Interbank Networks in the Shadows of the Federal Reserve Act

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Abstract

Central banks offer public liquidity to banks (through lending facilities and promises of bailouts) with the intention of stabilizing the financial system. However, shadow banks may receive access to that liquidity through an interbank system. We build a model that shows that the public liquidity provision of the Federal Reserve Act increased systemic risk through three channels: by reducing aggregate liquidity, by expanding the whole-sale funding market, and by crowding out the private insurance that had previously served to smooth cross-regional liquidity shocks. Then, using unique data on Virginia state banks that contain detailed disaggregated information on interbank deposits and short-term funds, we show that the introduction of the Federal Reserve System changed the structure and nature of the overall interbank network in ways that are consistent with the model.

Keywords: Dual Banking System, Federal Reserve Act, Shadow Banking, Interbank Networks, Systemic Risk

JEL Classification: G20, E50, N22

1 Introduction

Since the global financial crisis of 2007-2009, the inability of nonbank financial intermediaries (shadow banks) to access the Federal Reserve System's traditional lending facilities and the consequent impact on financial stability has been widely discussed. Should the Federal Reserve have extended access to its traditional lending facilities to shadow banks before the crisis? Was it the right policy to use emergency lending programs to provide liquidity to shadow banks when the crisis broke out? Is it appropriate for central banks to provide liquidity directly to shadow banks during the ongoing coronavirus pandemic?

These questions are not new. They were also prominent during the initial years of the Federal Reserve System's existence. The Federal Reserve System was created by the 1913 Federal Reserve Act (Act) to offer liquidity to member banks through a discount window, but with the trade-off that members would have to follow stricter regulations. The Act made membership compulsory for national banks but voluntary for state banks, most of which decided not to join the Federal Reserve System.

Thus, the Federal Reserve Act created what we call a shadow banking system, using the term to mean a system of banks, commercial or other, that is not under federal regulation. The shadow banking system of the 1920s consisted of nonmember banks that operated under relaxed regulations compared with the regulations imposed on member banks and that lacked direct access to central bank liquidity. Instead, these nonmember banks accessed public liquidity indirectly, by borrowing from city correspondent banks that were members of the Federal Reserve System (CQ Researcher (1923)).[1] These nonmember banks are similar to nonbank financial intermediaries (such as money market funds, investment banks, nonbank mortgage originators etc.) that are not considered banks in the traditional sense and are the so-called shadow banks today. They provide services similar to traditional commercial banks, but operate outside normal banking regulations.

The existence of a large number of banks operating outside the Federal Reserve System constrained the System's ability to implement monetary policy, prevent crises, and steer the banking system during a recovery. Several studies have shown that the inability of nonmember banks to access central bank liquidity magnified the severity of banking crises during the Great Depression, leading to the creation of new and more extensive lending facilities, such as the Reconstruction Finance Corporation (Wicker (2000), Anbil and Vossmeyer (2017)).

While much can be learned from studying the early years of the Federal Reserve System, few studies have investigated how the central bank's provision of liquidity affected the aggregate

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[1]Banks placing deposits in other banks were called respondents and banks receiving deposits were called correspondents. Correspondent banks were generally located in financial centers.

liquidity in the banking system or how banks' linkages-as affected by the liquidity provision-affected the system's stability. Some studies examine the effect of the liquidity provision on seasonal liquidity pressures in the banking system.[2] Others document the changes in the structure of interbank deposit networks that resulted from the creation of the Federal Reserve System and study how these changes contributed to the severity of banking crises during the Great Depression.[3] Yet, little research has been done to understand how the founding of the Fed altered the nature and structure of interbank networks and affected financial stability following the National Banking Era. Recently, Jaremski and Wheelock (2019), using information about correspondent linkages from the *Rand McNally Directory*, document changes in the structure of the overall interbank network that followed passage of the Federal Reserve Act. However, they focus on changes at the extensive margin without incorporating balance sheet information.

As a first step in answering these questions as to how the public liquidity provision of the Act affected both aggregate liquidity in the banking system and the system's stability, we build a model to understand how introducing public liquidity affected (1) the overall liquidity of the banking system, (2) the structure of the interbank network as a whole and the behavior of its participants, (3) the possibility and direction of contagion, and (4) the buildup of systemic vulnerabilities. In our model, we have reserve-city banks and country banks (footnote 6 explains the categories of banks). Banks in a reserve city (New York) have investment projects, collect deposits from country banks, and pay interest in return. This captures the nature of the national core-periphery structure, based in New York City, of the pre-Federal Reserve monetary system. Banks in the core had projects, and banks in the periphery had liquidity.

We show that the creation of the Federal Reserve System may have introduced a new source of fragility into the banking system by contributing to two factors: a decline in aggregate liquidity and an increase in interbank short-term borrowing activity between member and nonmember banks. The decline in aggregate liquidity after the introduction of public liquidity assistance made the system vulnerable because member banks were not the only ones that reduced their cash holdings; nonmember banks reduced theirs as well-yet nonmember banks had continued exposure to deposit withdrawal risk, since unlike member banks, they lacked direct access to the discount window. The increase in interbank short-term borrowing activity between member and nonmember banks extended the possibility of contagion by making the overall network more complex.

The model also shows that the public liquidity provision may have changed the structure of

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[2]See Miron (1986), Mankiw et al. (1987), Bernstein et al. (2010), and Carlson and Wheelock (2018b)).

[3]See Mitchener and Richardson (2019) and Carlson and Wheelock (2018b).

the interbank system by decentralizing it geographically (and thereby crowding out private insurance). At the same time that the concentration of reserves in New York City was deemed a source of financial instability, it also allowed banks to smooth local liquidity shocks: since New York City banks pooled the reserves of a large number of banks across different regions, the interbank network was able to diversify regional shocks that were not correlated (Gilbert (1983)). With the introduction of public liquidity, however, country banks were induced to rely more on their local correspondents at lower costs (the costs were lower because distances were shorter, information was better, relations were stronger, and so forth), and the emergence of decentralized interbank relationships made the banking system more vulnerable to regional liquidity shocks. In short, the role of financial-center banks was transformed, as they went from being a *provider of private liquidity insurance* to being a *conduit for public liquidity insurance.*

To test these implications of the model, we had to overcome a lack of detailed balance sheet information on financial networks at about the time the Federal Reserve Act was passed. The lack of such detailed information about networks is a shortcoming of existing studies. Most bank balance sheets for the period in question report only the total amounts of interbank balance sheet items, without disaggregating them by individual debtor or creditor correspondent bank.[4] The existence of these items on bank balance sheets documents the existence of interbank relationships but does not provide much quantitative information about the nature of such relationships. Commercial bank directories such as *Rand McNally* and *Polk*, for instance, provide information on self-reported correspondent linkages but not on the types of interbank transactions or the amounts associated with these transactions. These directories provide the names of counterparties but not any information on the strength and nature of the relationships.

To overcome these limitations, we construct various datasets. First, we obtain yearly bank balance sheets for state and national banks aggregated at the state level from 1910 to 1929. This information gives us an aggregated view of (1) how the Federal Reserve Act's liquidity provision changed the aggregate amount of private liquidity in the system, and (2) what the differences in balance sheets were between members and nonmembers. This differentiation is important because the public liquidity provision would almost automatically reduce private liquidity for member banks, but its effect on nonmembers is less obvious.[5] The model strongly

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[4]The existence of interbank relationships is indicated by "Notes and bills rediscounted," "bills payable," "due from other banks," and "due to other banks."

[5]The Federal Reserve Act lowered reserve requirements for member banks, while leaving state banks unaffected. In addition, member banks could access the Federal Reserve's discount window whereas nonmember banks could not. Some state regulators reduced reserve requirements, but we show that our results are robust despite changes in liquidity regulation at the state level.

suggests that liquidity is reduced by nonmember banks' ability to access liquidity through member banks.

Although these aggregate balance sheet data provide useful information, they still do not show how the creation of the Federal Reserve System changed the overall interbank structure. To solve this problem, we collect state bank examination reports for Virginia state banks for the years 1911 and 1922 (that is, before and after passage of the Federal Reserve Act). The examination reports provide balance sheet statements for banks as well as detailed information on the interbank system. In particular, on the asset side of the balance sheet we obtain the deposits of each bank on other banks (identified by correspondent identity), and on the liability side we obtain the short-term loans with the names of providers. This detailed information enables us to examine the nature and intensity of these payment and funding relationships and also to investigate how the creation of the Federal Reserve changed the structure and nature of interbank relationships.

Consistent with the findings from the aggregate balance sheet data, we find that the creation of the Federal Reserve System reduced the aggregate liquidity (cash and deposits) held by banks in Virginia, not only for members but also for nonmembers, and strengthened the short-term borrowing relations across banks. The ability of nonmember banks to borrow indirectly from the Federal Reserve through member banks increased nonmember banks' reliance on short-term funding, which in turn decreased their need to hold liquid assets.

Consistent with Virginia's detailed information on the interbank deposit and short-term borrowing networks, we find that the interbank system became more dispersed after the Federal Reserve System was created. The interbank deposit network became more local as banks reduced correspondent deposits in New York and increased them in local financial centers, such as Richmond and Norfolk. The interbank borrowing network also became more local because banks borrowed from member banks in nearby towns instead of larger banks in financial cities. In other words, the creation of the Federal Reserve System produced a decentralized interbank network, which was likewise consistent with the endogenous network response predicted by our model.

Our study has important implications for policy today. It seems naive to presume that providing public liquidity to (traditional) banking insulates them from potential risks that arise from shadow banks not being subject to federal regulations. Banks that are not subject to regulatory constraints can access public liquidity indirectly by changing their interbank operations and potentially create the fragility that the public liquidity provision tries to solve.

Related Literature: Our paper contributes to several strands of the relevant literature. First, it adds to the literature on the creation of the Federal Reserve System. Previous

studies have found that the creation of the Federal Reserve reduced financial volatility by smoothing seasonal liquidity pressures on the banking system (See Miron (1986), Mankiw et al. (1987), Bernstein et al. (2010), Carlson and Wheelock (2018b)). We show that the creation of the Federal Reserve may have created stability but at the cost of relying too much on public funds and guarantees, and that-by reducing the liquidity of the banking system, increasing contagion risk, and building up systemic pressures-creation of the System may have increased the underlying tail risk of a large collapse. The importance of contagion and the buildup of systemic risk during the Great Depression are discussed in recent empirical studies (Mitchener and Richardson (2019), Calomiris et al. (2019)).

Our paper also adds to the literature on the relationship between shadow banking and the central bank liquidity provision. Some scholars have studied the effect that the provision of public liquidity to nonbanks had on financial stability during the financial crisis of 2007-2009 (Fleming (2012), Duygan-Bump et al. (2013)). Others have studied the transmission channels of monetary policy in the presence of shadow banking (See Adrian and Shin (2009), Chen et al. (2018)). We contribute to this literature by showing that the provision of public liquidity affects not only the structure of the shadow banking system but also the way in which the resulting structure could have unanticipated effects on systemic risk.

On the theoretical front, we apply a network structure to understand how interlinkages (both intensively on the degree of borrowing and extensively on the existence and anatomy of links) react to government interventions. There is recent literature that endogenizes the effects of public interventions to the functioning of banking networks. Erol and Ordoñez (2017), for example, study the reactions of an interbank network to banking regulations. They show that liquidity and capital requirements that are intended to provide stability to the system may make the system unstable because these requirements can destroy a network structure that insures against financial shocks. In this paper we study how facilities that lend to certain banks may harm both network functionality and total stability.

The remainder of the paper is organized as follows: Section 2 provides information on the overall interbank system before and after passage of the Federal Reserve Act. Section 3 presents a model that not only introduces an external agent (central bank) that provides liquidity in a banking setting but also shows how the provision affects the holding of liquidity among banks, including their linkages and relations. Section 4 presents empirical evidence of (1) a reduction in aggregate liquidity (for both Federal Reserve members and nonmembers), (2) an increase in short-term borrowing and in the possibilities of contagion, and (3) changes in the geographical properties of the core-periphery network. We conclude with some final remarks.

2 Historical Background

During the National Banking Era, the banking system exhibited seasonal spikes in loan interest rates and frequent episodes of banking panics. Short-term interest rates displayed strong seasonal fluctuations due to large increases in the supply of deposits during agricultural harvest seasons and the demand for credit during agricultural planting seasons. As a result, banks faced liquidity pressures in spring and fall, and panics occurred at times of the year in which these pressures peaked.

The interbank system of the period, through the network of correspondent deposits and short-term funding, played an important role in reducing liquidity pressures. The interbank deposit network was characterized by a three-tier pyramid structure.[6] Country banks held deposits in reserve-city banks, which in turn kept deposits in New York City banks. The concentration of interbank deposits in New York City banks allowed these banks to reallocate liquidity across regions. When country banks in agricultural regions faced seasonal demands, they withdrew their interbank deposits from financial centers, with those funds coming from other banks in areas where seasonal demands were less pressing. The geographical regional differences in demand produced somewhat offsetting flows of interbank deposits in New York City banks, which effectively provided private insurance across regions (see, for instance, Kemmerer (1910)). The interbank system helped banks meet seasonal liquidity pressures not only by allowing banks to cross-share deposits but also by allowing them to borrow short-term funds from correspondents. Country banks borrowed the most, reserve-city banks borrowed rarely, and central reserve-city banks borrowed hardly at all.

But although the interbank system helped soften the seasonal demands on banks, it did not create additional liquidity. As a result, the cash demands of country banks drained cash balances held in New York City banks and led to seasonal spikes in interest rates. Contemporaries thought these seasonal swings contributed to bank panics and instability, and this belief prompted calls for reform to create an *elastic currency* that would make the reallocation of funds across regions less dependent on interbank relationships (Sprague (1910)).

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[6]The interbank system developed to overcome branching restrictions and facilitate interregional payments of goods and services. The National Banking Act institutionalized the interbank system by setting up a location-based three-tier system of national banks: central reserve-city banks (those located in New York City, Chicago, or St. Louis), reserve-city banks (banks in selected other large cities), and country banks (banks in all other locations). Central reserve-city banks were required to hold cash reserves equal to 25% of their deposits. Reserve-city banks were also required to hold reserves equal to 25% of their deposits, of which one-half could be deposits with a correspondent bank in a central reserve city. Country banks were required to hold reserves equal to 15% of their deposits, but they could keep three-fifths of the 15% as deposits with a correspondent bank in reserve and/or central reserve cities. State bank regulators subsequently passed similar laws.

In response to this financial landscape, the Federal Reserve System was created in 1913 (under the Federal Reserve Act) with three primary objectives: to eliminate the concentration of bank reserves in New York City banks by establishing 12 regional reserve banks; to create an elastic currency and thereby reduce seasonal volatility; and to prevent panics (Calomiris (1994)). To achieve these goals, the Federal Reserve offered a discount window to member banks through the 12 regional Federal Reserve Banks, but required members to meet new reserve requirements by placing deposits in those Federal Reserve Banks instead of in reserve-city and central reserve-city banks.[7]

The Federal Reserve Act retained for member banks the three-tier classification of central reserve-city banks, reserve-city banks, and country banks, but changed their reserve requirements. Member banks were required to hold 13%, 10% and 7%, respectively, of *demand deposits* within the regional Federal Reserve Banks. All member banks were required to hold 3% of *time deposits* within the Federal Reserve Banks. The Federal Reserve did not pay interest on any of these deposits.[8]

Although the Federal Reserve Act succeeded in reducing the volume of interbank deposits, it failed to eliminate interbank network linkages. Deposits with the Federal Reserve Banks did not pay any interest, whereas deposits at city correspondents paid 2% interest. Hence, member banks continued to hold some deposits with correspondents, both to earn interest and to diversify their asset portfolios (CQ Researcher (1923), Carlson and Wheelock (2018b)).

Becoming a member, however, was partly voluntary. The Act made it compulsory for national banks to join, but for state banks, joining was voluntary. The creators of the Federal Reserve System hoped to bring state banks under a more unified system of regulation and supervision, but only a small fraction of state banks became members: by June 1915, only 17 state banks had chosen to join. This reluctance had two causes. First, banks that did not become members could continue earning interest on all of their interbank deposits. Second, even though the Act prohibited member banks from using interbank deposits to meet reserve requirements, state regulators allowed state banks to do so (CQ Researcher (1923)). Indeed, more banks joined after a 1917 revision that lowered capital and reserve requirements.

Figure 1 shows the proportions of national, state member, and state nonmember banks from

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[7]Even though only member banks were given access to Federal Reserve services, including the discount window, the Act made it possible for the central bank to extend the discount window to nonmember banks in special circumstances with the approval of the Federal Reserve Board of Governors. Before 1923, for instance, the Board allowed member banks to discount eligible paper acquired from nonmember banks (See Hackley (1973), p. 119). Thereafter, the Board limited the extension of credit to nonmember banks in exceptional circumstances (See Carlson and Wheelock (2013)).

[8]The reserve requirements were first introduced in 1913 and took effect in 1914. They were amended in 1917.

1914 to 1929. With only 5% of state banks choosing to become a member, more than 60% of all banks remained outside the Federal Reserve System. Membership grew slowly, eventually reaching a peak of 1,648 state banks (compared with 19,141 nonmember banks) in 1922 (Committee on Branch and Banking (1935)).[9]

Figure 1: Share of Federal Reserve Member Banks, 1914-1929
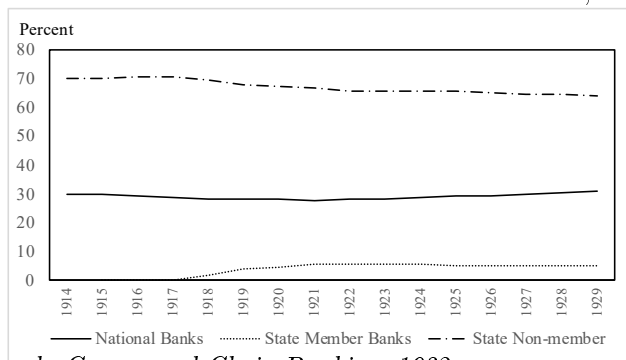
Source: Committee on Branch, Group, and Chain Banking, 1932.

Figure 2 presents the rate of state bank membership by state in 1920, showing some heterogeneity across states, but still an overall low participation at the Federal Reserve Bank.

Figure 2: Federal Reserve System Participation Rate of State Banks - 1920
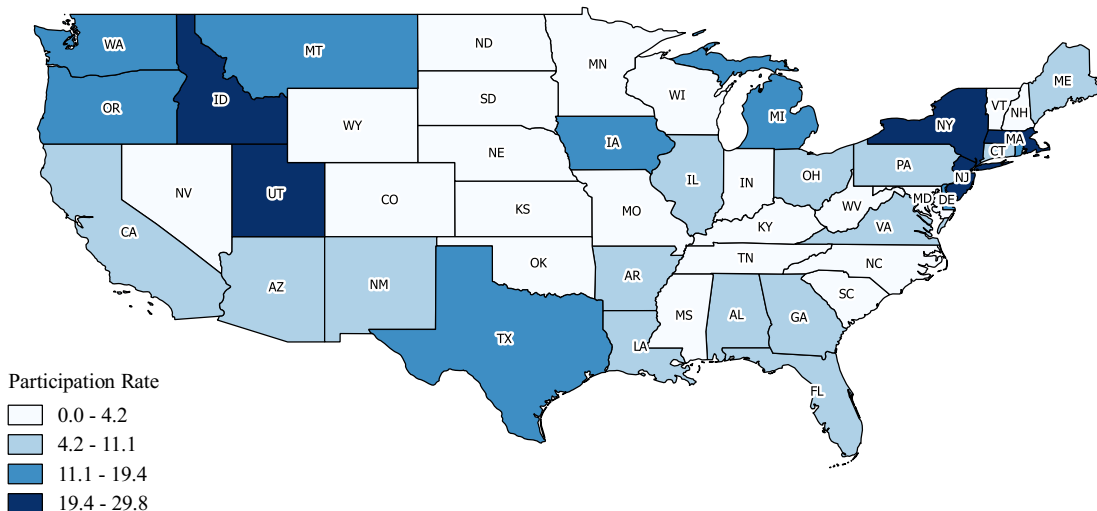
Source: Annual Report of the Board of Governors of the Federal Reserve System.

