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Abstract

This paper proposes a new Lagrange multiplier (LM) based unit root test for panel data allowing for heterogeneous structural breaks in both the intercept *and* slope of each cross-section unit in the panel. We note that panel unit root tests allowing for breaks in the slope will critically depend on the nuisance parameters indicating the size and location of breaks. Any panel tests that ignore this dependency on the nuisance parameter will be subject to serious size distortions. To address this problem, our test employs a method that renders the asymptotic distribution of the panel tests invariant to nuisance parameters. We derive the asymptotic properties of our test and also examine its finite-sample properties. In addition, our test easily can be modified to correct for the presence of cross-correlations in the innovations of the panel. We illustrate this by applying the cross-sectionally augmented ADF (CADF) procedure of Pesaran (2007) to our test statistic.

JEL Classification: C12, C15, C22

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1. Introduction

It now is well known that ignoring existing structural breaks when testing for a unit root in a single time series can lead to a significant loss of power, as was first shown by Perron (1989). To that end, Perron developed a unit root test based on the Dickey-Fuller (DF) framework that allowed for a structural break in the data. Perron's work in this area is most relevant for applied researchers since many economic time series tend to display structural breaks.

Over the past decade and a half, there has been increasing interest on the part of researchers to extend the univariate unit root tests to the panel framework, partly as a means of trying to increase the inherently low power of the unit root test and partly due to the desire to take advantage of the rich collection of data that now is available to researchers. The papers by Levin, Lin and Chu (2002) and Im, Pesaran and Shin (2003, IPS hereafter) have been the most popular examples of such tests, and there have been numerous applications and extensions of these tests. Recent developments in panel unit root tests have focused a great deal of attention on correcting for the presence of cross-correlations in the innovations of the panel.

However, extensions of the univariate unit root test with breaks to the panel framework have been rather limited. As one would expect, the loss of power that results when ignoring existing structural breaks in the univariate unit root tests also exists in panel unit root tests if existing structural breaks are ignored. That is, there will be a considerable loss of power in any panel unit root tests that do not properly control for structural changes. As a result, one may consider extending the work of Perron (1989) to produce a counterpart panel unit root test that allows for breaks. For example, one may be tempted to modify the IPS test, or other tests, to include dummy variables in the regression for each cross section unit in the panel in order to control for the effects of structural changes. However, this approach is problematic. In order to apply an IPS-type test in the situation with structural changes, one would need to compute the expected values and variances of the DF t-statistics for *all possible* different break locations in the sample. This task would be extremely cumbersome in practice. The source of the problem in this case is that the distribution of the individual test statistic, say, for instance, Perron's augmented DF type t-statistic, depends on the nuisance parameters indicating the location of the break(s). In the case of the univariate unit root test, Perron handled the location parameter by simulating critical values for his test at various break-point locations. Although this provides a good solution to handle the location parameter in the case of one break in the univariate setting, it would be extremely difficult, perhaps impossible, to simulate critical values in the panel framework for all possible combinations of break-point locations for each cross section unit. This difficulty has been one of the main reasons for the lack of development of panel unit root tests that allow for structural changes.

Some researchers simply have ignored this "nuisance parameter" problem and have used the same expected values and variances of the IPS tests, regardless of where the breaks are located. But this approach is invalid. Alternatively, in the case of the univariate unit root tests some researchers circumvented the "nuisance parameter" problem by assuming that breaks can occur *only* under the alternative, not under the null. Under this scenario one may be tempted to conclude that the tests do not depend on the break location parameters under the null. But this conclusion is invalid, for two reasons. First, when not allowing for a break under the null, a rejection of the null does not guarantee that the series in question is stationary. Instead, it can imply a unit root with breaks under the null; see Nunes, Newbold and Kuan (1997) and Lee and Strazicich (2001). Second, the aforementioned authors also showed that not allowing for breaks under the null can lead to serious size distortions and spurious rejections under the null. In light of Perron (1989) and others, any valid unit root tests must not be affected by the presence or absence of breaks under the null. Otherwise, the tests will not be invariant to the magnitude of breaks and the critical values of the tests will be dependent upon these magnitudes. These findings extend, similarly, to unit root tests in the panel framework; any panel unit root test that depends on a nuisance parameter will be subject to the same spurious rejections of the unit root null.

Amsler and Lee (1995) proposed a reasonable solution to the nuisance parameter problem in the special case of a univariate unit root test with level shifts. They showed that the asymptotic distribution of the Lagrange Multiplier (LM) test does not depend on the size or location of any level shifts and, thus, is free of nuisance parameters. This is true even when a finite number of dummy variables for level shifts are included in the LM unit root testing regression. This so-called "invariance property" of the LM test makes it unnecessary to simulate critical values for the test at all possible break-point locations, as must be done in any DF-based unit root test. Im, Lee and Tieslau (2005, ILT hereafter) further extended this work to derive a panel LM unit root test that allows for a finite number of level shifts. As with the univariate LM-based unit root tests, the panel LM-based unit root test of ILT offers an operating advantage over the DF-based panel unit root tests in that the test is free from the nuisance parameter problem.

However, there is a problem of using this approach in the panel framework when allowing for breaks in the *trend* of a series: the "invariance property" of the LM test does not hold if the series under investigation exhibits breaks in its trend. Thus, there are no appropriate panel unit root tests that allow for breaks in *both* the intercept *and slope* of a series. This leaves a tremendous void for researchers using time series variables since many such variables typically display breaks in both their level and trend. In the presence of trend shifts, the popular unit root tests are subject to the nuisance parameter problem—both the DF-type tests and the LM-type tests (although it is clear that the DF-type tests are known to be *more* sensitive to the nuisance parameters than the LM-type tests; see Nunes (2004) for example). To the best of our knowledge, there is no panel unit root test that allows for trend-breaks that also is invariant to the nuisance parameter. The key issue we face in constructing such a test is to find a way to make the unit root test statistic invariant to nuisance parameters.

