Note: Estimated networks from data simulations with 3 common factors when using generalized forecast error variance decompositions (GFEVD) instead of contributions.

Figure B.2: Spring Plots of Simulated Networks with GIRF

Note: Estimated networks from data simulations with 3 common factors when using the absolute value of generalized impulse response functions (GIRF) instead of GFEVc.

Figure B.3: Spring Plots of Simulated Networks with Adaptive Elastic Net
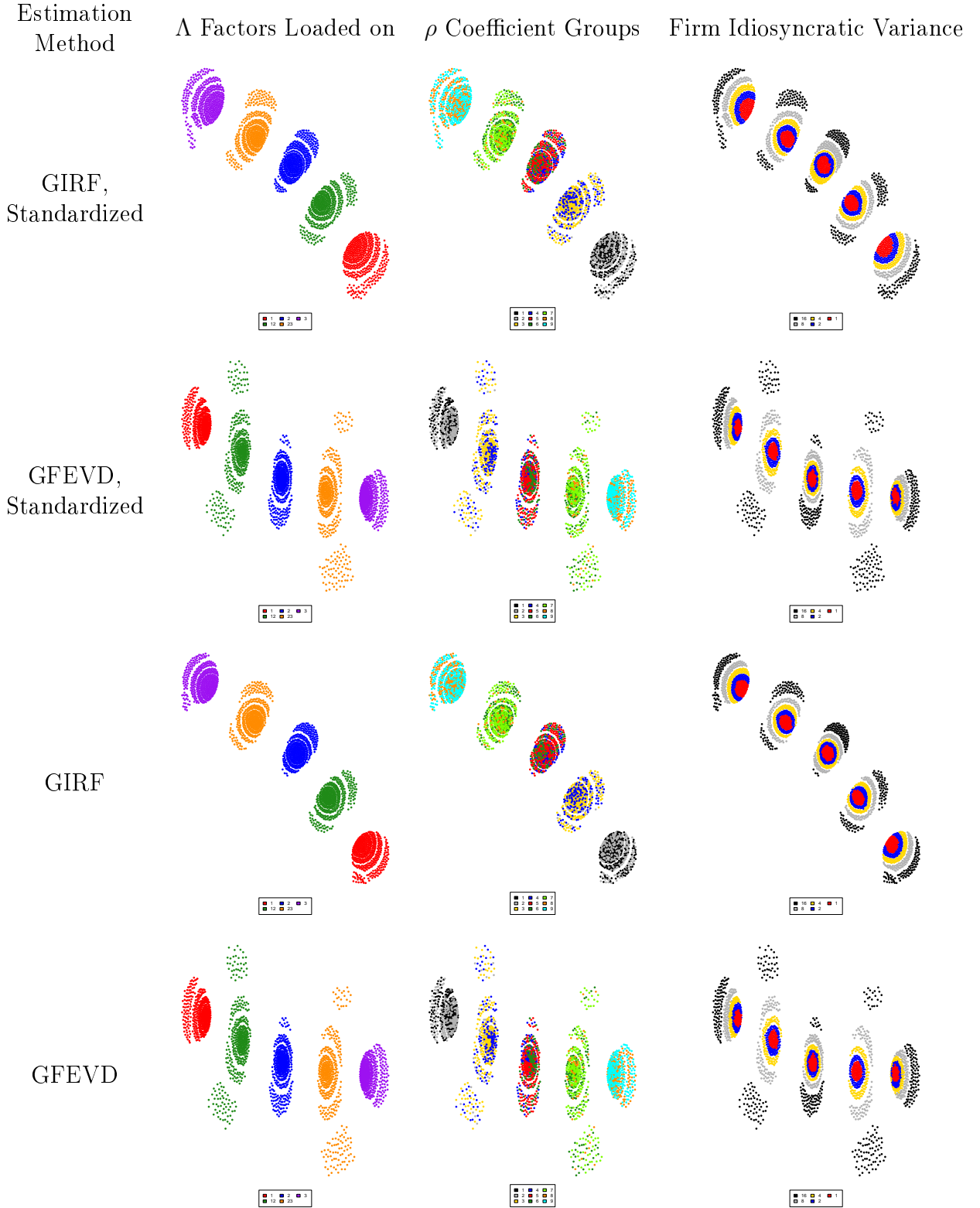
Note: Generalized forecast error variance decomposition (GFEVD) and generalized impulse response function (GIRF) networks calculated from VAR estimated using adaptive elastic net instead of OCMT for model selection.

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C U.S. 1989-2017 Factors & Loadings Details

Figure C.1: U.S. Factor Λ Coefficient Distributions by Industry (1989-2017)
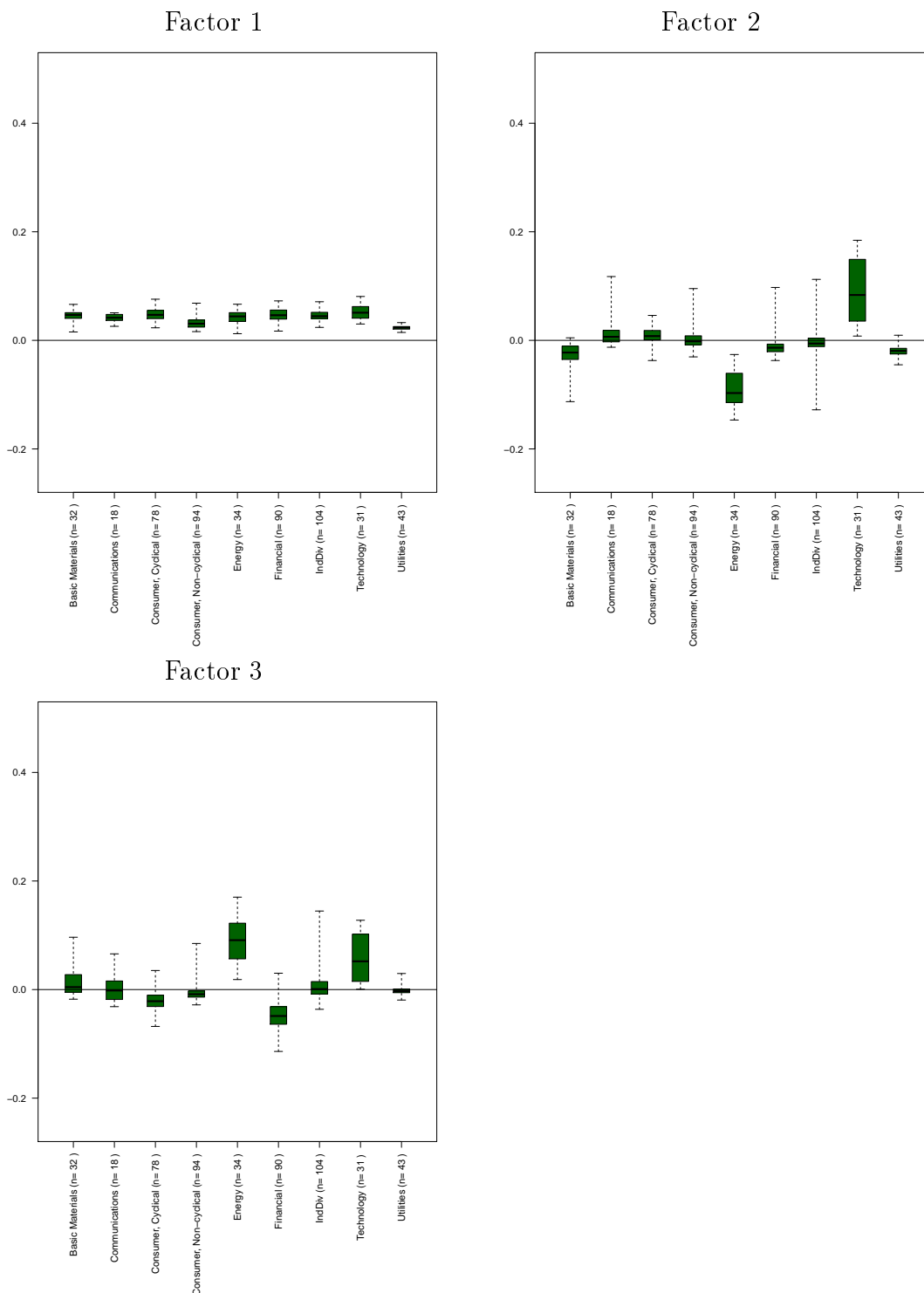
Note: Loadings on the first three factors extracted by principal component analysis on the covariance matrix of U.S. daily log equity returns for the portion of our sample continuously traded from 1989 through 2017 (T = 7,424 and N = 524).

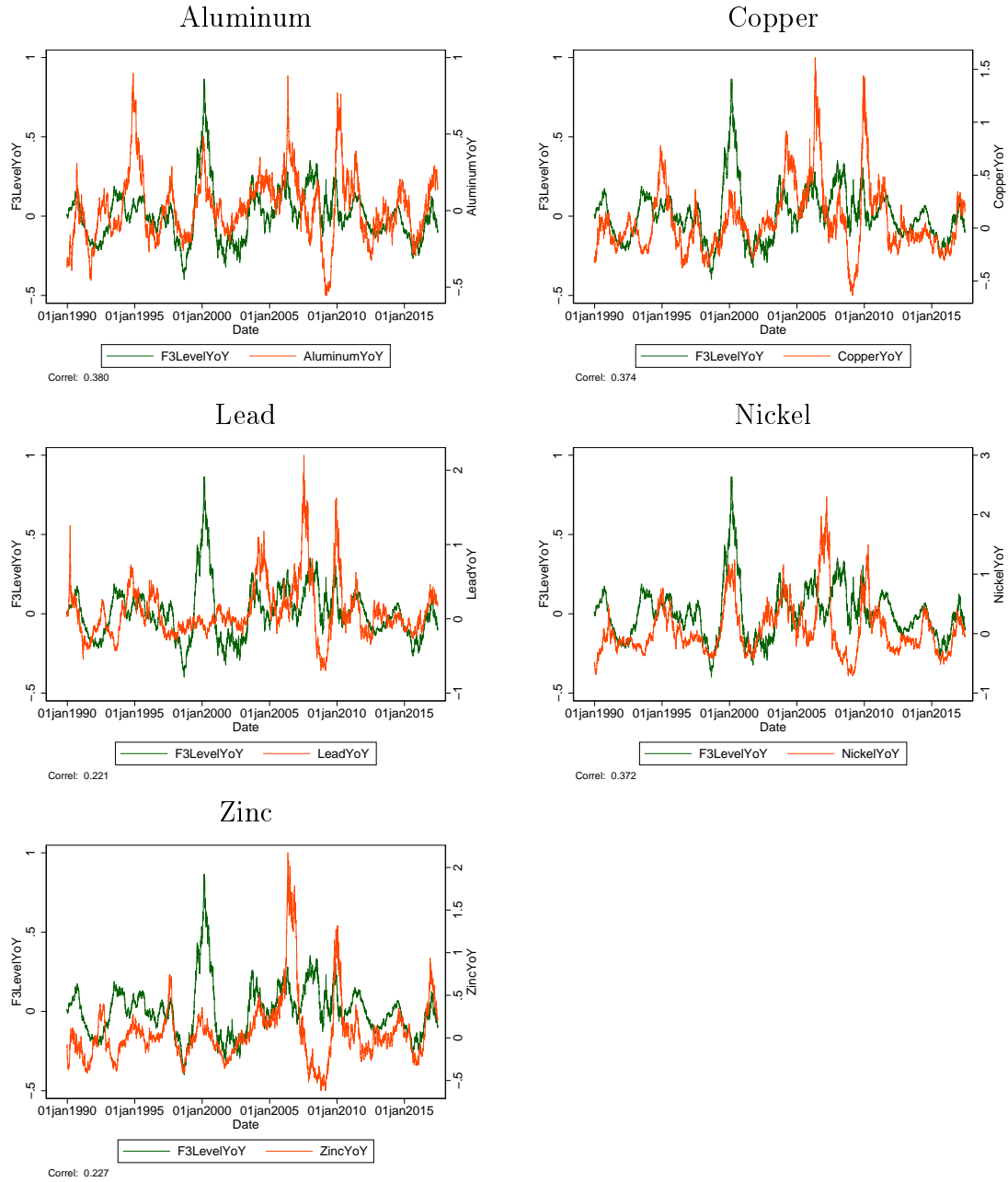Figure C.2: Third Factor & Energy Commodities, Year-over-Year Plots
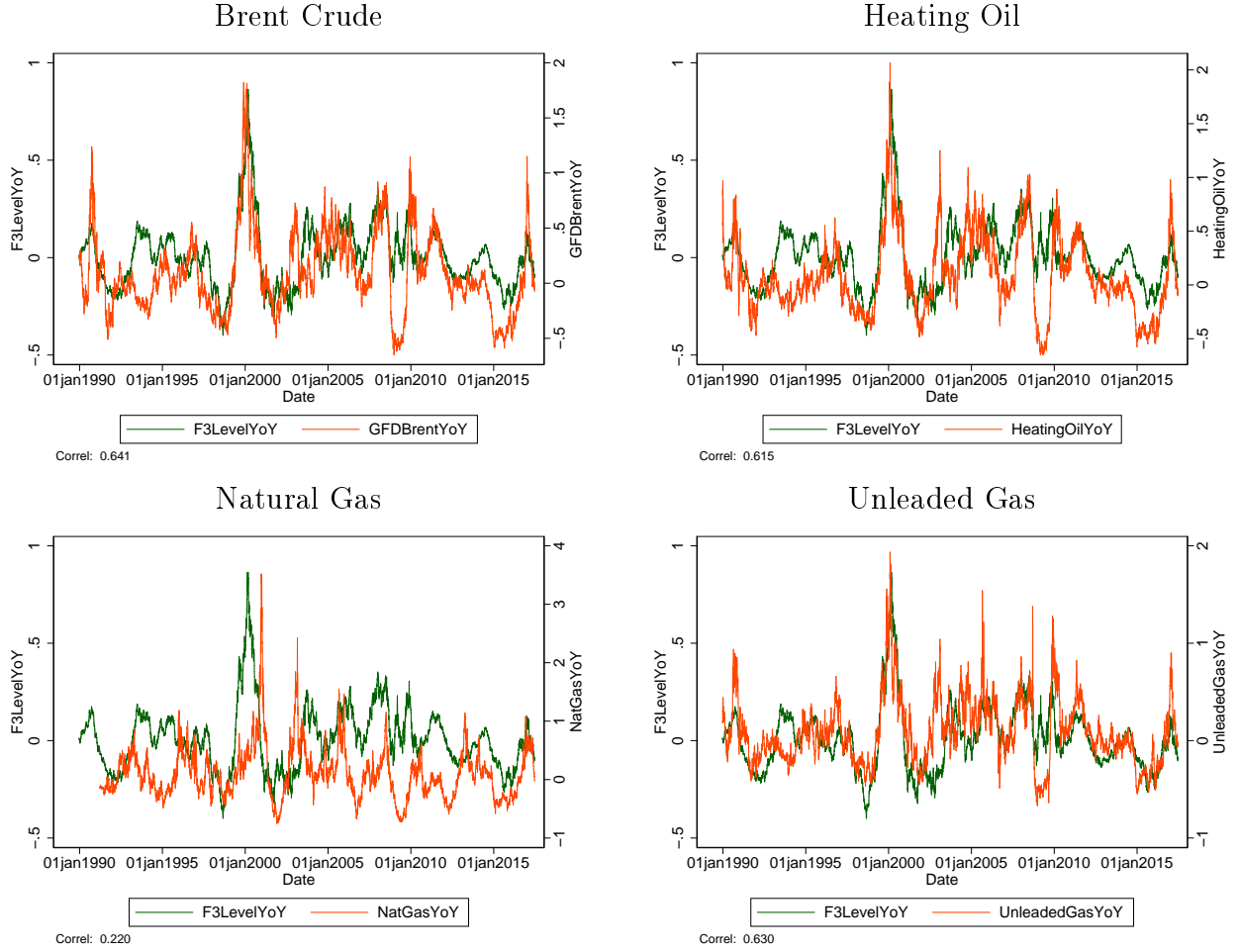
Note: Cumulative sum of the third factor extracted by principal component analysis on the variance-covariance matrix of U.S. daily log equity returns. (1989-2017, T = 7,424 and N = 524).

Figure C.3: Third Factor & Agriculture Commodities, Year-over-Year Plots
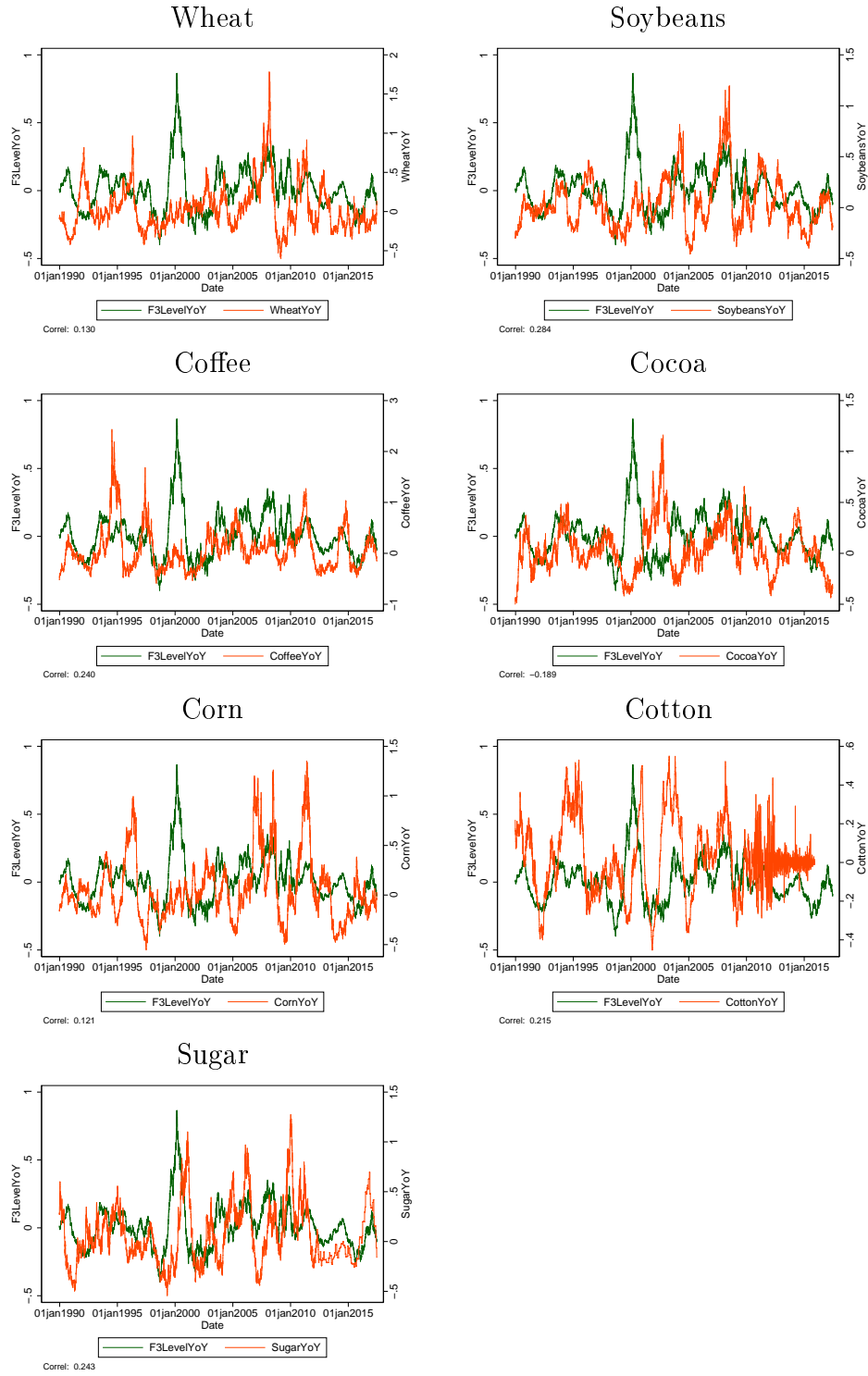
Wheat

Soybeans
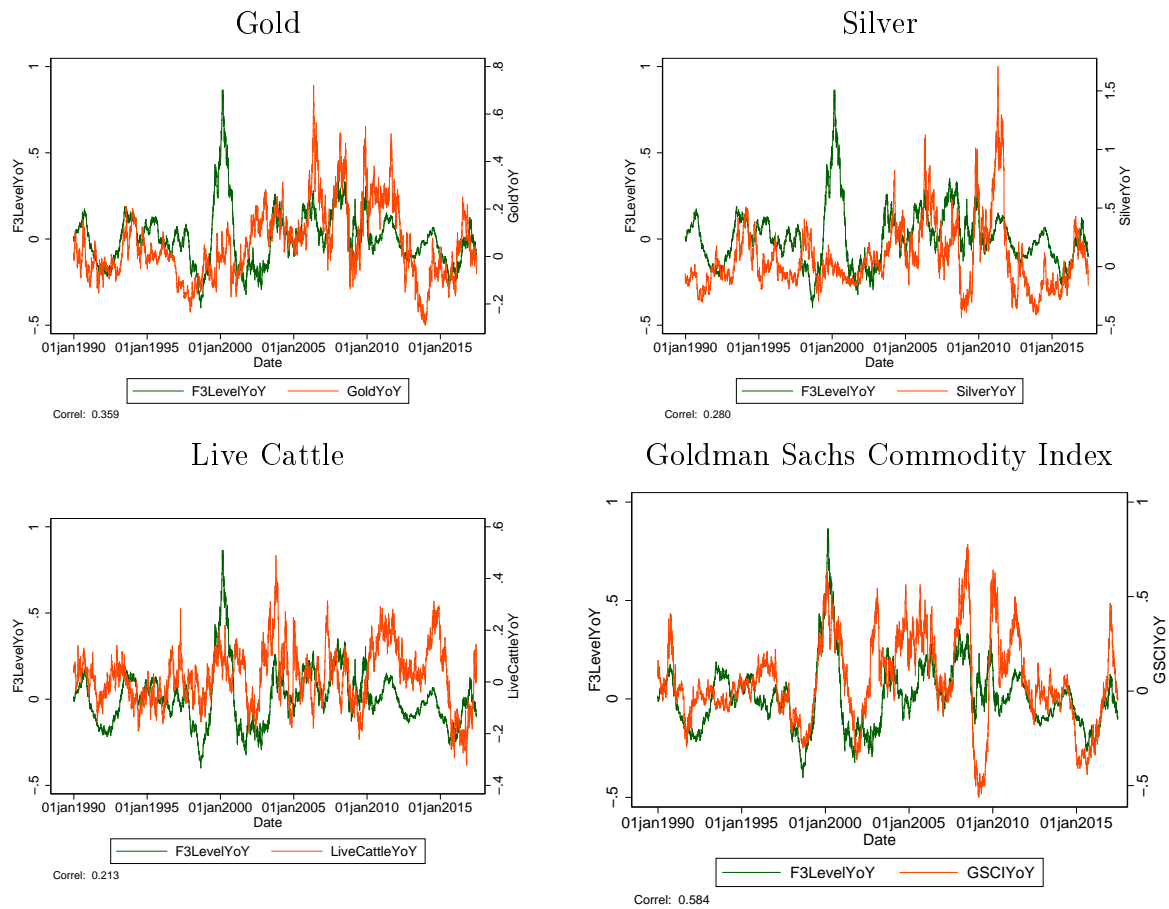
Coffee

Cocoa

Corn

Cotton

Sugar

Note: Cumulative sum of the third factor extracted by principal component analysis on the variance-covariance matrix of U.S. daily log equity returns. (1989-2017, T = 7,424 and N = 524).