Having a large fraction of state banks outside the Federal Reserve System had major implications for the nature of the interbank system. In what follows we show that this led to a large number of banks accessing the System's discount window indirectly-through their

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[9]In terms of relative size, member banks tended to be larger than nonmembers but nonmembers still held a sizable fraction of total deposits. In 1923, for instance, nonmember banks held more than a third of total U.S. commercial bank deposits ($10.6 billion of a total of $37.7 in the whole system).

correspondents. Before the Federal Reserve System was established, country banks borrowed short-term funds from their correspondents. After the System was established, country banks started relying more intensively on their correspondents in financial centers to borrow for short periods, with the understanding that when city correspondents ran out of funds, they would go to the Federal Reserve Bank and rediscount their own eligible paper to replenish their liquidity positions.

3   Model

As we have just noted, although the Federal Reserve System was to provide liquidity to the banking system, many state banks chose not to join the System. As we have also just noted, even though nonmember banks were not allowed to access the Federal Reserve's discount window directly, they did it indirectly through their relations with member banks in financial centers. The goal of our model is to illustrate how nonmember banks' indirect access to the central bank's liquidity affects (1) the aggregate liquidity of the banking system, (2) the nature of interbank exposures, and (3) the structure of the interbank network.

More broadly, our model helps us understand the behavior of financial intermediaries (such as money market funds, investment banks, etc.) that are not considered or regulated as banks in the traditional sense (and are the so-called shadow banks). These institutions do not have direct access to lending facilities and bailouts, but they can access public liquidity indirectly using interbank connections. Our model highlights the importance of understanding banking networks in order to understand shadow banking.

We begin with an environment containing two banks to study how the introduction of central bank liquidity affects aggregate liquidity and interbank exposures by incorporating the incentives of banks. We then add more banks to study the structure of the interbank network.

3.1   Environment

The economy is composed by two banks, x (nonmember bank) and y (member bank in a reserve city). Bank x accepts D household deposits and has access to a project that pays a net rate of return $r_x > 0$. Bank y does not have deposits and has a project that pays a net rate of return $r_y > 0$. Projects can be liquidated in full at any time to recover the original investment. We use this simple setting to capture a country bank in the periphery (bank x) that makes deposits at a reserve-city bank in the core (bank y).

Reserves and investments   After investments, some depositors may need the funds and withdraw from x before projects reach maturity (*liquidity shocks*). Accordingly, x wants to keep reserves to fulfill those needs, and may do so by holding cash or by depositing at bank y, earning net interest r, which we assume is low relative to the projects' returns.[10]

Denoting Φ_x the reserves that x keeps as cash, and L the amount that x deposits at y, bank x invests $I_x = D - \Phi_x - L$. Assuming bank y is subject to reserve requirements in the form of holding a fraction ϕ of liabilities in cash, and denoting Φ_y the reserves that y keeps in cash, $\Phi_y \geq \phi L$. This implies that y invests $I_y = L - \Phi_y$. We call I_x and I_y *investments*, Φ_x and Φ_y *cash reserves*, and L the *interbank deposits*. The transactions and obligations described thus far, absent liquidity shocks, are shown in Figure 3. Liquidity shocks caused by depositors withdrawing early can disrupt this flow of funds by depositors withdrawing early. We assume that full liquidation of projects always covers original investments. This last assumption allows us to focus on liquidity crises and not solvency crises, as depositors can always recover D regardless of shocks.

Figure 3: Transactions absent Liquidity Shocks
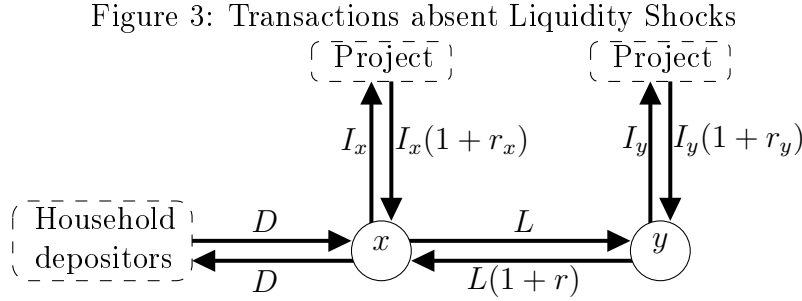
Figure 3 shows the flow of funds in the system absent a liquidity shock. Household depositors lend D to x. x invests I_x, makes interbank deposits L at y, and keeps the rest of D as cash reserves. y invests I_y and keeps the rest of L as cash reserves. After projects mature, y's project returns $I_y(1+r_y)$. y pays $L(1+r)$ of this to x. x's project returns $I_x(1+r_x)$. Finally, x returns D to households.

Liquidity shocks   We denote early withdrawals by $\zeta \in [0, Z]$, where Z is the upper bound on possible withdrawals and ζ is drawn randomly from a distribution with CDF denoted by S. We call ζ the *liquidity shock*. In terms of projects' liquidations, there are various scenarios that can materialize depending on the size of the liquidity shock. The several possibilities are as follows:

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[10]More specifically, we will assume $2r < (1-\phi)r_x$ and $r < (1-\phi)r_y$, where $\phi < 0.5$ are reserve requirements. During the National Banking Era, state regulators allowed state banks to keep reserves at reserve cities to meet reserve requirements, and reserve city banks paid 2% (and no more than 2%) interest on these deposits, which justify our assumption that r is exogenous (See James (1978)).

1. If $\zeta \leq \Phi_x + \Phi_y$, the combined cash reserves from x and y are sufficient to meet the liquidity shock.

   (a) If $\zeta \leq \Phi_x$, withdrawals are met by x's cash in vault.

   (b) If $\Phi_x < \zeta \leq \Phi_x + \Phi_y$, x's cash reserves are not enough and x borrows $\zeta - \Phi_x$ short-term from y to cover the withdrawals.[11]

2. If $\zeta > \Phi_x + \Phi_y$, the combined cash reserves from x and y are not enough to cover the liquidity shocks, in which case x must either liquidate its own project or withdraw its deposits from y to an extent that exceeds y's cash reserves. The latter forces y to liquidate its project. These are the possibilities:

   (a) If $\Phi_x + \Phi_y < \zeta \leq \Phi_x + L$, the deposits of x at y are enough to cover the liquidity needs, together with x's cash. Then x withdraws L from y, who has to liquidate its project. In principle, x could liquidate its own project I_x before withdrawing from y, but we will show that in equilibrium it is optimal for x to maintain its investment.[12]

   (b) If $\Phi_x + L < \zeta \leq \Phi_x + \Phi_y + I_x$, x must also liquidate its own project as deposits at y are insufficient. The liquidation recovers I_x, which, together with cash reserves Φ_x and Φ_y, suffices for x to ride out the shock. In this case, x can keep its deposits at y.

       i. If $\Phi_x + L < \zeta \leq \Phi_x + I_x$, x does not borrow short-term from y.

       ii. $\Phi_x + I_x < \zeta \leq \Phi_x + \Phi_y + I_x$, then x borrows $\zeta - \Phi_x - I_x$ short-term from y.

   (c) If $\Phi_x + \Phi_y + I_x < \zeta$, neither I_x from the liquidation of the project, nor deposits L at y suffice by themselves, hence x liquidates its project and withdraws its deposits from y. In this case, x makes no profit.

Given these possible states, ex-post short-term borrowing from by x from y is

$$
b = \begin{cases} \zeta - \Phi_x & \text{if } \Phi_x < \zeta \leq \Phi_x + \Phi_y \\ \zeta - \Phi_x - I_x & \text{if } \Phi_x + I_x < \zeta \leq \Phi_x + I_x + \Phi_y \\ 0 & \text{otherwise} \end{cases}
$$

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[11] Such lending is risk-free so we assume for the sake of simplicity that y does not charge an interest. Given this, whether x borrows $\Phi_x + \Phi_y - \zeta$ or Φ_y is inconsequential. In what follows, we assume that x borrows the smallest amount that suffices for it to ride out the shock, which is robust to the existence of small borrowing costs.

[12] Bank x can withdraw any amount between $\Phi_x + L - \zeta$ and L, but since y is forced to liquidate the whole project upon withdrawal, then we assume x withdraws the full amount L from y.

and x ex-post profit is

$$
\pi_x = \begin{cases}
I_x r_x + Lr & \text{if } \zeta \le \Phi_x + \Phi_y \\
I_x r_x & \text{if } \Phi_x + \Phi_y < \zeta \le \Phi_x + L \\
Lr & \text{if } \Phi_x + L < \zeta \le \Phi_x + \Phi_y + I_x \\
0 & \text{if } \Phi_x + \Phi_y + I_x < \zeta
\end{cases}
$$

To define ex-ante short-term borrowing and ex-ante profits, we can define as $\Gamma \equiv S\left[\Phi_x + L\right]$ the probability that x's project is not liquidated and as

$$
\Delta \equiv S[\Phi_x + \Phi_y] + (S\left[\Phi_x + \Phi_y + I_x\right] - S\left[\Phi_x + L\right])
$$

the probability that y's project is not liquidated. Then bank x's expected profits are

$$
\Pi_x = \mathbb{E}[\pi_x] = \Gamma I_x r_x + \Delta Lr
$$

and following similar arguments, bank y's expected profits are

$$
\Pi_y = \Delta \left(I_y r_y - Lr\right).
$$

Timing and optimality Given the expected profits, bank x chooses L to lend to y, its cash reserves Φ_x and investment I_x.[13] Then y chooses its cash reserves $\Phi_y \ge \phi L$ and investment I_y. Finally, liquidity shocks materialize. This timeline is summarized in Figure 4.

Figure 4: Timeline of events

D	L	I_x, Φ_x, I_y, Φ_y	ζ		
Accept household deposits	Make Interbank deposits	Decide on cash reserves and investments	Face liquidity shocks	Liquidate and withdraw interbank deposits	Project matures and bank repays depositors

Upstream contagion Consider a realized shock ζ. If $\zeta \le \Phi_x + \Phi_y$, there is no spillover from x to y. If $\Phi_x + L < \zeta \le \Phi_x + I_x + \Phi_y$, x liquidates its own project. In these two cases, there is no contagion from x to y in terms of forcing y's project liquidation.

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[13]We assume that x makes a "take it or leave it" offer L to y, which is always accepted because y's outside option is 0.

If $\Phi_x + \Phi_y < \zeta \leq L + \Phi_x$, then x withdraws its deposits L. x's project matures but y's project gets liquidated. If $\Phi_x + \Phi_y + I_x < \zeta$, then both projects get liquidated. In both of these cases, y's project gets liquidated. We call this situation *upstream contagion from x to y*. The probability of upstream contagion is then $1 - \Delta$.

Parametric specifications. To have closed-form results, we assume that ζ is drawn from a truncated uniform distribution with support $[0, Z]$ with $Z \geq D$.[14] For some $\alpha \in [0, 1]$, there is $1 - \alpha$ probability that there is no liquidity withdrawal and $\zeta = 0$. There is α probability that ζ is drawn from $U[0, Z]$. Then the expected profits of x and y are

$$\Pi_x = \underbrace{\left(1 - \alpha \frac{I_x}{Z}\right) I_x r_x}_{\Gamma} + \underbrace{\left(1 - \alpha \frac{2L - 2\Phi_y}{Z}\right) Lr}_{\Delta}$$

$$\Pi_y = \left(1 - \alpha \frac{2L - 2\Phi_y}{Z}\right) \left(L\left(r_y - r\right) - \Phi_y r_y\right).$$

Expected short-term borrowing is

$$B = \mathbb{E}[b] = \frac{\alpha \Phi_y^2}{Z}.$$

Bank y chooses $\Phi_y \in [\phi L, L]$ to maximize Π_y. Bank x chooses I_x and L to maximize Π_x subject to $I_x, L \geq 0$ and $I_x + L \leq D$, given the optimal continuation strategy of y.

In what follows we focus on the case in which $\alpha \leq \overline{\alpha} = \frac{Z}{Z + \rho D}$, with $\rho = \max\{0, 1 - 2\phi - \frac{r}{r_y}\}$. When α is not too large, in equilibrium, reserve requirements bind for y and $\Phi_y = \phi L$. In other words, when it is unlikely that banks suffer early withdrawals (α is not too large), banks will be less induced to hold cash buffers in order to prevent the liquidation of projects. We prove this result in the Appendix C.

Although it is standard to model partial liquidations that incur some costs, we assume that projects can be liquidated only fully at no cost (the investment can always can be recovered). We use this assumption to model "precautionary cash holdings." Banks want a significant cash buffer, so that they are not forced to liquidate a large investment.

Proposition 1. *If* $\overline{\alpha} \geq \alpha \geq \left(1 + \frac{D}{Z} \frac{1-2\phi}{3-2\phi}\right)^{-1}$, *the equilibrium quantities are given by*

$$L = \frac{D + Z_\alpha}{4(1 - \phi)}, \qquad I_x = \frac{D + Z_\alpha}{2}, \qquad \Phi_x = D - I_x - L, \qquad \Phi_y = \phi L.$$

---

[14]That a bank faces more withdrawals than deposits implies additional legacy liabilities by an amount $Z - D \geq 0$. This extension avoids kinks in the solution once we introduce public liquidity, but $Z > D$ is irrelevant in this part of the paper. One can simply take $Z = D$ for now.

If $\left(1 + \frac{D}{Z}\frac{1-2\phi}{3-2\phi}\right)^{-1} > \alpha > \left(1 + \frac{D}{Z}\frac{r_x+r}{r_x-r}\right)^{-1}$, the equilibrium quantities are given by

$$L = \frac{D\left(r_x + r\right) - Z_\alpha\left(r_x - r\right)}{2\left(r_x + 2(1-\phi)r\right)}, \qquad I_x = D - L, \qquad \Phi_x = 0, \qquad \Phi_y = \phi L.$$

where $Z_\alpha \equiv \frac{Z(1-\alpha)}{\alpha}$.

These cases are instructive about the pecking order on allocating funds. Whereas y's investments are protected by cash reserves $\Phi_x + \Phi_y$, x's investments are protected by both cash reserves and interbank reserves, $\Phi_x + \Phi_y + L$. Even though y's investments generate higher returns, the fact that there are more reserves protecting I_x and that I_x earns a higher return induces more investment in I_x.

The proposition also shows how this allocation of funds changes in response to the probability of a liquidity shock α. In Figure 5 we show that, as α increases (that is, liquidity shocks become more likely), all instruments for dealing with these shocks increase (more cash reserve, more expected borrowing, and more interbank deposits). An increase in liquid assets is offset by a decline in illiquid investments.

Figure 5 plots the equilibrium allocation as a function of the probability of a liquidity shock α in line with Proposition 1. Illiquid investments decrease with risk. Since reserves protect the illiquid investments, reserves go up. Before risk becomes too high, interbank deposits are the preferred form of reserves compared with idle cash reserves. At high risk, the returns promised on interbank deposits also need protection, and cash reserve substitute interbank deposits. Interbank deposits start falling yet the total reserves keep increasing.

Figure 5: Investments, Interbank Deposits, Short-Term Borrowing, and Cash Reserves
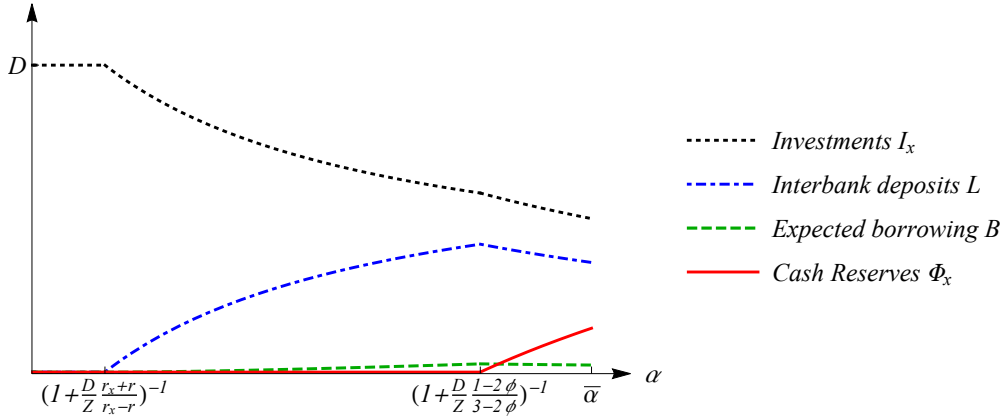
Equilibrium allocation as a function of the probability of liquidity shock α, as described in Proposition 1.

3.2 Introducing Central Bank Liquidity Provision

In this section we show that provision of public liquidity reduces aggregate private liquidity in the banking system, including the private liquidity of banks that do not have direct access to public liquidity. In addition, public liquidity provision can make the banking system more vulnerable to regional shocks because banks reduce their connectivity to core banks, and such connectivity provides a private tool to smooth out cross-regional liquidity shocks.

Suppose there is a *central bank* that provides short term liquidity to y, for a maximum amount m, which we refer to as the public liquidity provision ($m = 0$ is the baseline case of no liquidity provision of the previous section). Although bank x is not a member of the Federal Reserve System, it can indirectly access the Federal Reserve's liquidity facilities though its interbank relation with y. We are interested in how the ability of x to indirectly access the central bank's liquidity affects x' reserve holdings, and in turn affects contagion and systemic risk.

Regardless of the maximum amount of public liquidity m, bank y does not want to keep reserves and $\Phi_y = \phi L$. For bank x, using idle reserves Φ_x or borrowing at most m from the central bank via y are substitutes. For bank x, therefore, any shock ζ below m can be met at no cost just by borrowing short-term from the member bank. In contrast, a shock above m will require banks to use their own reserves or to liquidate projects, as above.

Formally, from the viewpoint of bank x, future shocks are $\zeta' = (\zeta - m)_+$, with ζ' equal to 0 with probability $1 - \alpha + \alpha \frac{m}{Z}$, drawn from $U[0, Z - m]$ with probability $\alpha \frac{Z-m}{Z}$. We focus on the values of $m < Z - D$ so that public liquidity does not eliminate liquidity risk in the financial sector when liquidity shocks are large.

$$\Pi_{x,m} = \underbrace{\left(1 - \alpha \frac{I_x - m}{Z}\right) I_x r_x}_{\Gamma_m} + \underbrace{\left(1 - \alpha \frac{2L - 2\Phi_y - m}{Z}\right) Lr}_{\Delta_m}$$

Proposition 2. *If* $\overline{\alpha} \geq \alpha \geq \left(1 + \frac{D}{Z} \frac{1-2\phi}{3-2\phi} - \frac{m}{Z}\right)^{-1}$ *the equilibrium quantities are given by*

$$L = \frac{D + Z_\alpha + m}{4(1 - \phi)}, \qquad I_x = \frac{D + Z_\alpha + m}{2}, \qquad \Phi_x = D - I_x - L, \qquad \Phi_y = \phi L.$$

If $\left(1 + \frac{D}{Z} \frac{1-2\phi}{3-2\phi} - \frac{m}{Z}\right)^{-1} \geq \alpha > \left(1 + \frac{D}{Z} \frac{r_x+r}{r_x-r} - \frac{m}{Z}\right)^{-1}$ *the equilibrium quantities are given by*

$$L = \frac{D(r_x + r) - (Z_\alpha + m)(r_x - r)}{2(r_x + 2(1 - \phi)r)}, \qquad I_x = D - L, \qquad \Phi_x = 0, \qquad \Phi_y = \phi L.$$

where $Z_\alpha \equiv \frac{Z(1-\alpha)}{\alpha}$.

Several effects of m on balance sheet allocation are worth highlighting. First, the combined reserves of bank x, $\Phi_x + L$, decrease in m in both cases. This is simply because the indirect access to public liquidity reduces the need for holding reserves privately. Second, interbank deposits L become an investment for x. For low levels of m (first parametric case in the previous proposition), I_x and L increase with m because both are treated as investments. This leads to a steep reduction in cash reserves. When m becomes large enough (second parametric case), x will not keep any cash reserves and will keep only interbank deposits. Then, as m goes up, x starts reducing interbank deposits L as it shifts its asset portfolio from low paying investment L to high paying investment I_x.