Given this situation, we adopt an approach in this paper that makes the univariate unit root test invariant to trend-shifts. To do so, we adopt a simple transformation approach with relevant asymptotic results in order to obtain a modified test statistic whose asymptotic distribution depends on neither the size nor the location of trend-shifts. For this task, we utilize an LM-type unit root test that will depend only on the number of breaks, not their size or location. Then, we extend the testing procedure to the panel framework with trend-shifts. Thus, although the ILT panel tests are valid for level shifts, our newly proposed panel LM unit root test offers the distinct advantage of also being invariant to nuisance parameters in the case of heterogeneous trend-shifts. In addition, our test can correct for the presence of cross-correlations in the innovations of the panel, although this is not the main focus of the present paper. Any of the popular methods to correct for this correlation can be used along with our proposed test. In this regard, we demonstrate how to apply the cross-sectionally augmented ADF (CADF) procedure of Pesaran (2007) to our tests as one possible means of correcting for cross-correlations.

The rest of the paper is organized as follows. In Section 2, we consider the transformed univariate LM unit root test with trend shifts to render the resulting test statistic invariant to the nuisance parameter. In Section 3, we extend the univariate LM test to the panel framework. We provide Monte Carlo simulation results in Section 4. Section 5 provides an empirical example wherein we use our panel LM test in an empirical example testing the inflation rates of 22 OECD countries. Section 6 provides concluding remarks.

2. The Transformed Univariate Unit Root Tests with Trend Shifts

This paper suggests an LM-based unit root test for panel data that allows for breaks in both the level and trend of the series under investigation. This panel LM unit root test is based on the univariate LM unit root test. Thus, we begin our analysis with a discussion of the univariate LM unit root test, following Lee and Strazicich (2009). It is possible to consider a DF-type test but we note that LM-based unit root tests are less sensitive to nuisance parameter problems than DF-based unit root tests. This is the motivation for our choice of the LM-based unit root test as the foundation for our panel data test.

We consider the following data generating process (DGP) based on the unobserved components representation:

$$y_t = \delta' Z_t + e_t, \quad e_t = \beta e_{t-1} + \varepsilon_t, \qquad (1)$$

where Z_t contains deterministic variables. The unit root null hypothesis is $\beta = 1$. If $Z_t = [1, t]'$, then the DGP is the same as that in the no-break test of Schmidt and Phillips (1992, hereafter SP). The level-shift only, or "crash," model can be described by $Z_t = [1, t, D_t]'$, where $D_t = 1$ for $t \geq T_B+1$ and zero otherwise, and T_B denotes the time period of the break. The LM version of the crash model was closely examined in Amsler and Lee (1995). The trend-break, or "changing growth" model, can be described by $Z_t = [1, t, DT_t^*]'$, where $DT_t^* = t - T_B$ for $t \geq T_B+1$, and zero otherwise. Finally, when $Z_t = [1, t, D_t, DT_t^*]'$ we have the most general model with level and trend breaks. This more general model is the most widely utilized in applied works, and will be the focus of our paper.

To consider multiple breaks, we can include additional dummy variables such that:

$$Z_t = [1,\ t,\ D_{1t},..,\ D_{Rt},\ DT_{1t}^*,..,\ DT_{Rt}^*]\ , \tag{2}$$

where $D_{jt} = 1$ for $t \geq T_{Bj+1}$, j=1,..,R, and zero otherwise, and $DT_{jt}^* = t - T_{Bj}$ for $t \geq T_{Bj+1}$ and zero otherwise. Following the LM (score) principle, we impose the null restriction $\beta = 1$ and consider in the first step the following regression in differences:

$$\Delta y_t = \delta' \Delta Z_t + u_t\ , \tag{3}$$

where $\delta = [\delta_1,\ \delta_2,\ \delta_{3j}',\ \delta_{4j}']'$, j=1,..,R. The unit root test statistics are then obtained from the following regression:

$$\Delta y_t = \delta' \Delta Z_t + \phi \tilde{S}_{t-1} + e_t\ , \tag{4}$$

where $\tilde{S}_t$ denotes the de-trended series

$$\tilde{S}_t = y_t - \tilde{\psi} - Z_t \tilde{\delta}\ . \tag{5}$$

Here, $\tilde{\delta}$ is the coefficient in the regression of Δy_t on ΔZ_t in (3), and $\tilde{\psi}$ is the restricted MLE of ψ. That is, $\tilde{\psi} = y_1 - Z_1 \tilde{\delta}$. Subtracting $\tilde{\psi}$ in (5) makes the initial value of the de-trended series begin at zero with $\tilde{S}_1 = 0$, but letting $\tilde{\psi} = 0$ leads to the same result. It is important to note that in the de-trending procedure (5), the de-trending coefficient $\tilde{\delta}$ was obtained in regression (3) using first differenced data.[1] Also note that ΔZ_t is used in (4), rather than Z_t in which case the DF type test

[1] This two-step detrending method also is used in the DF-GLS type tests of Elliott, Rothenberg and Stock (1996). When (3) is replaced with $y_t^* = \delta' Z_t^* + u_t$, where $y_t^* = y_t - (1\text{-}c/T)y_{t-1}$, $Z_t^* = Z_t - (1\text{-}c/T)Z_{t-1}$, and c takes on a small value, we can obtain the DF-GLS type tests. As a special case when $c = 0$, we have the LM type test. The DF-GLS type test also can be considered and it can be marginally more powerful than

can be obtained. Then, the dummy variable D_t becomes $\Delta D_t = B_t$, which is a point dummy variable. The point dummy variable will not affect the asymptotic distribution of the test, but it should not be omitted in the testing regression. However, as we will discuss later, the dependency on nuisance parameters cannot be removed with this de-trending procedure in the model with trend breaks.

The LM unit root test statistic is defined by:

$$\tilde{\tau} = t\text{-statistic for the null hypothesis } \phi = 0. \tag{6}$$

To allow for serially correlated and heterogeneously distributed innovations, one can include the terms $\Delta\tilde{S}_{t-j}$, j=1,.,k in (4) to correct for serial correlation in the usual augmented type tests:

$$\Delta y_t = \delta' \Delta Z_t + \phi \tilde{S}_{t-1} + \sum_{j=1}^{k} d_j \Delta\tilde{S}_{t-j} + e_t . \tag{7}$$

Lee and Strazicich (2009) showed that the asymptotic distribution of the test statistics is obtained from the following result.