Figure C.4: Third Factor & Industrial Metals Commodities, Year-over-Year Plots
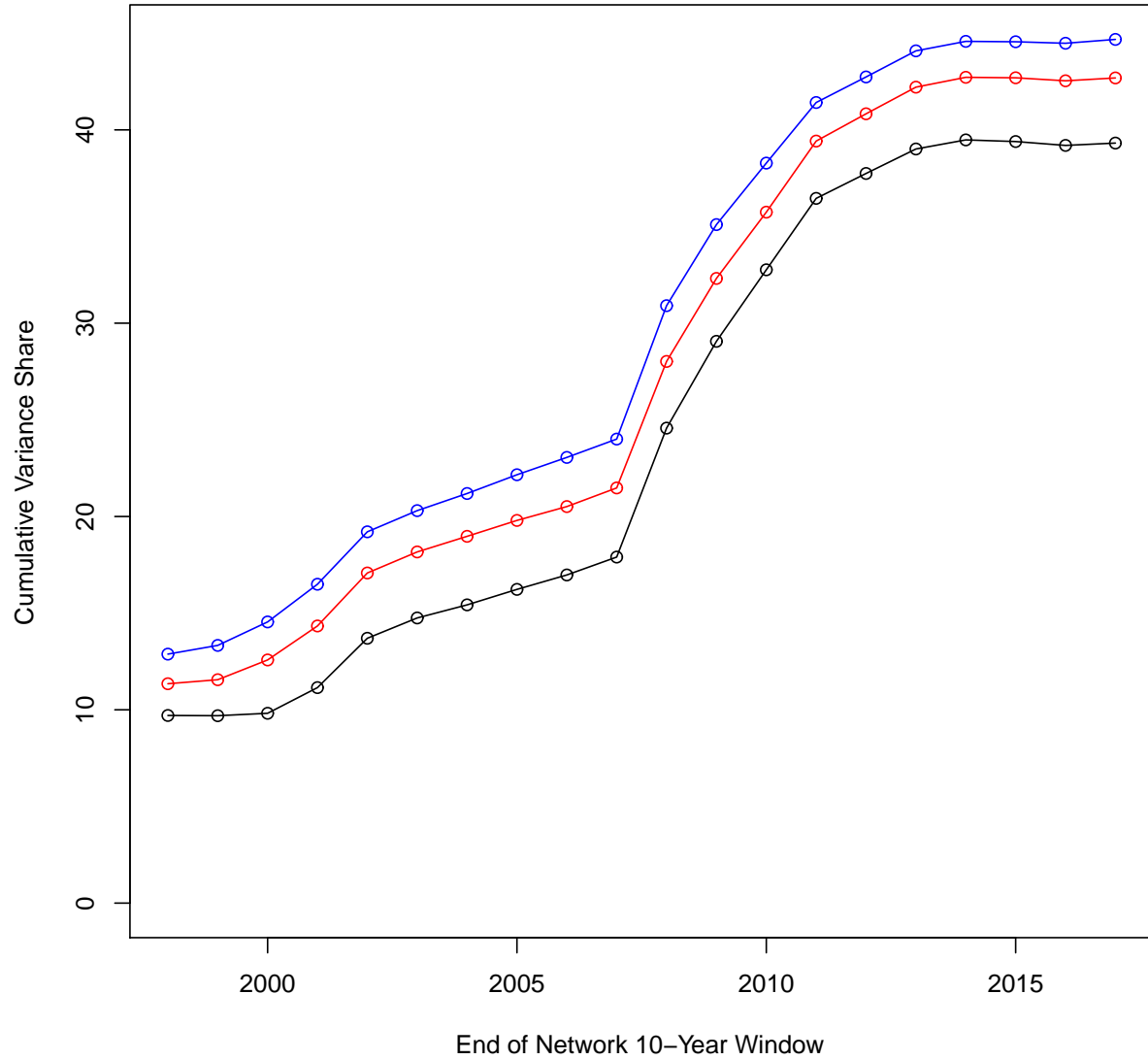

Note: Cumulative sum of the third factor extracted by principal component analysis on the variance-covariance matrix of U.S. daily log equity returns (1989-2017, T = 7,424 and N = 524).

Figure C.5: Third Factor & Miscellaneous Commodities, Year-over-Year Plots

Gold

Silver

Live Cattle

Goldman Sachs Commodity Index

Note: Cumulative sum of the third factor extracted by principal component analysis on the variance-covariance matrix of U.S. daily log equity returns (1989-2017, T = 7,424 and N = 524).

Figure C.6: Variance Share of Top 3 Factors, Balanced Firm Sample

Note: Factor variances for rolling 10-year samples, with factors extracted by principal component analysis on the variance-covariance matrix of U.S. daily log equity returns (1989-2017). (T = 7,424 and N = 524).

Table C.1: Top Firm Loadings on First Factor

Panel A: Highest Factor Loadings

	Λ	Name	Industry	Industry Subgroup	BEA Sector	BEA Subgroup	Country
1	0.08	XCERRA CORP	Technology	Semiconductor Equipment	Manufacturing	Computer and electronic products	US
2	0.08	KULICKE & SOFFA	Technology	Semiconductor Equipment	Manufacturing	Computer and electronic products	SGP
3	0.08	HOVNANIAN ENT-A	Consumer, Cyclical	Bldg-Residential/Commer	Construction	Construction	US
4	0.07	SAFEGUARD SCIENT	Financial	Venture Capital	Finance	Securities, commodity contracts, and investments	US
5	0.07	TEREX CORP	Industrial	Machinery-Constr&Mining	Manufacturing	Machinery	US
6	0.07	LINCOLN NATL CRP	Financial	Life/Health Insurance	Finance	Securities, commodity contracts, and investments	US
7	0.07	OFFICE DEPOT INC	Consumer, Cyclical	Retail-Office Supplies	Retail	General merchandise stores	US
8	0.07	ENZO BIOCHEM INC	Consumer, Non-cyclical	Medical-Biomedical/Gene	Manufacturing	Chemical products	US
9	0.07	MICRON TECH	Technology	Electronic Compo-Semicon	Manufacturing	Computer and electronic products	US
10	0.07	MBIA INC	Financial	Financial Guarantee Ins	Finance	Securities, commodity contracts, and investments	US

Panel B: Lowest Factor Loadings

	Λ	Name	Industry	Industry Subgroup	BEA Sector	BEA Subgroup	Country
1	0.01	NORTH EURO OIL	Energy	Oil-US Royalty Trusts	Mining	Oil and gas extraction	US
2	0.01	SOUTHERN CO	Utilities	Electric-Integrated	Utilities	Utilities	US
3	0.02	NEWMONT MINING	Basic Materials	Gold Mining	Mining	Mining, except oil and gas	US
4	0.02	SABINE ROYALTY	Energy	Oil-US Royalty Trusts	Mining	Oil and gas extraction	US
5	0.02	GENERAL MILLS IN	Consumer, Non-cyclical	Food-Misc/Diversified	Manufacturing	Food and beverage and tobacco products	US
6	0.02	CONS EDISON INC	Utilities	Electric-Integrated	Utilities	Utilities	US
7	0.02	DYNEX CAPITAL	Financial	REITS-Mortgage	Finance	Real estate	US
8	0.02	WEC ENERGY GROUP	Utilities	Electric-Integrated	Utilities	Utilities	US
9	0.02	HORMEL FOODS CRP	Consumer, Non-cyclical	Food-Meat Products	Manufacturing	Food and beverage and tobacco products	US
10	0.02	KELLOGG CO	Consumer, Non-cyclical	Food-Misc/Diversified	Manufacturing	Food and beverage and tobacco products	US

Note: Sample includes the 524 firms continuously traded over 1989-2017.

Table C.2: Top Firm Loadings on Second Factor

Panel A: Highest Factor Loadings

	Λ	Name	Industry	Industry Subgroup	BEA Sector	BEA Subgroup	Country
1	0.18	LAM RESEARCH	Technology	Semiconductor Equipment	Manufacturing	Computer and electronic products	US
2	0.18	XCERRA CORP	Technology	Semiconductor Equipment	Manufacturing	Computer and electronic products	US
3	0.17	INTEGRAT DEVICE	Technology	Semicon Compo-Intg Circu	Manufacturing	Computer and electronic products	US
4	0.17	KULICKE & SOFFA	Technology	Semiconductor Equipment	Manufacturing	Computer and electronic products	SGP
5	0.17	KLA-TENCOR CORP	Technology	Semiconductor Equipment	Manufacturing	Computer and electronic products	US
6	0.16	SKYWORKS SOLUTIO	Technology	Electronic Compo-Semicon	Manufacturing	Computer and electronic products	US
7	0.16	TERADYNE INC	Technology	Semiconductor Equipment	Manufacturing	Computer and electronic products	US
8	0.15	ANALOG DEVICES	Technology	Semicon Compo-Intg Circu	Manufacturing	Computer and electronic products	US
9	0.15	CYPRESS SEMICON	Technology	Semicon Compo-Intg Circu	Manufacturing	Computer and electronic products	US
10	0.15	MICRON TECH	Technology	Electronic Compo-Semicon	Manufacturing	Computer and electronic products	US

Panel B: Lowest Factor Loadings

|	Λ	Name	Industry	Industry Subgroup	BEA Sector	BEA Subgroup	Country
1	-0.15	ENSCO PLC-CL A	Energy	Oil&Gas Drilling	Mining	Oil and gas extraction	UK
2	-0.14	UNIT CORP	Energy	Oil Comp-Explor&Prodtn	Mining	Oil and gas extraction	US
3	-0.14	NOBLE CORP PLC	Energy	Oil&Gas Drilling	Mining	Oil and gas extraction	UK
4	-0.14	ROWAN COMPANIE-A	Energy	Oil&Gas Drilling	Mining	Oil and gas extraction	US
5	-0.14	NABORS INDS LTD	Energy	Oil&Gas Drilling	Mining	Oil and gas extraction	US
6	-0.13	TIDEWA-TER INC	Industrial	Transport-Marine	Transport	Air transportation	US
7	-0.12	PARKER DRILLING	Energy	Oil&Gas Drilling	Mining	Oil and gas extraction	US
8	-0.12	HELMERICH & PAYN	Energy	Oil&Gas Drilling	Mining	Oil and gas extraction	US
9	-0.12	BAKER HUGHES INC	Energy	Oil-Field Services	Mining	Oil and gas extraction	US
10	-0.11	APACHE CORP	Energy	Oil Comp-Explor&Prodtn	Mining	Oil and gas extraction	US

Note: Sample includes the 524 firms continuously traded over 1989-2017.

Table C.3: Top Firm Loadings on Third Factor

Panel A: Highest Factor Loadings

	Λ	Name	Industry	Industry Subgroup	BEA Sector	BEA Subgroup	Country
1	0.17	ENSCO PLC-CL A	Energy	Oil&Gas Drilling	Mining	Oil and gas extraction	UK
2	0.17	NOBLE CORP PLC	Energy	Oil&Gas Drilling	Mining	Oil and gas extraction	UK
3	0.16	UNIT CORP	Energy	Oil Comp-Explor&Prodtn	Mining	Oil and gas extraction	US
4	0.16	NABORS INDS LTD	Energy	Oil&Gas Drilling	Mining	Oil and gas extraction	US
5	0.16	ROWAN COMPANIE-A	Energy	Oil&Gas Drilling	Mining	Oil and gas extraction	US
6	0.16	PARKER DRILLING	Energy	Oil&Gas Drilling	Mining	Oil and gas extraction	US
7	0.14	TIDEWATER INC	Industrial	Transport-Marine	Transport	Air transportation	US
8	0.13	HELMERICH & PAYN	Energy	Oil&Gas Drilling	Mining	Oil and gas extraction	US
9	0.13	HALLIBURTON CO	Energy	Oil-Field Services	Mining	Oil and gas extraction	US
10	0.13	INTEGRAT DEVICE	Technology	Semicon Compo-Intg Circu	Manufacturing	Computer and electronic products	US

Panel B: Lowest Factor Loadings

|	Λ	Name	Industry	Industry Subgroup	BEA Sector	BEA Subgroup	Country
1	-0.11	HUNTINGTON BANC	Financial	Super-Regional Banks-US	Finance	SCCI	US
2	-0.11	FIFTH THIRD BANC	Financial	Super-Regional Banks-US	Finance	SCCI	US
3	-0.10	REGIONS FINANCIA	Financial	Commer Banks-Southern US	Finance	SCCI	US
4	-0.10	SUNTRUST BANKS	Financial	Super-Regional Banks-US	Finance	SCCI	US
5	-0.09	KEYCORP	Financial	Super-Regional Banks-US	Finance	SCCI	US
6	-0.09	SYNOVUS FINL	Financial	Commer Banks-Southern US	Finance	SCCI	US
7	-0.09	ZIONS BANCORP	Financial	Commer Banks-Western US	Finance	SCCI	US
8	-0.09	WELLS FARGO & CO	Financial	Super-Regional Banks-US	Finance	SCCI	US
9	-0.09	FIRST HORIZON NA	Financial	Commer Banks-Southern US	Finance	SCCI	US
10	-0.08	MBIA INC	Financial	Financial Guarantee Ins	Finance	SCCI	US

Note: Sample includes the 524 firms continuously traded over 1989-2017. SCCI: Securities, commodity contracts, and investments.

Figure C.7: Aggregate Industry Edge Weights R^I & Factors to R & Factors Networks

(a) Sum of Industry & Factor Weights Out to All Firms
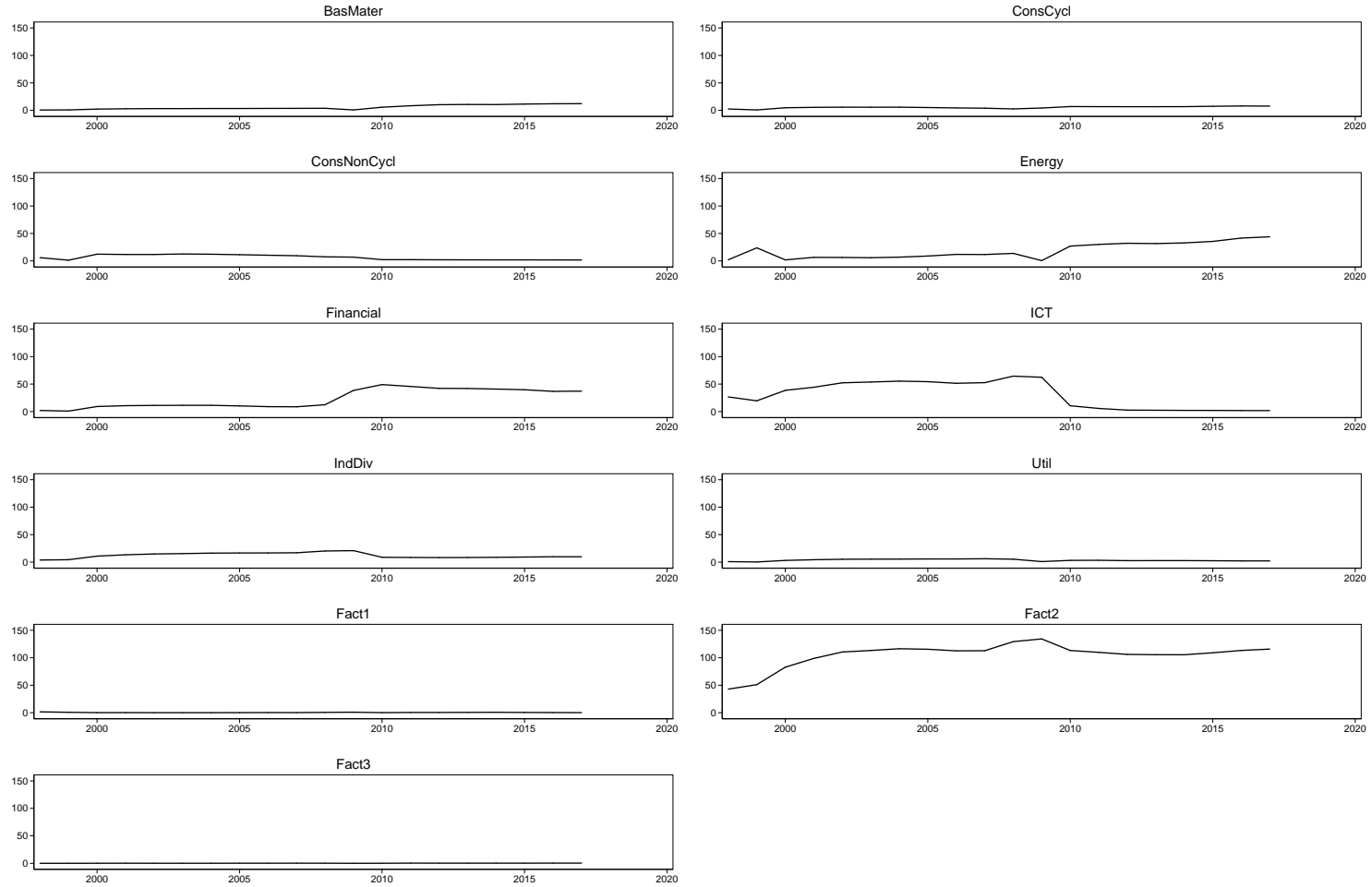

(b) Net Industry & Factor Edge Weights
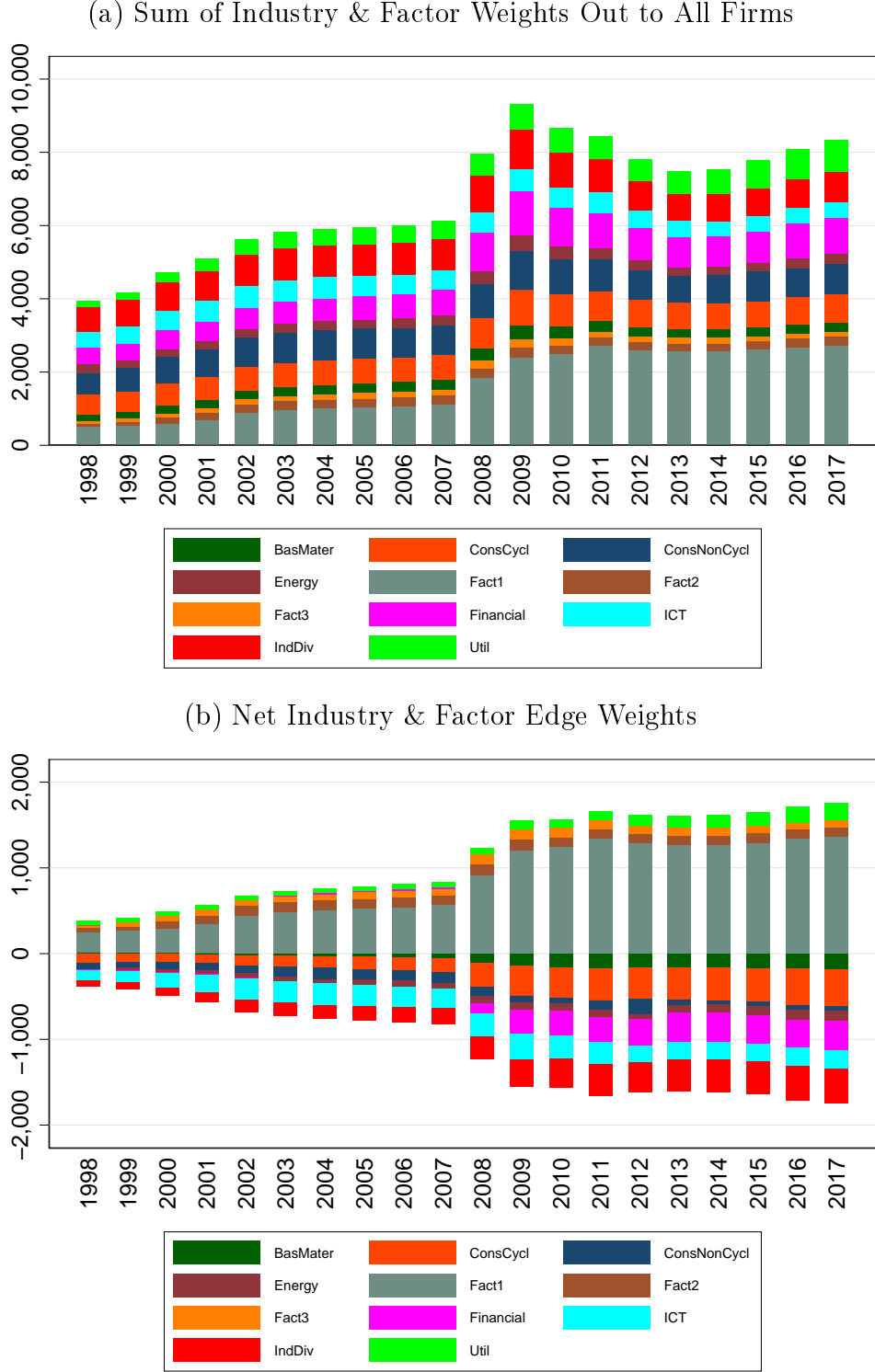
Note: Aggregate edge weights of the R^I & Factors to R & Factors GFEVc networks for the portion of our sample continuously traded from 1989 through 2017 with 3 common factors (T = 7,424 and N = 524). Factors extracted by principal component analysis on the covariance matrix of the daily log equity returns.