Next we describe how short-term borrowing reacts to m. The ex-post amount of x's short-term borrowing from y is

$$
b = \begin{cases}
\zeta - \Phi_x & \text{if } \Phi_x < \zeta \leq \Phi_x + \Phi_y + m \\
\zeta - L - \Phi_x & \text{if } \Phi_x + \max\{L, \Phi_y + m\} < \zeta \leq \Phi_x + L + m \\
\zeta - I_x - \Phi_x & \text{if } \Phi_x + \max\{I_x, L + m\} < \zeta \leq \Phi_x + I_x + \Phi_y + m \\
0 & \text{otherwise}
\end{cases}
$$

These cases lead to the following proposition

Proposition 3. *Expected short term borrowing is*

$$
B = \frac{\alpha}{Z}\left(2(m + \Phi_y)^2 + m^2 - \max\{0, m + \Phi_y - L\}^2 - \max\{0, m + L - I_x\}^2\right)
$$

which is strictly increasing in m in equilibrium.

Figure 6 summarizes the previous results for different levels of the public liquidity provision, m. Bank x increases both its own investments I_x and its indirect investments through interbank deposits L. At the same time, it reduces cash reserves at a much faster rate. Compared with the baseline, for medium m, idle cash reserves Φ_x are not necessary to protect the investment L, since there are enough public funds to do so. Given this enhanced flexibility, bank x increases its holdings of the high return investments I_x and reduces its interbank deposits L, which are no longer needed to protect its investments.

This simple analysis highlights the effect of public liquidity provision on the investments and private reserves of shadow banks. Compared with the case of no public provision of liquidity ($m = 0$), shadow banks always invest more in illiquid assets and hold less in cash reserves.

Figure 6: Investments, Interbank Deposits, Short-Term Borrowing, and Cash Reserves
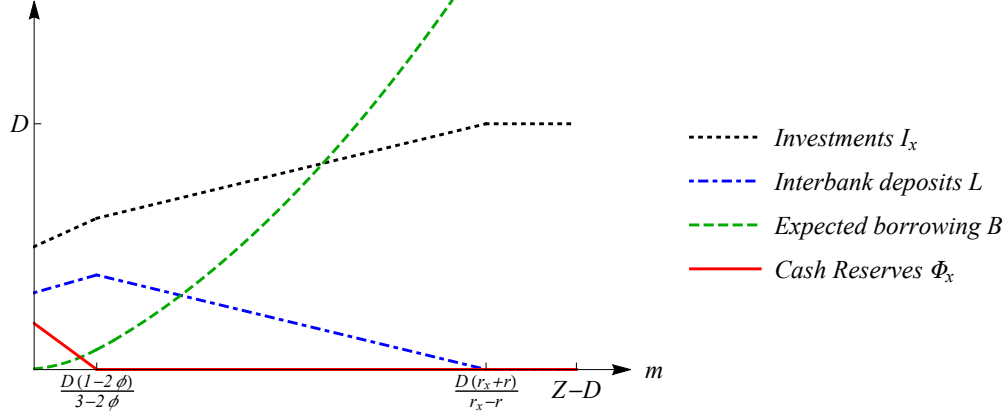
Equilibrium allocation as a function of central bank liquidity m in line with Propositions 2 and 3.

3.3 Systemic Fragility and Vulnerability

We have shown how, for a given public liquidity m, banks adjust their portfolios and choose private liquidity in the system. Even though they reduce private liquidity, its reduction is offset by the provision of public liquidity. Hence, they will not need to liquidate projects when they face liquidity shocks. If public liquidity is costless, central banks may provide an unlimited amount of public liquidity.

Although banks expect a public provision of liquidity by the central banks, they do not know the exact amount of such public liquidity. If they overestimate the availability of public liquidity, they will hold too much in illiquid assets and may have to liquidate their investments. We model this uncertainty with stochastic m. Suppose that m is random between 0 and $Z - D$. Then regardless of the level of public liquidity, there is always a shock high enough to require the liquidation of both projects. Therefore, all of our earlier analyses go through simply by replacing m with $\mathbb{E}[m]$ in the equilibrium quantities.

To discuss these issues with closed-form results, suppose that m is 0 w.p. β and $U[0, \frac{2m^*}{1-\beta}]$ w.p. $1 - \beta$ where $m^* < \frac{1-\beta}{2}(Z - D)$. This distribution implies that m has mean m^*.

There are different ways to categorize risks in the financial system. The first category involves the identity of projects that need liquidation. *Direct risk* refers to the probability that the project of x gets liquidated as a consequence of the (direct) liquidity shock to x. *Contagion risk* refers to the probability that the project of y gets liquidated as a consequence of the shock to x, which prompts x to withdraw its interbank deposits from y, which, in turn makes y liquidate its project. *Systemic risk* refers to the probability that all projects get liquidated.

The second category is based banks' demand for public liquidity and their use of it. *Fragility*

refs to the liquidation risk of portfolios chosen for expected public liquidity m^*, with respect to the random liquidity shock ζ and the random level of public liquidity m. Fragility takes into account all sources of liquidity as expected. For example, fixing m^* and increasing β increases fragility. A fragile economy, for instance is an economy that is more likely to have less than expected public liquidity (for political or macroeconomic shocks) that forces project liquidation *Vulnerability* refers to the liquidation risk of portfolios chosen for the average, m^*, with respect to the liquidity shock ζ, conditional on $m = 0$. Vulnerability takes into account only private liquidity. A vulnerable economy, for instance, would be one with very large projects and very few private reserves.

We can also view fragility and vulnerability as banks' reliance on public liquidity. One can think of a hypothetical economy with deterministic public liquidity m^*. Then fragility refers to risk that projects are indeed liquidated, with respect to the liquidity shock, conditional on the total amount of private and public liquidity. Vulnerability then refers to the risk that projects would have been in distress and forced to liquidate if it were not for public liquidity.

Expected public liquidity m^* will affect different combinations of these categorizations in the identified by Proposition 4:

Proposition 4. *Direct vulnerability is increasing in m^*. Systemic vulnerability and contagion vulnerability are increasing in m^* under $m^* < D\frac{1-2\phi}{3-2\phi} - Z_\alpha$ and decreasing in m^* under $m^* > D\frac{1-2\phi}{3-2\phi} - Z_\alpha$. All notions of fragility are decreasing in m^*.*

Fragility has two components. An *injection effect* of public liquidity, which always reduces the likelihood of public liquidity shortages, and an *equilibrium effect*, which increases the likelihood of private liquidity shortages. Notice that the equilibrium effect in the evaluation of fragility is, in fact, vulnerability.

$$\text{Fragility} = \text{Vulnerability - Injection Effect}$$

When there is no expectation that central banks will provide liquidity support (this is $m^* = 0$), fragility and vulnerability are the same. In that situation, projects that are vulnerable *because* they may be liquidated without public liquidity support will indeed be liquidated when there is no expectation that central banks will provide liquidity support. The larger the expected injection of public liquidity, the lower is the fragility given a level of system vulnerability.

Intuitively, this explains why all measures of fragility decline in m^* given a level of vulnerability in Proposition 4 (all projects are less likely to be liquidated when there are large amounts of public liquidity in the system).

Vulnerability, however, measures the exposure of the system to the need for liquidation. Direct vulnerability is increasing in m^* because bank x reduces the buffer $L + \Phi_x$ that protects I_x when it expects large public liquidity support. As the project of bank x becomes more reliant on public liquidity, its direct vulnerability increases.

Systemic vulnerability and contagion vulnerability are also increasing with m^* but only when bank x's cash buffer responds to changes in m^*. When not much public liquidity is expected (low m^*), bank x still holds cash as an insurance against liquidation risk. As m^* increases, cash declines and the investments channeled to bank y increase, thereby increasing the vulnerability of its project. Once bank x expects high enough levels of public liquidity, it will decide not to hold cash anymore. As m^* increases in this case, bank x cannot reduce cash further, so the bank would reduce L to increase I_x. This makes bank y's project less vulnerable and reduces contagion and systemic vulnerability.

3.4   Networks

In this section, we extend our framework to study how the structure of the interbank network changes in response to the provision of public liquidity. We show that banks move their interbank relations towards counterparts that are less costly to maintain but provide less liquidity insurance. We model banks' desire to connect less to central reserve cities and more to their connections to regional reserve cities. As a result, public insurance crowds out private insurance that smooths out cross-regional liquidity shocks.

We extend our analysis to several banks. As a first step we focus on four banks in two pairs. More specifically, banks x_1 and y_1 are linked as described in the baseline, and the same is true for banks x_2 and y_2. We assume that banks x_1 and x_2 have household deposits and projects. In contrast, banks y_1 and y_2 have interbank deposits received from x_1 and x_2, and projects. We call $\{x_1, x_2\}$ *the periphery* and $\{y_1, y_2\}$ *the core*. As a next step we generalize the functioning of banks in the core.

We introduce these generalizations in Section 3.4.1. In Section 3.4.2 we study how core banks can coinsure each other through forming a sort of clearinghouse, as was the case with large New York banks historically. Finally, in Section 3.4.3, we allow periphery banks located in different regions to choose their correspondents from among two groups of banks: those that have greater coinsurance possibilities but may be farther away (say, banks in New York) and those that have fewer coinsurance possibilities but may be closer (say, banks located in regional reserve cities). This allows us to study the effect of the central bank's liquidity provision m on the network structure. We show that central bank liquidity induces a shift

of links from the far core (New York City) to the close core (regional reserve cities), thereby crowding out the private insurance that the system is able to provide.

3.4.1 Extended setting with two pairs of two banks

We assume that each of the core banks y_1 and y_2 has access to central bank liquidity, capped at (deterministic) m in total. We also assume that the shocks faced by x_1 and x_2 are negatively correlated, so we rule out competition over central bank liquidity.

Denote $\theta = \frac{\alpha}{2} \leq 0.5$ the probability that the shock ζ_1 is drawn from $U[0, Z]$ and that the shock $\zeta_2 = 0$. The parameter θ is also the symmetric probability that the shock ζ_2 is drawn from $U[0, Z]$ and that the sock $\zeta_1 = 0$. There is, then a probability $1 - 2\theta = 1 - \alpha$ that there is no shock, and $\zeta_1 = \zeta_2 = 0$. This specification implies that only one bank needs liquidity at a time and that we do not need to model the priorities of the central bank over which bank to provide liquidity to, and how much. In other words, we abstract from aggregate liquidity shocks in the system, meaning all shocks are purely endogenous and induced by contagion.

Given this shock structure, the ex-ante profit of x_i is

$$\Pi_{x_i} = \left(1 - \theta\frac{I_{x_i} - m}{Z}\right) I_{x_i} r_x + \left(1 - \theta\frac{2(1 - \phi)L_i - m}{Z}\right) L_i r.$$

Proposition 5. *If $m < D\frac{r_x + r}{r_x - r} - Z\frac{1 - \theta}{\theta}$, equilibrium quantities are given by*

$$L_i = \frac{D(r_x + r) - (Z_\theta + m)(r_x - r)}{2(r_x + 2(1 - \phi)r)}, \qquad I_{x_i} = D - L_i, \qquad \Phi_{x_i} = 0, \qquad \Phi_{y_i} = \phi L_i.$$

Otherwise, $L_i = 0$ and $I_{x_i} = D$.

3.4.2 Liquidity coinsurance in the network

Now we allow core banks y_1 and y_2 to insure each other against liquidity shocks coming from banks x by reallocating liquidity between the two. When x_i faces a liquidity shock, it can borrow from y_i, which can borrow from y_j as well as from the central bank. Now ex-ante profits are given by

$$\Pi_{x_i} = \left(1 - \theta\frac{I_{x_i} - \phi L_j - m}{Z}\right) I_{x_i} r_x + \left(1 - \theta\frac{2(1 - \phi)L_i - \phi L_j - m}{Z}\right) L_i r.$$

Liquidity coinsurance, captured in the term ϕL_j, reduces the liquidation risk of the interbank deposits of x_i.

Proposition 6. *If* $m < D\frac{r_x + r}{r_x - r} - Z\frac{1-\theta}{\theta}$, *equilibrium quantities are given by*

$$L_i = \frac{D\left(r_x + r\right) - \left(Z_\theta + m\right)\left(r_x - r\right)}{2\left(r_x + 2(1-\phi)r\right) + \phi(r_x - r)}, \qquad I_{x_i} = D - L_i, \qquad \Phi_{x_i} = 0, \qquad \Phi_{y_i} = \phi L_i.$$

Otherwise, $L_i = 0$ *and* $I_{x_i} = D$.

3.4.3 Endogenous network

Here we extend the framework to show that the provision of public liquidity insurance crowds out the provision of private liquidity insurance. This happens when the periphery banks' choices of correspondents changes under the central bank liquidity provision and leads to the formation of a new network structure.

Let x_i represent a bank in region i which can place deposits in a local reserve-city bank y_i^C or a New York City bank y_i^N. Similarly, let x_j represent a bank in region j, which can place deposits in a local reserve-city bank y_j^C or a New York City bank y_j^N. For both banks x_i and x_j, placing deposits in New York City banks incurs a higher cost than placing deposits in regional reserve-city banks because of the geographical distance between respondents and correspondents. As discussed above, two New York City banks y_i^N and y_j^N insure each other against liquidity shocks by reallocating liquidity in the system. In the absence of the central bank, this framework, x_i and x_j will choose y_i^N and y_j^N in order to reduce their exposure to local liquidity shocks. Since liquidity shocks are not perfectly correlated between regions i and j, x_i and x_j can smooth local liquidity shocks by adjusting their interbank deposits in New York City.

Now, let us introduce central bank liquidity, m. Since x_i can mitigate local liquidity shocks by borrowing from a regional correspondent y_i^C directly, we study conditions under which it will choose to connect to y_i^C rather than y_i^N because connecting to the former is cheaper. Similarly, x_j will choose to connect to y_j^C rather than y_j^N. There are two options for equilibria. Banks can either connect to New York City banks for private insurance but pay higher costs, or they can connect to regional reserve-city banks.

From the analysis in Section 3.4.1 above, we know that if both banks connect to their regional correspondents, in equilibrium,

$$\Pi_{x_i}^C = \left(1 - \theta\frac{(D - L_C) - m}{D}\right)(D - L_C)\,r_x + \left(1 - \theta\frac{2\left(1 - \phi\right)L_C - m}{D}\right)Lr$$

where L_C is given by Proposition 5.

From the analysis in Section 3.4.2 above, if both banks connect to NY, in equilibrium,

$$\Pi_{x_i}^N = \left(1 - \theta\frac{D - L_N - \phi L_N - m}{D}\right)(D - L_N)\,r_x + \left(1 - \theta\frac{2\,(1-\phi)\,L_N - \phi L_N - m}{D}\right)L_N r - c$$

where L_N is given by Proposition 6.

The next Lemma shows that the relative gain to connect with core banks decline with the volume of public liquidity offered by the Federal Reserve System.

Lemma 1. *If* $0 \le m < D\frac{r_x + r}{r_x - r} - Z\frac{1-\theta}{\theta}$

$$\frac{d\left(\Pi_{x_i}^C - \Pi_{x_i}^N\right)}{dm} > \frac{r(r_x - r)\phi}{2\,(r_x + 2(1-\phi)r) + \phi(r_x - r)}\frac{\theta D}{Z} > 0.$$

Corollary 1. *For all* $c > 0$*, there exists* $m_c \in [0, D\frac{r_x + r}{r_x - r} - Z\frac{1-\theta}{\theta}]$ *such that the following hold. For all* $m \in (0, m_c)$*, both regional banks deposit at their NYC correspondent. For all* $m \in (m_c, D\frac{r_x + r}{r_x - r} - Z\frac{1-\theta}{\theta})$*, both regional banks deposit at their local reserve-city correspondents. For* $m > D\frac{r_x + r}{r_x - r} - Z\frac{1-\theta}{\theta}$*, regional banks do not deposit at correspondents.*

This discussion shows that, with high enough public liquidity (more specifically, when $m > D\frac{r_x + r}{r_x - r} - Z\frac{1-\theta}{\theta}$), there is no lending to NYC banks regardless, as banks do not rely explicitly on cross-regional insurance to save their own projects. With lower levels of public liquidity, however (more specifically, $m < D\frac{r_x + r}{r_x - r} - Z\frac{1-\theta}{\theta}$), the extensive margin of lending switches. Even after accounting for endogenous deposit levels, the marginal benefit decreases as the amount of central bank liquidity increases.

This discussion is summarized in Proposition 7.

Proposition 7. *There exists* m_c *such that, for* $m < m_c$*, banks both regions deposit their reserves at NYC banks, and for* $m > m_c$ *banks in both regions deposit their reserves in their corresponding reserve-cities.*[15]

Because public liquidity increases the ability of banks to absorb local liquidity shocks, x_i and x_j reduce their reliance on New York City banks and rely on banks in regional reserve cities. A new network structure emerges as the concentration of links decreases. These changes are illustrated in Figure 7.

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[15]We use stability as our equilibrium concept, which allows for x_1 and x_2 to deviate together.

Figure 7: Network Reactions to Public Liquidity Provision
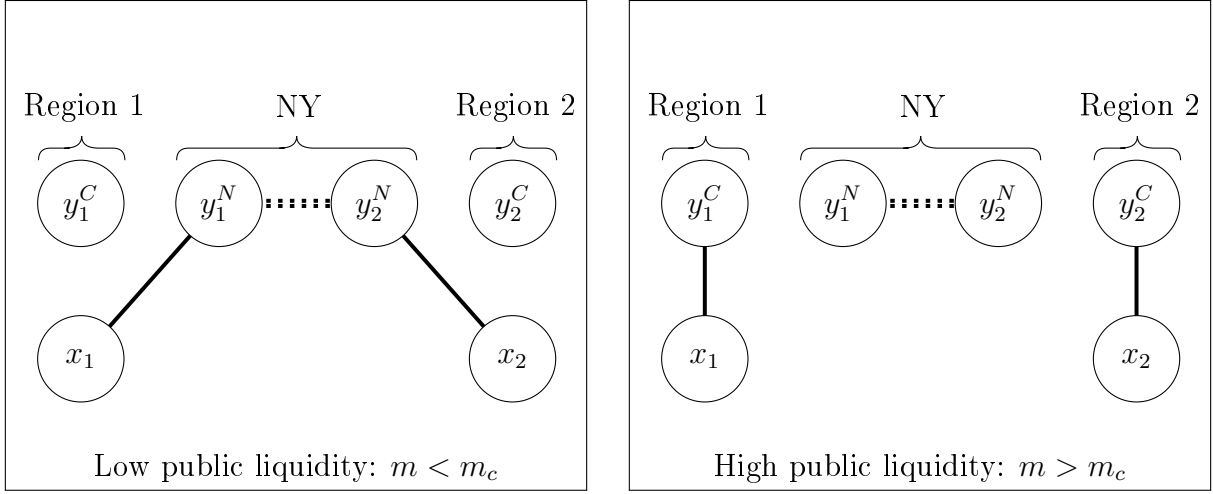
Change in the structure of the regional interbank network.

3.5 Summary

Our simple model highlights three testable predictions of public liquidity provision, m, which correspond to implications for the allocation of funds in the economy and the shape of interbank linkages. As a summary, these are

1. *An increase in public liquidity provision (m) reduces aggregate private liquidity.* Private liquidity holdings (cash and interbank deposits) decline for both member and nonmember banks.

2. *An increase in public liquidity provision (m) intensifies interbank relations.* With more interconnections (in terms of short-term borrowing) there is an increase in the possibility of contagion, which increases the system's vulnerability to regional liquidity shocks.

3. *An increase in public liquidity provision (m) dissipates the overall interbank network.* The network structure changes from a geographically concentrated core to a dissipated core, crowding out private insurance for cross-regional shocks.

4 Empirical Evidence

In this section we provide empirical evidence to support our theoretical predictions and their implications. We document how the advent of the Federal Reserve's discount window changed aggregate liquidity in the banking system as well as the nature of interbank relations and the structure of interbank networks.

4.1 Data Sources

We collect data from two sources. The first source is the Annual Report of the Comptroller of the Currency (OCC) from 1910 through 1929. This source provides aggregated balance sheet information for national and state banks, and because these data are aggregated, our dataset allows us to test the implications in terms of aggregate private liquidity and short-term borrowing in the nation as a whole. The second source is bank-specific: it consists of state bank examination reports for all state-chartered banks in Virginia in 1911 and 1922. This source provides individual balance sheets and detailed information on interbank linkages.