Proposition 1: Suppose that the data generating process implies (1) with $\beta = 1$ and $Z_t = [1, t, D_{1t},.., D_{Rt}, DT_{1t}^*,.., DT_{Rt}^*]$ for the model with level and trend-breaks. We define $V_i^*(r)$, which is the weak limit of the partial sum residual process $\tilde{S}_t$ in (5), as follows:

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the corresponding LM test. However, note that both the LM and DF-GLS tests adopt the same detrending method using (3), which is the main source of power gains. One technical difficulty of the DF-GLS test in the panel setting is that we would need to obtain different values of c for different model specifications (different combinations of N and T, and different break locations) in order to obtain valid critical values in finite samples. This task is somewhat troublesome. Moreover, the value of c is often obtained asymptotically when T is 500 or 1,000. Therefore, the focus of our paper is on the LM-type test.

$$V_i*(r)=\begin{cases}\sqrt{\lambda_1^*}\,V_1(r/\lambda_1) & for\ r\le\lambda_1\\ \sqrt{\lambda_2^*}\,V_2\,[(r-\lambda_1)/(\lambda_2-\lambda_1)] & for\ \lambda_1<r\le\lambda_2\\ \vdots & \\ \sqrt{\lambda_{R+1}^*}\,V_{R+1}\,[(r-\lambda_R)/(1-\lambda_R)] & for\ \lambda_R<r\le 1\end{cases}. \quad (8)$$

Then, we have

$$\tilde{\tau}\to -\frac{1}{2}\,\omega\,[\sum_{j=1}^{R+1}\lambda_i^{*2}\int_0^1 \underline{V}_i(r)^2dr]^{-1/2}\ , \quad (9)$$

where λ_i^* denotes the fraction of sub-samples in each regime such that $\lambda_1^* = T_{B1}/T$, $\lambda_i^* = (T_{Bi} - T_{Bi-1})/T$, $i = 2,..,R$, and $\lambda_{R+1}^* = (T - T_{BR})/T$. Here, $\underline{V}_i$ (r) is the projection of the process $V_i(r)$ on the orthogonal complement of the space spanned by the trend break function $dz(\lambda^*, r)$, as defined over the interval $r \in [0, 1]$. $V_i(r) = W_i(r) - rW_i(1)$ with a Wiener process $W_i(r)$ for $i = 1,.., R$.[2]

Proof: See Lee and Strazicich (2009).

The result in (9) shows that, in contrast to the model with level shifts in Amsler and Lee (1995), the asymptotic distributions of the test statistics with trend breaks depend on the nuisance parameters, λ_i^*. However, Lee and Strazicich (2009) adopt an approach similar to that in Park and Sung (1994) where the dependency of the test statistic on the nuisance parameter can be removed with the following transformation:

$$\tilde{S}_t^*=\begin{cases}\frac{T}{T_{B_1}}\tilde{S}_t & for\ t\le T_{B_1}\\ \frac{T}{T_{B_2}-T_{B_1}}\tilde{S}_t & for\ T_{B_1}<t\le T_{B_2}\\ \vdots & \\ \frac{T}{T-T_{B_R}}\tilde{S}_t & for\ T_{B_R}<t\le T\end{cases} \quad (10)$$

[2] In the above, the argument $r_i^* = (r-\lambda_i)/(\lambda_i-\lambda_{i-1})$ is defined over the range between λ_{i-1} and λ_i, which has been transformed into r defined over the range 0 to 1.

We then replace $\tilde{S}_{t\text{-}1}$ with $\tilde{S}_{t\text{-}1}^{\;*}$ in the testing regression and change (7) to:

$$\Delta y_t = \delta'\Delta Z_t + \phi\,\tilde{S}_{t\text{-}1}^{\;*} + \sum_{j=1}^{k} d_j\,\Delta\tilde{S}_{t\text{-}j} + e_t\,. \tag{11}$$

Theorem 1: Let $\tilde{\tau}^*$ be the *t*-statistic for $\phi = 0$. Then, the asymptotic distributions of these test statistics will be invariant to the nuisance parameter λ:

$$\tilde{\tau}^* \rightarrow -\frac{1}{2}\,[\sum_{j=1}^{R+1}\int_0^1 \underline{V}_i(r)^2 dr]^{-1/2}\,, \tag{12}$$

where $\underline{V}_i(r)$ is defined in Proposition 1.

Proof: See Lee and Strazicich (2009).

The above result shows that the transformed unit root test statistic $\tilde{\tau}^*$ no longer depends on the nuisance parameter λ_j in the trend-break model, although information on λ is required to construct the test statistic. Following the transformation, the asymptotic distribution of $\tilde{\tau}^*$ only depends on the number of trend breaks, since the distribution is given as the sum of *R* independent stochastic terms. With one trend-break (R = 1), the distribution of $\tilde{\tau}^*$ is the same as that of the untransformed test $\tilde{\tau}$ using $\lambda = 1/2$, regardless of the initial location of the break(s). Similarly, with two trend-breaks (R = 2), the distribution of $\tilde{\tau}^*$ is the same as that of the untransformed test $\tilde{\tau}$ using $\lambda_1 = 1/3$ and $\lambda_2 = 2/3$. In general, for the case of *R* multiple breaks, the same analogy holds: the distribution of $\tilde{\tau}^*$ is the same as that of the untransformed test $\tilde{\tau}$ using $\lambda_j = j/(R+1)$, j = 1,.., *R*. Therefore, we do not need to simulate new critical values at all possible break point combinations. Instead, we only need critical values that correspond to the number of breaks, *R*. In Table 1, we reproduced and provided the critical values of exogenous tests for R = 1,.., 4 and T = 50, 100, 200, 500 and 1,000, respectively, which are obtained in Lee and Strazicich (2009), via Monte Carlo simulations. These critical values can be used when the break locations are known, or they are estimated consistently.[3]

As we will see, the above invariance results will prove helpful in constructing panel LM unit root tests with unknown trend breaks. However, the invariance result does not mean that one can adopt an incorrect number and/or placement of breaks, even under the null. In fact, one should include the correct number of breaks and their correct placement when performing unit root tests. This is true for two reasons. First, unit root tests lose power under the stationary alternative hypothesis if the number and/or placement of breaks is incorrect. Second, as noted by Perron (1989), the usual augmented DF tests will be biased against rejecting the null when the stationary alternative is true and a structural break is ignored. This also will hold for LM tests with trend breaks.