Figure C.8: Total Industry Edge Weights in from First Factor, Rolling 10-Year R^I & Factors to R & Factors Networks
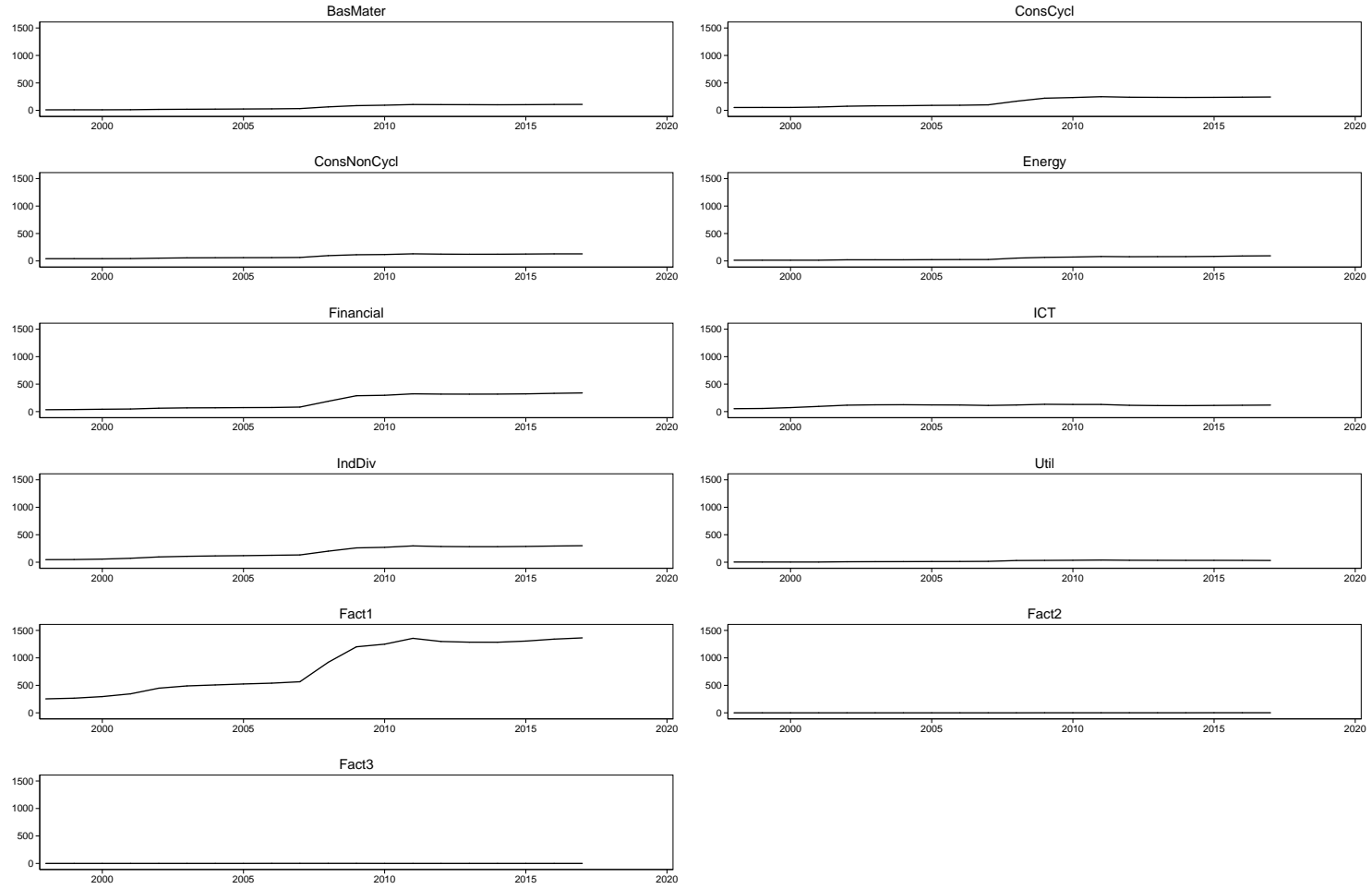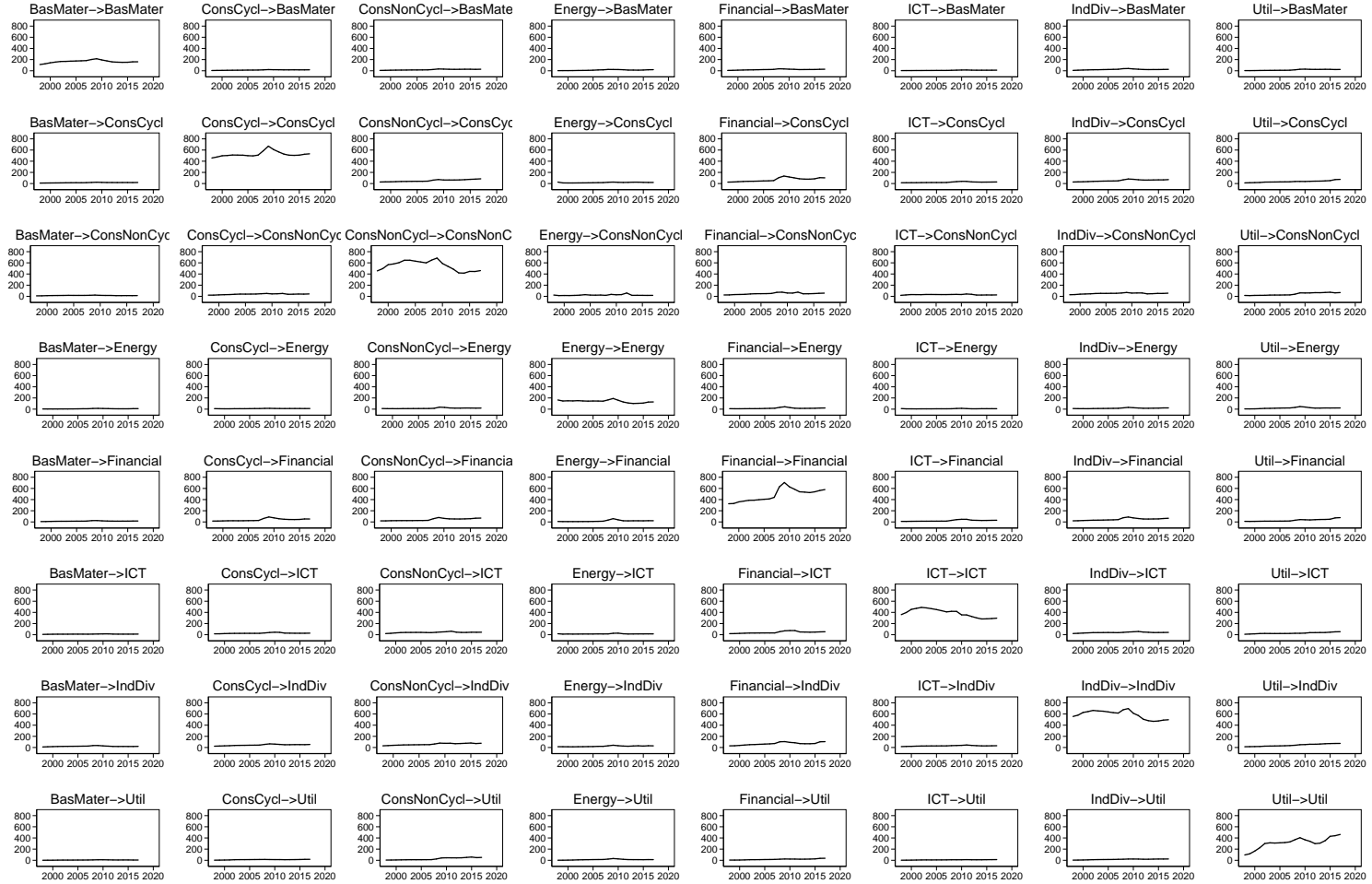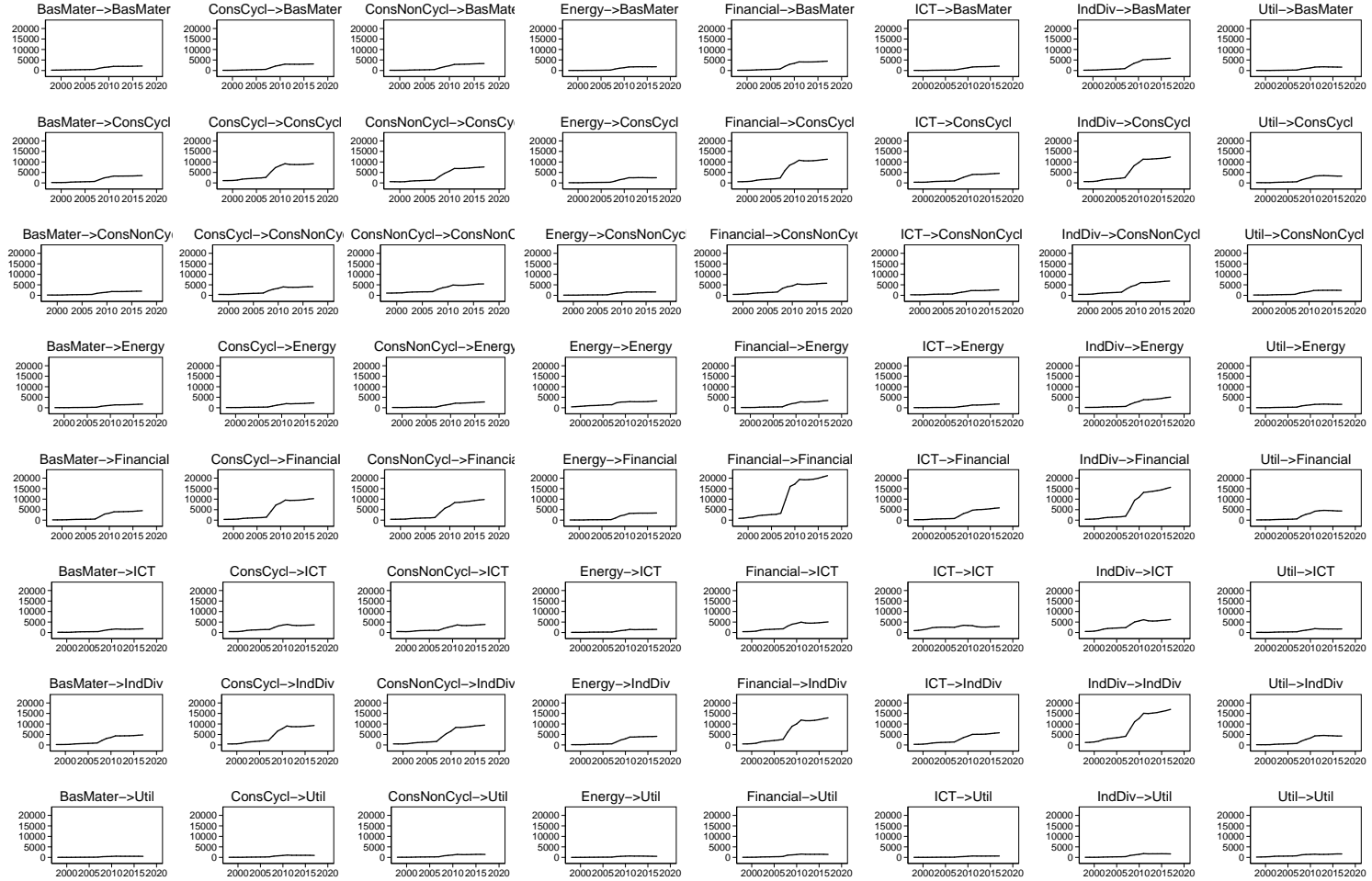

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Note: Rolling 10-year windows of the porton of our sample continuously traded from 1989-2017. R^I & Factors to R & Factors GFEVc networks sum of weights in from first factor with 3 common factors (T = 7,424 and N = 524).

Figure C.9: Total Industry Edge Weights in from Second Factor, Rolling 10-Year R^I & Factors to R & Factors Networks

Note: Rolling 10-year windows of the porton of our sample continuously traded from 1989-2017. R^I & Factors to R & Factors GFEVc networks sum of weights in from second factor with 3 common factors (T = 7,424 and N = 524).

Figure C.10: Total Industry Edge Weights in from Third Factor, Rolling 10-Year R^I & Factors to R & Factors Networks
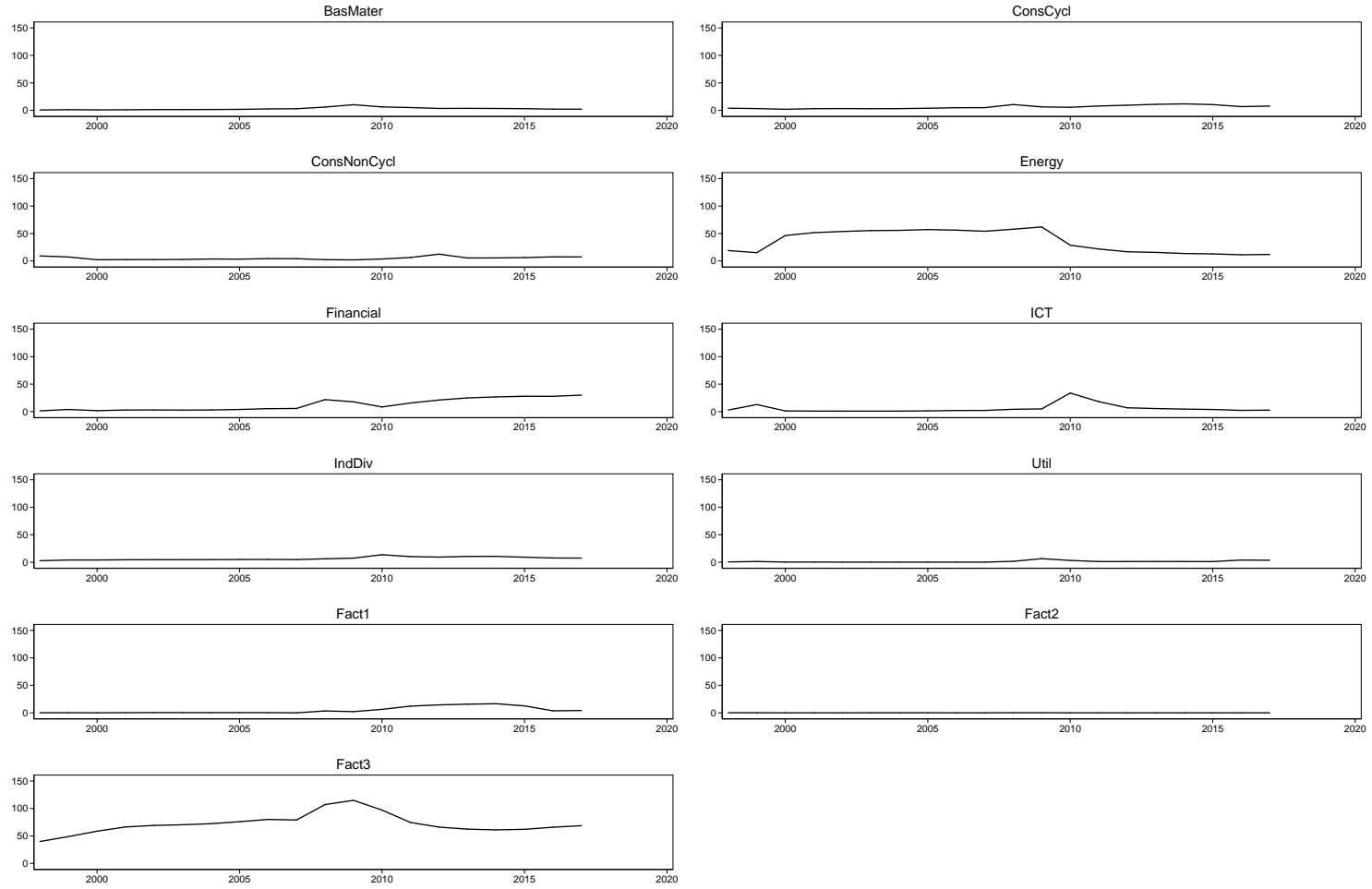
Note: Rolling 10-year windows of the porton of our sample continuously traded from 1989-2017. R^I & Factors to R & Factors GFEVc networks sum of weights in from third factor with 3 common factors (T = 7,424 and N = 524).

Figure C.11: Bilateral Industry Edge Weight Sums, Rolling 10-Year R^I & Factors to R & Factors Networks

Note: Rolling 10-year windows of the porton of our sample continuously traded from 1989-2017. R^I & Factors to R & Factors GFEVc networks with 3 common factors (T = 7,424 and N = 524).

Figure C.12: Bilateral Industry Edge Weight Sums, Rolling 10-Year R to R Networks

Note: Rolling 10-year windows of the porton of our sample continuously traded from 1989-2017. R^I & Factors to R & Factors GFEVc networks with 3 common factors (T = 7,424 and N = 524).

Table C.4: U.S. Top Firms by Network Out Weights (1989-2017): "R to R"

| Rank	Ticker	Name	Industry	Industry Group	Sum Weights Out
1	BEN	FRANKLIN RES INC	Financial	Diversified Finan Serv	337.25
2	DOV	DOVER CORP	IndDiv	Miscellaneous Manufactur	324.07
3	GE	GENERAL ELECTRIC	IndDiv	Miscellaneous Manufactur	323.75
4	JPM	JPMORGAN CHASE	Financial	Banks	323.54
5	NTRS	NORTHERN TRUST	Financial	Banks	320.45
6	CMA	COMERICA INC	Financial	Banks	319.47
7	PCH	POTLATCH CORP	Financial	REITS	319.02
8	LNC	LINCOLN NATL CRP	Financial	Insurance	318.1
9	AXP	AMERICAN EXPRESS	Financial	Diversified Finan Serv	317.81
10	PPG	PPG INDS INC	BasMater	Chemicals	316.39
11	EMR	EMERSON ELEC CO	IndDiv	Electrical Compo and Equip	315.61
12	LM	LEGG MASON INC	Financial	Diversified Finan Serv	314.18
13	STI	SUNTRUST BANKS	Financial	Banks	310.72
14	L	LOEWS CORP	Financial	Insurance	310.47
15	BK	BANK NY MELLON	Financial	Banks	308.07
16	TMK	TORCHMARK CORP	Financial	Insurance	306.14
17	ETN	EATON CORP PLC	IndDiv	Miscellaneous Manufactur	305.44
18	WFC	WELLS FARGO AND CO	Financial	Banks	302.69
19	TROW	T ROWE PRICE GRP	Financial	Diversified Finan Serv	302.25
20	IR	INGERSOLL-RAND	IndDiv	Miscellaneous Manufactur	299.68
21	KEY	KEYCORP	Financial	Banks	298.88
22	WRI	WEINGARTEN RLTY	Financial	REITS	298.32
23	PCAR	PACCAR INC	ConsCycl	Auto Manufacturers	297.32
24	TRN	TRINITY INDUSTRI	IndDiv	Miscellaneous Manufactur	295.99
25	ITW	ILLINOIS TOOL WO	IndDiv	Miscellaneous Manufactur	293.81

Note: Sample includes the 524 firms continuously traded over 1989-2017. GFEVc networks with 3 common factors (T = 7,424 and N = 524). Self-loops not included.

Figure C.13: U.S. Firm Network Aggregate Industry Edge Weights (1989-2017), R to R

(a) Sum of Industry Weights Out to All Firms
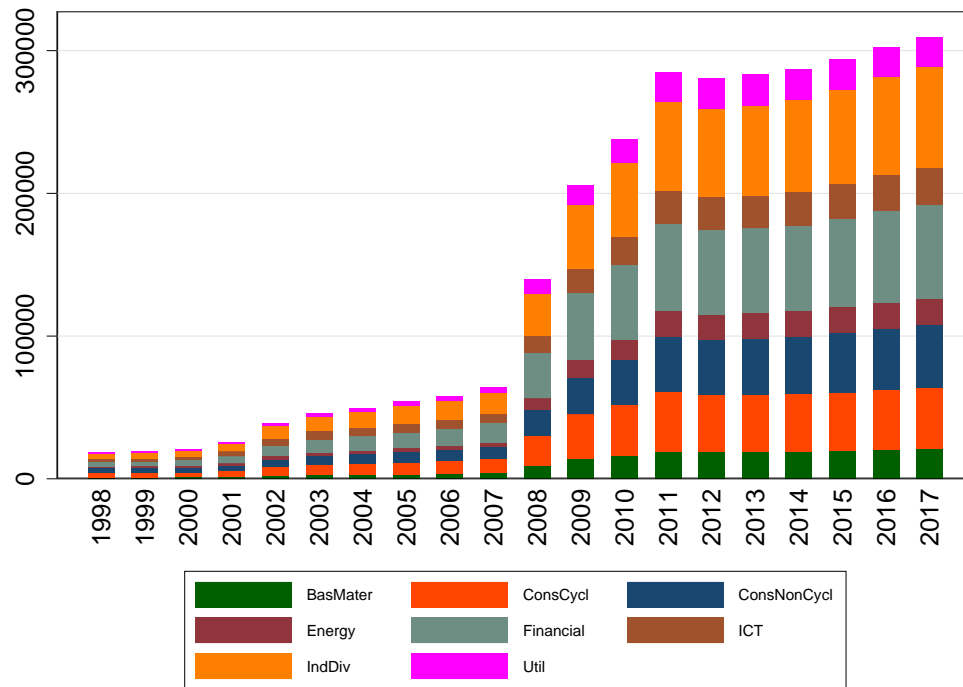
(b) Net Industry Edge Weights
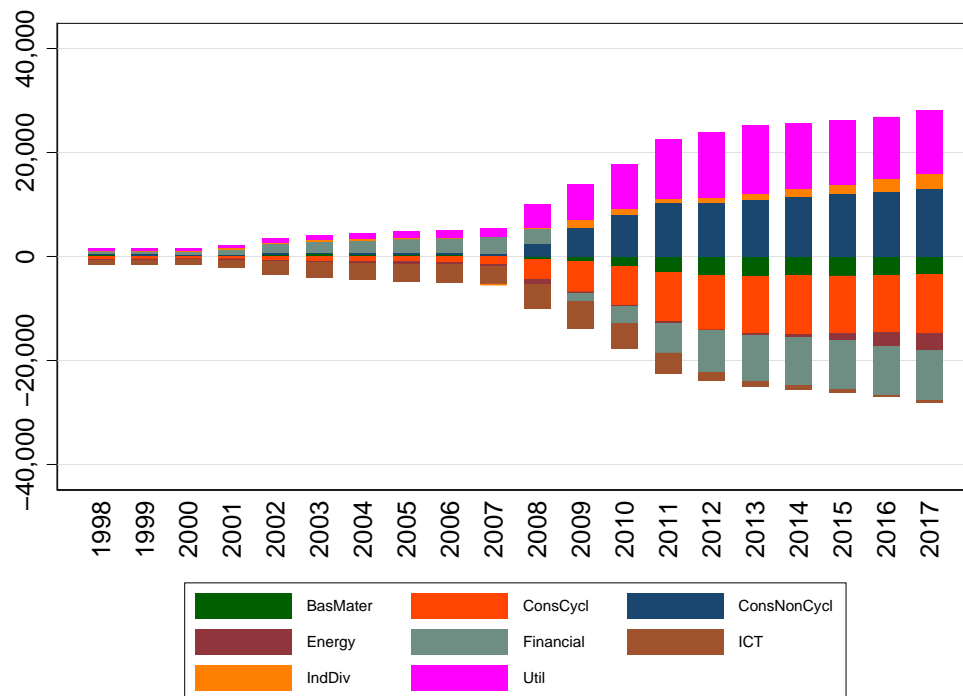
Note: Aggregate edge weights of the R to R GFEVc networks for the firms in our sample continuously traded over 1989-2017 with 3 common factors (T = 7,424 and N = 524). Factors extracted by principal component analysis on the covariance matrix of U.S. daily log equity returns.
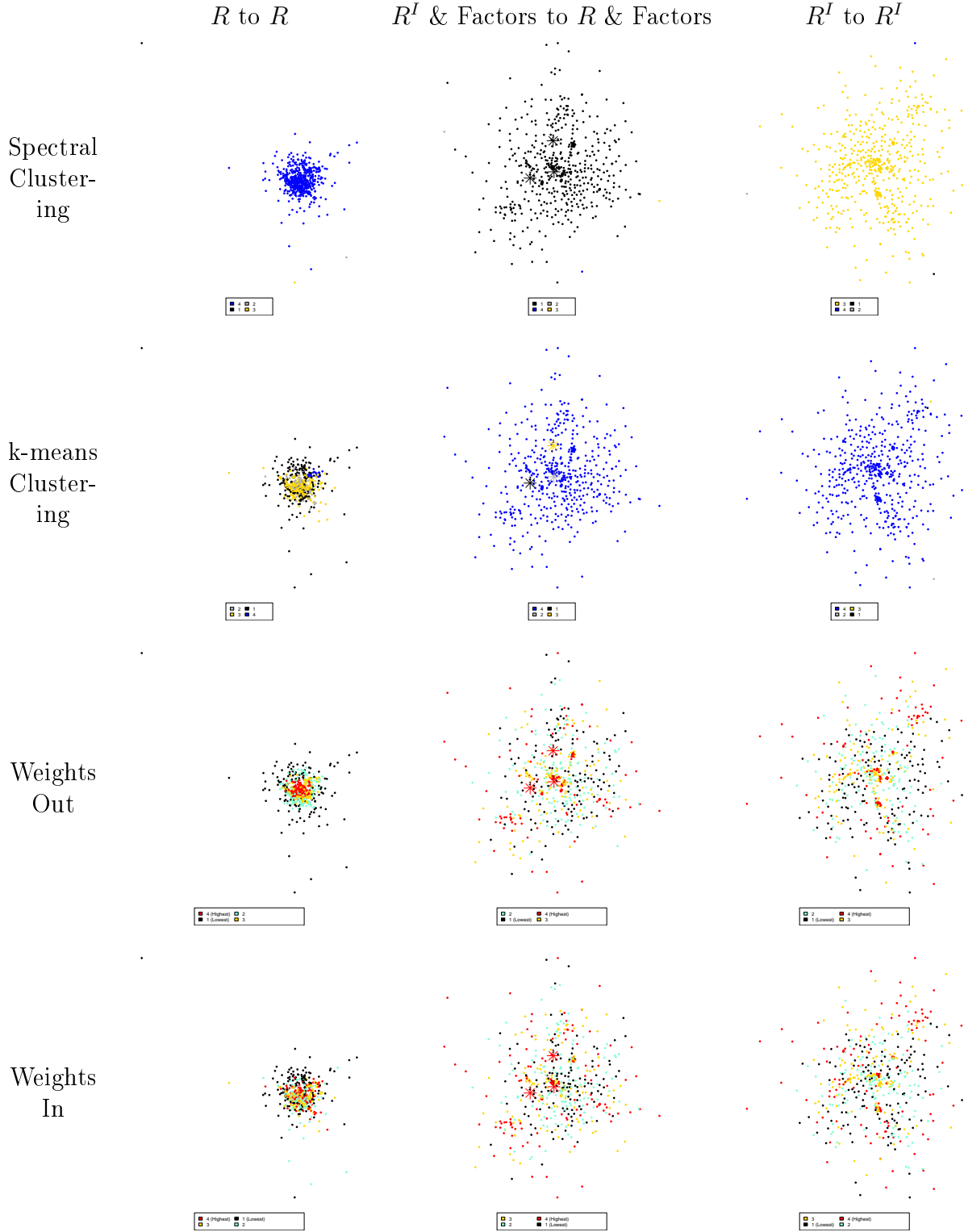
D U.S. 1989-2017 Estimated Network Robustness & Simulations