4.1.1 Aggregate Balance Sheet Data

First, using the *Annual Report of the Comptroller of the Currency*, we collect balance sheet information for national and state banks from 1910 to 1929 and construct balance sheet data aggregated at the state level. During this period, banks were divided into three subcategories based on size and location: central reserve-city banks, reserve-city banks, and country banks. For national banks, the OCC report provides data for three groups of banks separately, but for state banks it does not.[16]

With these aggregate balance sheet data, we examine the effect that the creation of the Federal Reserve had on member versus nonmember banks at the aggregate level. Because all national banks were members of the Federal Reserve System and few state banks became members (Figure 1), we treat national banks generically as a proxy for member banks and state banks as a proxy for nonmember banks.

As noted, we examine national and state banks from 1910 to 1929-but with a gap between 1918 and 1920 for three reasons. First, in 1917 the Federal Reserve provided a three-year phase-in period allowing member banks to adjust to new reserve requirements. Second, in 1917 Congress amended the 1913 legislation and lowered reserve requirements in order to attract more state banks. Third, after the nation's entrance into World War I (April 1917), the Federal Reserve offered a preferential discount rate on loans secured by government debt to support the war effort, but between 1920 and 1921 it removed this preferential rate, raising its discount rate and tightening banks' access to the discount window.

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[16]For 3 central reserve cities (New York, Chicago, and St. Louis), the OCC constructs data at the city level, and it does likewise for 17 reserve cities (Albany, NY; Baltimore, MD; Boston, MA; Cincinnati, OH; Cleveland, OH; Detroit, MI; Kansas City, MO; Louisville, KY; Milwaukee, WI; Minneapolis, MN; New Orleans, LA; Omaha, NE; Philadelphia, PA; Pittsburgh, PA; Saint Joseph, MO; Saint Paul, MN; and San Francisco, CA). For the country banks it regulates, however, the OCC constructs data at the state-level.

4.1.2 Bank-Level Balance Sheet Data

To capture the testable implications in terms of the structure of interbank relations, we collect state bank examination reports for all state-chartered banks in Virginia for the years 1911 and 1922-thus, before and after passage of the Federal Reserve Act. Virginia State bank examiners inspected all banks and trust companies with a state charter and filed reports once or twice a year. We collected 222 of these examination reports for 1911 and 327 examination reports for 1922. In Virginia there were 248 and 334 state banks in 1911 and 1922, respectively. Hence, our dataset provides comprehensive information on Virginia state banks-most especially, for our purposes, on their balance sheets and their counterparties-before and after passage of the Federal Reserve Act. We focus on nonmember banks, however, because in Virginia only 11 state banks joined the Federal Reserve System.[17]

The banks' balance sheet statements and detailed information on interbank relationships (see Appendix Figure A1) allow us to examine the connections that exist between the role of interbank relationships in the payments system and in funding. For a given bank, the dataset reports three types of interbank relationships: deposits *due from* other bank, deposits *due to* other bank, and short-term borrowing from another bank.

The examiners recorded detailed information on interbank deposits to verify whether state banks were holding enough interbank deposits to meet regulatory reserve requirements. As mentioned in the section on historical background, above, state bank regulators allowed state nonmember banks to hold interbank deposits to satisfy reserve requirements, which member banks were not allowed to do. In Virginia, nonmember banks could hold up to $7/12$ of required reserves in the form of interbank deposits with approved reserve agents. Member bank reserve requirements differed based on geographic classification of banks. In most cases, nonmember bank had similar requirements. However, Virginia state bank regulators did not make reserve requirements differently for Richmond and country banks. In our analysis we do not divide the sample into Richmond banks and country banks, because the Virginia banking department imposed the same reserve requirements for both groups of banks.[18]

The examiner reported the amount that was *due from* each correspondent bank and the name of each of those correspondent banks. "Due from banks" are then assets. Deposits *due to* other banks are deposits that other banks hold with a correspondent bank and are thus liabilities of the bank. Although only balances with reserve agents could be used to satisfy the legal reserve requirement, the examiner reported *all* the balances due from other banks. Because due-to deposits constituted a small fraction of country banks' liabilities, we focus

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[17]Although we collected information on 2 of these 11 banks, we drop them from our analysis.

[18]Richmond was a reserve city, and in 1922 it became home to the district's Federal Reserve Bank.

throughout on due-from deposits, and this information provides us with a complete picture of the *payment networks* of state banks in Virginia during the two years in question.

The examination reports also provide information on whether a bank borrowed on a collateralized basis from its correspondents, the amounts of the borrowed money, and the identity of the lender. These short-term borrowings took the form of rediscounts and bills payable. "Bills rediscounted" were loans sold with recourse. "Bills payable" consisted of either promissory notes of the borrowing bank or borrowing from Federal Reserve Banks. This information provides us with a complete picture of the *funding networks* of Virginia state banks.

The data for 1911 (the first year when Virginia's banking department released examination reports) capture bank behavior before passage of the Act. The data for 1922, capturing bank behavior after passage of the Act, covers state nonmember banks. In the data for 1922, moreover, we expect to find that the structure of the interbank network stabilized after the end of the war.

4.2 Balance Sheet Analysis

To determine how the advent of the Federal Reserve System affected the liquidity of the banking system, we begin with an analysis of balance sheet ratios of national (member) and state (nonmember) banks using state-level aggregate bank balance sheets. Then we drill down to bank-level balance sheet data to compare macro- and micro-level patterns. We focus on trends in our testable counterparts: cash, due from other banks (deposits in other banks), and borrowed money (short-term borrowing).

4.2.1 Aggregate balance sheet analysis

In Table 1, to show the effect that creation of the Federal Reserve System had on member versus nonmember banks at the aggregate level, we focus on the comparison of short-term borrowing, cash, and deposits in other banks.

We begin by examining the volume of short-term borrowing by banks. Before the advent of the Federal Reserve, short-term borrowing was not large and national banks borrowed less than state banks (roughly 1% versus 2%).[19] After the advent of the Federal Reserve, both types of banks increased their borrowing significantly, but national banks increased theirs more than state banks did: whereas state banks almost doubled their relative borrowing,

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[19]These patterns are driven by the fact that reserve-city and central reserve-city banks generally borrowed less often than country banks (See Carlson and Wheelock (2018b)).

Table 1: Balance Sheet Ratios, U.S. Aggregate, 1910-1917 and 1921-1929

	National Banks		State Banks	
	1910-1917	1921-1929	1910-1917	1921-1929
Cash to assets	6.056	1.769	4.257	2.392
	(2.887)	(0.705)	(1.688)	(1.442)
Duefroms to assets	14.36	7.615	14.26	7.914
	(6.167)	(3.487)	(4.915)	(5.524)
Equity to liabilities	18.9	13.2	19.84	13.86
	(9.467)	(2.92)	(5.776)	(3.202)
Deposits to liabilities	59.3	69.94	72.57	77.75
	(24.16)	(9.05)	(8.823)	(6.706)
Duetos to liabilities	13.76	8.39	3.062	2.452
	(13.91)	(8.545)	(2.237)	(2.164)
Borrowing to liabilities	1.142	3.422	2.165	3.553
	(2.359)	(3.544)	(2.989)	(3.395)
Obs.	567	630	384	432

Table 1 displays summary statistics for national and state banks during the period 1910-1929. Cash is composed of specie and legal tender notes. Duefroms are interbank deposits due from other banks. Equity is composed of paid in capital and surplus. Duetos are interbank deposits due to other banks. Borrowing is short-term borrowing from other banks or the Federal Reserve Bank.
Source: *Annual report of the Comptroller of the Currency.*

national banks more than tripled theirs. This is consistent with our model in that after public liquidity was provided, banks relied more on short-term borrowing to face short-term liquidity needs.

Now we turn attention to the most liquid asset on the balance sheet-cash (Φ_x in our model). We look at the share of bank assets held in the form of vault cash for both national and state banks from 1910 to 1929. Although all banks held less cash after the Federal Reserve was created, the reduction was larger for national banks (from 6.1% to 1.8%) than for state banks (from 4.3% to 2.4%). While the reduction for national banks resulted directly from access to public liquidity and lower reserve requirements, the reduction for state banks is largely explained by indirect access to public liquidity, which is consistent with our model.

Even though state banks were not members, they held less liquidity because they were able to access public liquidity through their member correspondents.

Next, we examine the movement of interbank deposits (L in our model). On the asset side, for both national and state banks the relative deposits in other banks declined roughly by 50% (in both cases, dropping from around 14% to around 8%). The liability side of the balance sheets shows that deposits due to other banks decreased as well. In this case, the large decline was mostly experienced by national banks that used to be correspondents and received most of the interbank deposits in the system. For national banks, a reduction in the volume of interbank deposits due from and due to other banks was likely a direct consequence of the National Banking Act, which prohibited the use of interbank deposits to meet reserve requirements. For state banks, in contrast, a decline in the volume of due-from deposits was less mechanical, for those banks were still able to meet reserve requirements by holding interbank deposits.

Because interbank deposits were an important source of financial contagion, we decompose the holding of interbank deposits by national banks. In Figure 8, we plot interbank deposits for national banks by geographical classification and examine separately the relationships with national and with state banks.[20] Panel (a) of Figure 8 shows national banks' deposits due from other national banks and due from state banks. Until 1914, national banks held most of their deposits in other national banks and very little in state banks. After creation of the Federal Reserve, for all three groups of national banks the share of bank assets held in the form of deposits due from other national banks declined significantly. This suggests that among national banks, private insurance was reduced after creation of the central bank.

In Panel (b) of Figure 8, we look at national banks' deposits due to other national banks and due to state banks. After creation of the Federal Reserve, national banks in both reserve and central reserve cities saw a large reduction in the volume of their deposits due to other national banks. However, they did not see a significant decline in the volume of their deposits due to state banks. In other words, despite the reduction of private cross-insurance, national banks in financial centers were still vulnerable to runs by state banks.

4.2.2 Bank-Level balance sheet analysis

In this section, we use individual balance sheets of Virginia state banks to examine the consistency of the results between state-level balance sheet data and aggregated bank-level

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[20]Note that the gap is larger in these two figures when compared with the rest because the OCC did not separate "deposits due from other banks" into "deposits due from other national banks" and "deposits due from other state banks" between 1915 and 1917. Similarly, it did not report "deposits due to other national banks" and "deposits due to other state banks" separately.

Figure 8: Interbank Deposits, National Banks in Tier Groups, 1910-1914 and 1921-1928

(a): Deposits due from other Banks

Due from National Banks	Due from State Banks

(b): Deposits due to other Banks

Due to National Banks	Due to State Banks
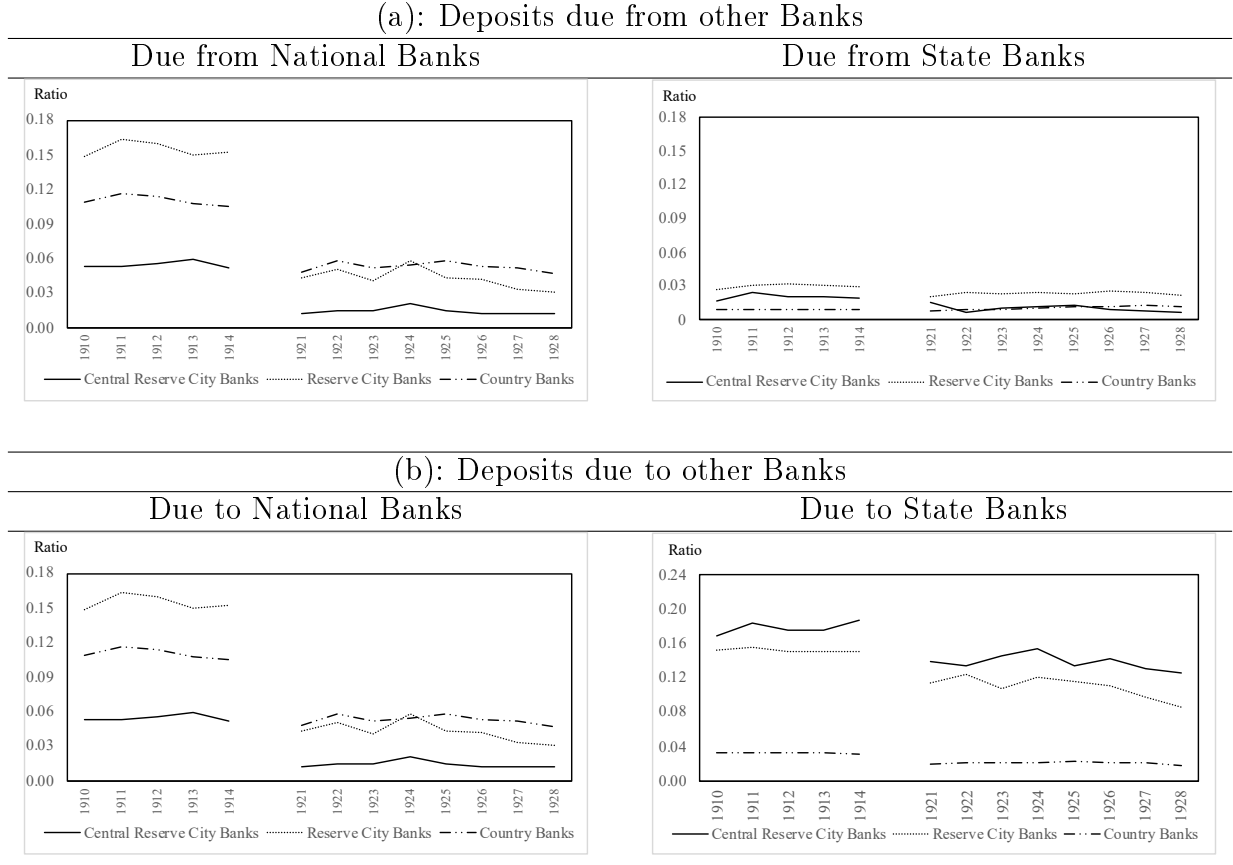
Figure 8 plots the ratio of deposits to other banks against total liabilities for national banks in each tier group. The table examines national banks' deposits due to other national banks and their deposits due to other state banks separately. Data for country banks are aggregated across all U.S. states; data for reserve city banks are aggregated across 17 reserve cities; and data for central reserve cities are aggregated across the three central reserve cities.
Source: Annual report of the Comptroller of the Currency.

balance sheet data, and we focus on the behavior of nonmember banks. In Table 2, we compute balance sheet ratios for the years 1911 and 1922. First, we examine the share of short-term borrowing by banks. In 1911, before creation of the Federal Reserve, country banks' short-term borrowing accounted for 4% of country bank liabilities. In 1922, after creation of the central bank, country banks' short-term borrowing increased to 6%. These patterns suggest that in the absence of direct access to the discount window, nonmember banks were able to increase the use of short-term funds by borrowing from member banks. These patterns are consistent with findings from the aggregated data and also with the prediction of the model.

Second, we show that in addition to increasing short-term borrowing, Virginia state banks reduced liquid asset holdings. The share held in the form of vault cash (specie and legal tender

Table 2: Balance Sheet Ratios, Virginia State Banks, 1911 and 1922

	1911	1922
Cash to assets	4.607	3.159
	(2.913)	(2.782)
Duefroms to assets	12.38	8.880
	(7.597)	(7.018)
Equity to liabilities	25.65	22.22
	(10.63)	(11.85)
Deposits to liabilities	68.38	70.19
	(14.80)	(16.61)
Duetos to liabilities	1.608	1.224
	(6.972)	(6.015)
Borrowing to liabilities	3.805	5.764
	(6.309)	(7.675)
Obs.	220	320

Cash is composed of specie and legal tender notes. Duefroms are interbank deposits due from other banks.
Equity is composed of paid in capital and surplus. Duetos are interbank deposits due to other banks.
Borrowing is short-term borrowing from other banks.
Source: *Virginia State Bank Examination Reports*.

notes) declined from 4.7% in 1911 to 3.1% in 1922. In addition, the share of correspondent deposits in other banks declined roughly by 30% (from around 12% to around 8%). The bank-level data yield exactly the same results as the aggregate data and the direction predicted by the model.

To summarize, both aggregate balance sheet data for all banks and more-detailed balance sheet data from Virginia state banks indicate that the advent of the Federal Reserve reduced liquidity (in the form of cash and interbank deposits) and intensified the funding role of interbank relationships (in the form of higher short-term borrowing) between member and nonmember banks.

4.3 Network Analysis

The effect we have seen on the allocation of funds at the bank level does not, however, provide insights into the concentration of interbank relations at the geographical level. In this section, we study how the creation of the Federal Reserve System affected the nature and structure of the interbank system in Virginia, a state where we have detailed extensive and intensive margins of banking interrelations. The balance sheet analysis shows that the creation of the central bank weakened the role played by these relations in the payments system but strengthened the relations' role in funding. We examine the due-from deposits and short-term borrowing networks at extensive and intensive margins.

Figure 9 maps all respondent banks (Virginia state banks) and correspondent banks for the years 1911 and 1922. The respondent (corresponding) banks that *only* placed (received) deposits are in blue, while banks that *both* placed (received) deposits and borrowed (lent) short-term funds are in red. There were 1,033 and 1,056 unique due-from relationships in 1911 and 1922, respectively. In addition, there were 150 and 309 unique short-term borrowing relationships in 1911 and 1922. These maps provide the first clues that the funding roles of the network became more prevalent after passage of the Federal Reserve Act.

Table 3 shows the number of correspondent relationships in 1911 and 1922. Three patterns can be discerned. First, state banks held multiple correspondent accounts in multiple banks, whereas they borrowed from one or two banks each. Second, the average number of correspondents in which a bank had deposits decreased from 4.8 to 3.3. Third, the average number of correspondents from which a bank borrowed short term remained at around 1.8.

Before providing detailed information on the overall network before and after passage of the Federal Reserve Act, we present a specific example to illustrate banks' relations. The banking relations of the Bank of Warm Springs in Warm Springs, Virginia, are depicted in

Figure 9: Respondent and Correspondent Banks, 1911 and 1922

Respondent banks	
1911	1922

Correspondent banks	
1911	1922
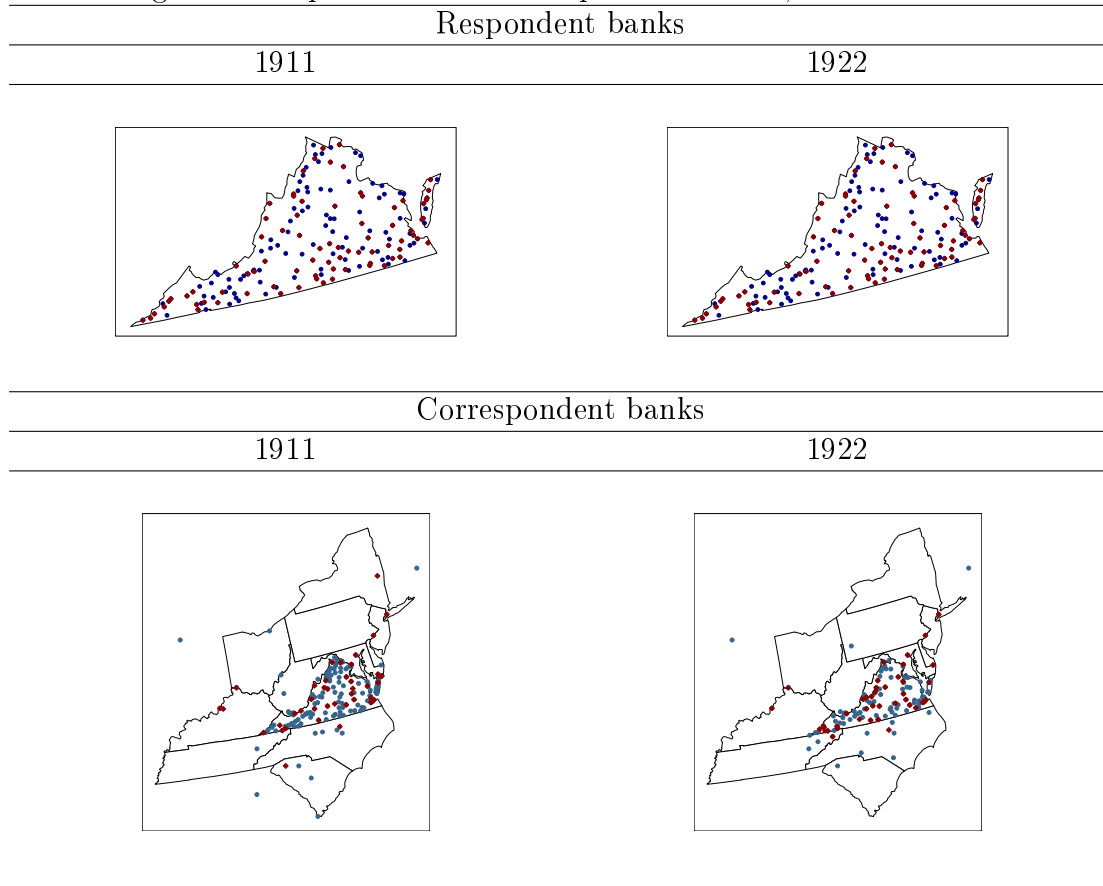
Table 3: Number of Correspondent Relationships, 1911 and 1922

	1911			1922		
	Number of Banks	Mean	SD	Number of Banks	Mean	SD
Due-from	218	4.71	4.13	323	3.27	2.41
Borrowing	89	1.71	1.05	172	1.80	1.06

Table 3 displays the average number of correspondent relationships per bank. "Due-from" indicates the average number of other banks from which a bank had amounts due. Similarly, "Borrowing" indicates the average number of correspondent banks that lent short-term funds to a respondent bank.
Source: Virginia State Bank Examination Reports.