3. The Transformed Panel Unit Root Tests with Trend Shifts

We now develop a new panel LM test statistic with trend shifts. Our testing procedure is similar to that of ILT, but we utilize the transformed LM unit root statistic given in (11). Since our test is set in the panel framework, we add the subscript "i" to equation (11), highlighting the fact that we run the testing regression for each cross-section unit:

[3] In this paper, as in ILT (2005), we do not consider using the critical values of the endogenous break tests. We maintain the assumption that the break location parameters can be estimated consistently. The usual method of minimizing the sum of squared residuals and the standard information criteria can be used in this context. A recent development in the literature also shows that break estimates are consistent both under the null and alternative; see Perron and Zhu (2005). Furthermore, it has been shown that univariate unit root tests assuming consistency of the estimated breaks perform better than endogenous tests; see Perron (2006), and Kim and Perron (2009) for example. Using the critical values of the endogenous tests in a panel setting would yield aggravated size distortions since small biases in the estimates of the break parameters in the univariate tests would add up in the panel setting as N increases, even if these biases are negligible in the univariate setting.

$$\Delta y_{i,t} = \delta_i' \Delta Z_{i,t} + \phi_i \tilde{S}_{i,t\text{-}1}{}^{*} + \sum_{j=1}^{k} d_{ij}\, \Delta\tilde{S}_{i,t\text{-}j} + e_{it}\,, \quad i = 1,.., N \tag{11$'$}$$

In this expression, $\tilde{S}_{i,t-1}{}^{*}$ is as defined in (8). We then denote the resulting test statistic as $\tilde{\tau}_i^{\,*}$. The test statistic is based on the following null hypothesis,

H$_0$: $\phi_i = 0$ for all i

against the alternative hypothesis

H$_1$: $\phi_i < 0$, for some i.

We can construct the t-bar statistic using the average of the test statistics. The panel LM statistic for the above hypothesis can be obtained as the standardized statistic of the following average test statistic:

$$\bar{t} = \frac{1}{N}\sum_{i=1}^{N} \tilde{\tau}_i^{\,*}. \tag{13}$$

The distribution of $\bar{t}$ depends on T, but is free of other parameters under the null hypothesis. We denote the expected value and variance of $\bar{t}$ under the null hypothesis as E($\bar{t}$) and V($\bar{t}$). Note that the critical values of $\bar{t}$ do not vary much over N. In fact, our simulation results confirm that the critical values are almost invariant to different values of N in the data generating process. However, we note that the effect of the autocorrelation structure cannot be downplayed. As such, we compute the values of E($\bar{t}$) and V($\bar{t}$) for various combinations of N and T (sample size), p (truncation lag), and R (number of breaks), via stochastic simulations using 500,000 replications. These are reported in Table 2. As noted above, our test statistics do not depend on the location of breaks. As such, we do not need these mean and variance values for different locations of breaks. This is the key feature of our proposed panel test statistic; under any other method, it is not easy to formulate a valid test statistic if the test depends on the location(s) of the break(s). Formally, our panel test statistic follows a standard normal distribution:

$$LM(\tilde{\tau}^*) = \frac{\sqrt{N}\left[\bar{t} - \tilde{E}(\bar{t})\right]}{\sqrt{\tilde{V}(\bar{t})}} \tag{14}$$

where $\tilde{E}(\bar{t})$ and $\tilde{V}(\bar{t})$ are the estimated values of the average of the means and variances of $\bar{t}$, say E($\bar{t}$) and Var($\bar{t}$), as reported in Table 2, which correspond to the estimated parameter values of p and R. That is, we compute these values as:

$$\tilde{E}(\bar{t}) = \frac{1}{N}\sum_{i=1}^{N} E(\bar{t}(\tilde{R}_i, \tilde{p}_i)) \tag{15}$$

and:

$$\tilde{V}(\bar{t}) = \frac{1}{N}\sum_{i=1}^{N} Var(\bar{t}(\tilde{R}_i, \tilde{p}_i)),$$

where $(\tilde{R}_i, \tilde{p}_i)$ are the estimated values of the number of breaks and the number of truncation lags in the testing regression for the *i*-th cross-section unit. Thus, we allow for different numbers of breaks and truncation lags in different cross-section units. Note that we utilize the transformed test statistic $\tilde{\tau}_i^*$ using the estimated break locations and the number of breaks ($\tilde{R}_i$) for each cross-section unit. Therefore, our suggested panel statistic utilizes endogenously determined values of all parameters and it is free of nuisance parameters. In the next section, we employ various forms of DGPs to confirm that the suggested tests are robust to different locations of trend-breaks.

On the Issue of Cross-Correlations

The earliest panel unit root tests assumed zero correlations in the innovations across the panel. Such correlation is very likely to exist in cross-country studies and, therefore, such an assumption is highly unrealistic. Several methods have been proposed as a means of dealing with such correlations. To begin with, IPS proposed cross-section demeaning of each series in the panel as a way of partially dealing with cross correlations in the panel. However, this approach may not be effective in the presence of pair-wise correlations among cross-section units. As such, Choi (2006) generalized the de-meaning procedure and proposed a two-way error components model as a means of controlling for cross-correlations in the panel. The method suggested by Choi (2006) easily can be adopted in our framework. In the case of heterogeneous panels, however, the two-way error components model might be too restrictive. This led Bai and Ng (2002), Phillips and Sul (2003) and Moon and Perron (2004) to propose common factor models as a means of correcting for cross correlations. Others have suggested the use of seemingly unrelated regression to correct this problem. Alternatively, one may consider using the cross-sectionally augmented ADF (CADF) procedure suggested by Pesaran (2007). Each of the above-mentioned methods for dealing with cross correlations in the innovations of the panel has both merits and caveats, depending on the situation. Any one of these methods could be used in conjunction with our proposed test without affecting the properties and relative performance of the test. However, providing the details of each of these extensions (or comparing the performance of these tests in correcting for cross-correlations) is beyond the scope of this paper.