Figure D.1: U.S. Firm Network Spring Plots (1989-2017): Correlation PCA
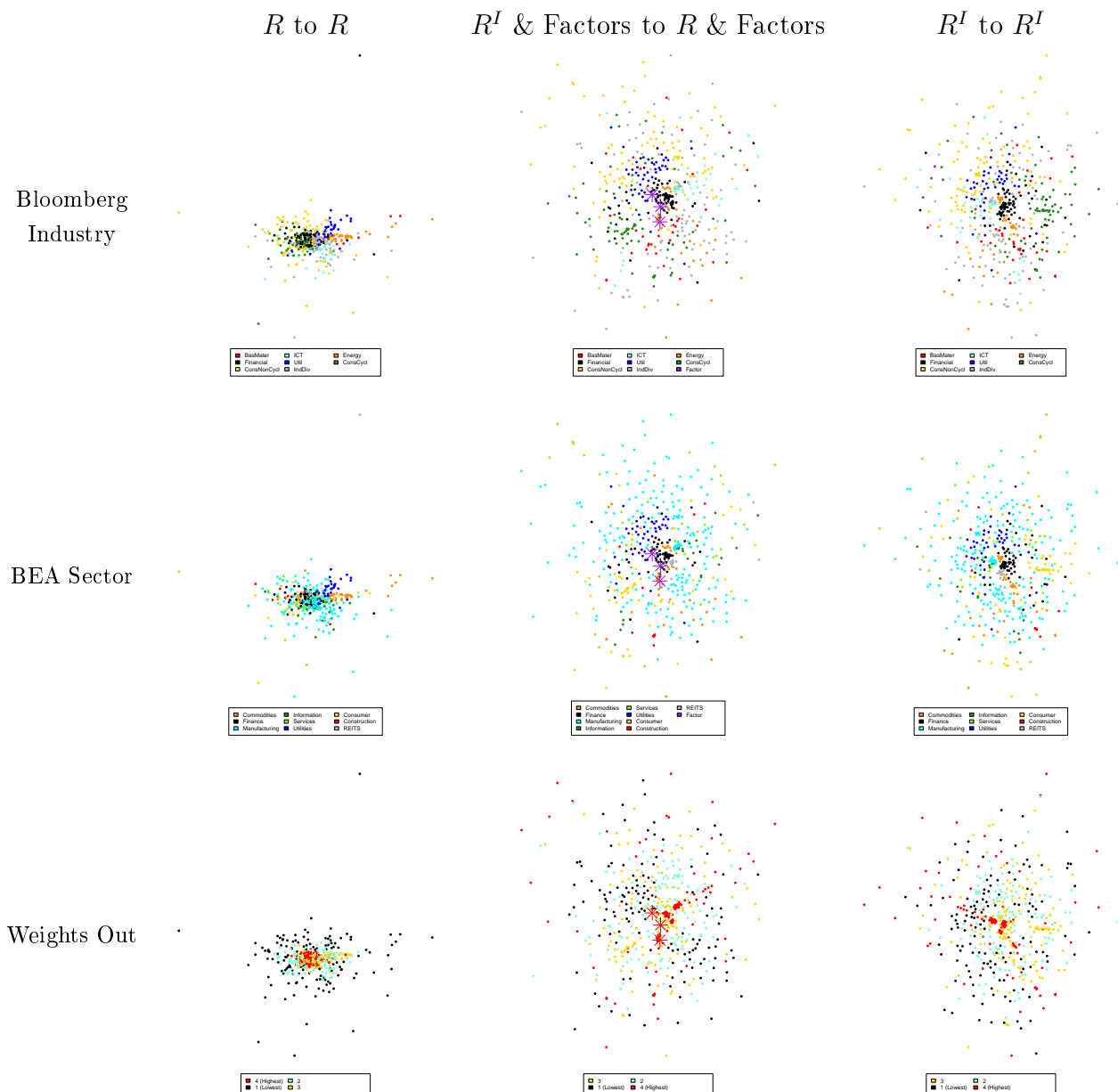
Note: Sample includes the firms continuously traded over 1989-2017 with factors estimated by principal component analysis (PCA) on the correlation matrix of U.S. log equity returns with 3 common factors (T = 7,424 and N = 524).
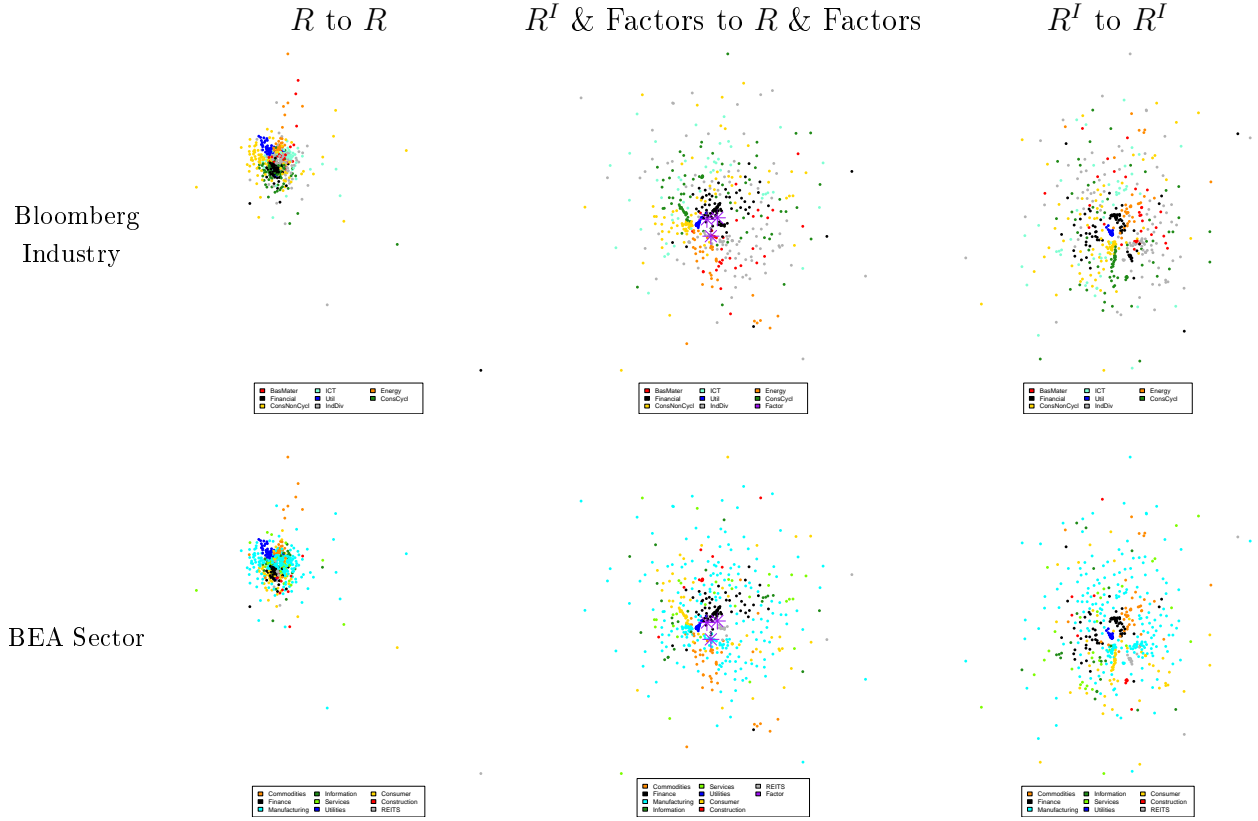
Figure D.2: U.S. Firm Network Detail Spring Plots (1989-2017)

R to R R^I & Factors to R & Factors R^I to R^I

Spectral
Cluster-
ing

k-means
Cluster-
ing

Weights
Out

Weights
In

Note: Sample includes the 524 firms continuously traded over 1989-2017. GFEVc networks with 3 common factors (T = 7,424 and N = 524).
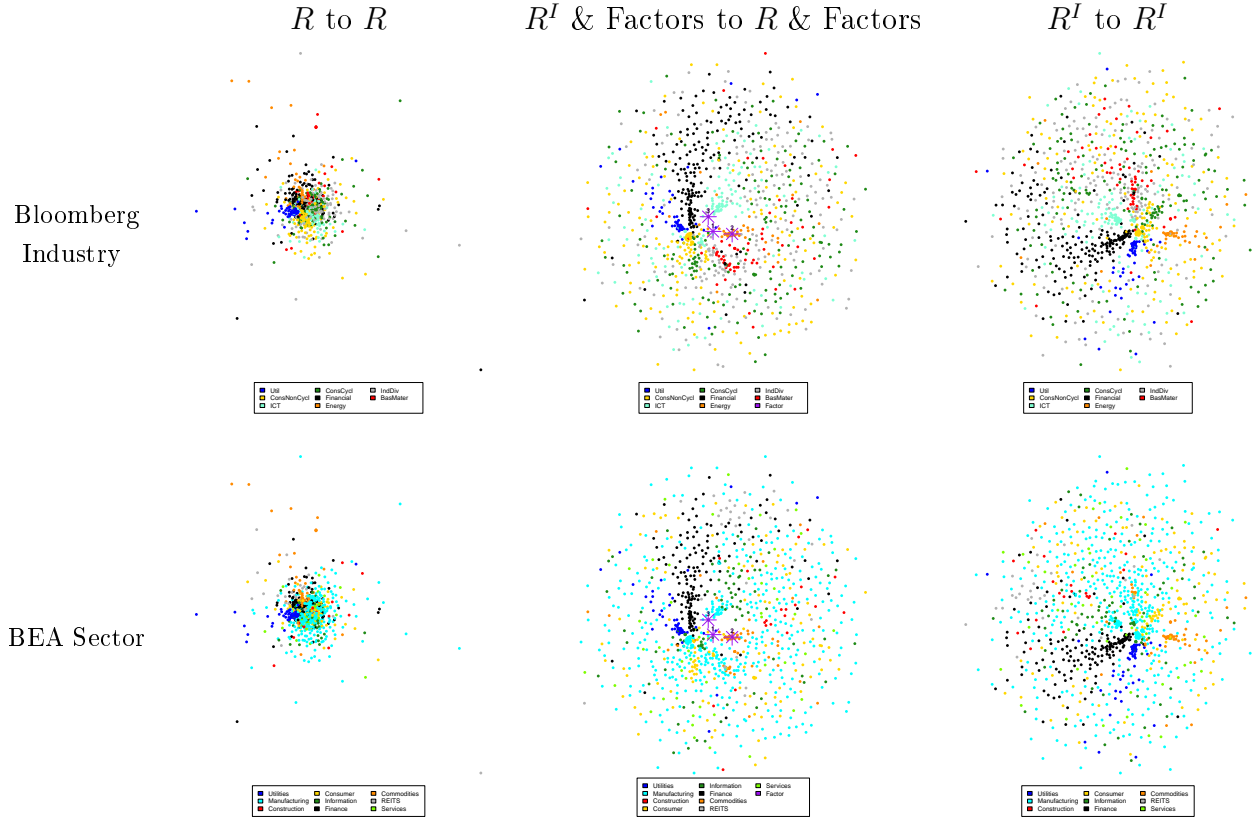
D.1 GFEVD Networks

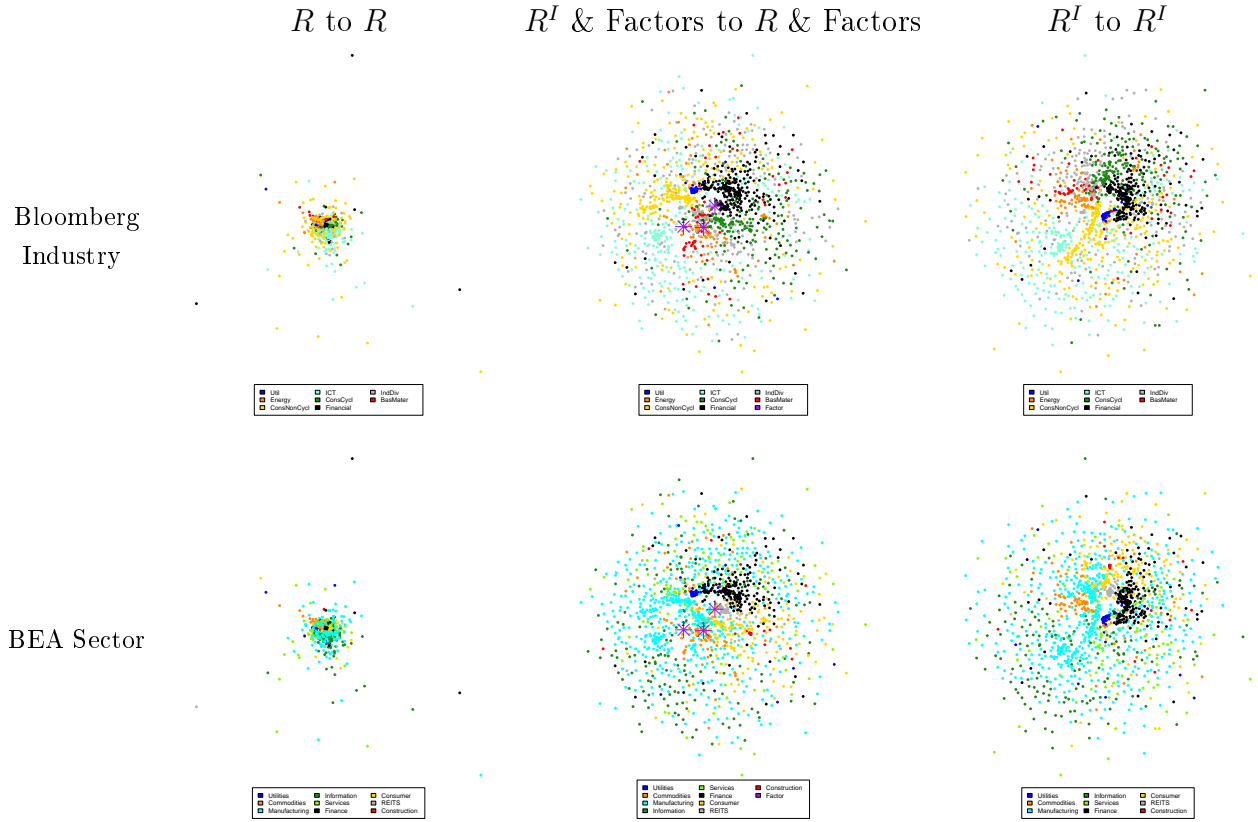Figure D.3: U.S. Firm Network Spring Plots (1989-2017): GFEVD

Note: Sample includes the firms continuously traded over 1989–2017 estimated using generalized forecast error variance decompositions (GFEVD) instead of contributions. (T = 7,424 and N = 524)

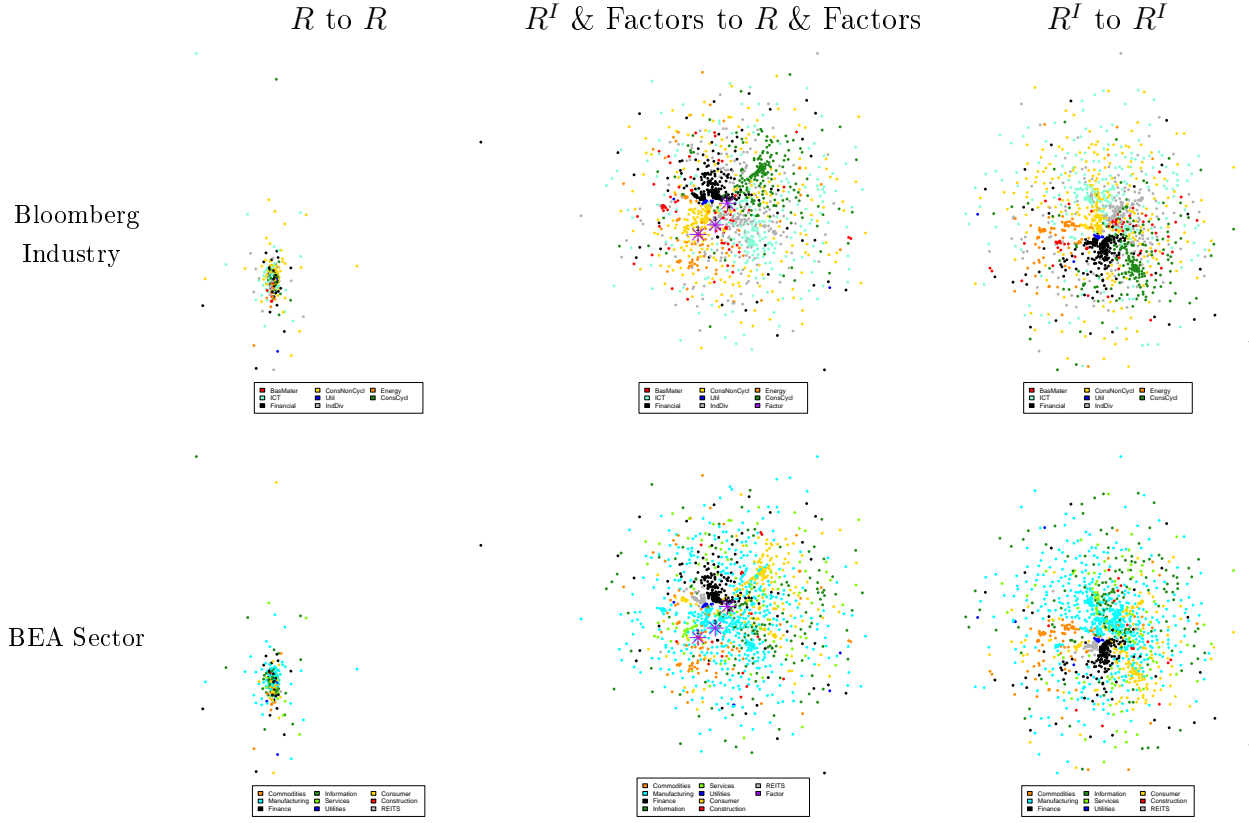# Figure D.4: U.S. Firm Network Spring Plots (1990-1999): GFEVD

	R to R	R^I & Factors to R & Factors	R^I to R^I
Bloomberg Industry			
BEA Sector			

Note: Sample includes the firms continuously traded over 1990-1999 estimated using generalized forecast error variance decompositions (GFEVD) instead of contributions.

Figure D.5: U.S. Firm Network Spring Plots (2000-2009): GFEVD

R to R	R^I & Factors to R & Factors	R^I to R^I

Bloomberg Industry

BEA Sector

Note: Sample includes the firms continuously traded over 2000-2009 estimated using generalized forecast error variance decompositions (GFEVD) instead of contributions.
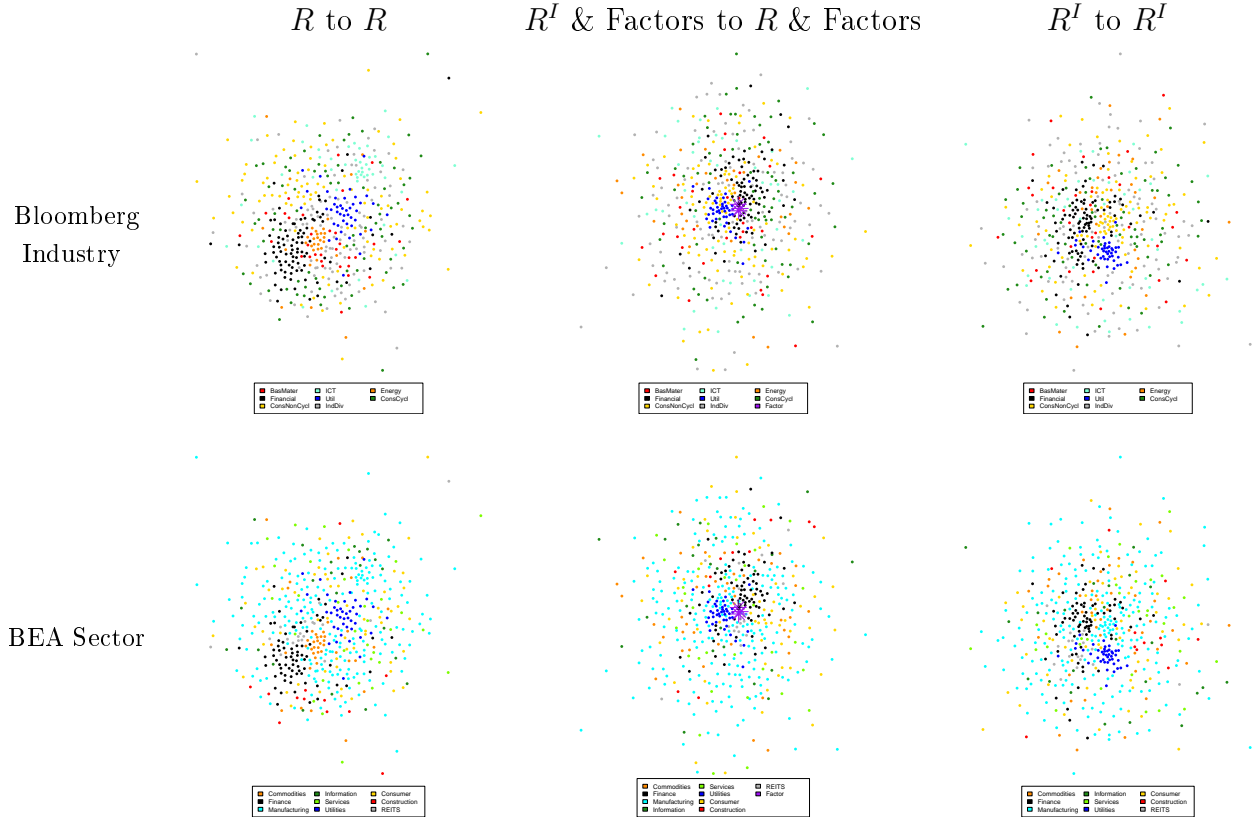
Figure D.6: U.S. Firm Network Spring Plots (2010-2017): GFEVD

R to R R^I & Factors to R & Factors R^I to R^I

Bloomberg
Industry

BEA Sector

Note: Sample includes the firms continuously traded over 2010-2017 estimated using generalized forecast error variance decompositions (GFEVD) instead of contributions.
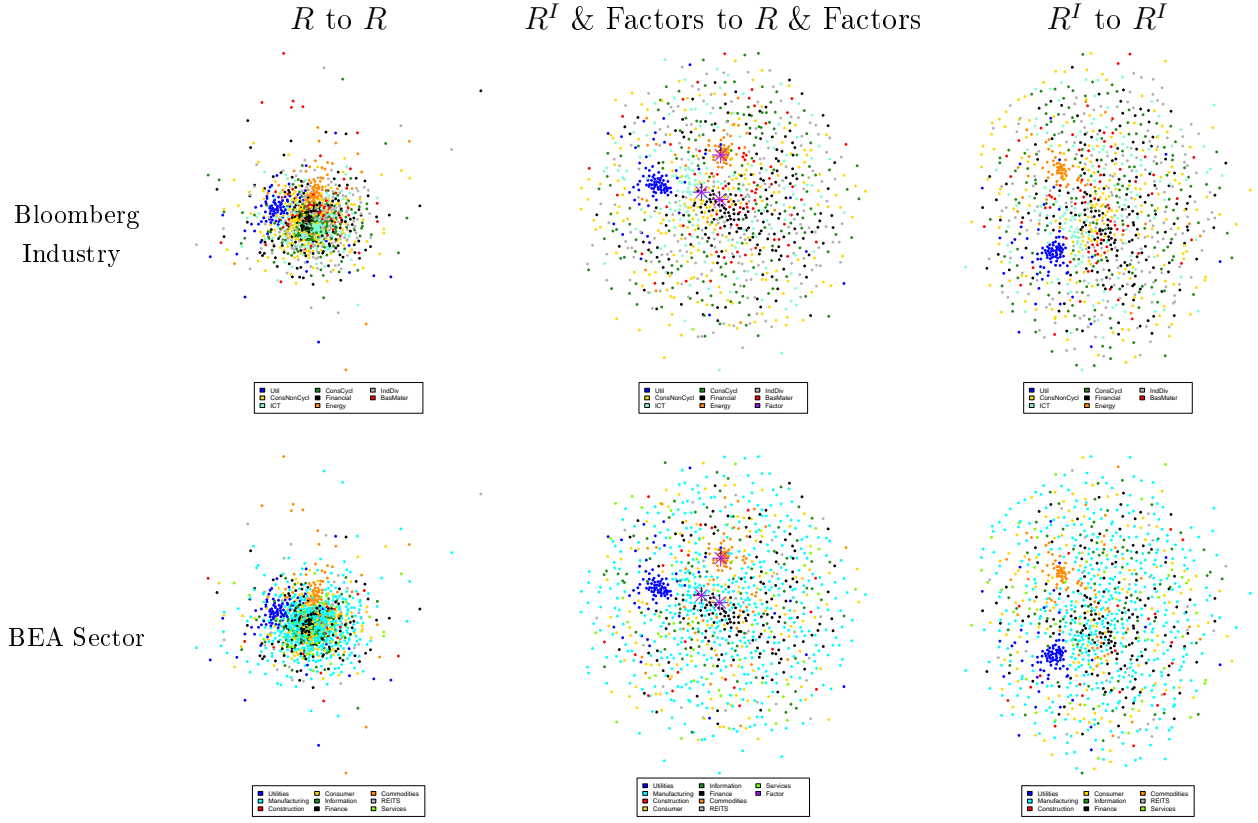
D.2 GIRF Networks

Figure D.7: U.S. Firm Network Spring Plots (1989-2017): GIRF

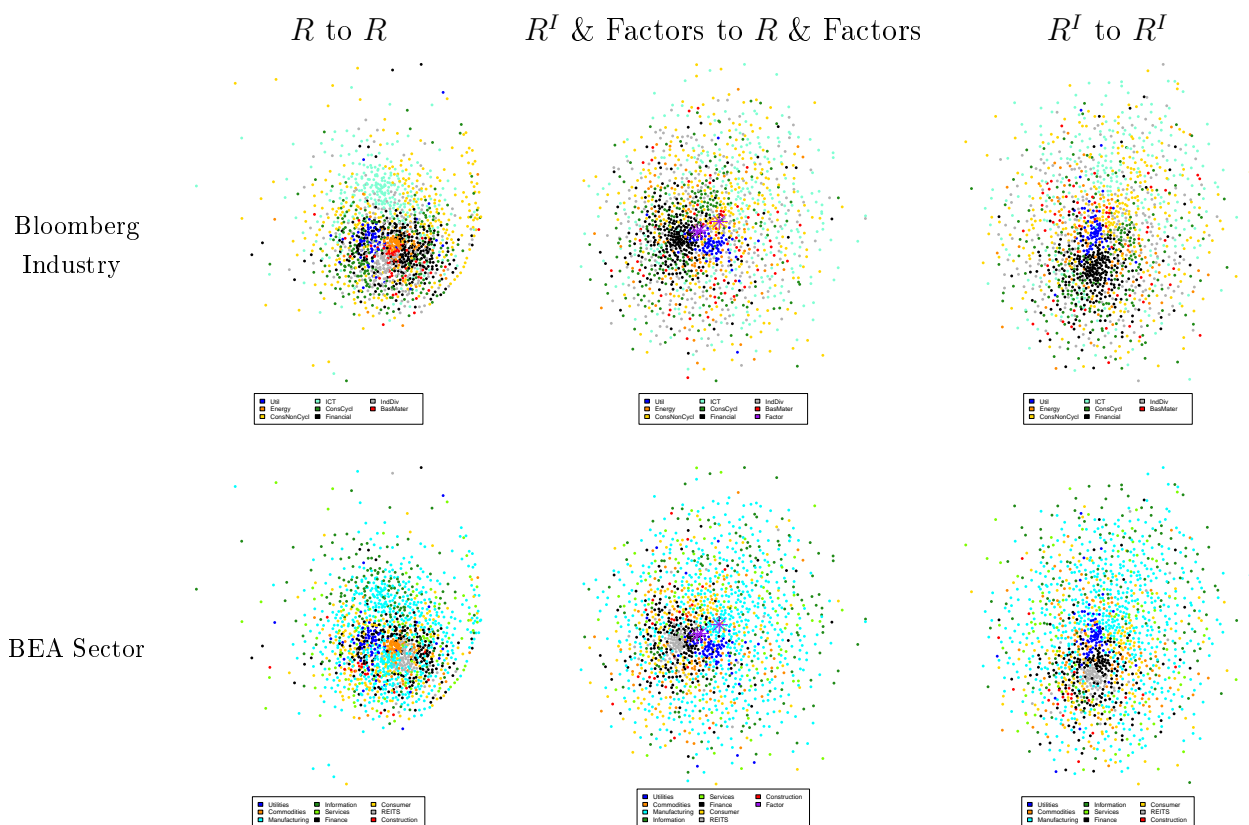|	R to R	R^I & Factors to R & Factors	R^I to R^I

Bloomberg Industry

BEA Sector

Note: Sample includes the firms continuously traded over 1989-2017 estimated using the absolute value of generalized impulse response functions (GIRF) instead of GFEVc.