Figure 10. The correspondent banks that received *only* deposits from the Bank of Warm Springs are in blue and the ones that *both* received deposits and lent short-term to the Bank of Warm Springs are in red. In the tabular component of the map, we provide detailed information about these correspondent relationships. Columns (1) and (2) provide the names and locations of the correspondent banks of the Bank of Warm Springs. Columns (3) and (4) show the amount of interbank deposits due from these banks and the amount of short-term funds borrowed from them.

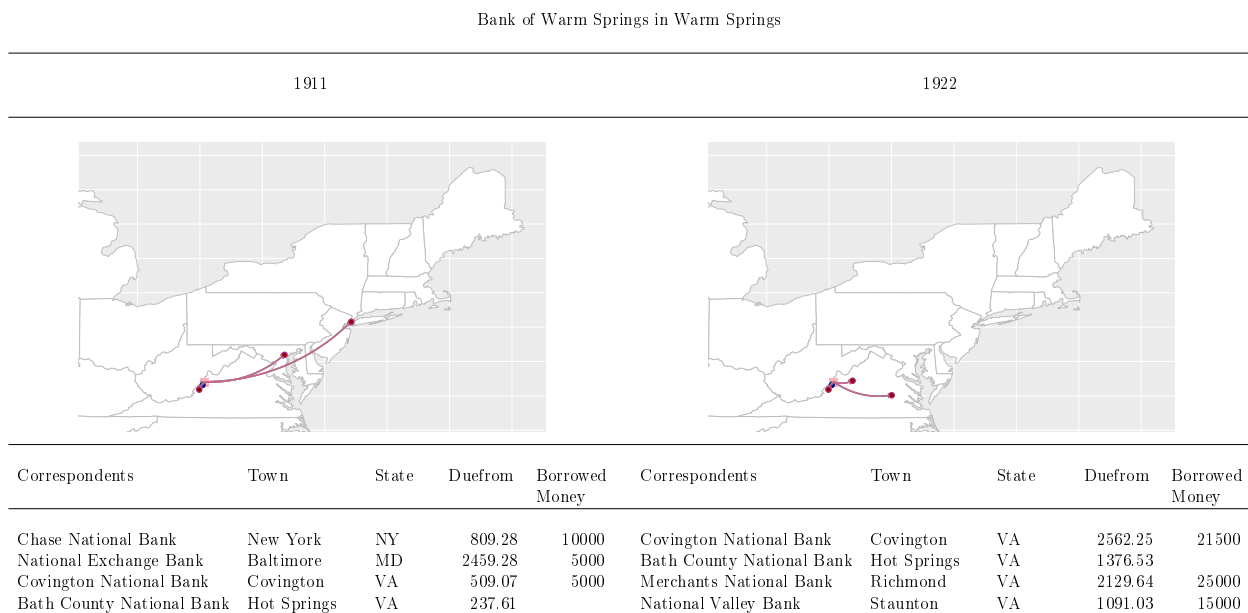Figure 10 shows how the structure and nature of the bank network for that specific bank changed after passage of the Federal Reserve Act. First, correspondent relationships became more local. In 1911, Bank of Warm Springs maintained correspondent banking relationships in New York and Baltimore, but by 1922 it had dissolved these relationships and opened new ones with banks in Richmond and Staunton, which were in close proximity. In addition, after the Act was passed the bank placed large correspondent deposits in local banks, whereas previously it had held a majority of its interbank deposits in Baltimore. Similarly, after passage of the Act the bank relied on Virginia banks for short-term borrowing, whereas previously it had borrowed from a New York City correspondent.

Figure 10: Bank Network for Bank of Warm Springs

Bank of Warm Springs in Warm Springs

1911   1922

Correspondents	Town	State	Duefrom	Borrowed Money	Correspondents	Town	State	Duefrom	Borrowed Money
Chase National Bank	New York	NY	809.28	10000	Covington National Bank	Covington	VA	2562.25	21500
National Exchange Bank	Baltimore	MD	2459.28	5000	Bath County National Bank	Hot Springs	VA	1376.53	
Covington National Bank	Covington	VA	509.07	5000	Merchants National Bank	Richmond	VA	2129.64	25000
Bath County National Bank	Hot Springs	VA	237.61		National Valley Bank	Staunton	VA	1091.03	15000

Notes: Figure 10 provides information for the Bank of Warm Springs in Warm Springs. Columns (1) and (2) provide information about the names and locations of correspondent banks. Columns (3) and (4) provide information about the amount of interbank deposits due from these banks and the amount of short-term funds borrowed from them.

Source: *Virginia State Bank Examination Reports.*

The changes made by Bank of Warm Springs are representative of the general patterns that characterize interbank networks before and after creation of the Federal Reserve System. In Tables 4 through 7 we present more systematically the interbank relationships for all Virginia state banks. Tables 4 and 5 show the structure of the interbank system at extensive and intensive margins, and Tables 6 and 7 show the distance in miles between respondent and correspondent banks.

Table 4 shows the distribution of state banks' due-from deposits (payment network). We find that the creation of the central bank encouraged banks to rely more heavily on local correspondents. Before the advent of the Federal Reserve, banks relied more on correspondent relationships with banks outside Virginia; for example, many banks had city correspondents in New York City and Baltimore. After the Federal Reserve's creation, however, the due-from deposit network became more dispersed. Banks also shifted their relationships away from New York and Baltimore and into other country banks in Virginia. These results are consistent at both extensive and intensive margins.

Table 5 shows the nature of the short-term borrowing network (funding network). We find that creation of the central bank encouraged more local short-term borrowing relationships. Before the agency's creation, 40% of country banks borrowed short-term funds from their correspondents, particularly from Richmond banks. After the agency's creation, banks borrowed more heavily from other country banks in Virginia instead of from Richmond banks.

To identify more clearly the change in the geographical concentration of the interbank system, we compute the distances in miles between respondent and correspondent banks. Table 6 shows the distance between state banks and the correspondent banks with which they placed deposits, and Table 7 shows the distance between state banks and the correspondent banks from which they borrowed short-term funds. In placing deposits after creation of the central bank, Virginia state banks chose to reduce their connectivity to New York City and placed their deposits in local banks. Likewise in borrowing short-term funds after creation of the central bank, those same banks chose to borrow from banks located in close geographic proximity, for any member bank independent of location could access the central bank's discount window.

The shift in the network structure toward geographically closer links suggests that the existence of the Federal Reserve System enabled banks to rely more on the provision of public liquidity and less on the provision of private liquidity. The importance of New York City banks in providing private insurance arrangements for regional liquidity shocks before the advent of the Federal Reserve has been well documented (Carlson and Wheelock (2018a)). By pooling bank reserves from banks in different regions, New York City banks had been able

Table 4: "Due from" Relationships, Virginia State Banks, 1911 and 1922.

	Extensive Margin (Links)		Intensive Margin (Amount)	
	1911	1922	1911	1922
New York	19.37	12.47	10.63	6.884
	(19.02)	(16.55)	(16.93)	(13.91)
Chicago	0.105	0.0163	0.0564	0.01
	(0.791)	(0.293)	(0.471)	(0.0908)
Baltimore	9.126	6.816	10.33	6.948
	(17.73)	(16.39)	(23.45)	(19.70)
Washington, DC	2.226	1.719	1.812	1.556
	(7.787)	(9.725)	(7.506)	(10.73)
Richmond	20.81	22.22	28.09	26.85
	(20.14)	(27.98)	(32.56)	(34.53)
Reserve Cities in Other States	2.592	3.404	2.554	3.987
	(7.774)	(13.46)	(8.465)	(16.68)
Country Banks in VA	43.07	50.59	42.46	51.76
	(29.04)	(34.87)	(37.56)	(40.74)
Country Banks in Other States	2.701	2.769	3.174	1.704
	(10.70)	(10.25)	(14.80)	(9.389)
Obs.	218	323	218	323

Notes: Rows indicate the location of correspondent banks. New York was a central reserve city. Baltimore and Washington, DC, were reserve cities. Richmond was not a reserve city in 1911 but became one in 1922. Columns indicate the location of respondent banks. Extensive margins are the proportions of links in each location against total links. Intensive margins are proportions of correspondent deposits held at different locations against total due-from deposits.
Source: *Virginia State Bank Examination Reports*.

Table 5: "Short-term Borrowing" Relationships, Virginia State Banks, 1911 and 1922.

	Extensive Margin (Links)		Intensive Margin (Amount)	
	1911	1922	1911	1922
New York	8.427	8.992	7.661	8.376
	(23.11)	(22.35)	(22.40)	(22.57)
Baltimore	11.33	7.045	11.35	6.936
	(28.43)	(22.98)	(29.23)	(23.18)
Washington, DC	2.060	1.599	1.814	1.589
	(11.74)	(11.52)	(11.20)	(11.64)
Richmond	32.68	21.57	32.14	21.10
	(41.73)	(35.19)	(42.40)	(35.41)
Reserve Cities in Other States	3.464	4.186	3.684	4.162
	(13.92)	(17.39)	(15.14)	(17.39)
Country Banks in VA	38.58	52.42	36.56	49.66
	(42.67)	(43.64)	(42.87)	(44.43)
Country Banks in Other States	3.464	4.186	2.340	4.127
	(14.37)	(18.05)	(12.51)	(18.04)
Obs.	89	172	91	172

Notes: Rows indicate the location of correspondent banks. Extensive margins provide information on the proportions of links in each location against total links. Intensive margins provide information on the proportions of borrowed money from correspondents at different locations against total borrowed money. *Source*: *Virginia State Bank Examination Reports*.

to accommodate liquidity transfers between regions, thereby smoothing interregional flows (Gilbert (1983), James and Weiman (2010)). As our analysis shows, however, the provision of liquidity by the central bank reduced the relevance of New York City banks in the U.S. interbank network. In this way, the central bank liquidity provision crowded out previous private liquidity insurance, plausibly at the cost of using public funds to cover such public insurance.

To summarize, the introduction into the banking system of liquidity provided by a central bank changed the structure of the interbank system for nonmember banks as well as for member banks. By injecting public liquidity into the banking system, the existence of the Federal Reserve reduced the need for nonmember banks to maintain correspondent relationships across multiple cities and outside the state. In addition, the provision of public liquidity eliminated the role of New York City banks as the ultimate liquidity provider and allowed country banks to rely on local banks to access liquidity. The shift of correspondent relationships away from New York and toward local banks transformed what had been a *national core-periphery structure* based in New York City into a *regional core-periphery structure* based in reserve cities.

Table 6: Distance between Respondent and Correspondent Banks: Due froms.

	1911	1922
Longest Distance	347.1	253.3
	(156.8)	(267.5)
Shortest Distance	20.76	31.08
	(34.29)	(224.1)
Mean Distance	134.4	111.0
	(63.72)	(227.7)
Median Distance	105.8	84.64
	(74.10)	(229.0)
Total Distance	1107.8	587.1
	(1125.3)	(698.9)
Number of banks	218	323
Obs.	997	1047

For due-froms, Table 6 provides information on geographical distance between respondent and correspondent banks in miles. It shows that the existence of the Federal Reserve led banks to choose correspondents located in close geographic proximity.
Source: *Virginia State Bank Examination Reports*.

Table 7: Distance between Respondent and Correspondent Banks: Short-term borrowing.

	1911	1922
Longest Distance	187.9	167.7
	(147.5)	(427.0)
Shortest Distance	64.16	67.68
	(83.45)	(414.9)
Mean Distance	118.3	108.2
	(95.23)	(415.0)
Median Distance	107.2	91.78
	(97.07)	(416.1)
Total Distance	307.2	238.5
	(313.9)	(457.3)
Number of Banks	86	169
Obs.	145	303

For short-term borrowing, Table 7 provides information on geographical distance between respondent and correspondent banks in miles. It shows that the presence of the Federal Reserve led banks to choose correspondents located in close geographic proximity.

Source: *Virginia State Bank Examination Reports*.

5   Conclusion

The provision of public liquidity by the Federal Reserve System was the subject of heated debate among academics and policymakers when emergency lending facilities were put in place during the financial crisis of 2007-2009, and there was much discussion about whether shadow banks should access those facilities. How much liquidity should the Federal Reserve provide? To whom? Under what conditions? As the answers to these recent questions may be "contaminated" by closeness to the events and by the complexity and variety of modern banking, in this paper we study a unique historical event. How did the creation of the Federal Reserve System affect the structure, functioning, and stability of the banking system? Our findings cast new light on some of the issues central to the recent discussion of reforms.

In 1913 the Federal Reserve Act was passed to provide liquidity to member banks that satisfied reserve requirements. It did so by providing public liquidity insurance for the banking system. While this public insurance came at the social cost of taxation, it also brought the benefit of regulating and supervising members. In addition, however, as we have shown, this intervention-the provision of public liquidity insurance-produced three unintended consequences.

First, it reduced the incentives of banks, member and nonmember alike, to hold liquid assets. Member banks reduced liquid assets in the expectation of borrowing from the Federal Reserve, and nonmembers also reduced liquid assets by obtaining indirect access to public liquidity through their connections with members. As a result the whole banking system became less liquid.

A second unintended consequence was that a new source of contagion arose within banking networks: Banks reduced their holdings of interbank deposits, but their interbank borrowing increased, with a concomitant increase in the intensity of interbank relations. This intensity induced instability by making the entire network more vulnerable to shocks and therefore more exposed to contagion.

The third unintended consequence of the provision of public liquidity insurance was that, by altering the structure of the system of interbank relationships, public liquidity insurance crowded out private insurance. Before the creation of the Federal Reserve System, New York City had been at the center of the interbank system, and the city's centrality ensured private cross-regional liquidity insurance. After the Federal Reserve came into being, however, the interbank system with a common center was transformed into a system with a diffuse set of relationships. Thus, a third major repercussion of the public liquidity provision of the Federal Reserve Act was that public liquidity came to crowd out the private insurance that

had existed previously.

These three changes-a decline in aggregate liquidity, more-intense networks associated with a higher risk of contagion and vulnerability, and the disappearance of an overall concentrated network that helped the banking system manage cross-regional liquidity shocks privately-suggest that from the outset, the provision of public liquidity may have hindered the functioning of interbank relations at the cost of taxpayer funds. In addition, the provision of public liquidity created a system of "shadow banks" (the nonmember banks that, between 1914 and 1934, operated outside federal regulation and supervision). These results have natural implications for the current policy discussion and for assessing post-reform attempts to prevent nonbanks from accessing public liquidity. As our results show, restricting "official" access to public liquidity does not prevent "real" access to public liquidity, and an attempt to prevent real access by restricting official access may indeed backfire by creating a landscape favorable to the flourishing of a shadow banking system, operating with larger and more-illiquid assets and generating greater systemic risk.
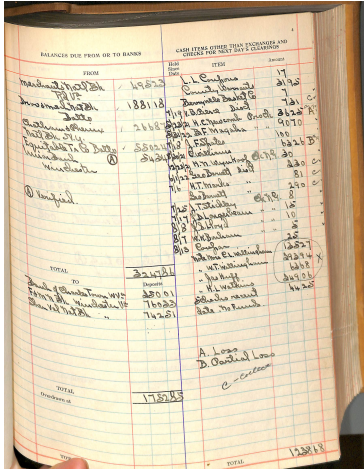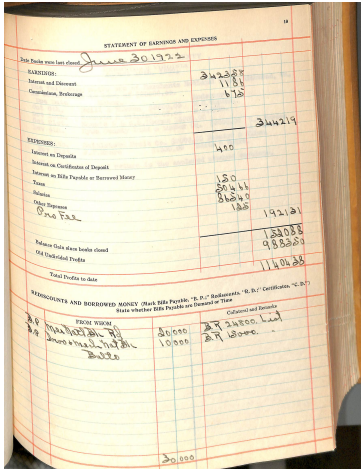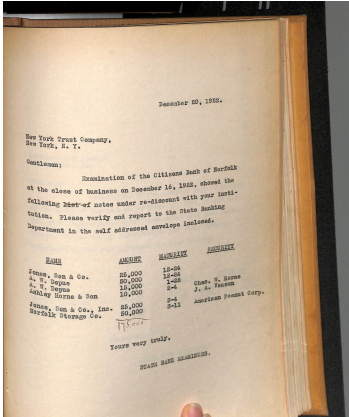
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A   Virginia State Bank Examination Reports

In Figure A1 we present images of representative pages in the state bank examination reports used for this study. The reports provide information on three types of interbank relationships: on the asset side of the balance sheet, the amounts due from other banks by individual debtor banks; on the liability side of the balance sheet, the amounts due to other banks by individual creditor banks; and the amounts of borrowed money and the provider of these short-term loans. In some cases, the reports provide information on collateral used for securing short-term funds.

Figure A1: Virginia State Bank Examination Reports

Interbank Deposits	Short-term Borrowing	Collateral for Borrowing

B Aggregate Balance Sheet Analysis in More Detail

In the main text, we provide summary statistics of the balance sheet data aggregated at the state level. In Figure B2, we plot the movement of balance sheet ratios from 1910 to 1929. Figure B2 shows that in the 1920s, short-term borrowing increased and liquid assets declined.

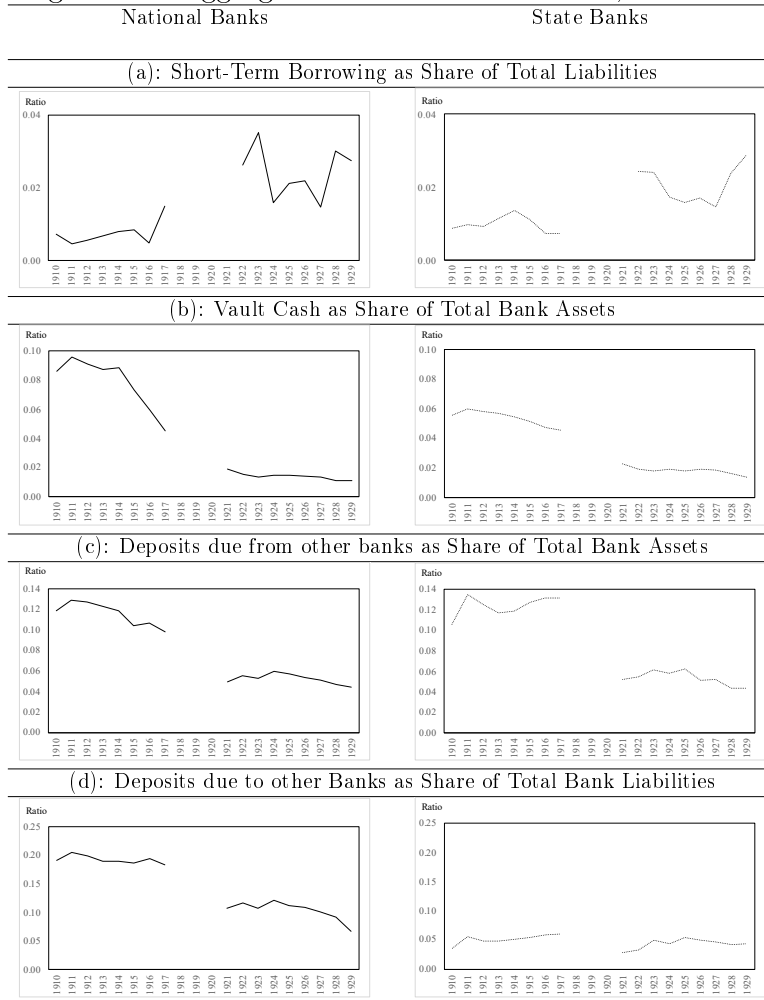### Figure B2: Aggregate Balance Sheet Ratios, 1910-1929

Figure B2 plots the ratio of short-term borrowing to total liabilities for national and state banks. All data are aggregated by the OCC: data for national banks are aggregated across all states, 17 reserve cities, and 3 central-reserve cities; data for state banks are aggregated across all states, all reserve cities, and reserve cities.