We do, however, wish to illustrate the application of the CADF procedure of Pesaran (2007) to our testing framework. This procedure is simpler but most effective. Specifically, we assume that the error term in (11)′ has the single-factor structure

$$e_{it} = \gamma_i f_t + u_{it} \tag{16}$$

where f_t is the unobserved common effect. Then, we consider the following testing regression which is augmented by the cross-section averages of lagged levels and first-differences of the individual series

$$\Delta y_{i,t} = \delta_i'\Delta Z_{i,t} + \phi_i \widetilde{S}^*_{i,t-1} + g\,\overline{S}^*_{t-1} + h\Delta\overline{S}_t^* + \sum_{j=1}^{p} g_{ij}\Delta\overline{S}^*_{t-j} + \sum_{j=1}^{p} d_{ij}\Delta\widetilde{S}_{i,t-j} + u_{it}\,. \tag{17}$$

We use the t-statistic on ϕ_i, denoted as $\widetilde{\tau}_i^{**}$, to construct the mean statistic $\bar{t}$ as in (13).

Finding critical values that do not depend on nuisance parameters is again the key issue. Note that Pesaran (2007) suggests using the mean statistic, as given in (13), for a formal panel test statistic, and provides its critical values for various combinations of N and T. Similarly, we suggest using the critical values for various combinations of N and T, but we utilize both the means and variances of $\bar{t}$.[4] Then, one can use the standardized statistic in (14), which follows a standard normal distribution. There is one more technical issue that will affect the performance of the test: the means and variances of the test statistic vary significantly over different AR truncation orders (p). As such, it is important to use the means and variances of $\bar{t}$ for each different value of p, as in IPS (2003) and ILT (2005). Accordingly, we simulate new critical values for the means and variances for various combinations of N and T and for different numbers of beaks, R. These values are provided in the Appendix and were obtained via stochastic simulations using 50,000 replications.

4. Simulation Results

In this section, we provide finite sample Monte Carlo simulation results on the Panel LM unit root tests with trend breaks. Our goal is to verify the theoretical results presented above, and to examine the general performance of the tests. To perform our simulations, pseudo-iid $N(0,1)$ random numbers were generated using the Gauss procedure RNDNS and all calculations were conducted using the Gauss software version 8.0. The DGP used in the simulations has the form in (1). For simplicity, we let $Z_t = [1, t, D_t, DT_t^*]'$ and $\delta = [\delta_1, \delta_2, \delta_3, \delta_4]'$ so that δ_3 is the level shift coefficient and δ_4 is the trend break coefficient. We also let $\lambda = T_B/T$ denote the fraction of the

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[4] We observe that the mean statistic $\bar{t}$ in (13) does not vary over different values of N in our case. On the other hand, we observe that the variance changes significantly over different combinations of N and T. Thus, we suggest using the critical values of both the means and variances of $\bar{t}$ for various combinations of N and T.

series before the break occurs at $t = T_B + 1$. The initial values y_0 and ε_0 are assumed to be random, and we assume that $\sigma_\varepsilon^2 = 1$. All simulation results are calculated using 20,000 replications. The size (frequency of rejections under the null when $\beta = 1$) and power (frequency of rejections under the alternative when $\beta = 0.9$) of the tests are evaluated using 5% critical values.

In Table 3, we report the size and power properties of the univariate unit root tests for different break magnitudes and locations. In each case, we wish to examine how the transformed test ($\tilde{\tau}^*$) and untransformed test ($\tilde{\tau}$) behave under the null and alternative hypotheses. In particular, we wish to examine if the transformed test ($\tilde{\tau}^*$) is invariant to the size and location of breaks. The results reported in Table 3 show that both tests have reasonably good size under the null. While they show mild size distortions in some cases, there is no clear pattern of significant size distortions. This is an encouraging finding and supports our proposition that the size properties are fairly invariant to different locations and magnitudes of level and trend breaks. Comparing the size properties of the transformed test with the untransformed test we see little difference, although the transformed test has marginally more accurate size than the untransformed test. Thus, the (untransformed) LM test is fairly less sensitive to the location of the trend break location. In this regard, Nunes (2004) might suggest ignoring the location of the trend shift in the LM version test. However, it is evident that the transformed test performs better and the effect of ignoring the trend shift will be more problematic in the panel version of the test, as we will see the result below. Most importantly, we see only trivial variation in size for the transformed test as the size and location of the trend break varies. Since we are using the same critical values in the transformed test regardless of the size and location of the breaks, these results are gratifying and confirm our theoretical prediction that the transformed test with trend break is invariant to the level and trend break.

In Table 4, we report the size and power properties of the panel unit root tests for different break magnitudes and locations. The point of this simulation is the same as that of the univariate tests. We wish to confirm that the transformed panel tests $LM(\tilde{\tau}^*)$, which are based on the transformed univariate tests, $\tilde{\tau}_i^*$, are robust to different break locations. To show this, we set the break location parameter $\lambda = 0.3$. We also consider the untransformed tests $LM(\tilde{\tau})$, which are based on the untransformed test statistics $\tilde{\tau}$. Since $LM(\tilde{\tau})$ depends on the break location, we will need to use the mean and variances that correspond to correct break locations. However, we still use the same mean and variances of the transformed test statistics to see how they behave under the null and alternative hypotheses. Moreover, we also examine the panel LM unit root tests without breaks under the heading of "LM no break" as well as the IPS tests without breaks under the heading of "DF no break," respectively. The results reported in Panel A of Table 4 show that the transformed test $LM(\tilde{\tau}^*)$ has reasonably good size under the null. In addition, the power property seems reasonable. While the test shows mild size distortions in some cases, there is no clear pattern of significant size distortions. These results are gratifying and confirm our theoretical prediction that the transformed panel LM test with trend breaks is invariant to breaks. In particular, this finding supports our proposition that the size properties are invariant to different locations and magnitudes of level and trend breaks. Comparing the size properties of the transformed test with the untransformed test we see significant differences. The size distortion problem is evident from the untransformed tests. It is also clear that the loss of power is significant for the panel tests that ignore breaks. The power is nearly zero for both the panel LM and the DF tests without breaks.