Figure D.8: U.S. Firm Network Spring Plots (1990-1999): GIRF

R to R　　　　R^I & Factors to R & Factors　　　　R^I to R^I
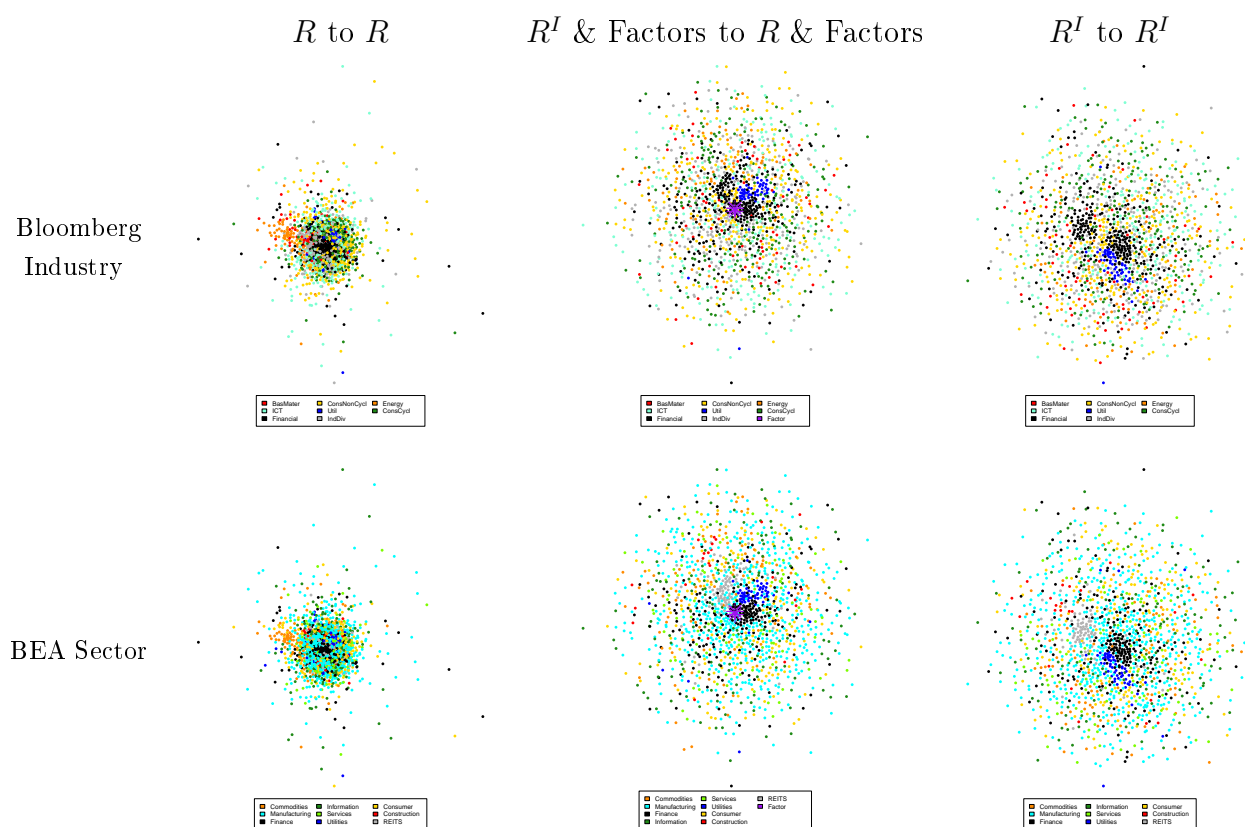
Bloomberg Industry

BEA Sector

Note: Sample includes the firms continuously traded over 1990-1999 estimated using the absolute value of generalized impulse response functions (GIRF) instead of GFEVc.

Figure D.9: U.S. Firm Network Spring Plots (2000-2009): GIRF

R to R $\qquad$ R^I & Factors to R & Factors $\qquad$ R^I to R^I

Bloomberg
Industry

BEA Sector

Note: Sample includes the firms continuously traded over 2000-2009 estimated using the absolute value of generalized impulse response functions (GIRF) instead of GFEVc.

Figure D.10: U.S. Firm Network Spring Plots (2010-2017): GIRF

R to R R^I & Factors to R & Factors R^I to R^I

Bloomberg Industry

BEA Sector

Note: Sample includes the firms continuously traded over 2010-2017 estimated using the absolute value of generalized impulse response functions (GIRF) instead of GFEVc.
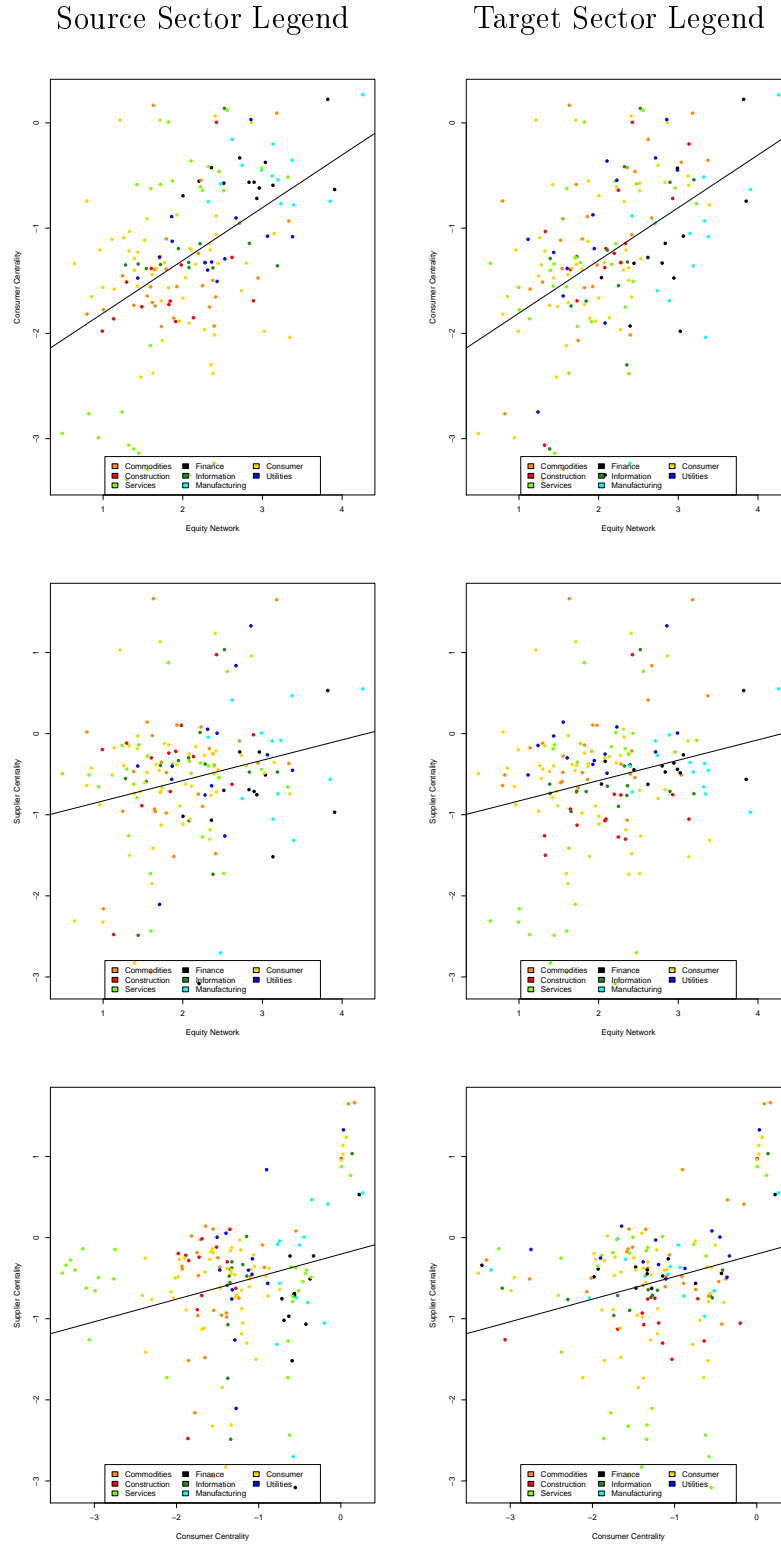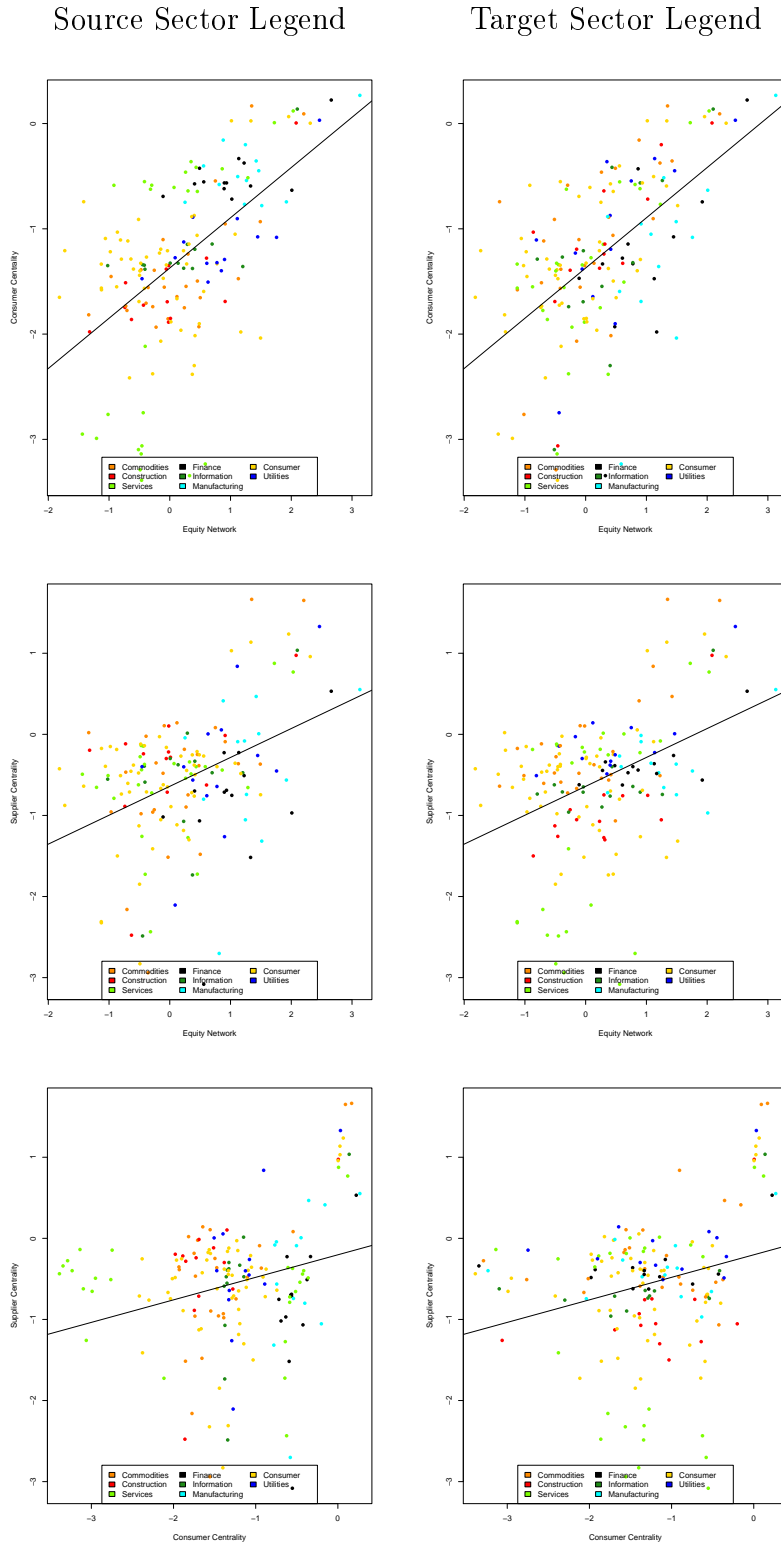
D.3 Network Centralities & Network Edge Weights Comparisons

Figure D.11: Centralities Spring Plots (1989-2017): R to R

Note: Logs of network edge weights of 1989-2017 generalized forecast error variance contribution networks with 3 common factors against 2001 input-output matrix based centralities.

Figure D.12: Centralities Spring Plots (1989-2017): R^I to R^I

Source Sector Legend

Target Sector Legend

Note: Logs of network edge weights of 1989-2017 generalized forecast error variance contribution networks with 3 common factors against 2001 input-output matrix based centralities.

Table D.1: Firm Equity Return Bilateral Correlation Networks vs. Equity & I/O Based Networks

Panel A: 1989-2017 Network

R to R, Firm Level	R to R, Sector Level	R^I to R^I, Firm Level	R^I to R^I, Sector Level	Raw IO	IO Output Normalized	Leon-tief Inverse	Up-stream Expo-sure	Down-stream Exposure	Upstream + Down-stream Exposure
0.63***	0.99***	0.30***	0.83***	0.79***	0.47**	0.35**	0.40***	0.02	0.04

Panel B: Average Across 10-Year Networks with Maximum Number of Firms Ending 1998-2017

R to R, Firm Level	R to R, Sector Level	R^I to R^I, Firm Level	R^I to R^I, Sector Level	Raw IO	IO Output Normalized	Leon-tief Inverse	Up-stream Expo-sure	Down-stream Exposure	Upstream + Down-stream Exposure
0.54	0.98	0.19	0.85	0.75	0.44	0.34	0.41	0.01	0.03

Note: The firms in each sample are those that are continuously traded throughout the corresponding period. Bilateral firm equity return correlation networks. GFEVc 3 factor networks compared in first four columns. In the top panel, *** p<0.01, ** p<0.05, * p<0.1. Note that the results for the R^I & Factors to R & Factors, and R^I to R^I networks are identical by construction, so we only include results for the latter.

Table D.2: Firm Monthly Equity Return vs. Input-Output Based Networks Over Time

EQ Network Period	IO Year	R to R Network Correlations					R^I to R^I Network Correlations				
		IO Output Normalized	Leontief Inverse	Upstream Exposure	Downstream Exposure	Upstream + Downstream Exposure	IO Output Normalized	Leontief Inverse	Upstream Exposure	Downstream Exposure	Upstream + Downstream Exposure
1989-1998	1997	0.54**	0.42**	0.46***	0.06	0.08	0.54**	0.46**	0.49***	0.08	0.11
1990-1999	1997	0.55**	0.43**	0.48***	0.08	0.10	0.54**	0.45**	0.48***	0.07	0.09
1991-2000	1997	0.55**	0.43**	0.48***	0.08	0.10	0.54**	0.45**	0.48***	0.07	0.09
1992-2001	1997	0.54**	0.43**	0.47***	0.07	0.09	0.54**	0.45**	0.48***	0.07	0.09
1993-2002	1998	0.54**	0.42**	0.46***	0.06	0.08	0.54**	0.45**	0.48***	0.06	0.09
1994-2003	1999	0.53**	0.41**	0.45***	0.06	0.08	0.53**	0.44**	0.47***	0.06	0.08
1995-2004	2000	0.52**	0.41**	0.45***	0.05	0.08	0.52**	0.44**	0.47***	0.06	0.09
1996-2005	2001	0.51**	0.40**	0.43***	0.06	0.08	0.51**	0.43**	0.46***	0.06	0.09
1997-2006	2002	0.51**	0.40**	0.43***	0.06	0.08	0.51**	0.43**	0.46***	0.06	0.08
1998-2007	2003	0.50**	0.39**	0.43***	0.05	0.07	0.50**	0.43**	0.46***	0.06	0.09
1999-2008	2004	0.50**	0.38**	0.43***	0.03	0.05	0.51**	0.43**	0.47***	0.07	0.09
2000-2009	2005	0.48**	0.38**	0.43***	0.02	0.05	0.51**	0.44**	0.49***	0.07	0.10
2001-2010	2006	0.49**	0.37**	0.43***	0.02	0.04	0.51**	0.45**	0.49***	0.07	0.10
2002-2011	2007	0.49**	0.38**	0.44***	0.02	0.05	0.52**	0.46**	0.50***	0.09	0.12
2003-2012	2008	0.47**	0.37**	0.43***	0.02	0.05	0.49**	0.46**	0.50**	0.10	0.13
2004-2013	2009	0.46**	0.35**	0.41***	0.02	0.04	0.48**	0.44**	0.48***	0.11	0.14
2005-2014	2010	0.48**	0.36**	0.42***	0.02	0.05	0.48**	0.45**	0.50**	0.13	0.16
2006-2015	2011	0.49**	0.37**	0.44**	0.02	0.05	0.49**	0.46**	0.50**	0.14	0.17
2007-2016	2012	0.48**	0.37**	0.44***	0.02	0.05	0.49**	0.45**	0.50**	0.14	0.17
2008-2017	2013	0.50**	0.38**	0.45***	0.04	0.07	0.49**	0.46**	0.50**	0.16	0.18
1989-2017	2001	0.49**	0.39**	0.42***	0.04	0.07	0.52**	0.45**	0.48***	0.09	0.12

Note: Rolling GFEVc 3 factor monthly log return networks of 524 firms continuously traded over 1989-2017. *** p<0.01, ** p<0.05, * p<0.1. Results for the R^I & Factors to R & Factors, and R^I to R^I networks are identical by construction, so we only include results for the latter.

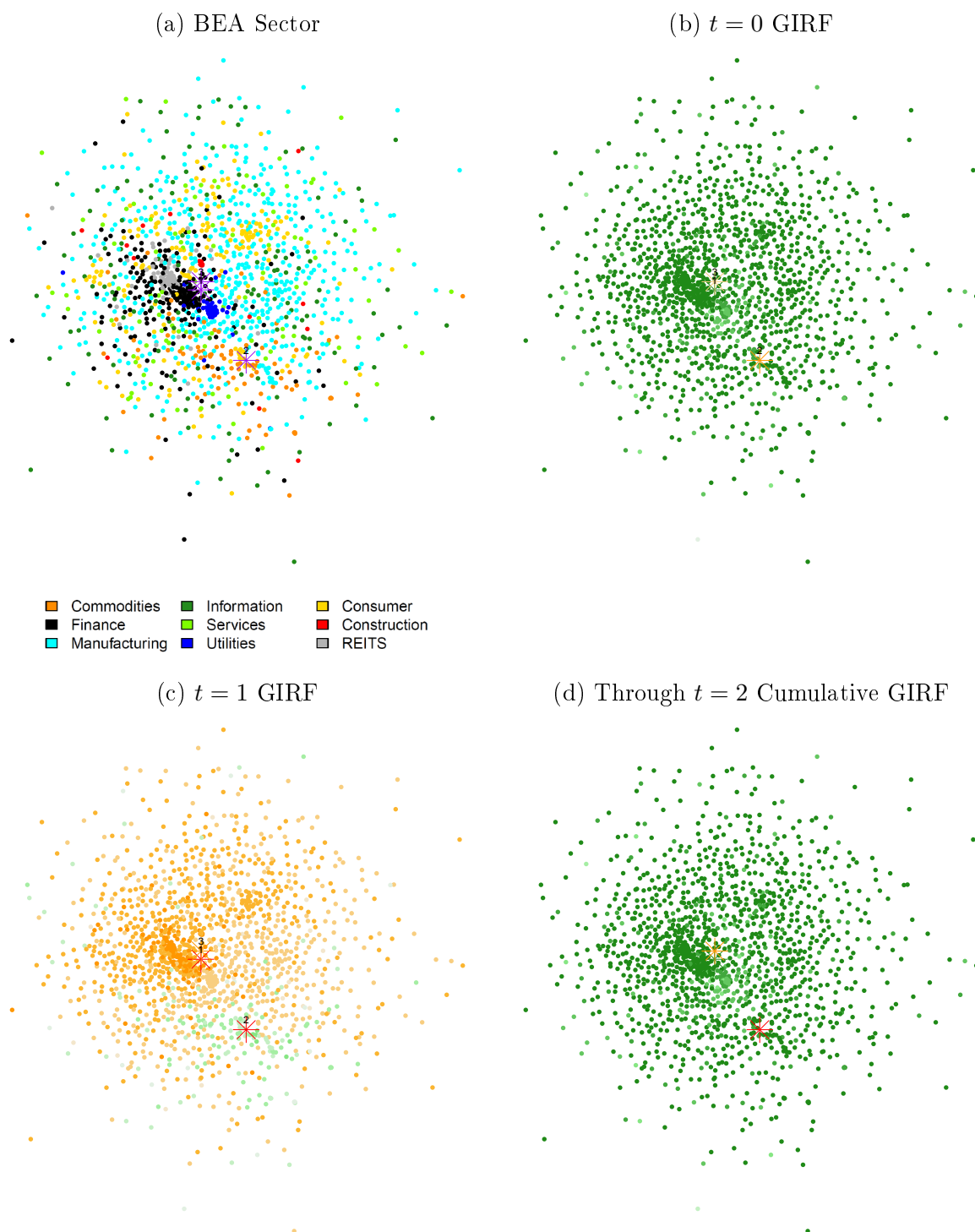## D.4 Simulations: Market Beta & Commodity Factor Shocks

For these simulations, we combine our network estimation with visualization algorithms to analyze responses to common shocks, with the novel addition of including common factors as separate nodes in the networks. We focus on the network of $1,416$ firms continuously traded over 2008-2017. In the spring plots in Appendix Figures D.13 and D.14 the location of each firm is the same, though the colors are scaled based on their expected log returns over the given period following a shock to one of the common factors, with the darkest green for returns over 1% and the darkest red for those below -1%. Most of the price movement comes on impact, with some moderate returns for a few firms the day after, and virtually zero impact in the following days, as equity markets are quick to incorporate new information. The spring plots with BEA sector legends are also included for reference.