Source: Annual Report of the Comptroller of the Currency.

In addition, we check the robustness of our findings by restricting the data in two dimensions. First, we restrict our sample using state bank participation rate. As shown in Figure 2, states with financial and manufacturing sectors displayed a higher proportion of state bank membership than agricultural states. Given the irregular geographic distribution of membership, one might be concerned that the described changes were generated by state member banks

and that therefore our classifying all state banks as nonmembers clutters the analysis. To alleviate this concern, we restrict our sample and compare the asset composition of member and nonmember banks only in states where the membership ratio of state banks was under 10% in 1920.

Second, we restrict our sample using state-level reserve requirements. Changes in the liquidity of the state banking system might be driven by changes in reserve requirements by state regulators rather than by voluntary liquidity changes. To rule out this possibility, we divided states into three groups: (1) states that decreased their reserve requirements, (2) states that increased their reserve requirements, and (3) states that did not change their reserve requirements. Between 1910 and 1929, 22 states reduced reserve requirements, 10 states increased reserve requirements, and 16 states kept reserve requirements unchanged.[21]

For states where the state bank participation rate was below 10%, Figure B3 plots the fraction of total assets that state banks in those states held in borrowing, cash, and interbank deposits. In all cases, and regardless of the change in reserve requirements, nonmember banks reduced cash and interbank deposits and increased borrowing after the Federal Reserve came into existence (in 1914).

To summarize, we find that the existence of the Federal Reserve reduced liquidity (in the form of cash and interbank deposits) and intensified interbank relations (in the form of higher short-term borrowing) for both member and nonmember banks. Furthermore, member banks significantly reduced their relations with other member banks, but not their relations with nonmember banks. These factors suggest less private cross-insurance but still exposure to withdrawals, which contributed to the possibility of more contagion and greater vulnerability of the financial system.

---

[21]See White (2014) for information on state reserve requirements. We classify CA, DE, GA, IN, KS, KY, LA, MI, MN, MT, NM, NY, OK, OR, PA, SD, TX, VA, WA, WI, WV as states with decreasing reserve requirements. In addition, we classify AR, CO, IA, MD, MS, NH, SC, TN, VT, WY as states with increasing reserve requirements. Last, we classify AL, CT, FL, ID, IL, MA, ME, MO, NC, ND, NE, NJ, NV, OH, OK, UT as states that did not change reserve requirements.

Figure B3: Bank Liquidity and Changes in State-Level Reserve Requirements, 1910-1929
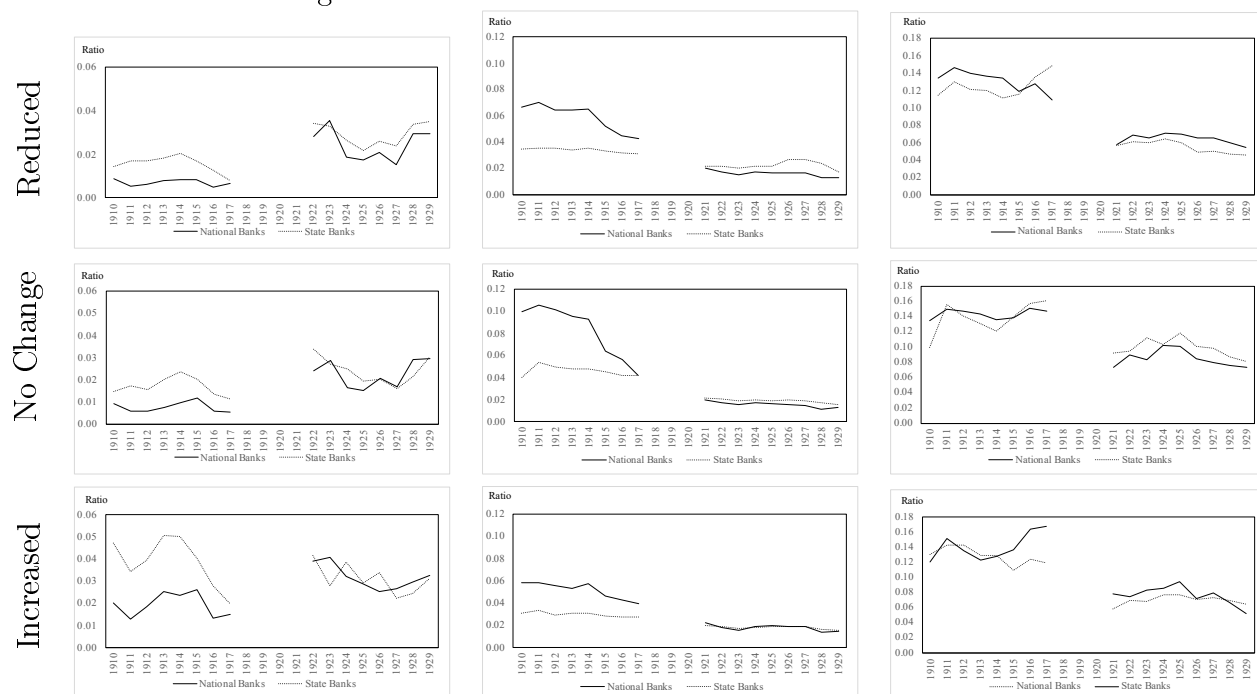

Figure B3 the share of short-term borrowing against total liabilities, the share of vault cash against total assets, and the share of deposits due from other banks against total assets for states with different reserve requirements. Data are further restricted for states where the Federal Reserve membership ratio of state banks was under 10% in 1920. All data are aggregated by the OCC: data for national banks are aggregated across all states, 17 reserve cities, and 3 central-reserve cities; data for state banks are aggregated across all states, all reserve cities, and reserve cities.

Source: Annual Report of the Comptroller of the Currency.

ONLINE APPENDIX

C   Remarks on the model and assumptions

C.1   Model

We assume that the liquidity shock can exceed D so we do not deal with the corner solutions. In particular, the liquidity shock ζ is 0 w.p. $1 - \alpha$ and $U[0, Z]$ w.p. α where $Z > D$. The story is as follows. There are legacy assets and liabilities. M captures the sum of legacy liabilities and K captures the sum of returns from illiquid legacy assets. These are safe but the the return time for legacy assets and withdrawal time for legacy liabilities are random. $K \geq M$ so there is no solvency issue. There can be an illiquidity issue. At the time of the liquidity shock, if the return so far from legacy assets is k and the amount of legacy liabilities realized so far is m, and the realized liquidity withdrawal from depositors (who have seniority) is $d \in [0, D]$ then the actual liquidity need at the time of the liquidity at the time of the shock is $l = d + m - k$. We assume that l has distribution $U[-K, D + M]$. Now denote $\alpha = \frac{D+M}{D+M+K}$ and $Z = D + M$. Then $l \leq 0$ w.p. $1 - \alpha$ and $l \sim U[0, Z]$ w.p. α. Now let $\zeta = l_+$ the liquidity need. (We use the notation $z_+ = \max\{z, 0\}$.) Then $\zeta = 0$ w.p. $1 - \alpha$ and $U[0, Z]$ w.p. α.

We also assume that y has Φ_0 reserves in hand at the time of the liquidity shock. (This is only for generality and we take $\Phi_0 = 0$ in the main text.) This is fixed can not be invested. One can think of $\Phi_0 = (1 - \phi)D_y$ for some deposit level D_y that the bank y had early on and invested in a different project. Or Φ_0 can be thought of some extra reserve that is received after the investments are made. One can very well take $\Phi_0 = 0$. When there is central bank liquidity m, the effect of m will be to make Φ_0 become $\Phi_m = \Phi_0 + m$. So the two cases are unified this way. Denote Φ_m is the extra liquidity available to y at the time of shocks. The no central bank case is $m = 0$. Accordingly, the liquidity shortage is $(\zeta - \Phi_m)_+$.

Going forward, the fundamentals of the model are r_x, r_y, r for the return rates, α, Z, ζ, for shocks, D, Φ_0, m for liquidity. Going forward, we denote $Z_{\alpha,m} = \frac{Z(1-\alpha)}{\alpha} + \Phi_m$. For a random variable X, F_X denotes its CDF. Also, $f \underset{z}{\propto} g$ means that f and g are monotone transformations of each other as functions of z.

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C.2 Assumptions

We will take Z to be large enough compared to D and Φ_m in order to avoid corner issues in the algebra. In particular, $Z > \Phi_m + D$ so that even the entire liquidity in the system may not suffice, although this event has small probability. This way, we do not need to worry about cumbersome corner solutions in the algebra. This, in a way, "convexifies" the problem.

Assumption 1. $0 \leq m \leq Z - D - \Phi_0$.

Also, for technical reasons and for the simplicity of algebra, we will restrict attention to $\alpha \leq \overline{\alpha}$ for some appropriate $\overline{\alpha}$ described as follows. Denote

$$\rho = \max\left\{0, 1 - 2\phi - \frac{r}{r_y}\right\}$$

Assumption 2. $\alpha \leq \overline{\alpha} = \frac{Z}{Z+(\rho D-\Phi_0)_+}$.

The major role of this assumption is to make sure that the reserve requirements bind and $\Phi_y = \phi L$.

Finally, we assume that r_x and r_y are relatively large compared to r.

Assumption 3. $(1 - \phi)r_x > 2r$ and $(1 - \phi)r_y > r$.

The condition on r_y is innocuous. If $(1 - \phi)r_y$ were less than r, y would not borrow. The condition on r_x deserves some discussion. One might think, at first, that by $r_x > r$, bank x's own project is a better investment than the "interbank investment" of lending to y. Since each investment provides a buffer against liquidation of the other, each investment would be non-zero under sufficiently high risk. But by $r_x > r$, I_x would be larger than L. But this simple logic is missing a critical point. Bank y pays interest on the full loan L, not the investment size I_y. At least ϕL is kept by y as reserves, which is a source of short term liquidity for x at the time of shocks. That is, an interbank investment has an extra benefit above and beyond its investment value and diversification value. This complicates proofs. For this reason we make a simplifying assumption $(1 - \phi)r_x > 2r$ that makes sure there is a pecking order: first priority is the project of bank x, then the interbank investment.

D   An auxiliary result

D.1   Liquidation stage

We have $F_\zeta(0) = 1 - \alpha$ and $F_\zeta(x) = F_\zeta(0) + x\frac{\alpha}{Z}$ for $x \in (0, Z]$. At the time of liquidations, all values of $I_x, \Phi_x, L, I_y, \Phi_y$ are set. Denote $\zeta' = (\zeta - (\Phi_x + \Phi_y + \Phi_m))_+$ and $Z' = Z - (\Phi_x + \Phi_y + \Phi_m)$. ζ' is the liquidity shortage given the liquidity buffers. Note that the maximum size of the shortage is $Z' > 0$ as $Z > D + \Phi_m$. In fact, we have

$$Z' = Z - \Phi_x - \Phi_m - \Phi_y = Z - D - \Phi_m + I_x + I_y > I_x + I_y \geq \max\{I_x, I_y\}$$

Below are the cases of shocks and corresponding optimal liquidations by x:

- $\zeta' = 0 \implies$ no liquidation.

- $0 < \zeta' \leq \min\{I_x, I_y\} \implies$ liquidate I_x if $I_x r_x < Lr$, liquidate L if $Lr < I_x r_x$.

- $\min\{I_x, I_y\} < \zeta' \leq \max\{I_x, I_y\} \implies$ liquidate $\max\{I_x, L\}$.

- $\max\{I_x, I_y\} < \zeta' \implies$ liquidate both.

The distribution of ζ' is given by $F_{\zeta'}(0) = 1 - \alpha\frac{Z'}{Z}$ and $F_{\zeta'}(z) = F_{\zeta'}(0) + \frac{\alpha}{Z}z$ for $z \in (0, Z']$. Accordingly, for all $z \in [0, Z]$

$$F_{\zeta'}(z) = \frac{\alpha}{Z}\left(Z_{\alpha,m} + \Phi_x + \Phi_y + z\right)$$

The expected payoff of x is

$$
\begin{aligned}
\Pi_x =& F_{\zeta'}(0)\left(I_x r_x + Lr\right) \\
& + \left(F_{\zeta'}(\min\{I_x, I_y\}) - F_{\zeta'}(0)\right)\max\left\{Lr, I_x r_x\right\} \\
& + \left(F_{\zeta'}(\max\{I_x, I_y\}) - F_{\zeta'}(\min\{I_x, I_y\})\right)\min\{I_x, I_y\}r_{\arg\min_z I_z} \\
\underset{(I_x, L)}{\propto}& \left(Z_{\alpha,m} + D - I_x - I_y\right)\left(I_x r_x + Lr\right) = \\
& + \min\{I_x, I_y\}\max\left\{Lr, I_x r_x\right\} \\
& + \left(\max\{I_x, I_y\} - \min\{I_x, I_y\}\right)\min\{I_x, I_y\}r_{\arg\min_z I_z}
\end{aligned}
$$

This is maximized under constraints $L \geq 0$, $I_x \geq 0$, $L + I_x \leq D$ given the continuation play by y.

The expected payoff of y is

$$
\begin{aligned}
\Pi_y = (I_y r_y - Lr) & \Big[F_{\zeta'}(0) + \mathbf{1}_{I_x r_x < Lr} \left(F_{\zeta'}(\min\{I_x, I_y\}) - F_{\zeta'}(0) \right) \\
& + \mathbf{1}_{I_x < L} \left(F_{\zeta'}(\max\{I_x, I_y\}) - F_{\zeta'}(\min\{I_x, I_y\}) \right) \Big] \\
\underset{I_y}{\propto} (I_y r_y - Lr) & \Big[(Z_{\alpha,m} + D - I_x - I_y) + \mathbf{1}_{I_x r_x < Lr} \min\{I_x, I_y\} \\
& + \mathbf{1}_{I_x < L} \left(\max\{I_x, I_y\} - \min\{I_x, I_y\} \right) \Big]
\end{aligned}
$$

This is maximized under constraint $0 \leq I_y \leq (1 - \phi)L$.

D.2 Two lemmas

Before we find the optimal portfolio choice for bank y, we start with two auxiliary lemmas. The first one is about the problem of x.

Lemma 2. *Let Ω be the set of subgames (I_x, L) at which y's best response is $I_y = L(1 - \phi)$. Take any $(I_x, L) \in Int(\Omega)$. If $L(1 - \phi) > I_x$, then (I_x, L) is locally suboptimal.*

Proof. Take any locally optimal $(I_x, L) \in Int(\Omega)$. Suppose that $L(1 - \phi) > I_x$ (i.e. $I_y > I_x$). Then liquidations are

- $\zeta' = 0 \implies$ no liquidation.

- $0 < \zeta' < I_x \implies$ liquidate I_x if $I_x r_x < Lr$, liquidate L if $Lr < I_x r_x$.

- $I_x < \zeta' < L(1 - \phi) \implies$ liquidate L.

- $L(1 - \phi) < \zeta' \implies$ liquidate both.

Also, $F_{\zeta'}(0) = \frac{\alpha}{Z} (Z_{\alpha,m} + D - I_x - L(1 - \phi))$. Then the expected profit of x is

$$
\begin{aligned}
\Pi_x = & F_{\zeta'}(0) (I_x r_x + Lr) \\
& + (F_{\zeta'}(I_x) - F_{\zeta'}(0)) \max \{Lr, I_x r_x\} \\
& + (F_{\zeta'}(L(1 - \phi)) - F_{\zeta'}(I_x)) I_x r_x \\
\underset{(I_x,L)}{\propto} & (Z_{\alpha,m} + D - I_x - L(1 - \phi)) (I_x r_x + Lr) \\
& + I_x \max \{Lr, I_x r_x\} \\
& + (L(1 - \phi) - I_x) (I_x r_x)
\end{aligned}
$$

If $I_x r_x \neq Lr$, then the partial derivatives must be 0. The F.O.C. w.r.t. I_x is

$$0 = - (I_x r_x + Lr) + r_x \left(Z_{\alpha,m} + D - I_x - L(1 - \phi) \right)$$

$$+ \begin{cases} Lr & \text{if } Lr > I_x r_x \\ 2I_x r_x & \text{if } Lr < I_x r_x \end{cases}$$

$$(L(1 - \phi) - 2I_x) r_x$$

$$= r_x \left(Z_{\alpha,m} + D - 2I_x \right) - \begin{cases} 2I_x r_x & \text{if } Lr > I_x r_x \\ Lr & \text{if } Lr < I_x r_x \end{cases}$$

$$\implies Z_{\alpha,m} + D = 2I_x + \begin{cases} 2I_x & \text{if } Lr > I_x r_x \\ \frac{Lr}{r_x} & \text{if } Lr < I_x r_x \end{cases}$$

The F.O.C. w.r.t. to L is

$$0 = - (1 - \phi) (I_x r_x + Lr) + r \left(Z_{\alpha,m} + D - I_x - L(1 - \phi) \right)$$

$$+ \begin{cases} I_x r & \text{if } Lr > I_x r_x \\ 0 & \text{if } Lr < I_x r_x \end{cases}$$

$$+ (1 - \phi) I_x r_x$$

$$= r \left(Z_{\alpha,m} + D - I_x - 2L(1 - \phi) \right)$$

$$+ \begin{cases} I_x r & \text{if } Lr > I_x r_x \\ 0 & \text{if } Lr < I_x r_x \end{cases}$$

$$\implies Z_{\alpha,m} + D = I_x + 2L(1 - \phi) - \begin{cases} I_x r & \text{if } Lr > I_x r_x \\ 0 & \text{if } Lr < I_x r_x \end{cases}$$

Combining the two, we get

$$2I_x + \begin{cases} 2I_x & \text{if } Lr > I_x r_x \\ \frac{Lr}{r_x} & \text{if } Lr < I_x r_x \end{cases}$$

$$= I_x + 2L(1 - \phi) - \begin{cases} I_x r & \text{if } Lr > I_x r_x \\ 0 & \text{if } Lr < I_x r_x \end{cases}$$

$$\implies 0 = I_x - 2L(1 - \phi) + \begin{cases} 3I_x & \text{if } Lr > I_x r_x \\ \frac{Lr}{r_x} & \text{if } Lr < I_x r_x \end{cases}$$

Under $Lr < I_x r_x$, we get

$$0 = I_x - 2L(1 - \phi) + \frac{Lr}{r_x} < I_x - 2L(1 - \phi) + I_x < 0$$

So we must have $Lr > I_x r_x$. Then the condition is $2I_x = L(1 - \phi)$. But then $2I_x r_x = Lr_x(1 - \phi) > 2Lr$ because by Assumption 3. This is a contradiction.