The results in Panel B of Table 4 are based on the case where the break location is given randomly at any location. We use the uniform distribution for the break location (λ) between 0.15 and 0.85. The pattern of the results is not much different from that in Panel A of Table 4. The transformed panel LM tests are mostly robust to different break locations and the untransformed tests show size distortions and loss of power. The panel tests without breaks exhibit a considerable loss of power.

5. Empirical Application

We now apply our panel LM unit root test to the CPI inflation rates of 22 OECD countries[5] to address the question of whether or not the inflation rate is stationary. It is especially important to address this question while allowing for changes in the slope of inflation since this series typically exhibits changing trends as the series responds to various shocks in the economy. For example, many countries experienced periods of markedly increasing inflation rates in the postwar period—during the first oil shock, for example, while this trend reversed itself in more recent decades with much of the global economy experiencing relatively low rates of inflation—for example, during the "oil glut" of the early 1980s and the low global inflation rates of the 1990s.

The question of whether inflation is I(0) or I(1) has important policy implications for many aspects of macroeconomics and finance. For example, the validity of the Phillips curve, the ability to forecast inflation or conduct inflation targeting, the analysis of real interest rates, the

[5] The complete list of countries is given in Table 5. The data were taken from the OECD's Main Economic Indicators CD ROM, September 2007. Inflation rates were computed by first differencing the logged CPI series.

conduct of monetary policy, and the validity of the capital asset pricing model critically hinge on the long-run properties of inflation. Nonetheless, there still is some debate as to whether inflation should be considered as a stationary or non-stationary series.

The earliest research on the stationarity of inflation rates considered this issue without allowing for breaks, and supported the notion that inflation is non-stationary. This includes Nelson and Schwert (1977), Barsky (1987), Ball and Cecchetti (1990), and Kim (1993). With the publication of Perron's seminal paper in 1989, researchers began to recognize the importance of allowing for breaks when testing for unit roots. When allowing for breaks, some researchers began to conclude that inflation is stationary, although this finding may be called into question. Perron (1989), for example, concluded that the US CPI inflation rate is non-stationary, even when allowing for an exogenous level shift. Zivot and Adrews (1992) were the first to analyze inflation rates while allowing for an endogenously determined break in the level of the series, and they were able to reject the null of a unit root in US CPI inflation. However, their test does not allow for a break under the null and, thus, their rejection of the null does not guarantee that the series is stationary. Similarly, Lumsdaine and Papell (1997), when allowing for breaks under only the alternative hypothesis, were able to reject the null of a unit root in inflation. Nunes, Newbold and Kuan (1997), however, were not able to reject the null of a unit root in inflation when allowing for breaks under both the null and alternative hypotheses. It is important to note that no one has yet considered this issue while allowing for trend breaks under both the null and alternative while using a test statistic that is invariant to the break-point location. This is a crucial shortcoming of all previous analyses. We hope to improve on prior studies by applying our new test to this issue.

Our testing procedure begins by computing the transformed univariate LM unit root test statistics (allowing for trend breaks) for the inflation rate series of each country. We apply the two-step procedure suggested by Lee and Strazicich (2009) which begins by jointly determining from the data whether or not breaks exist and, if they do, their location, while also determining the optimal lag length "*p*" that is needed to correct for autocorrelation in the errors. This procedure makes use of the "maximum F test," which is described in Lee and Strazicich (2009), assuming the data generating process specified in equation (1), where $\beta_i = 1$. We begin by allowing for two trend breaks[6] and perform a grid search over all possible break point locations, while eliminating 10% of the end points of the sample. If two trend breaks are found to exist in the inflation series of a given country, we then move to step 2 of the testing procedure in which we test for a unit root in the series using the transformed two-break LM unit root test statistic.

If we can reject the presence of two trend breaks in the inflation series of a given country, we repeat the entire 2-step procedure again where, now, step 1 determines whether or not a single trend break exists and, if so, where it is located (while simultaneously determining the optimal lag length "*p*"). If one trend break is found to exist in the inflation series of a given country, we then move to step 2 and test for a unit root in this series using the transformed one-break LM unit root test statistic.

If no trend breaks are found to exist in the inflation series of a given country, we then employ the procedure suggested by Lee and Strazicich (2003), which tests for a unit root while allowing for up to two breaks in the *level* of the series. Since the distribution of the LM unit root test is invariant to the location of any existing level shifts, there is no need to employ the transformation of Park and Sung (1994) in this instance. If no breaks of any kind are found to

[6] A series with more than two breaks might best be modeled as a non-linear process and, thus, we restrict our analysis to consider only up to two breaks.

exist in the inflation series of a given country, we compute the no-break LM unit root test of Schmidt and Phillips (1992) for that country.

After computing all of the univariate LM unit root tests allowing for the optimal number and type of breaks, we then use these statistics to compute the standardized panel LM unit root test statistic, as described in equation (14). Since the asymptotic distribution of this test statistic is standard normal, the critical values of the test follow the usual z-scores of the standard normal distribution.