Appendix Figure D.13 shows that financial, consumer cyclical and commodity companies would be most affected by a one standard deviation positive market beta shock at $t = 0$. Visually, this is reflected in the fact that financial firms are at the center of the network in close proximity to the first factor. Additionally, a positive move in the first factor is correlated with a negative move in the second factor, which over this period reflects a commodity price increase in that factor. The second factor here has a year-over-year growth correlation of -0.79 with Brent crude oil and -0.74 with the Goldman Sachs Commodity Index. These plots show which firms are most driven by the market beta factor — both directly and indirectly through other firms and factors — and that the central finance firms have a high degree of influence on the network, in agreement with our findings that the majority of the top 25 firms by total return network weights out are financial firms (see Appendix Table C.4). Further, all of the firms' cumulative returns through $t = 2$ are positive, consistent with the findings in Figure C.1 that firms load positively on the first factor.

Appendix Figure D.14 shows the effect of a commodity price drop as measured by a one standard deviation shock to the second factor. Lower commodity prices would unsurprisingly most adversely affect energy and base materials companies. In fact, the top 10 declining equities following the shock are for firms in the oil and gas extraction sub-sector. On the other hand, United Continental Holdings — the parent company of United Airlines — would be expected to have the largest positive response. This result likely reflects the high fuel costs faced by airlines, and recognizing this could be used by an airline's managers as an indicator that it should hedge its commodity exposure. The other firms in the top ten highest expected equity returns are banks and REITs, possibly because lower commodity prices benefit firms in other sectors of the economy that would be passed onto them. It is not only these central sectors mentioned that increase, but the consumer cyclical firms also have positive returns, supporting this interpretation. Overall, the responses to a shock to commodity prices have more variation in firm returns than those to the first factor, with some cumulative returns positive and some negative after two days.
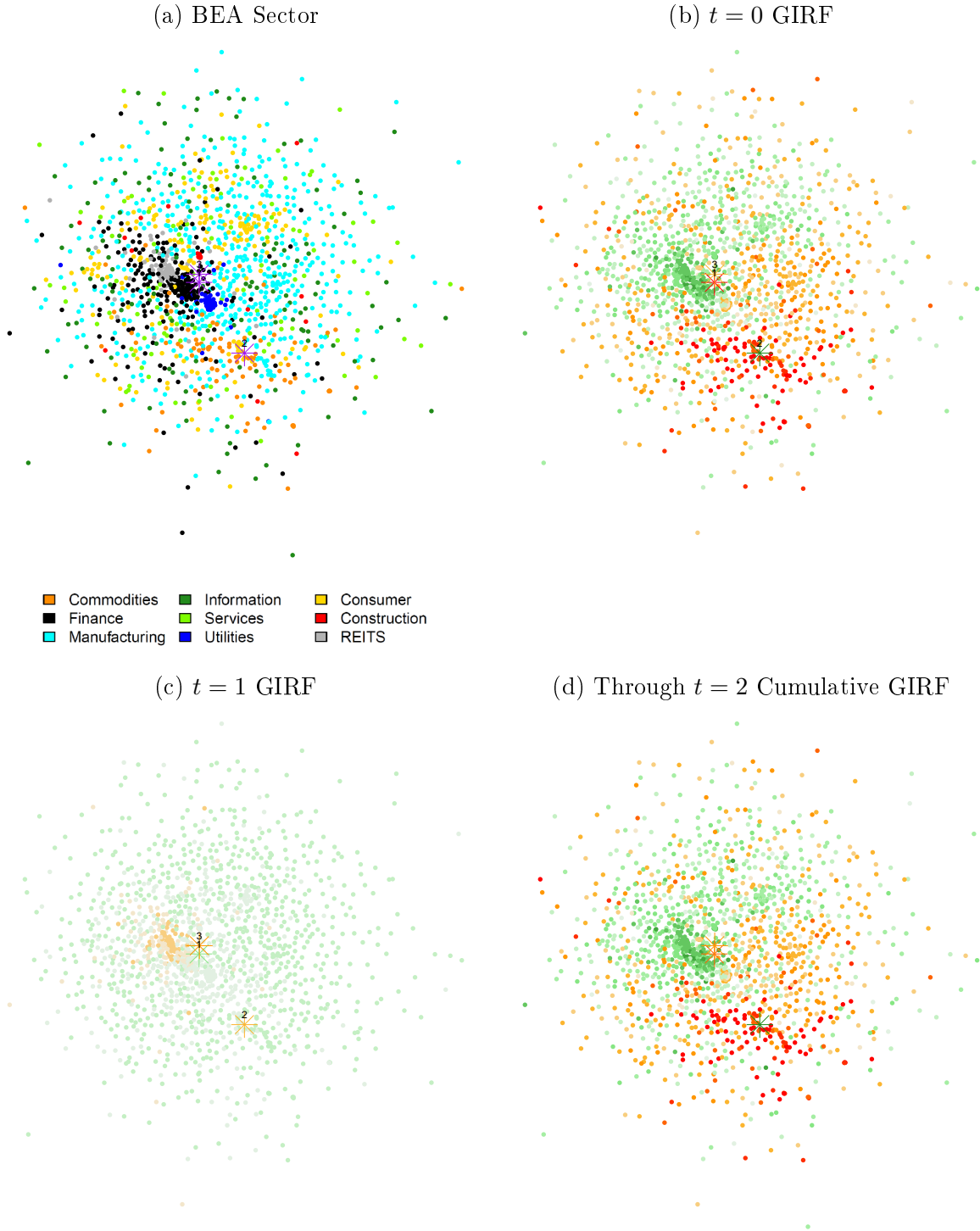
These observations of the expected inter-firm dynamics can help managers and policy-makers analyze potential exposures to common shocks, and inform investors' diversification choices to confront the systemic risk they face, as in the case of a fund that is long airlines recognizing the latent commodity risk factor in that investment by also going long oil and gas extraction firms.

Figure D.13: Positive Market Beta Shock for the U.S. Firm Network (2008-2017)

(a) BEA Sector

(b) $t = 0$ GIRF

Commodities   Information   Consumer
Finance   Services   Construction
Manufacturing Utilities   REITS

(c) $t = 1$ GIRF

(d) Through $t = 2$ Cumulative GIRF

Note: Networks are for the portion of our sample continuously traded from 2008-2017 with 3 factors. Factors extracted by covariance PCA on the matrix of daily log equity returns. The shock is a positive one to the first factor, correlating with a positive market beta shock. The node colors represent the return impact on each firm from $> 1\%$ in dark green to $< -1\%$ in red.

Figure D.14: Commodity Price Decline Shock for the U.S. Firm Network (2008-2017)

(a) BEA Sector

(b) $t = 0$ GIRF

Commodities | Information | Consumer
Finance | Services | Construction
Manufacturing | Utilities | REITS

(c) $t = 1$ GIRF

(d) Through $t = 2$ Cumulative GIRF

Note: Networks are for the portion of our sample continuously traded from 2008-2017 with 3 factors. Factors extracted by covariance PCA on the matrix of daily log equity returns. The shock is a positive one to the second factor, which over this period correlates with a commodity price decline. The node colors represent the return impact on each firm from $> 1\%$ in dark green to $< -1\%$ in red.

E DSGE Model Extension: TFP, Commodities & Finance

This section describes the model environment for an extension of our basic model, and studies the analytical solution of the model in response to productivity and taste shocks. The model has two sets of agents acting in discrete time: a unit continuum of identical households; and firms divided across J sectors, each producing a single differentiated product.

Each period proceeds in three stages. In the first stage at the beginning of the period, the households — who are the sole owners of inter-period capital — determine how to allocate their labor, and capital holdings across renting to the firms and a technology to produce further capital for tomorrow. Each firm determines how much labor, raw commodities and capital to employ, the amount of other firms' output to use as intermediate inputs in its production process, and the prices it will charge. In the second stage, production of goods and next period capital occur, and the productivity and consumer taste shocks for $t+1$ are realized. In the final stage, the firms pay the households their wages, return on capital, and profits as equity dividends. The households make their consumption and firm equity purchases.

E.1 Household's Problem

The representative household maximizes expected discounted utility:

$$E_0 \sum_{t=0}^{\infty} \psi_t U_t$$

where

$$U_t = \left(\sum_{j=1}^{J} \beta_{tj}^{\frac{1}{\sigma}} c_{tj}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}.$$

U_t is the total consumption index, the c_{tj} are industry composite consumption indexes, the β_{tj} are consumer taste weights across industries' goods and services, and σ is the inter-industry elasticity of substitution. The industry composite consumption indexes are defined as:

$$c_{tj} = \left(M_j^{-\psi_j} \int_{M_j} c_t(j,i)^{\frac{\varphi_j-1}{\varphi_j}} di \right)^{\frac{\varphi_j}{\varphi_j-1}},$$

where $c_t(j,i)$ is consumption of the output from firm i in industry j, M_j is the mass of firms in industry j, and φ_j is the intra-industry elasticity of substitution across varieties. $M_j^{-\psi_j}$ controls the love of variety effect: if $\psi_j = \frac{1}{\varphi_j}$ then there is no love of varieties, and $\psi_j = 0$ yields the standard Dixit and Stiglitz (1977) demand system.

Entering the period the household has capital holding K_t. Each unit of capital can only be applied to one use in the current period, so the household must choose how to allocate it across investment in further capital for tomorrow, I_t, and the amount to rent to firms, K_t^r, at a rate of r_t so that:

$$K_t = I_t + K_t^r.$$

Capital depreciates at a rate δ, and the law of motion for capital is:

$$K_{t+1} = \alpha_t I_t^\phi + (1 - \delta)K_t$$

where the first term on the right side reflects the technology for producing further capital. The household inelastically supplies one unit of labor each period. The labor and capital markets are both perfectly competitive, with all participants taking prices as given.

The second stage is production, during which the state for period $t + 1$ is drawn, determining the productivity (v_{t+1j}, A_{t+1j}) and taste shocks (β_{t+1j}). Post-production, the household makes its consumption and firm equity purchases. The household budget constraining these choices is:

$$\sum_{j=1}^{J} \int_{M_j} p_t(j, i) c_t(j, i) di = B_t$$

where

$$B_t \equiv w_t + K_t^r r_t + \sum_{j=1}^{J} \int_{M_j} s_t(j, i) q_t(j, i) di - \sum_{j=1}^{J} \int_{M_j} s_{t+1}(j, i)[q_t(j, i) - \pi_t(j, i)] di$$

and $p_t(j, i)$ is the price of the good from firm i in industry j. The $q_t(j, i)$ are the cum-dividend firm equity prices, $s_t(j, i)$ are the holdings of those equities and $\pi_t(j, i)$ are the profits repaid as dividends to the equity holders. There is a unit supply of each firm's equity with initial equal holdings across the households.

E.2   Firms' Problem

Within each sector j there is a measure M_j continuum of firms taking part in monopolistic competition, and the firms use labor, raw commodities, capital, and other firms' goods as inputs to their production processes. These inputs must be paid for in advance of production, requiring the firms to borrow short-term to cover their costs at an externally determined industry interest rate i_{tj}. Since the firms do not have any inter-period choice variables they solve a series of independent problems each period seeking to maximize profits:

$$\pi_t(j, i) = p_t(j, i) \left[c_t(j, i) + D_t(j, i) \right]$$
$$-i_{tj} \left[w_t L_t(j, i) + r_t K_t(j, i) + m_t d_t(j, i) + \sum_{l=1}^{J} \int_{M_l} p_t(l, n) x_t(j, i, l, n) dn \right].$$

Firm i in industry j makes its intermediate input decision for purchases of good n from industry l, $x_t(j, i, l, n)$, and agrees on prices for those, $p_t(l, n)$, before production occurs. $D_t(j, i) \equiv \sum_{l=1}^{J} \int_{M_l} x_t(l, n, j, i) dn$ is the total demand for good i from other firms to use as an intermediate input. The amounts of labor, $L_t(j, i)$, capital, $K_t(j, i)$, and raw commodities, $d_t(j, i)$, employed by firm i are also decided upon in the first stage. The price of the raw commodities, m_t, is determined exogenously in the world market.

The firm's production function is:

$$y_t(j, i) = A_{tj} \bar{y}_t(j, i) = A_{tj} \left[v_{tj}^{\frac{1}{\sigma}} \left(K_t(j, i)^\gamma L_t(j, i)^{1-\gamma} \right)^{\frac{\sigma-1}{\sigma}} + \sum_{l=1}^{J} \omega_{jl}^{\frac{1}{\sigma}} x_t(j, i, l)^{\frac{\sigma-1}{\sigma}} + \theta_{tj}^{\frac{1}{\sigma}} d_t(j, i)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

where $x_t(j, i, l)$ is the intermediate input index of goods from industry l used by firm i in industry

j:

$$x_t(j,i,l) = \left(M_l^{-\psi_l} \int_{M_l} x_t(j,i,l,n)^{\frac{\varphi_l-1}{\varphi_l}} \, dn \right)^{\frac{\varphi_l}{\varphi_l-1}}.$$

The A_{tj}, v_{tj}, and θ_{tj} are industry productivity shocks realized during production in the prior period. The ω_{jl} are share parameters for the goods of industry l in production for industry j, and the $J \times J$ matrix of these entries, Ω, drives the real firm network of the economy.

Finally, the amount that firm i sells to the households will be its final output minus the quantity it already sold to other firms:

$$c_t(j,i) = A_{tj}\bar{y}_t(j,i) - D_t(j,i).$$

E.3   Solution to the Household's Problem

Working backwards from the end of a period, the solution to the household's problem allows us to derive the demand for goods given prices, and the capital investment decision rule. To begin, the household demand for the goods of firm i in industry j is:

$$c_t(j,i) = \beta_{tj} U_t M_j^{-\psi_j\varphi_j} \left(\frac{p_t(j,i)}{P_{tj}} \right)^{-\varphi_j} \left(\frac{P_{tj}}{P_{ct}} \right)^{-\sigma}$$

where P_{tj} is the industry j price index:

$$P_{tj} \equiv \left(M_j^{-\psi_j\varphi_j} \int_{M_j} p_t(j,i)^{1-\varphi_j} \, di \right)^{\frac{1}{1-\varphi_j}},$$

and P_{ct} is the aggregate consumption price index:

$$P_{ct} \equiv \left(\sum_{j=1}^{J} \beta_{tj} P_{tj}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}.$$

Also, note that the household utility equals the units of the consumption aggregate that it can buy:

$$U_t = \frac{B_t}{P_{ct}}.$$

The cum-dividend equity price equation — derived by iterating the household's first order condition for $s_{t+1}(j,i)$ and utilizing its equity transversality condition — is:

$$q_t(j,i) = P_{ct} E_t \sum_{\tau=0}^{\infty} \frac{\psi_{t+\tau}}{\psi_t} \frac{\pi_{t+\tau}(j,i)}{P_{c,t+\tau}}.$$

Moving on to the first stage quantities, the capital investment decision rule trades off the real rent to be made from leasing the capital to firms this period from the marginal real rent in future periods:

$$\frac{r_t}{P_{ct}} = \alpha_t \phi I_t^{\phi-1} \sum_{\tau=1}^{\infty} \frac{\psi_{t+\tau}}{\psi_t} (1-\delta)^{\tau-1} E_t \frac{r_{t+\tau}}{P_{c,t+\tau}}.$$

E.4   Solution to the Firms' Problem

We begin the solution to the firms' problem by examining their cost minimization problem. For simplicity, we assume the symmetric equilibrium where firms within the same industry all make identical choices each period. This is helpful both for calculating the demand for intermediate inputs and in simplifying the firms' profit maximization problem. Specifically, the marginal (and average) cost for firm i in industry j to increase its $\bar{y}_t(j,i)$ unit output is:

$$\lambda_{tj} = i_{tj} \left[v_{tj} \tilde{z}_t^{\sigma-1} \tilde{R}_t^{1-\sigma} + \theta_{tj} m_t^{1-\sigma} + \sum_{l=1}^{J} \omega_{jl} P_{tl}^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$$

where the labor and capital aggregate, $\tilde{z}_t$, is equal to:

$$\tilde{z}_t \equiv K_t^{r\gamma} L_t^{1-\gamma} = K_t^{r\gamma}.$$

Each firm will utilize a share, $sh_t(j,i)$ of capital and labor, and therefore that same share of this aggregate. $\tilde{R}_t$ is the price for this labor-capital composite:

$$\tilde{R}_t \equiv r_t K_t^r + w_t L_t = \frac{w_t L_t}{1-\gamma} = \frac{1}{1-\gamma}$$

setting $w_t = 1$ as the numeraire of the economy. Firm i in industry j will spend a total of $sh_t(j,i)\tilde{R}_t$ on capital and labor. Firms will choose this share such that:

$$sh_t(j,i)\tilde{R}_t i_{tj} = v_{tj} \tilde{z}_t^{\sigma-1} \tilde{R}_t^{1-\sigma} i_{tj}^{1-\sigma} \lambda_{tj}^\sigma \bar{y}_t(j,i).$$

Similarly, the demand for the raw commodity satisfies:

$$d_t(j,i) m_t i_{tj} = \theta_{tj} m_t^{1-\sigma} i_{tj}^{1-\sigma} \lambda_{tj}^\sigma \bar{y}_t(j,i).$$

The demand of firm i in industry j for the good of firm n from industry l, $x_t(j,i,l,n)$, is:

$$x_t(j,i,l,n) = \omega_{jl} M_l^{-\psi_l \varphi_l} \left(\frac{p_t(l,n)}{P_{tl}} \right)^{-\varphi_l} \left(\frac{P_{tl} i_{tj}}{\lambda_{tj}} \right)^{-\sigma} \bar{y}_t(j,i)$$

From this we can calculate the total demand for firm i's goods from other firms:

$$D_t(j,i) = M_j^{-\psi_j \varphi_j} p_t(j,i)^{-\varphi_j} P_{tj}^{\varphi_j - \sigma} \sum_l M_l \omega_{lj} \lambda_{tl}^\sigma i_{tl}^{-\sigma} \bar{y}_t(l,n).$$

The firms choose inputs and set prices at the beginning of the period knowing the $c_t(j,i)$ and $D_t(j,i)$ household and firm demand curves:

$$\max_{p_t(j,i)} \pi_t(j,i) = \left[p_t(j,i) - \frac{\lambda_{tj}}{A_{tj}} \right] [c_t(j,i) + D_t(j,i)]$$

$$\Longrightarrow$$

$$p_t(j,i) = \frac{\mu_j \lambda_{tj}}{A_{tj}}; \quad \mu_j \equiv \frac{\varphi_j}{\varphi_j - 1} \tag{E.1}$$

$$\Longrightarrow$$

$$\pi_t(j,i) = \frac{1}{\varphi_j M_j} P_{tj} y_{tj}$$

$$\text{where} \quad y_{tj} \equiv \left(M_j^{-\psi_j} \int_{M_j} y_t(j,i)^{\frac{\varphi_j-1}{\varphi_j}} di \right)^{\frac{\varphi_j}{\varphi_j-1}}.$$

There are two important measures of centrality for each firm, that as a consumer of inputs and as a supplier to the households which we will derive next.

E.4.1 Consumer Centrality

Using within industry symmetry, and the ratio between firm prices and marginal costs, the prices that firms charge can be related to those of other firms and of the raw inputs:

$$A_{tj}^{1-\sigma} \mu_j^{\sigma-1} \tilde{M}_j^{1-\sigma} P_{tj}^{1-\sigma} = v_{tj} \tilde{z}_t^{\sigma-1} \tilde{R}_t^{1-\sigma} i_{tj}^{1-\sigma} + \theta_{tj} m_t^{1-\sigma} i_{tj}^{1-\sigma} + i_{tj}^{1-\sigma} \sum_{l=1}^{J} \omega_{jl} P_{tl}^{1-\sigma}$$

where $\tilde{M}_j = M_j^{\frac{\psi_j \varphi_j - 1}{1 - \varphi_j}}$. Stacking these equations for each industry, the following matrix system solves for the prices given the quantity and prices of the raw inputs, interest rates, and model parameters:

$$P_t^{1-\sigma} = \underbrace{\left[I_J - i_t^{1-\sigma} A_t^{\sigma-1} \mu^{1-\sigma} \tilde{M}^{\sigma-1} \Omega \right]^{-1} i_t^{1-\sigma} A_t^{\sigma-1} \mu^{1-\sigma} \tilde{M}^{\sigma-1}}_{\equiv \Psi_{dt}} \left[v_t \tilde{z}_t^{\sigma-1} \tilde{R}_t^{1-\sigma} + \theta_t m_t^{1-\sigma} \right]$$

where P_t, v_t and θ_t are vectors of the associated quantities, and μ, $\tilde{M}$, i_t and A_t are square matrices with the associated quantities on the diagonal. Let $\tilde{\alpha}_t \equiv \Psi_{dt} v_t$ and $\tilde{\nu}_t \equiv \Psi_{dt} \theta_t$ be vectors of the consumer centralities for the labor-capital aggregate and the raw commodity, respectively. The price index for industry j then satisfies:

$$P_{tj}^{1-\sigma} = \left[\tilde{\alpha}_{tj} \tilde{z}_t^{\sigma-1} \tilde{R}_t^{1-\sigma} + \tilde{\nu}_{tj} m_t^{1-\sigma} \right].$$

E.4.2 Supplier Centrality

The supplier centrality can be calculated by examining the total demand for a firm's goods from other industries and consumers. Using within industry symmetry, the prices that firms charge can be related to the amounts that other firms wish to produce:

$$P_{tj}^{\sigma} D_{tj} = \sum_l M_l \omega_{lj} \lambda_{tl}^{\sigma} i_{tl}^{-\sigma} \bar{y}_t(l,n)$$

$$\text{where} \quad D_{tj} \equiv \left(M_j^{-\psi_j} \int_{M_j} D_t(j,i)^{\frac{\varphi_j-1}{\varphi_j}} di \right)^{\frac{\varphi_j}{\varphi_j-1}}.$$

The consumer's demand can also be related to the industry and total price indices:

$$P_{tj}^{\sigma} c_{tj} = \beta_{tj} P_{ct}^{\sigma} U_t.$$

From these two it can be found that:

$$P_{tj}^\sigma y_{tj} = \beta_{tj} P_{ct}^\sigma U_t + \sum_l \omega_{lj} \mu_l^{-\sigma} i_{tl}^{-\sigma} A_{tl}^{\sigma-1} \tilde{M}_l^{\sigma-1} P_{tl}^\sigma y_{tl}.$$

Let $Q_{tj} \equiv P_{tj}^\sigma y_{tj}$. The supplier centrality can then be determined from the following system of these stacked total demand equations:

$$Q_t' = \beta' \underbrace{\left[I_J - i_t^{-\sigma} A_t^{\sigma-1} \mu^{-\sigma} \tilde{M}^{\sigma-1} \Omega \right]^{-1}}_{\equiv \Psi_{St}} P_{ct}^\sigma U_t = \tilde{\beta}_t' P_{ct}^\sigma U_t$$

where $\tilde{\beta}_t \equiv \Psi_{St}'\beta$ is the vector of supplier centralities. It then holds that:

$$\tilde{\beta}_{tj} = \frac{P_{tj}^\sigma y_{tj}}{P_{ct}^\sigma U_t}.$$

E.4.3   Firm Profits

The firms' revenue and profits can both be derived in terms of the economic aggregates and centralities. To derive these relationships, first note that:

$$P_{tj} y_{tj} = P_{ct}^\sigma U_t \left[\tilde{\alpha}_{tj} \tilde{z}_t^{\sigma-1} \tilde{R}_t^{1-\sigma} + \tilde{\nu}_{tj} m_t^{1-\sigma} \right] \tilde{\beta}_{tj}.$$