So we must have $I_x r_x = Lr$. Then the right partial derivatives must be negative and left partial derivatives must be positive. In particular, for I_x, the right derivative is

$$r_x (Z_{\alpha,m} + D - 2I_x) - Lr$$

and the left derivative is

$$r_x (Z_{\alpha,m} + D - 2I_x) - 2I_x r_x$$

Then the left derivative is smaller than the right derivative. Contradiction. $\qquad \square$

Now we provide a lemma for regarding the problem of y. The cases of shocks under which I_y is not liquidated is given by

- $\zeta' = 0$

- $0 < \zeta' < \min\{I_x, I_y\}$ under $I_x r_x < Lr$

- $I_y < \zeta' < I_x$ under $I_x > L$.

Lemma 3. *Let Ω' be the set of all subgames (I_x, L) such that the best response of y satisfies $I_y > I_x$. Then for all $(I_x, L) \in \Omega'$, the best response of y is given by $I_y = (1 - \phi)L$.*

Proof. Take any $(I_x, L) \in \Omega'$. Let I_y be the best response. Note that $(I_x, L) \in \Omega'$ implies $L(1 - \phi) > I_x$ because of the feasibility constraint $I_y \leq (1 - \phi)L$. Also note that $I_y \geq I_x$ implies $L > I_y \geq I_x$. Then the no-liquidation cases for y are

- $\zeta' = 0$

- $0 < \zeta' \leq I_x$ under $I_x r_x < Lr$

Then the problem of y at (I_x, L) is to maximize

$$\Pi_y = F_{\zeta'}(I_x 1_{I_x r_x < Lr})(I_y r_y - Lr)$$

$$\underset{I_y}{\propto} (Z_{\alpha,m} + \Phi_x + \Phi_y + I_x 1_{I_x r_x < Lr}) \left(I_y - \frac{Lr}{r_y}\right)$$

$$= (Z_{\alpha,m} + \Phi_x + I_x 1_{I_x r_x < Lr} + L - I_y) \left(I_y - \frac{Lr}{r_y}\right)$$

The constraints are $I_y \leq L(1 - \phi)$ and $I_y \geq I_x$. Then the solution I_y^* is given by

$$I_y^* = \min \left\{ (1 - \phi)L, \max \left\{ I_x, \frac{1}{2} \left(Z_{\alpha,m} + \Phi_x + L + I_x 1_{I_x r_x < Lr} + \frac{Lr}{r_y} \right) \right\} \right\}$$

Note that

$$I_y^* = (1 - \phi)L$$

$$\Longleftarrow (1 - \phi)L < \frac{1}{2} \left(Z_{\alpha,m} + \Phi_x + L + I_x 1_{I_x r_x < Lr} + \frac{Lr}{r_y} \right)$$

$$\Longleftarrow L \left(2(1 - \phi) - \frac{r}{r_y} - 1 \right) < Z_{\alpha,m}$$

$$\Longleftarrow D \left(1 - 2\phi - \frac{r}{r_y} \right) < Z_{\alpha,m}$$

Under assumption 2, the last condition holds. This completes the proof of the lemma. $\qquad\square$

D.3   Regarding bank y's portfolio

Now, by Lemma 3, $\Omega' \subset \Omega$. Also note that Ω' is an open set due to the continuity of the optimizers of the objective functions. Thus $\Omega' \subset \text{int}(\Omega)$. Then by Lemma 2, for all locally optimal $(I_x, \Phi_x, L) \in \Omega'$, $L(1 - \phi) \leq I_x$. By definition of Ω', $I_y > I_x$ at such subgames. But by feasibility, $I_y \leq (1 - \phi)L$, which is a contradiction. Thus $\Omega' = \emptyset$. That is, the best response of y always satisfies $I_y \leq I_x$.

Therefore, we can introduce $I_y \leq I_x$ as a constraint on y without loss of generality. Then the cases of shocks under which I_y is not liquidated is given by

- $\zeta' = 0$

- $0 < \zeta' \leq I_y$ under $I_x r_x < Lr$.

- $I_y < \zeta' \leq I_x$ under $I_x > L$

Now we have three cases to consider for y's problem.

Case 1: Under $I_x r_x < Lr$ (which implies $I_x < L$) the condition is $\zeta' \leq I_y$. Note that $F_{\zeta'}(I_y) = \frac{\alpha}{Z}(\Phi_x + L)$. The probability term is constant, so the problem is equivalent to maximizing I_y, meaning $I_y^* = \min\{I_x, (1-\phi)L\}$.

Case 2: Under $I_x r_x > Lr$ and $I_x < L$, we have only $\zeta' = 0$ case. Then the objective is

$$
\begin{aligned}
\Pi_y &= F_{\zeta'}(0)(I_y r_y - Lr) \\
&\underset{I_y}{\propto} (Z_{\alpha,m} + \Phi_x + \Phi_y)\left(I_y - \frac{Lr}{r_y}\right) \\
&= (Z_{\alpha,m} + \Phi_x + L - I_y)\left(I_y - \frac{Lr}{r_y}\right)
\end{aligned}
$$

The constraint is $I_y \leq \min\{(1-\phi)L, I_x\}$. Then the solution is

$$
I_y^* = \min\left\{(1-\phi)L,\ I_x,\ \frac{1}{2}\left(Z_{\alpha,m} + \Phi_x + L + \frac{Lr}{r_y}\right)\right\}
$$

Note that

$$
\begin{aligned}
I_y^* &= \min\{(1-\phi)L, I_x\} \\
&\Longleftarrow (1-\phi)L \leq \frac{1}{2}\left(Z_{\alpha,m} + \Phi_x + L + \frac{Lr}{r_y}\right) \\
&\Longleftarrow D\left(2(1-\phi) - \frac{r}{r_y} - 1\right) < Z_{\alpha,m}
\end{aligned}
$$

which holds by Assumption 2. So $I_y^* = \min\{(1-\phi)L, I_x\}$.

Case 3: Under $I_x > L$ (which implies $I_x r_x > Lr$) the conditions are $\zeta' = 0$ or $I_y < \zeta' < I_x$. Then the objective is

$$
\begin{aligned}
\Pi_y &= (F_{\zeta'}(0) + F_{\zeta'}(I_x + L - I_y) - F_{\zeta'}(L))(I_y r_y - Lr) \\
&\underset{I_y}{\propto} (Z_{\alpha,m} + \Phi_x + \Phi_y + I_x - I_y)\left(I_y - \frac{Lr}{r_y}\right) \\
&= (Z_{\alpha,m} + D - 2I_y)\left(I_y - \frac{Lr}{r_y}\right)
\end{aligned}
$$

The constraint is $I_y \leq \min\left\{(1-\phi)L, I_x\right\} = (1-\phi)L$. Then the solution is

$$I_y^* = \min\left\{(1-\phi)L, \frac{1}{4}\left(Z_{\alpha,m} + D\right) + \frac{1}{2}\frac{Lr}{r_y}\right\}$$

Note that $I_x > L$ implies $L < \frac{D}{2}$. Then

$$
\begin{aligned}
I_y^* &= (1-\phi)L \\
&\Longleftarrow (1-\phi)L \leq \frac{1}{4}\left(Z_{\alpha,m} + D\right) + \frac{1}{2}\frac{Lr}{r_y} \\
&\Longleftarrow L\left(1 - \phi - \frac{1}{2}\frac{r}{r_y}\right) \leq \frac{1}{4}\left(Z_{\alpha,m} + D\right) \\
&\Longleftarrow \frac{D}{2}\left(1 - \phi - \frac{1}{2}\frac{r}{r_y}\right) \leq \frac{1}{4}\left(Z_{\alpha,m} + D\right) \\
&\Longleftrightarrow D\left(2(1-\phi) - \frac{r}{r_y} - 1\right) \leq Z_{\alpha,m}
\end{aligned}
$$

which holds by Assumption 2.

Therefore, the solution to y's problem is, in general, $I_y = \min\left\{L(1-\phi), I_x\right\}$.

D.4   Regarding bank x's portfolio

Now we know that $I_y = \min\left\{L(1-\phi), I_x\right\} \leq I_x$. Thus the optimal liquidations on path are:

- $\zeta' = 0 \implies$ no liquidation.

- $0 < \zeta' \leq \min\left\{L(1-\phi), I_x\right\} \implies$ liquidate I_x if $I_x r_x < Lr$, liquidate L if $Lr < I_x r_x$.

- $\min\left\{L(1-\phi), I_x\right\} < \zeta' \leq I_x \implies$ liquidate $\max\{I_x, L\}$.

- $I_x < \zeta' \implies$ liquidate both.

Consider the case of $I_x \leq L(1-\phi)$. Then the optimal liquidations on path are:

- $\zeta' \leq 0 \implies$ no liquidation.

- $0 < \zeta' \leq I_x \implies$ liquidate I_x if $I_x r_x < Lr$, liquidate L if $Lr < I_x r_x$.

- $I_x < \zeta' \implies$ liquidate both.

Since $I_x \le L(1-\phi)$, $I_y = \min\{L(1-\phi), I_x\} = I_x$ meaning that $\Phi_y = L - I_x$. Then

$$\zeta' = (\zeta - \Phi_m - \Phi_x - \Phi_y)_+ = (\zeta - \Phi_m - \Phi_x - I_x)_+ = (\zeta - \Phi_m - D + L)_+$$

Then the expected payoff of x is

$$
\begin{aligned}
\Pi_x =& F_{\zeta'}(0)\left(I_x r_x + Lr\right) \\
&+ \left(F_{\zeta'}(I_x) - F_{\zeta'}(0)\right)\max\{Lr, I_x r_x\} \\
\underset{(I_x, L)}{\propto}& \left(Z_{\alpha, m} + D - L\right)\left(I_x r_x + Lr\right) \\
&+ I_x \max\{Lr, I_x r_x\}
\end{aligned}
$$

It is clearly optimal to have $\Phi_x = 0$. Then x maximizes

$$\left(Z_{\alpha, m} + I_x\right)\left(I_x\left(r_x - r\right) + Dr\right) + I_x \max\{(D - I_x)r, I_x r_x\}$$

subject to $I_x \le D\frac{1-\phi}{2-\phi}$. Note that $I_x \le D\frac{1-\phi}{2-\phi} < \frac{D}{2}$. Also note that $I_x(D - I_x)$ is increasing on $I_x \le \frac{D}{2}$. Therefore, the objective is increasing in I_x and the solution is $I_x = D\frac{1-\phi}{2-\phi}$ and $L = D\frac{1}{2-\phi}$. This value falls also into the case of $I_x \ge L(1-\phi)$ and so we can impose $I_x \ge L(1-\phi)$ without loss of generality.

D.5   The auxiliary result

We have shown that we can impose $I_x \ge L(1-\phi)$ and $I_y = L(1-\phi)$ without loss of generality. Then by $(1-\phi)r_x > r$ and $I_x \ge L(1-\phi)$ we also have $I_x r_x > Lr$. Therefore, the optimal liquidations are given by

- $\zeta' \le 0 \implies$ no liquidation.

- $0 < \zeta' \le L(1-\phi) \implies$ liquidate L.

- $L(1-\phi) < \zeta' \le I_x \implies$ liquidate I_x.

- $I_x < \zeta' \implies$ liquidate both.

E   Equilibrium

E.1   Deterministic public liquidity

Ex-post profit for bank x is given by

$$\pi_x = \begin{cases} I_x r_x + Lr & if\ 0 \leq \zeta \leq \Phi_m + \Phi_x + L\phi \\ I_x r_x & if\ \Phi_m + \Phi_x + L\phi < \zeta \leq \Phi_m + \Phi_x + L \\ Lr & if\ \Phi_m + \Phi_x + L < \zeta \leq \Phi_m + \Phi_x + L\phi + I_x \\ 0 & if\ \Phi_m + \Phi_x + L\phi + I_x < \zeta \end{cases}$$

The expected profit is

$$\begin{aligned} \Pi_x =& \frac{\alpha}{Z} \left(Z_{\alpha,m} + \Phi_x + L\right) I_x r_x + \frac{\alpha}{Z} \left(Z_{\alpha,m} + (\Phi_m + \Phi_x + L\phi) + (L\phi + I_x - L)\right) Lr \\ =& \frac{\alpha}{Z} \left(Z_{\alpha,m} + D - I_x\right) I_x r_x + \frac{\alpha}{Z} \left(Z_{\alpha,m} + D - 2L(1 - \phi)\right) Lr \\ \underset{(I_x,L)}{\propto}& r_x \left(Z_{\alpha,m} + D - I_x\right) I_x + 2(1 - \phi)r \left(\frac{Z_{\alpha,m} + D}{2(1 - \phi)} - L\right) L \end{aligned}$$

The unconstrained maximizer is

$$L = \frac{Z_{\alpha,m} + D}{4(1 - \phi)}$$

$$I_x = \frac{Z_{\alpha,m} + D}{2}$$

At these values, $L, I_x \geq 0$ and $I_x \geq L(1 - \phi)$ hold. The remaining constraint is

$$\begin{aligned} D \geq L + I_x &\iff D \geq (Z_{\alpha,m} + D) \left(\frac{1}{2} + \frac{1}{4(1 - \phi)}\right) \\ &\iff D \geq (Z_{\alpha,m} + D) \frac{3 - 2\phi}{4(1 - \phi)} \\ &\iff D \left(1 - \frac{3 - 2\phi}{4(1 - \phi)}\right) \geq Z_{\alpha,m} \frac{3 - 2\phi}{4(1 - \phi)} \\ &\iff D \frac{1 - 2\phi}{3 - 2\phi} \geq Z_{\alpha,m} \\ &\iff \alpha \geq \frac{Z}{Z + D \left(\frac{1 - 2\phi}{3 - 2\phi}\right) - \Phi_m} \end{aligned}$$

(Note that this lower bound is less than $\bar{\alpha}$ for $m = 0$ if $\frac{r}{r_y} \geq \frac{2(1-2\phi)(1-\phi)}{3-2\phi}$, which makes this region of parameters is non-empty for $m = 0$. This guarantees that the following regions are also non-empty for $m = 0$. As m grows, it is natural that some regions become obsolete in the pecking order.)

Next consider $\alpha < \frac{Z}{Z + D\left(\frac{1-2\phi}{3-2\phi}\right) - \Phi_m}$ $(D\frac{1-2\phi}{3-2\phi} < Z_{\alpha,m})$. The constraint $I_x + L \leq D$ binds. Under constraint $I_x = D - L \in [0, D]$, the FOC gives

$$\frac{d\Pi_x}{dL} = 0 \implies 0 = -r_x \left(Z_{\alpha,m} + D - 2(D - L)\right) + 2(1 - \phi)r \left(\frac{Z_{\alpha,m} + D}{2(1 - \phi)} - 2L\right)$$

$$\implies 2L\left(r_x + 2(1 - \phi)r\right) = 2Dr_x - (Z_{\alpha,m} + D)(r_x - r)$$

$$\implies L = \frac{D(r_x + r) - Z_{\alpha,m}(r_x - r)}{2(r_x + 2(1 - \phi)r)}$$

$$\implies I_x = \frac{D(4(1 - \phi)r + r_x - r) + Z_{\alpha,m}(r_x - r)}{2(r_x + 2(1 - \phi)r)}$$

As $r_x > r$ we have $L \leq D$ and $I_x \geq 0$. On the other hand

$$L \geq 0 \iff 2Dr_x \geq (Z_{\alpha,m} + D)(r_x - r)$$

$$\iff D\frac{r_x + r}{r_x - r} \geq Z_{\alpha,m}$$

$$\iff \alpha \geq \frac{Z}{Z + D\frac{r_x + r}{r_x - r} - \Phi_m}$$

This also ensures $I_x \leq D$. The last constraint $I_x \geq L(1 - \phi)$ holds trivially.

Finally, under $\alpha < \frac{Z}{Z + D\frac{(r_x + r)}{(r_x - r)} - \Phi_m}$ $(D\frac{r_x + r}{r_x - r} < Z_{\alpha,m})$, we have $L = 0$ and $I_x = D$.

Summarizing these:

1. If $\bar{\alpha} \geq \alpha > \frac{Z}{Z + D\left(\frac{1-2\phi}{3-2\phi}\right) - \Phi_m}$ $(D\frac{1-2\phi}{3-2\phi} \geq Z_{\alpha,m})$,

$$I_x = \frac{D + Z_{\alpha,m}}{2}$$

$$L = \frac{D + Z_{\alpha,m}}{4(1 - \phi)}$$

$$\Phi_x = D - I_x - L > 0$$

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2. If $\frac{Z}{Z+D\left(\frac{1-2\phi}{3-2\phi}\right)-\Phi_m} > \alpha > \frac{Z}{Z+D\frac{(r_x+r)}{(r_x-r)}-\Phi_m}$ $\left(D\frac{r_x+r}{r_x-r} > Z_{\alpha,m} > D\frac{1-2\phi}{3-2\phi}\right)$ then

$$I_x = \frac{D\left(4(1-\phi)r + r_x - r\right) + Z_{\alpha,m}\left(r_x - r\right)}{2\left(r_x + 2(1-\phi)r\right)}$$

$$L = \frac{D\left(r_x + r\right) - Z_{\alpha,m}\left(r_x - r\right)}{2\left(r_x + 2(1-\phi)r\right)}$$

$$\Phi_x = 0$$

3. If $\frac{Z}{Z+D\frac{(r_x+r)}{(r_x-r)}-\Phi_m} > \alpha$ $\left(Z_{\alpha,m} > D\frac{r_x+r}{r_x-r}\right)$, then

$$I_x = D, \quad L = 0, \quad \Phi_x = 0.$$

E.2 Short-term borrowing

Note that there is some inconsequential multiplicity in the amount of ex-post short term borrowing. As the short-term borrowing is risk-free in the model, for simplicity, we have assumed away interest on it. For robustness, we assume the smallest amount of short-term borrowing to meet the shock takes place. If $\zeta < \Phi_x$, there is no need for short-term borrowing. For $\Phi_x < \zeta \le \Phi_x + \Phi_y + m$, y can lend the shortage $\zeta - \Phi_x$ to x to avoid liquidations. If $\zeta > \Phi_x + \Phi_y + m$, liquidation is inevitable. If $\Phi_x + L + m > \zeta > \Phi_x + \Phi_y + m$, x liquidates L. This gives L extra liquidity to x on top of its reserves Φ_x. Bank x can still borrow m from y in this case. But if $\zeta < L + \Phi_x$, x does not need to borrow from y. Only when $\zeta > L + \Phi_x$, there is borrowing from y at the amount of shortage $\zeta - L - \Phi_x$. Therefore, when $\Phi_x + L + m > \zeta > \max\{\Phi_x + \Phi_y + m, L + \Phi_x\}$, there is $\zeta - L - \Phi_x$ borrowing. Continuing with the same logic, we find that the ex-post amount of short-term borrowing by x from y under m is given by

$$b = \begin{cases} \zeta - \Phi_x & \text{if } \Phi_x < \zeta \le \Phi_x + \Phi_y + m \\ \zeta - L - \Phi_x & \text{if } \Phi_x + \max\{L, \Phi_y + m\} < \zeta \le \Phi_x + L + m \\ \zeta - I_x - \Phi_x & \text{if } \Phi_x + \max\{I_x, L + m\} < \zeta \le \Phi_x + I_x + \Phi_y + m \\ 0 & \text{otherwise} \end{cases}$$

The expectation of this w.r.t. ζ is

$$B = \frac{\alpha}{Z}\left(2(m + \Phi_y)^2 + m^2 - \max\{0, m + \Phi_y - L\}^2 - \max\{0, m + L - I_x\}^2\right)$$

Under $D\frac{1-2\phi}{3-2\phi} \geq Z_{\alpha,m}$, this is

$$B = \frac{\alpha}{Z}\left(2(m+\Phi_y)^2 + m^2 - \max\left\{0, m-(1-\phi)L\right\}^2 - \max\left\{0, m-(1-2\phi)L\right\}^2\right)$$

Note that $D\frac{1-2\phi}{3-2\phi} \geq Z_{\alpha,m}$ implies $L = \frac{D+Z_{\alpha,m}}{4(1-\phi)} > \frac{Z_{\alpha,m}}{1-2\phi} > \frac{m}{1-2\phi}$. So $B = \frac{\alpha}{Z}\left(2(m+\Phi_y)^2 + m^2\right)$ which is increasing in m.