The results of testing are shown in Table 5. For purposes of comparison, we report the results of the univariate LM unit root tests in Panel A of Table 5. We also report the optimal break point locations for each country and we note that all countries were found to have two trend breaks. The locations of the break dates identified by our procedure seem quite reasonable. For example, 17 of the 22 countries under investigation experienced their first break during the period from 1972 to 1977, a well-known period of high global inflation coinciding with the first oil crisis and the collapse of the Bretton Woods Accord. All of the remaining five countries experienced their first break during the period from 1981 to 1986, during the time of the so-called "oil glut." Not surprisingly, seven of the countries under investigation here experienced their second break during this same period, while Greece experienced its second break during the 1979 energy crisis—the beginning of a period of declining growth in Greece. Nine countries experienced their second break from 1990 to 1992. This is of interest since 1990 marked a high point of average global inflation rates over the postwar period and it was at this time when central banks across the globe began to embrace inflation targeting. World-wide average inflation rates declined significantly over the early 1990s. In particular, it is interesting to note that New Zealand experienced its second break in 1990, shortly after its central bank began its policy of inflation targeting. Similarly, Canada experienced its second break in 1991, which coincides with the period during which its central bank first adopted inflation targeting.

Based on the univariate LM unit root tests, the inflation rates for 16 of 22 countries were found to be stationary at the 10% level of significance or better. We conjecture that the failure to reject the null of a unit root in the remaining series may be due to the relatively low power of the univariate test, which may be improved by moving to the panel framework. To examine this issue, panel B of Table 5 reports the panel LM unit root test statistic. The null of a unit root is strongly rejected in this case, supporting the notion that these series are stationary with occasional trend breaks. Since the power of the unit root test greatly increases in the panel setting, it is not surprising that we find strong evidence of stationarity using the panel test statistic while we do not find this result uniformly in all series when testing in the univariate setting. This result supports the notion that inflation rates in these OECD countries are stationary with occasional trend breaks.

6. Conclusion

We have suggested a new panel LM unit root tests allowing for heterogeneous structural breaks in both the intercept and slope of each cross-section in the panel. Given that all existing unit root tests of this nature depend on the nuisance parameters that indicate the size and location of any breaks, we employ a method that makes the asymptotic distribution of our test invariant to these nuisance parameters. We derive the asymptotic properties of our test and also examine its finite-sample properties. Overall, we show evidence that our suggested tests are robust to different locations of trend-shifts. No existing tests have this crucially important feature.

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TABLE 1: CRITICAL VALUES FOR THE *TRANSFORMED* UNIVARIATE LM UNIT ROOT TEST

R	sig. level (%)	Sample Size *T*=50	*T*=100	*T*=200	*T*=500	*T*=1000
1	1	−4.604	−4.363	−4.261	−4.206	−4.176
	5	−3.950	−3.792	−3.716	−3.675	−3.662
	10	−3.635	−3.501	−3.443	−3.410	−3.402
2	1	−5.365	−4.980	−4.799	−4.698	−4.687
	5	−4.661	−4.379	−4.261	−4.191	−4.175
	10	−4.338	−4.097	−3.997	−3.934	−3.921
3	1	−6.092	−5.510	−5.302	−5.140	−5.127
	5	−5.362	−4.931	−4.752	−4.634	−4.620
	10	−5.019	−4.635	−4.484	−4.382	−4.361

Notes: *R* = # of breaks.

TABLE 2: MEANS AND VARIANCES FOR THE PANEL LM UNIT ROOT TEST

PANEL A: R=0

	p=0		p=1		p=2		p=3		p=4		p=5		p=6		p=7		p=8	
T	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.
25	−1.99	0.38	−1.99	0.39	−1.91	0.38	−1.90	0.41	−1.82	0.43	−1.80	0.47	−1.71	0.51	−1.69	0.58	−1.60	0.65
50	−1.98	0.36	−1.97	0.36	−1.93	0.35	−1.93	0.37	−1.89	0.37	−1.89	0.38	−1.84	0.38	−1.83	0.39	−1.78	0.40
100	−1.97	0.34	−1.97	0.34	−1.95	0.34	−1.95	0.34	−1.93	0.34	−1.93	0.34	−1.90	0.34	−1.90	0.35	−1.88	0.35
200	−1.98	0.34	−1.97	0.34	−1.96	0.34	−1.96	0.34	−1.95	0.34	−1.95	0.34	−1.94	0.34	−1.93	0.34	−1.93	0.34

PANEL B: R=1

	p=0		p=1		p=2		p=3		p=4		p=5		p=6		p=7		p=8	
T	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.
25	−2.69	0.40	−2.73	0.40	−2.67	0.37	−2.68	0.42	−2.59	0.50	−2.57	0.62	−2.44	0.73	−2.35	0.89	−2.18	1.04
50	−2.67	0.37	−2.68	0.36	−2.65	0.34	−2.67	0.34	−2.63	0.34	−2.64	0.36	−2.59	0.37	−2.58	0.41	−2.52	0.44
100	−2.65	0.34	−2.66	0.34	−2.64	0.33	−2.65	0.32	−2.63	0.32	−2.64	0.32	−2.62	0.31	−2.62	0.32	−2.60	0.32
200	−2.64	0.33	−2.64	0.33	−2.63	0.32	−2.64	0.32	−2.63	0.31	−2.63	0.31	−2.63	0.31	−2.63	0.31	−2.62	0.31

PANEL C: R=2

	p=0		p=1		p=2		p=3		p=4		p=5		p=6		p=7		p=8	
T	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.
50	−3.22	0.37	−3.27	0.35	−3.26	0.32	−3.30	0.33	−3.27	0.35	−3.28	0.40	−3.21	0.45	−3.18	0.53	−3.08	0.59
100	−3.19	0.34	−3.21	0.33	−3.21	0.31	−3.23	0.30	−3.23	0.30	−3.24	0.30	−3.23	0.29	−3.24	0.30	−3.22	0.31
200	−3.17	0.33	−3.18	0.32	−3.18	0.32	−3.19	0.31	−3.19	0.30	−3.20	0.30	−3.20	0.29	−3.21	0.29	−3.20	0.28

PANEL D: R=3

	p=0		p=1		p=2		p=3		p=4		p=5		p=6		p=7		p=8	
T	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.
50	−3.72	0.39	−3.82	0.35	−3.84	0.31	−3.90	0.35	−3.87	0.43	−3.86	0.55	−3.73	0.64	−3.64	0.76	−3.47	0.86
100	−3.66	0.35	−3.71	0.33	−3.72	0.30	−3.76	0.29	−3.76	0.28	−3.79	0.29	−3.78	0.29	−3.80	0.32	−3.77	0.36
200	−3.63	0.33	−3.65	0.32	−3.66	0.31	−3.68	0.30	−3.68	0.29	−3.71	0.28	−3.71	0.27	−3.72	0.27	−3.72	0.26