Plugging this into the final line of Equation (E.1) provides the following detailed profit equation:

$$\pi_t(j,i) = \underbrace{\frac{1}{\varphi_j M_j}}_{\substack{\text{Market} \\ \text{Structure}}} \underbrace{P_{ct} U_t}_{\substack{\text{GDP /} \\ \text{Aggregate} \\ \text{Demand}}} \underbrace{\left[\tilde{\alpha}_{tj} \tilde{z}_t^{\sigma-1} \left(\frac{\tilde{R}_t}{P_{ct}} \right)^{1-\sigma} + \tilde{\nu}_{tj} \left(\frac{m_t}{P_{ct}} \right)^{1-\sigma} \right]}_{\substack{\text{Resource availability relative} \\ \text{to Consumer Centrality /} \\ \text{Aggregate Supply}}} \underbrace{\tilde{\beta}_{tj}}_{\substack{\text{Supplier} \\ \text{Centrality}}}.$$

This equation provides the firm profits as a function of GDP, model parameters, and equilibrium price levels. We can further refine this by using the household budget constraint to solve for the economy's GDP $= P_{ct} U_t$:

$$P_{ct} U_t = w_t + K_t^r r_t + \sum_{j=1}^J \int_{M_j} \pi_t(j,i) di = \tilde{R}_t + \sum_{j=1}^J M_j \pi_t(j,i)$$
$$\Longrightarrow$$
$$P_{ct} U_t = \tilde{R}_t + \sum_{j=1}^J \frac{1}{\varphi_j} P_{ct} U_t \left[\tilde{\alpha}_{tj} \tilde{z}_t^{\sigma-1} \tilde{R}_t^{1-\sigma} + \tilde{\nu}_{tj} m_t^{1-\sigma} \right] \tilde{\beta}_{tj} P_{ct}^{\sigma-1}$$
$$\Longrightarrow$$
$$P_{ct} U_t = \frac{\tilde{R}_t}{1 - P_{ct}^{\sigma-1} \tilde{\beta}_t' \kappa \left[\tilde{\alpha}_t \tilde{z}_t^{\sigma-1} \tilde{R}_t^{1-\sigma} + \tilde{\nu}_t m_t^{1-\sigma} \right]}$$

where κ is a square matrix with $\frac{1}{\varphi_k}$ along the diagonal. We can further reduce this by substituting out the P_{ct} price index relative to the interest rate:

$$P_{ct}^{1-\sigma} = \sum_{j=1}^J \beta_{tj} P_{tj}^{1-\sigma} = \beta_t' P_t^{1-\sigma} = \beta_t' \left[\tilde{\alpha}_t \tilde{z}_t^{\sigma-1} \tilde{R}_t^{1-\sigma} + \tilde{\nu}_t m_t^{1-\sigma} \right]$$

Finally, substituting this back in yields:

$$P_{ct}U_t = \frac{\tilde{R}_t\beta_t'\left[\tilde{\alpha}_t\tilde{z}_t^{\sigma-1}\tilde{R}_t^{1-\sigma} + \tilde{\nu}_t m_t^{1-\sigma}\right]}{\beta_t'\left[I_J - \Psi_{St}\kappa\right]\left[\tilde{\alpha}_t\tilde{z}_t^{\sigma-1}\tilde{R}_t^{1-\sigma} + \tilde{\nu}_t m_t^{1-\sigma}\right]}$$

and

$$P_{ct}^{\sigma}U_t = \frac{\tilde{R}_t}{\beta_t'\left[I_J - \Psi_{St}\kappa\right]\left[\tilde{\alpha}_t\tilde{z}_t^{\sigma-1}\tilde{R}_t^{1-\sigma} + \tilde{\nu}_t m_t^{1-\sigma}\right]}.$$

E.5   Equity Price Dynamics

In this section we examine the derivatives of the firm equity prices with respect to productivity and demand shocks. We specifically want to focus on the idiosyncratic aspect of each firm's equity return, and what these can tell us about a firm's proximity through the inter-firm network to the source of the shock. To begin, let's assume that the economy is in steady-state. The equity price vector is then:

$$q_t = \frac{P_{ct}^{\sigma}U_t}{1-\psi}\hat{M}\left[\left(\tilde{\alpha}_t\tilde{z}_t^{\sigma-1}\tilde{R}_t^{1-\sigma} + \tilde{\nu}_t m_t^{1-\sigma}\right) \circ \tilde{\beta}_t\right]$$

where $\hat{M}$ is a square matrix with $\frac{1}{\varphi_j M_j}$ along the diagonal. To study the equity return response to a shock in the individual factor productivities or taste parameters we will examine the change in the log equity price of firm i in industry j:

$$ln(q_t(j,i)) = ln\left(\frac{1}{\varphi_j M_j(1-\psi)}\right) + ln(P_{ct}^{\sigma}U_t) + ln\left(\tilde{\alpha}_{tj}\tilde{z}_t^{\sigma-1}\tilde{R}_t^{1-\sigma} + \tilde{\nu}_{tj}m_t^{1-\sigma}\right) + ln(\tilde{\beta}_{tj}).$$

The first term is a constant that will be unaffected, and the second term represents the aggregate response that will be common across firms. Since we are interested in the idiosyncratic response of each firm's equity price we will ignore these and focus on the latter two terms. The log change in the equity price from these two, approximating the idiosyncratic equity return, would be:

$$dln(q_t^*(j,i)) = \frac{\iota_j'\Psi_{dt}\left[\tilde{z}_t^{\sigma-1}\tilde{R}_t^{1-\sigma}dv + m_t^{1-\sigma}d\theta\right]}{\tilde{\alpha}_{tj}\tilde{z}_t^{\sigma-1}\tilde{R}_t^{1-\sigma} + \tilde{\nu}_{tj}m_t^{1-\sigma}} + \frac{\iota_j'\Psi_{St}'d\beta}{\tilde{\beta}_{tj}}$$

where ι_j is a selection vector with a one in the jth position and zeroes elsewhere. Note that the effect of these shocks depends on the centrality of the source of the shock to the target firm as a ratio of the total centrality for that firm.

E.6   Market Clearing Conditions

The market clearing conditions for the labor and capital markets are:

$$1 = \sum_{j=1}^{J}\int_{M_j}L_t(j,i)di \quad \text{and} \quad K_t^r = \sum_{j=1}^{J}\int_{M_j}K_t(j,i)di.$$

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From these conditions one can calculate the pricing aggregate for the labor-capital composite, $\tilde{R}_t$, using:

$$\tilde{R}_t^\sigma \tilde{z}_t^{1-\sigma} = \sum_{l=1}^{J} M_l v_{tl} \lambda_{tl}^\sigma i_{tl}^{-\sigma} \bar{y}_t(l, n).$$
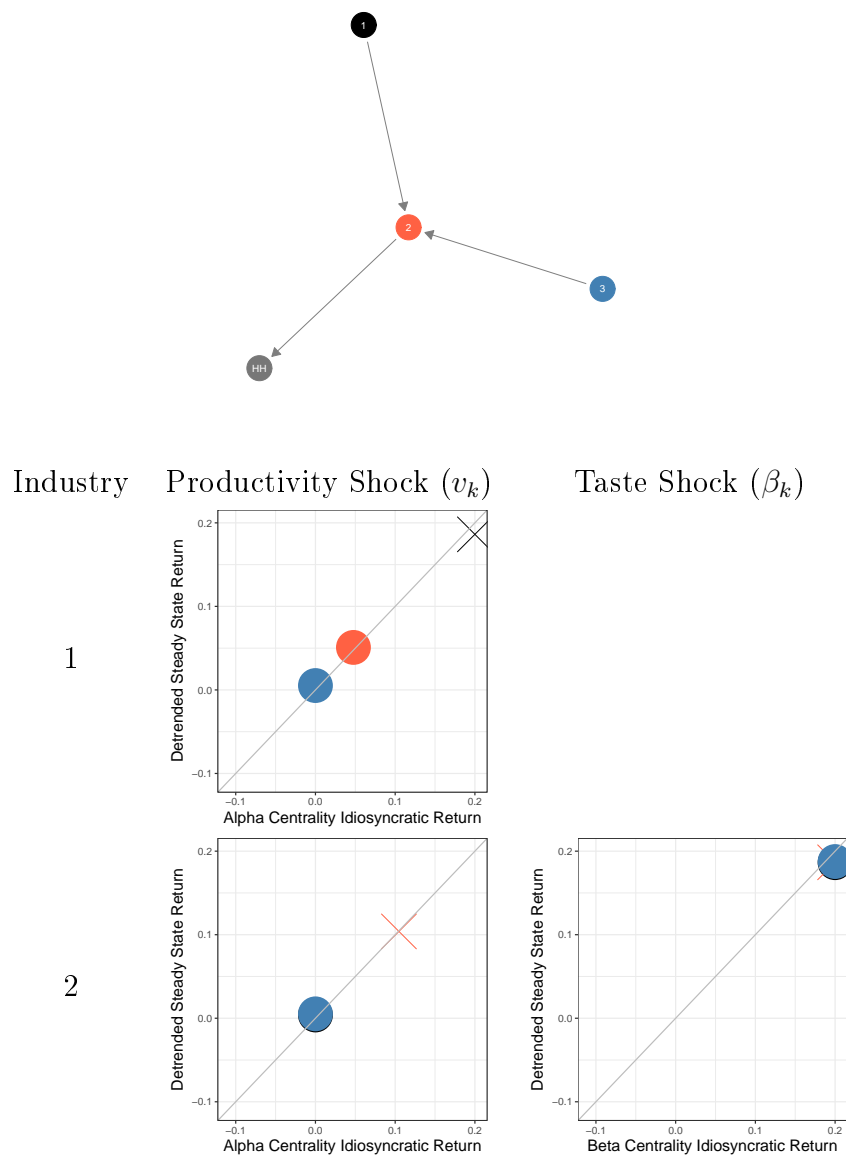
F   DSGE Model Simulations to Upstream & Downstream Shocks

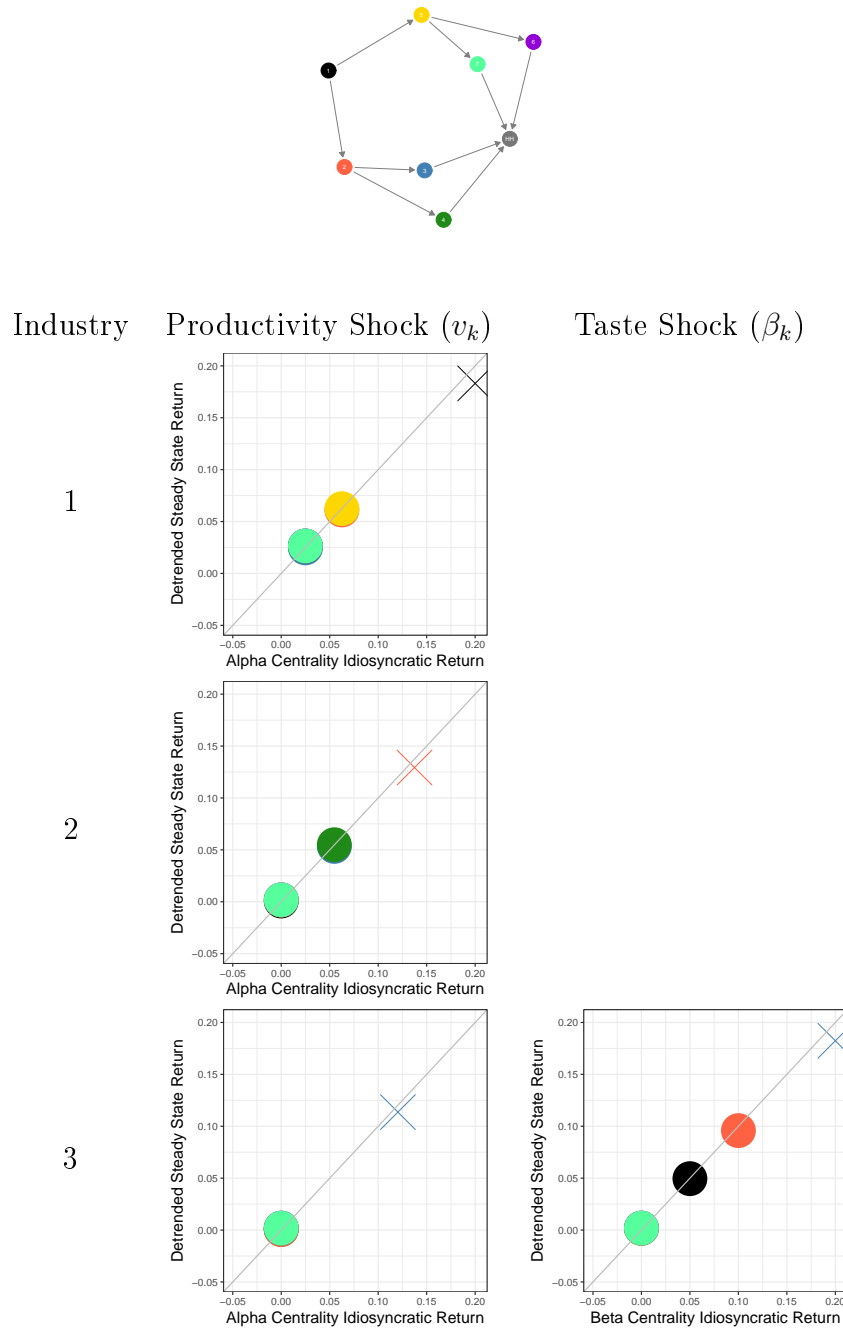Figure F.1: Simulated Idiosyncratic Equity Responses to Shocks by Exposures: Star Network
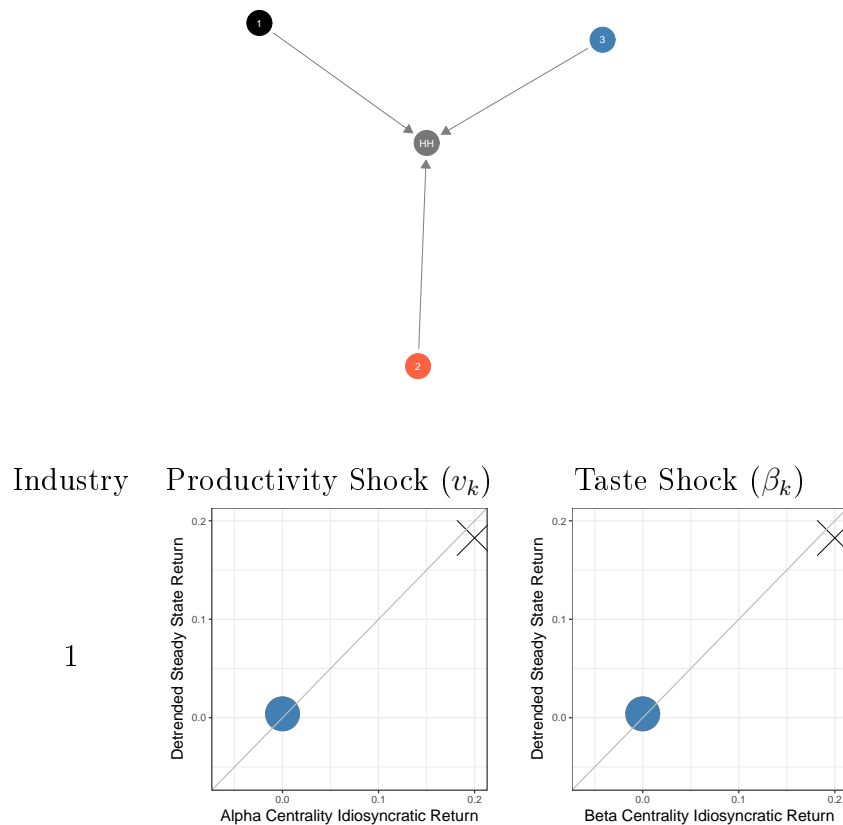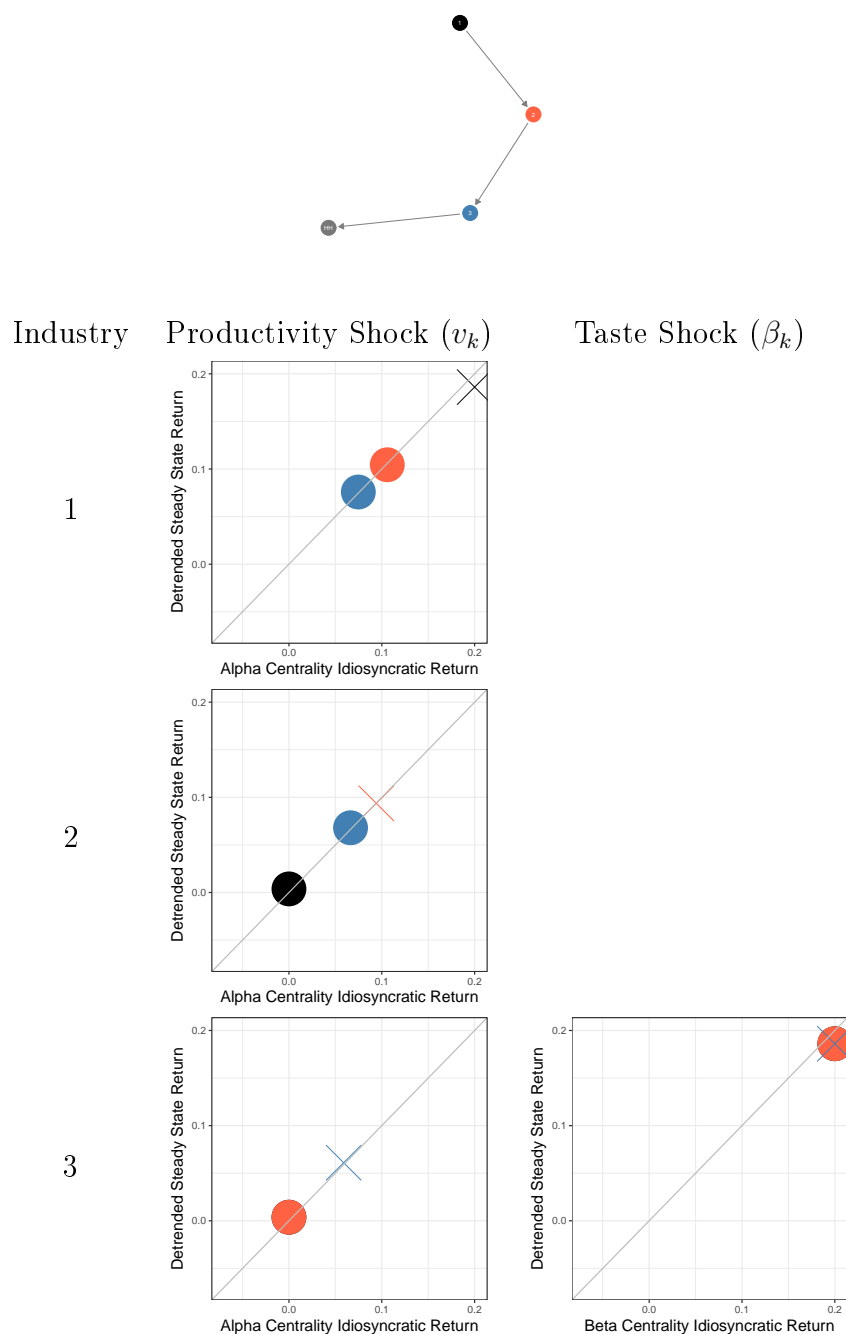

Note: The y-axis measures the idiosyncratic return from being in steady-state at the initial parameter levels to moving to a steady-state after the shock. The x-axis measures the upstream or downstream exposure to the source sector multiplied by the change in that sector's specified parameter. The lines in the lower plots are the 45-degree lines equating these two. The source firm is denoted with an x-marker. The industry legend is 1-black, 2-red, 3-blue, 4-green, and 5-yellow. Gray is the household.

Figure F.2: Simulated Idiosyncratic Equity Responses to Shocks by Exposures: Y-Network

Note: The y-axis measures the idiosyncratic return from being in steady-state at the initial parameter levels to moving to a steady-state after the shock. The x-axis measures the upstream or downstream exposure to the source sector multiplied by the change in that sector's specified parameter. The lines in the lower plots are the 45-degree lines equating these two. The source firm is denoted with an x-marker. The industry legend is 1-black, 2-red, 3-blue, 4-green, and 5-yellow. Gray is the household.

Figure F.3: Simulated Idiosyncratic Equity Responses to Shocks by Exposures: Nested Network
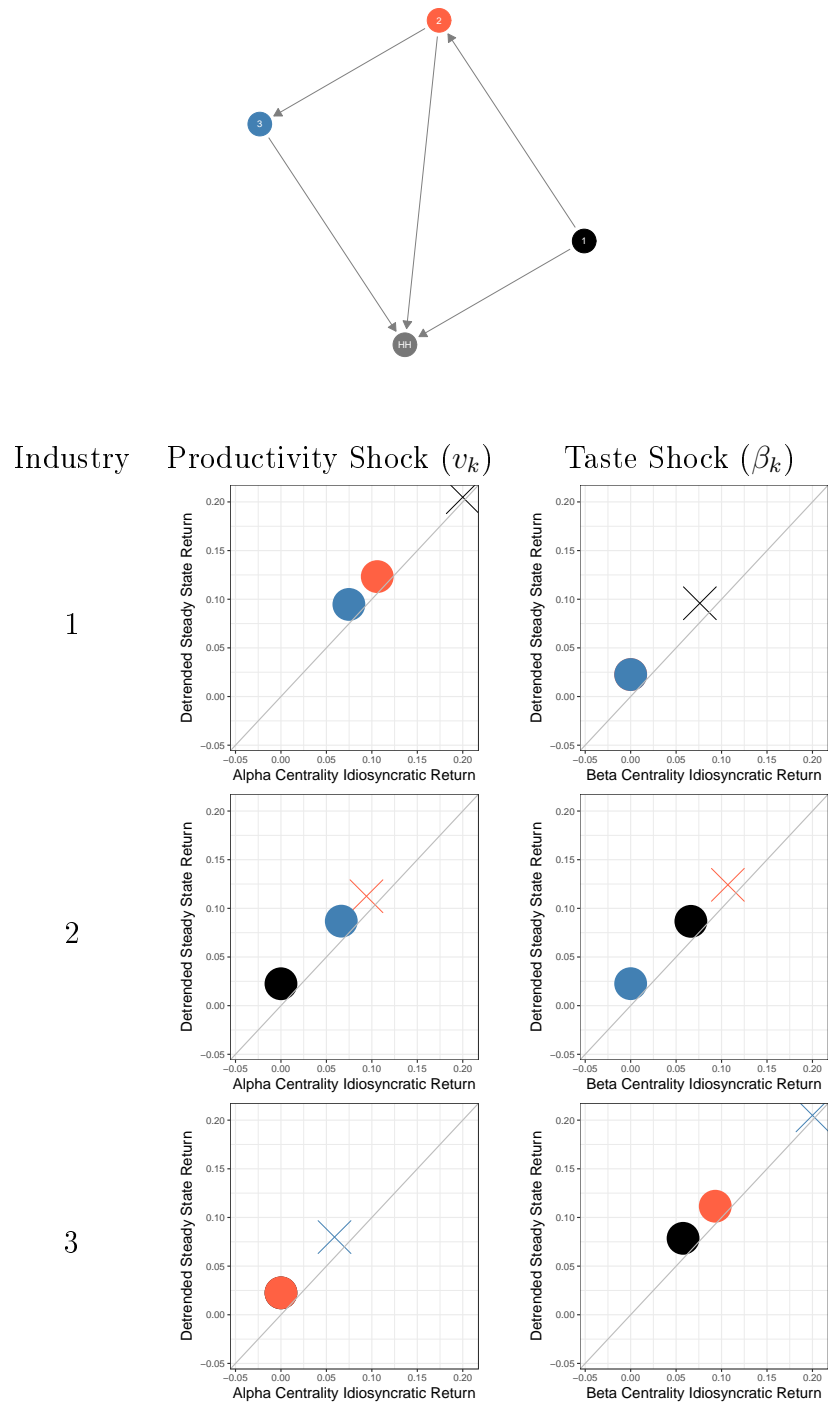

Note: The y-axis measures the idiosyncratic return from being in steady-state at the initial parameter levels to moving to a steady-state after the shock. The x-axis measures the upstream or downstream exposure to the source sector multiplied by the change in that sector's specified parameter. The lines in the lower plots are the 45-degree lines equating these two. The source firm is denoted with an x-marker. The industry legend is 1-black, 2-red, 3-blue, 4-green, and 5-yellow. Gray is the household.