For the case of $D\frac{1-2\phi}{3-2\phi} < Z_{\alpha,m}$, note that B is continuous in m. Also, the negative terms $\max\left\{0, m+\Phi_y-L\right\}$ and $\max\left\{0, m+L-I_x\right\}$ are increasing in m. So if

$$2(m+\Phi_y)^2 + m^2 - (m+\Phi_y-L)^2 - (m+L-I_x)^2$$

is increasing in m, then B is increasing in m. The derivative of this expression w.r.t. m is 2 times

$$2(m+\Phi_y)\left(1+\phi\frac{dL}{dm}\right) + m - (m+\Phi_y-L)\left(1-(1-\phi)\frac{dL}{dm}\right) - (m+L-I_x)\left(1+\frac{dL}{dm}-\frac{dI_x}{dm}\right)$$

Under $Z_{\alpha,m} > D\frac{r_x+r}{r_x-r}$ this is

$$2(m+\Phi_y) + m - (m+\Phi_y-L) - (m+L-I_x)$$
$$=m+\Phi_y+I_x > 0$$

Under $D\frac{r_x+r}{r_x-r} > Z_{\alpha,m} > D\frac{1-2\phi}{3-2\phi}$ this is

$$2(m+\Phi_y)\left(1+\phi\frac{dL}{dm}\right) + m - (m+\Phi_y-L)\left(1-(1-\phi)\frac{dL}{dm}\right) - (m+2L-D)\left(1+2\frac{dL}{dm}\right)$$
$$=m+D-L(1-\phi) + \frac{dL}{dm}\left(2(m+\Phi_y)\phi + (m+\Phi_y-L)(1-\phi) - 2(m+2L-D)\right)$$
$$=m+D-L(1-\phi) + \frac{dL}{dm}\left(-m(1-\phi) - L(5-2\phi) + 2D\right)$$
$$=m+D-L(1-\phi) + \frac{r_x-r}{2(r_x+2(1-\phi)r)}\left(m(1-\phi) + L(5-2\phi) - 2D\right)$$
$$>D\left(1 - \frac{r_x-r}{r_x+2(1-\phi)r}\right) + L\left(\frac{(5-2\phi)(r_x-r)}{2(r_x+2(1-\phi)r)} - (1-\phi)\right) > 0$$

Thus, B is continuous and increasing.

F Risk

F.1 Stochastic public liquidity

Now suppose that m is independently drawn from distribution F_m with support $[0, \overline{m}]$ and mean m^*. Assume $\overline{m} < Z - \Phi_0 - D$.

In principle, stochastic m could complicate the algebra dramatically. Shocks can always become larger than the existing shock. Hence, banks' portfolio choices are consistent under m^* instead of m. In order to formalize this, go back to the general liquidation cases in Section D.1. The last region of the shock where a project is liquidated is given by $\max\{I_x, I_y\} < \zeta' = \zeta - \Phi_m - \Phi_x - \Phi_y$. This is, $\zeta > \max\{I_x, I_y\} + \Phi_m + \Phi_x + \Phi_y$. By $\overline{m} < Z - \Phi_0 - D$, we have $\Phi_m < Z - D$. Then

$$\max\{I_x, I_y\} + \Phi_m + \Phi_x + \Phi_y < \max\{I_x, I_y\} + Z - D + \Phi_x + \Phi_y < Z$$

Therefore, there is positive probability that a project gets liquidated regardless of the portfolio. So, all regions of shocks in the cases for liquidations have positive probability. Then the expected payoffs are given by

$$
\begin{aligned}
\frac{Z}{\alpha}\mathbb{E}_m\left[\Pi_x\right] =& \mathbb{E}_m\Big[\left(Z_{\alpha,0} + m + D - I_x - I_y\right)\left(I_x r_x + Lr\right) = \\
& + \min\{I_x, I_y\}\max\left\{Lr, I_x r_x\right\} \\
& + \left(\max\{I_x, I_y\} - \min\{I_x, I_y\}\right)\min\{I_x, I_y\} r_{\arg\min_z I_z}\Big] \\
=& \left(Z_{\alpha,0} + m^* + D - I_x - I_y\right)\left(I_x r_x + Lr\right) = \\
& + \min\{I_x, I_y\}\max\left\{Lr, I_x r_x\right\} \\
& + \left(\max\{I_x, I_y\} - \min\{I_x, I_y\}\right)\min\{I_x, I_y\} r_{\arg\min_z I_z} \\
\frac{Z}{\alpha}\mathbb{E}_m\left[\Pi_x\right] =& \mathbb{E}_m\Big[\left(I_y r_y - Lr\right)\Big[\left(Z_{\alpha,0} + m + D - I_x - I_y\right) + \mathbf{1}_{I_x r_x < Lr}\min\{I_x, I_y\} \\
& + \mathbf{1}_{I_x < L}\left(\max\{I_x, I_y\} - \min\{I_x, I_y\}\right)\Big]\Big] \\
=& \left(I_y r_y - Lr\right)\Big[\left(Z_{\alpha,0} + m^* + D - I_x - I_y\right) + \mathbf{1}_{I_x r_x < Lr}\min\{I_x, I_y\} \\
& + \mathbf{1}_{I_x < L}\left(\max\{I_x, I_y\} - \min\{I_x, I_y\}\right)\Big]
\end{aligned}
$$

So the solution is identical, just by replacing m with m^* now.

For closed form results, we suppose that m is 0 w.p. β and $U[0, \frac{2m^*}{1-\beta}]$ w.p. $1 - \beta$ where

$m^* < \frac{1-\beta}{2}(Z - \Phi_0 - D)$. Note that this has mean m^*.

F.2 Systemic fragility

We first consider the event that all funded projects get liquidated, which we call systemic risk. This is, $\zeta' > I_x$. (Under $D\frac{r_x+r}{r_x-r} \geq Z_{\alpha,m^*}$ y's project is indeed funded. Otherwise, the only funded project is x's.) Systemic risk is

$$\frac{\alpha}{Z}(Z - \Phi_0 - D + (1-\phi)L - m^*)$$

$$\underset{(m^*,\beta)}{\propto} (1-\phi)L - m^*$$

$$= -m^* + (1-\phi) \begin{cases} \frac{D+Z_{\alpha,m^*}}{4(1-\phi)} & \text{if } D\frac{1-2\phi}{3-2\phi} \geq Z_{\alpha,m^*} \\ \frac{D(r_x+r)-(r_x-r)Z_{\alpha,m^*}}{2(r_x+2(1-\phi)r)} & \text{if } D\frac{r_x+r}{r_x-r} > Z_{\alpha,m^*} > D\frac{1-2\phi}{3-2\phi} \\ 0 & \text{if } Z_{\alpha,m^*} > D\frac{r_x+r}{r_x-r} \end{cases}$$

The first term $-m^*$ is the direct effect of the availability of public liquidity. This has a natural effect of reducing the risk of liquidations. The second term after the bracket is the equilibrium effect of public liquidity. The availability of public liquidity influences the availability of private liquidity in the system through the portfolio choices, in particular, through L. The equilibrium effect increases in m^* up to $D\frac{1-2\phi}{3-2\phi} - Z_{\alpha,0}$ and decreases afterwards. The net effect is always to reduce systemic risk.

F.3 Contagion fragility

Next consider contagion risk, the probability that the project of y gets liquidated. This event is the union of $\zeta' > I_x$ (systemic risk) and $0 < \zeta' \leq L(1-\phi)$, "only-contagion." The probability of only-contagion is $\frac{\alpha}{Z}L(1-\phi)$. This is increasing in m^* for $m^* < D\frac{1-2\phi}{3-2\phi} - Z_{\alpha,0}$ and decreasing afterwards m^*. We have already calculated systemic risk. Then contagion risk is

$$= \frac{\alpha}{Z}(Z - \Phi_0 - D + 2(1-\phi)L - m^*)$$

$$\underset{(m^*,\beta)}{\propto} 2(1-\phi)L - m^*$$

This is always decreasing in m^*.

F.4 Direct fragility

Now consider direct risk, the probability that the project of x gets liquidated. This event is given by $L(1-\phi) < \zeta'$. The part $I_x < \zeta'$ is the systemic risk. The part of $L(1-\phi) < \zeta' \leq I_x$ is "only-direct-risk." Only-direct-risk is given by

$$\frac{\alpha}{Z}\left(I_x - L(1-\phi)\right)$$

$$\underset{(m^*,\beta)}{\propto} = \begin{cases} \frac{D+Z_{\alpha,m^*}}{4} & \text{if } D\frac{1-2\phi}{3-2\phi} \geq Z_{\alpha,m^*} \\ \frac{D(3(1-\phi)r-r+\phi r_x)+(2-\phi)(r_x-r)Z_{\alpha,m^*}}{2(r_x+2(1-\phi)r)} & \text{if } D\frac{r_x+r}{r_x-r} > Z_{\alpha,m^*} > D\frac{1-2\phi}{3-2\phi} \\ D & \text{if } Z_{\alpha,m^*} > D\frac{r_x+r}{r_x-r} \end{cases}$$

This is always increasing in m^*. The public liquidity always increases the only-direct-risk. This is perhaps particularly relevant for the Great Depression. The combined direct-risk to x is

$$\frac{\alpha}{Z}\left(Z - \Phi_0 - D + (1-\phi)L - m^* + I_x - L(1-\phi)\right)$$

$$\underset{(m^*,\beta)}{\propto} - m^* + I_x$$

$$= -m^* + \begin{cases} \frac{D+Z_{\alpha,m^*}}{2} & \text{if } D\frac{1-2\phi}{3-2\phi} \geq m^* + Z_{\alpha,0} \\ \frac{D(4(1-\phi)r+r_x-r)+Z_{\alpha,m^*}(r_x-r)}{2(r_x+2(1-\phi)r)} & \text{if } D\frac{r_x+r}{r_x-r} > m^* + Z_{\alpha,0} > D\frac{1-2\phi}{3-2\phi} \\ D & \text{if } m^* + Z_{\alpha,0} > D\frac{r_x+r}{r_x-r} \end{cases}$$

This is always decreasing in m^*. The public liquidity always reduces the direct-risk to x.

F.5 Vulnerability

Finally, we consider vulnerability, that is, the risks conditional on $m = 0$. Systemic vulnerability is given by

$$\frac{\alpha}{Z}\left(Z - \Phi_0 - D + (1-\phi)L\right)$$

This is increasing in m^* for small m^* and decreasing for large m^*. Contagion vulnerability is

$$\frac{\alpha}{Z}\left(Z - \Phi_0 - D + 2(1-\phi)L\right)$$

This is also increasing in m^* for small m^* and decreasing for large m^*. Direct vulnerability is

$$\frac{\alpha}{Z}\left(I_x - L(1-\phi)\right)$$

This is always increasing in m^*.

G   Networks

G.1   Two separate pairs

Now, there is θ probability that x_i gets a shock. Then all earlier results go through by replacing α with θ. Note that $\frac{Z}{Z+D\left(\frac{1-2\phi}{3-2\phi}\right)-\Phi_m} > \frac{1}{2} > \theta$ so we do not have the region in which $\Phi_{x_i} > 0$. Then

1. If $\frac{1}{2} > \theta > \frac{Z}{Z+D\frac{(r_x+r)}{(r_x-r)}-\Phi_m}$ (i.e. $D\frac{r_x+r}{r_x-r} > Z_{\theta,m}$) then

$$I_x = \frac{D\left(4(1-\phi)r + r_x - r\right) + Z_{\theta,m}\left(r_x - r\right)}{2\left(r_x + 2(1-\phi)r\right)}$$

$$L = \frac{D\left(r_x + r\right) - Z_{\theta,m}\left(r_x - r\right)}{2\left(r_x + 2(1-\phi)r\right)}$$

2. If $\frac{Z}{Z+D\frac{(r_x+r)}{(r_x-r)}-\Phi_m} > \theta$, then

$$I_x = D, \quad L = 0.$$

G.2   Liquidity coinsurance

Now suppose that the the core banks can borrow each others reserves. We assume $Z > 2D+m$ so that the shock can always be larger than the total cash in the system and we can avoid corner cases. For the pair i, the cash reserves of y_i act as an addition to m. Also note that x_i and x_j do not keep reserves and so we do not need to worry about x_i short-term lending to y_i and y_i intermediating this to y_j. Thus, for x_i, the best response is given by

$$L_i = \left(\frac{D\left(r_x + r\right) - \left(Z_{\theta,m} + L_j\phi\right)\left(r_x - r\right)}{2\left(r_x + 2(1-\phi)r\right)}\right)_+$$

The symmetric equilibrium is given by

$$L = \left(\frac{D\left(r_x + r\right) - \left(Z_{\theta,m} + L\phi\right)\left(r_x - r\right)}{2\left(r_x + 2(1-\phi)r\right)} \right)_+$$

1. If $\frac{1}{2} > \theta > \frac{Z}{Z + D\frac{(r_x+r)}{(r_x-r)} - \Phi_m}$ $\left(D\frac{r_x+r}{r_x-r} > Z_{\theta,m}\right)$ then

$$L_{x_i} = \frac{D\left(r_x + r\right) - Z_{\theta,m}\left(r_x - r\right)}{2\left(r_x + 2(1-\phi)r\right) + \phi(r_x - r)}, \quad I_x = D - L$$

2. If $\frac{Z}{Z + D\frac{(r_x+r)}{(r_x-r)} - \Phi_m} > \theta$, $\left(D\frac{r_x+r}{r_x-r} < Z_{\theta,m}\right)$ then

$$L_{x_i} = 0, \quad I_{x_i} = D$$

G.3 Endogenous network

For $D\frac{r_x+r}{r_x-r} < Z_{\theta,m}$, $L_C = L_D = 0$. There is no network. So consider the region $D\frac{r_x+r}{r_x-r} > Z_{\theta,m}$. From the earlier analysis we know that if both banks connect to their regional correspondents, in equilibrium,

$$\frac{Z}{\alpha}\Pi^C_{x_i} = \left(Z_{\theta,m} + D - I_x\right)I_x r_x + \left(Z_{\theta,m} + D - 2(1-\phi)L\right)Lr$$

where

$$L_C = \frac{D\left(r_x + r\right) - Z_{\theta,m}\left(r_x - r\right)}{2\left(r_x + 2(1-\phi)r\right)}$$

If both regions connect to NY, in equilibrium,

$$\frac{Z}{\alpha}\Pi^N_{x_i} = \left(Z_{\theta,m} + \phi L + D - I_x\right)I_x r_x + \left(Z_{\theta,m} + \phi L + D - 2(1-\phi)L\right)Lr - c(L)$$

where

$$L_N = \frac{D\left(r_x + r\right) - Z_{\theta,m}\left(r_x - r\right)}{2\left(r_x + 2(1-\phi)r\right) + \phi(r_x - r)}$$

Note

$$\frac{d\left(\frac{Z}{\alpha}\Pi_{x_i}^C\right)}{dm} = \frac{d}{dm}\left\{(Z_{\theta,m} + D - I_{x,C})\,I_{x,C}r_x + (Z_{\theta,m} + D - 2(1-\phi)L_C)\,L_C r\right\}$$

$$= I_{x,C}r_x + L_C r + (Z_{\theta,m} + D - 2I_{x,C})\frac{dI_{x,C}}{dm}r_x + (Z_{\theta,m} + D - 4(1-\phi)L_C)\frac{dL_C}{dm}r$$

$$= I_{x,C}r_x + L_C r$$

and

$$\frac{d\left(\frac{Z}{\alpha}\Pi_{x_i}^N\right)}{dm} = \frac{d}{dm}\left\{(Z_{\theta,m} + \phi L_N + D - I_{x,N})\,I_{x,N}r_x + (Z_{\theta,m} + \phi L_N + D - 2(1-\phi)L_N)\,L_N r - c(L_N)\right\}$$

$$= \left(1 + \phi\frac{dL_N}{dm} - \frac{dI_{x,N}}{dm}\right)I_{x,N}r_x + (Z_{\theta,m} + \phi L_N + D - I_{x,N})\,r_x\frac{dI_{x,N}}{dm}$$

$$+ \left(1 + \phi\frac{dL_N}{dm} - 2(1-\phi)\frac{dL_N}{dm}\right)L_N r + (Z_{\theta,m} + \phi L_N + D - 2(1-\phi)L_N)\,r\frac{dL_N}{dm}$$

$$= \left(1 + \phi\frac{dL_N}{dm}\right)I_{x,N}r_x + (Z_{\theta,m} + \phi L_N + D - 2I_{x,N})\frac{dI_{x,N}}{dm}r_x$$

$$+ \left(1 + \phi\frac{dL_N}{dm}\right)L_N r + (Z_{\theta,m} + \phi L_N + D - 4(1-\phi)L_N)\frac{dL_N}{dm}r$$

$$= \left(1 + \phi\frac{dL_N}{dm}\right)I_{x,N}r_x + \left(1 + \phi\frac{dL_N}{dm}\right)L_N r$$

$$= (I_{x,N}r_x + L_N r)\left(1 + \phi\frac{dL_N}{dm}\right)$$

Denote $A = 2(r_x + 2(1-\phi)r)$ and $B = D(r_x + r) - Z_{\theta,m}(r_x - r)$. Then

$$
\frac{Z}{\alpha}\frac{d\left(\Pi_{x_i}^C\right)}{dm} = I_{x,C}r_x + L_C r
$$

$$
= Dr_x - (r_x - r)L_C
$$

$$
= Dr_x - (r_x - r)\frac{B}{A}
$$

$$
\frac{Z}{\alpha}\frac{d\left(\Pi_{x_i}^N\right)}{dm} = (I_{x,N}r_x + L_N r)\left(1 + \phi\frac{dL_N}{dm}\right)
$$

$$
= \left(Dr_x - (r_x - r)\frac{B}{A + \phi(r_x - r)}\right)\left(1 - \phi\frac{r_x - r}{A + \phi(r_x - r)}\right)
$$

$$
= \left(Dr_x - (r_x - r)\frac{B}{A + \phi(r_x - r)}\right)\left(\frac{A}{A + \phi(r_x - r)}\right)
$$

$$
\frac{Z}{\alpha}\left(\frac{d\left(\Pi_{x_i}^C\right)}{dm} - \frac{d\left(\Pi_{x_i}^N\right)}{dm}\right) = Dr_x - (r_x - r)\frac{B}{A}
$$

$$
- \left(Dr_x - (r_x - r)\frac{B}{A + \phi(r_x - r)}\right)\left(\frac{A}{A + \phi(r_x - r)}\right)
$$

$$
= Dr_x\left(1 - \frac{A}{A + \phi(r_x - r)}\right) - (r_x - r)\frac{B}{A}\left(1 - \left(\frac{A}{A + \phi(r_x - r)}\right)^2\right)
$$

$$
= \left(1 - \frac{A}{A + \phi(r_x - r)}\right)\left(Dr_x - (r_x - r)\frac{B}{A}\left(1 + \frac{A}{A + \phi(r_x - r)}\right)\right)
$$

$$
= \frac{\phi(r_x - r)}{A + \phi(r_x - r)}\left(Dr_x - (r_x - r)\frac{B}{A}\left(2 - \frac{\phi(r_x - r)}{A + \phi(r_x - r)}\right)\right)
$$

$$
> \frac{\phi(r_x - r)}{A + \phi(r_x - r)}\left(Dr_x - (r_x - r)\frac{D}{2}\left(2 - \frac{\phi(r_x - r)}{A + \phi(r_x - r)}\right)\right)
$$

$$
= \frac{\phi(r_x - r)}{A + \phi(r_x - r)}\left(Dr + (r_x - r)\frac{D}{2}\left(\frac{\phi(r_x - r)}{A + \phi(r_x - r)}\right)\right)
$$

$$
> \frac{Dr(r_x - r)\phi}{A + \phi(r_x - r)} > 0
$$

Since the difference in the derivative is bounded away from zero, as m grows, $\Pi_{x_i}^C$ exceeds $\Pi_{x_i}^N$ eventually. The switching point m_c depends on the fixed cost c as well. If the cost c is very large, the stable network is regional for all m. In this case, $m_c = D\frac{r_x + r}{r_x - r} - Z\frac{1-\theta}{\theta}$. If c is very small, the stable network is central for all m. Then $m_c = 0$. In between as c grows, m_c grows from 0 to $D\frac{r_x + r}{r_x - r} - Z\frac{1-\theta}{\theta}$.