Figure F.4: Simulated Idiosyncratic Equity Responses to Shocks by Exposures: Parallel Network
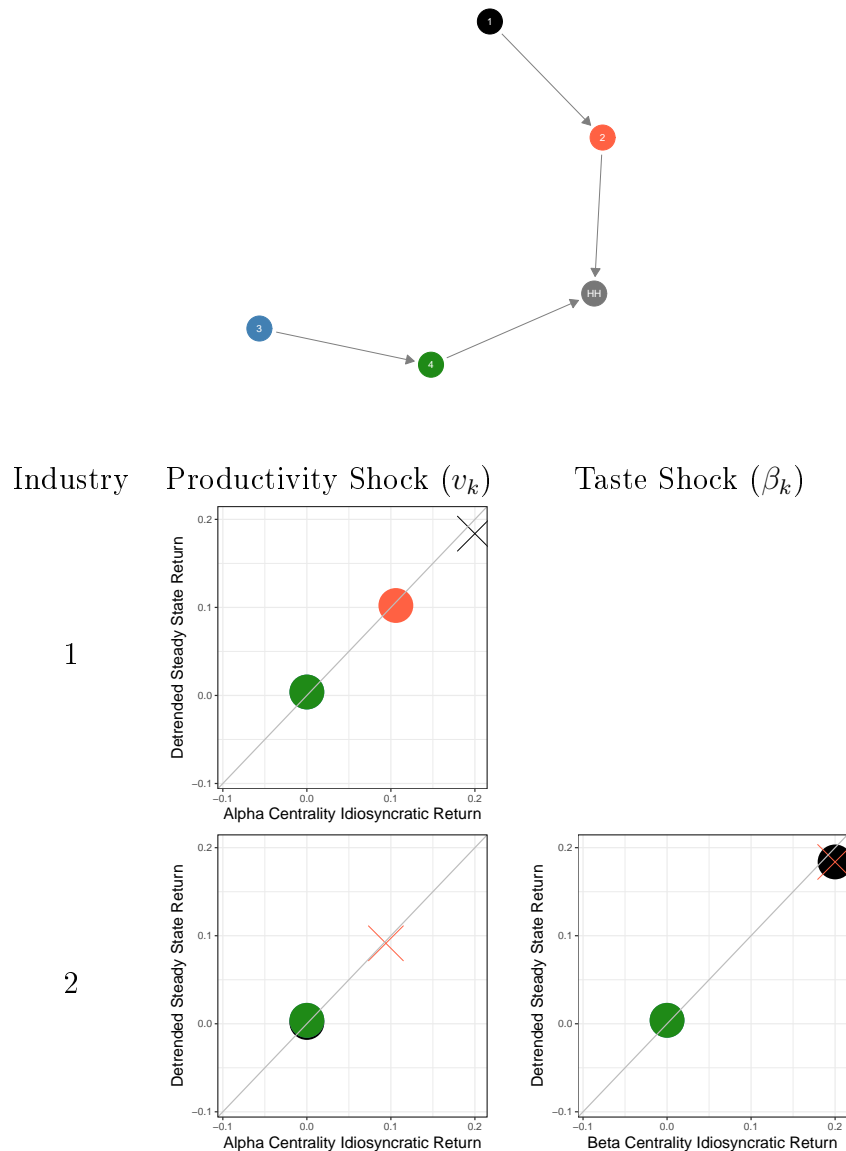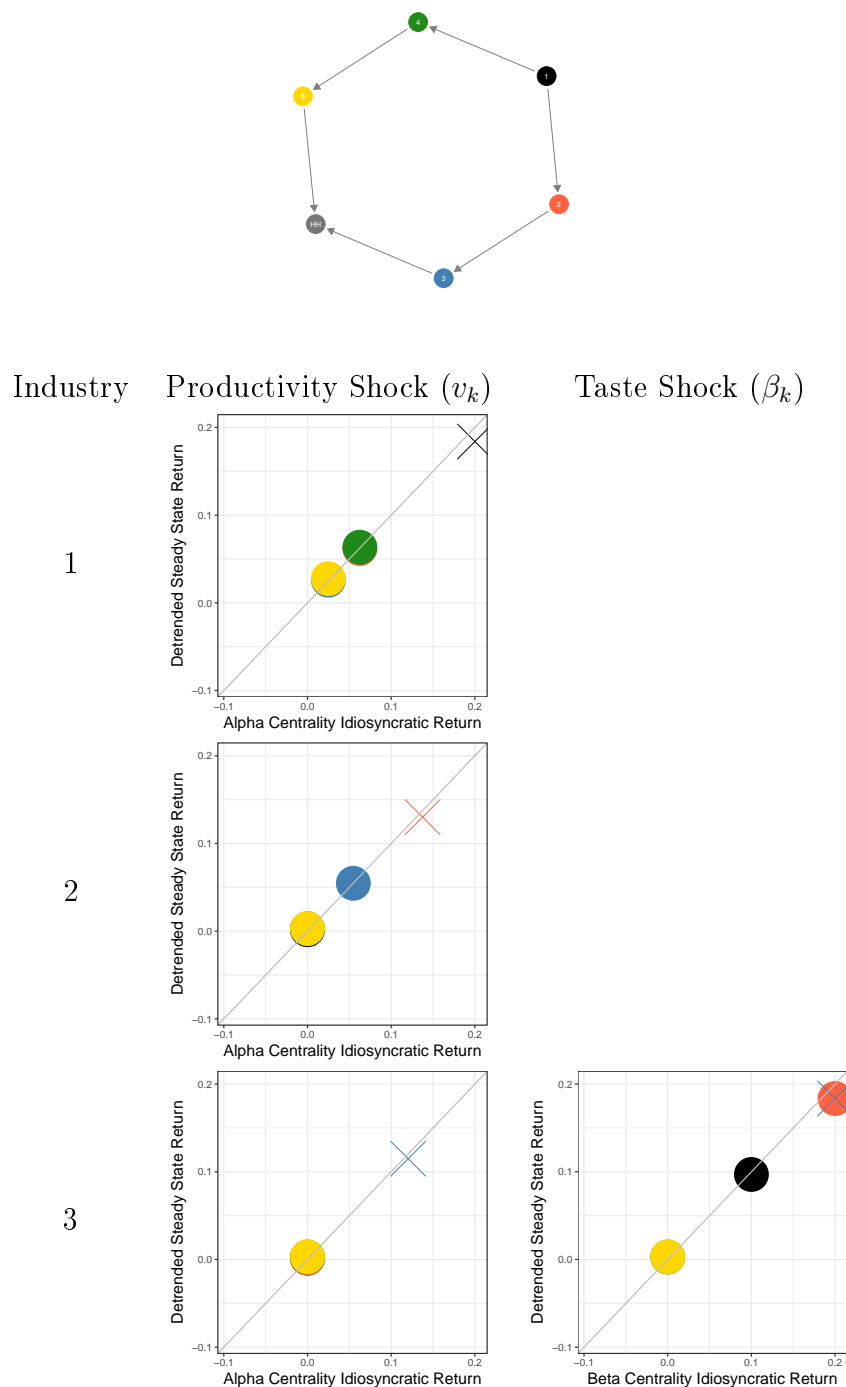

Note: The y-axis measures the idiosyncratic return from being in steady-state at the initial parameter levels to moving to a steady-state after the shock. The x-axis measures the upstream or downstream exposure to the source sector multiplied by the change in that sector's specified parameter. The lines in the lower plots are the 45-degree lines equating these two. The source firm is denoted with an x-marker. The industry legend is 1-black, 2-red, 3-blue, 4-green, and 5-yellow. Gray is the household.

Figure F.5: Simulated Idiosyncratic Equity Responses to Shocks by Exposures: Linear Network
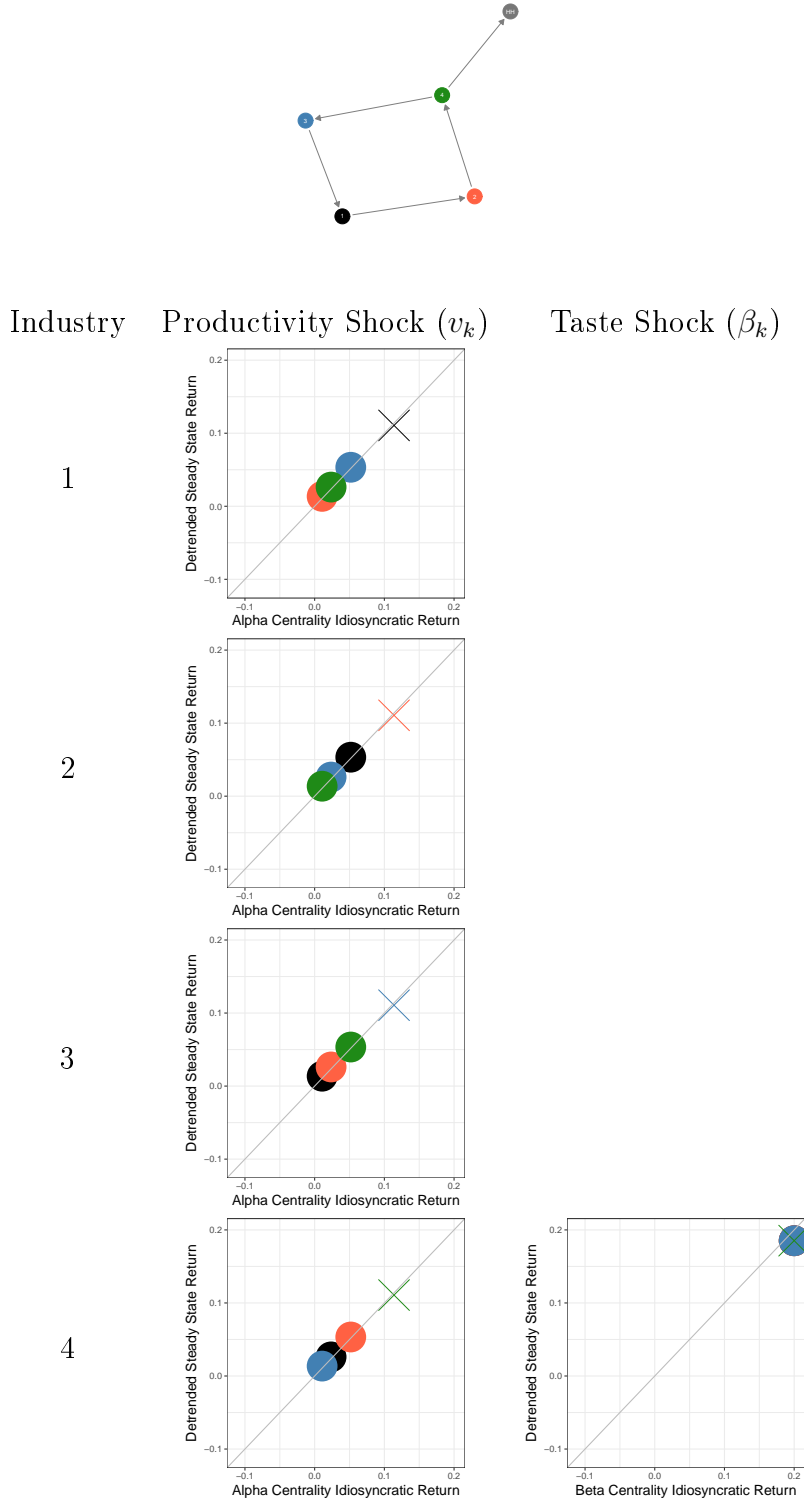

Note: The y-axis measures the idiosyncratic return from being in steady-state at the initial parameter levels to moving to a steady-state after the shock. The x-axis measures the upstream or downstream exposure to the source sector multiplied by the change in that sector's specified parameter. The lines in the lower plots are the 45-degree lines equating these two. The source firm is denoted with an x-marker. The industry legend is 1-black, 2-red, 3-blue, 4-green, and 5-yellow. Gray is the household.

Figure F.6: Simulated Idiosyncratic Equity Responses to Shocks by Exposures: Dense Linear Network
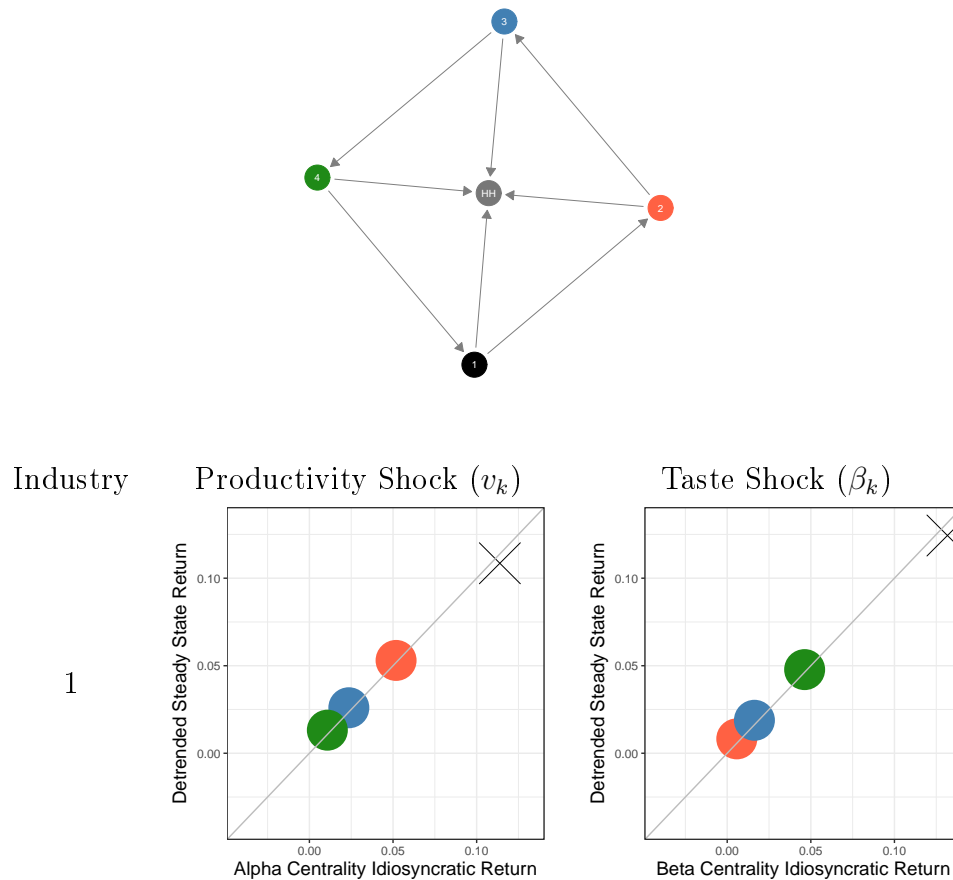

Note: The y-axis measures the idiosyncratic return from being in steady-state at the initial parameter levels to moving to a steady-state after the shock. The x-axis measures the upstream or downstream exposure to the source sector multiplied by the change in that sector's specified parameter. The lines in the lower plots are the 45-degree lines equating these two. The source firm is denoted with an x-marker. The industry legend is 1-black, 2-red, 3-blue, 4-green, and 5-yellow. Gray is the household.

Figure F.7: Simulated Idiosyncratic Equity Responses to Shocks by Exposures: Diamond Network

Note: The y-axis measures the idiosyncratic return from being in steady-state at the initial parameter levels to moving to a steady-state after the shock. The x-axis measures the upstream or downstream exposure to the source sector multiplied by the change in that sector's specified parameter. The lines in the lower plots are the 45-degree lines equating these two. The source firm is denoted with an x-marker. The industry legend is 1-black, 2-red, 3-blue, 4-green, and 5-yellow. Gray is the household.

Figure F.8: Simulated Idiosyncratic Equity Responses to Shocks by Exposures: Circle
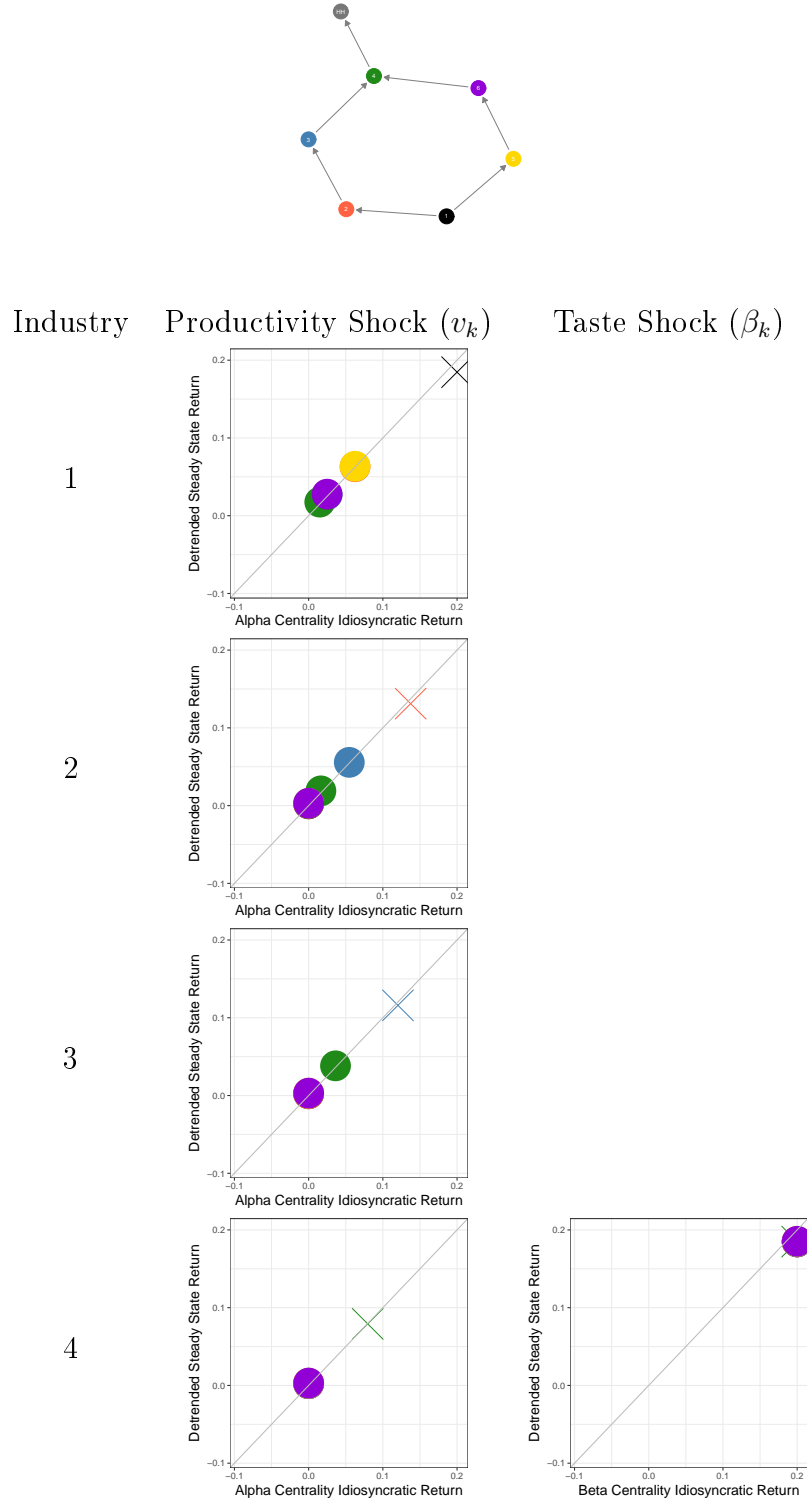
Note: The y-axis measures the idiosyncratic return from being in steady-state at the initial parameter levels to moving to a steady-state after the shock. The x-axis measures the upstream or downstream exposure to the source sector multiplied by the change in that sector's specified parameter. The lines in the lower plots are the 45-degree lines equating these two. The source firm is denoted with an x-marker. The industry legend is 1-black, 2-red, 3-blue, 4-green, and 5-yellow. Gray is the household.

Figure F.9: Simulated Idiosyncratic Equity Responses to Shocks by Exposures: Dense Circle Network
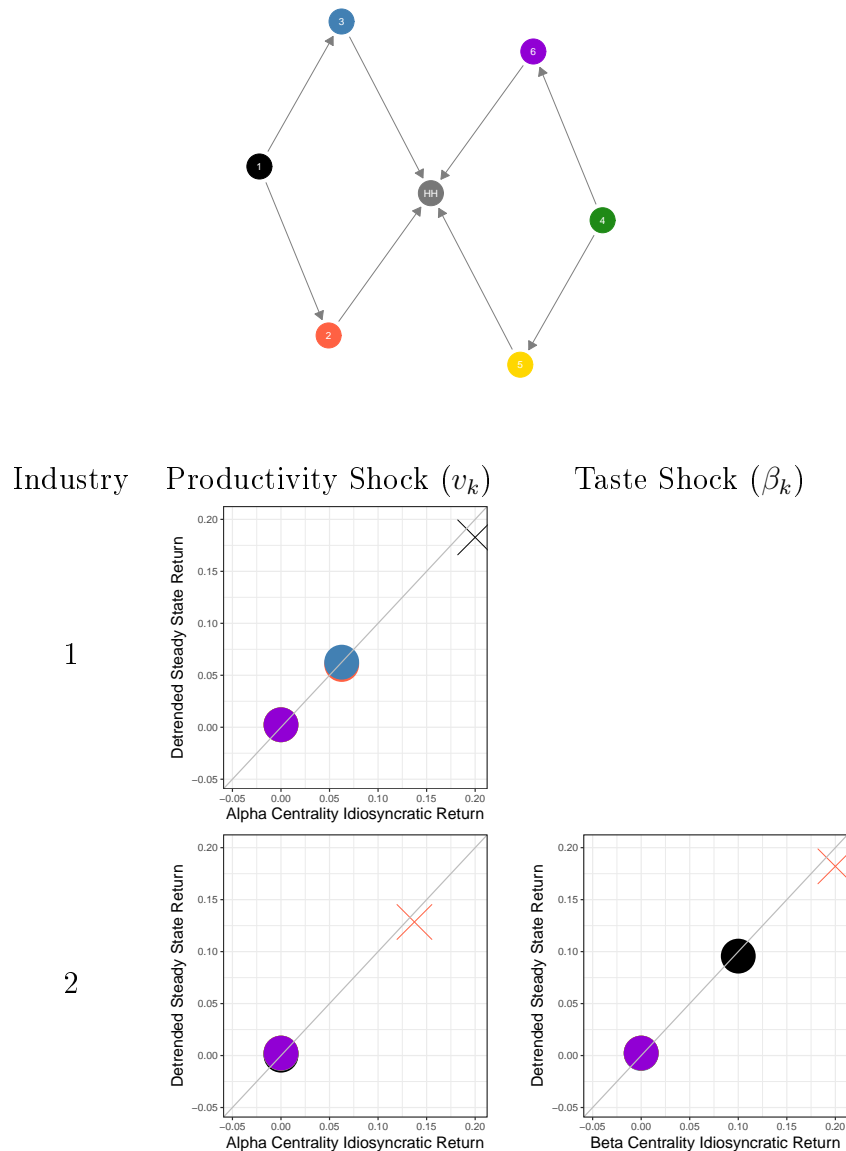

Note: The y-axis measures the idiosyncratic return from being in steady-state at the initial parameter levels to moving to a steady-state after the shock. The x-axis measures the upstream or downstream exposure to the source sector multiplied by the change in that sector's specified parameter. The lines in the lower plots are the 45-degree lines equating these two. The source firm is denoted with an x-marker. The industry legend is 1-black, 2-red, 3-blue, 4-green, and 5-yellow. Gray is the household.

Figure F.10: Simulated Idiosyncratic Equity Responses to Shocks by Exposures: 1-2-2-1

Note: The y-axis measures the idiosyncratic return from being in steady-state at the initial parameter levels to moving to a steady-state after the shock. The x-axis measures the upstream or downstream exposure to the source sector multiplied by the change in that sector's specified parameter. The lines in the lower plots are the 45-degree lines equating these two. The source firm is denoted with an x-marker. The industry legend is 1-black, 2-red, 3-blue, 4-green, and 5-yellow. Gray is the household.

Figure F.11: Simulated Idiosyncratic Equity Responses to Shocks by Exposures: 2 Nests Network

Note: The y-axis measures the idiosyncratic return from being in steady-state at the initial parameter levels to moving to a steady-state after the shock. The x-axis measures the upstream or downstream exposure to the source sector multiplied by the change in that sector's specified parameter. The lines in the lower plots are the 45-degree lines equating these two. The source firm is denoted with an x-marker. The industry legend is 1-black, 2-red, 3-blue, 4-green, and 5-yellow. Gray is the household.

G References

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