Notes: R = # of breaks; p = order of autocorrelation; T = # of time periods

TABLE 3: SIZE & POWER OF UNIVARIATE LM UNIT ROOT TEST
5% REJECTION RATES

			T=100				T=500			
DGP			Size (β=1.0)		Power (β=1.0)		Size (β=1.0)		Power (β=1.0)	
δ_3	δ_4	λ	$\tilde{\tau}^*$	$\tilde{\tau}$	$\tilde{\tau}^*$	$\tilde{\tau}$	$\tilde{\tau}^*$	$\tilde{\tau}$	$\tilde{\tau}^*$	$\tilde{\tau}$
2	0.5	0.3	0.043	0.039	0.138	0.137	0.045	0.044	0.973	0.991
		0.5	0.054	0.054	0.149	0.149	0.048	0.048	0.995	0.995
		0.8	0.040	0.034	0.120	0.120	0.050	0.038	0.874	0.996
5	0.5	0.3	0.049	0.042	0.131	0.128	0.041	0.036	0.976	0.991
		0.5	0.053	0.053	0.149	0.149	0.050	0.050	0.996	0.996
		0.8	0.039	0.031	0.130	0.134	0.044	0.028	0.872	0.996
2	1	0.3	0.042	0.041	0.134	0.132	0.049	0.041	0.970	0.990
		0.5	0.048	0.048	0.147	0.147	0.053	0.053	0.994	0.994
		0.8	0.043	0.030	0.125	0.128	0.048	0.029	0.880	0.994
5	1	0.3	0.046	0.040	0.129	0.133	0.044	0.040	0.975	0.993
		0.5	0.047	0.047	0.148	0.148	0.048	0.048	0.994	0.994
		0.8	0.040	0.031	0.122	0.125	0.048	0.033	0.870	0.997
5	1.5	0.3	0.043	0.038	0.137	0.127	0.051	0.042	0.972	0.992
		0.5	0.052	0.052	0.148	0.148	0.045	0.045	0.995	0.995
		0.8	0.043	0.033	0.120	0.132	0.050	0.031	0.873	0.998
10	1.5	0.3	0.048	0.038	0.136	0.132	0.050	0.045	0.972	0.990
		0.5	0.051	0.051	0.144	0.144	0.048	0.048	0.995	0.995
		0.8	0.044	0.034	0.121	0.122	0.051	0.033	0.870	0.994
5	3	0.3	0.046	0.040	0.137	0.134	0.050	0.041	0.974	0.992
		0.5	0.048	0.048	0.149	0.149	0.048	0.048	0.994	0.994
		0.8	0.041	0.029	0.131	0.126	0.054	0.034	0.876	0.997

Notes: T = # of time periods; δ_3 = intercept coefficient; δ_4 = break coefficient (magnitude of the break); λ = break location; $\tilde{\tau}^*$ = transformed univariate test statistic; $\tilde{\tau}$ = untransformed univariate test statistic.

TABLE 4.A: SIZE & POWER OF PANEL LM UNIT ROOT TEST
5% REJECTION RATES, δ_4 = 0.5, λ = 0.3

		T=25		*T*=50		*T*=100		*T*=250	
N	Test	size	power	size	power	size	power	size	power
10	$LM(\tilde{\tau}^*)$	0.047	0.068	0.036	0.148	0.044	0.659	0.051	1.000
	$LM(\tilde{\tau})$	0.035	0.059	0.025	0.109	0.015	0.613	0.018	1.000
	LM no break	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000
	DF no break	0.000	0.000	0.000	0.000	0.005	0.000	0.024	0.001
25	$LM(\tilde{\tau}^*)$	0.032	0.059	0.029	0.240	0.048	0.950	0.042	1.000
	$LM(\tilde{\tau})$	0.019	0.035	0.008	0.132	0.011	0.927	0.008	1.000
	LM no break	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	DF no break	0.000	0.000	0.000	0.000	0.006	0.001	0.131	0.468
50	$LM(\tilde{\tau}^*)$	0.017	0.051	0.026	0.370	0.035	0.999	0.041	1.000
	$LM(\tilde{\tau})$	0.008	0.027	0.003	0.169	0.003	0.997	0.003	1.000
	LM no break	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	DF no break	0.000	0.000	0.000	0.000	0.005	0.004	0.492	1.000
100	$LM(\tilde{\tau}^*)$	0.014	0.063	0.015	0.607	0.025	1.000	0.036	1.000
	$LM(\tilde{\tau})$	0.002	0.018	0.000	0.228	0.000	1.000	0.000	1.000
	LM no break	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000
	DF no break	0.000	0.000	0.000	0.000	0.017	0.039	0.904	1.000

Notes: δ_4 = break coefficient (magnitude of the break); λ = break location; T = # of time periods; N = # of cross sections; $LM(\tilde{\tau}^*)$ = transformed panel LM test statistic allowing for level and trend breaks; $LM(\tilde{\tau})$ = untransformed panel LM test statistic allowing for level and trend breaks; "LM no break" = a conventional LM-type panel unit root test without allowing for breaks; "DF no break" = a conventional Dickey-Fuller-type panel unit root test without allowing for breaks.

TABLE 4.B: SIZE & POWER OF PANEL LM UNIT ROOT TEST
5% REJECTION RATES, δ_4 = 0.5; λ = RANDOMLY DETERMINED

		T=25		T=50		T=100		T=250	
N	Test	size	power	size	power	size	power	size	power
10	$LM(\tilde{\tau}^*)$	0.034	0.047	0.050	0.165	0.043	0.676	0.054	1.000
	$LM(\tilde{\tau})$	0.024	0.035	0.026	0.127	0.020	0.640	0.026	1.000
	LM no break	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	DF no break	0.000	0.000	0.001	0.000	0.005	0.000	0.024	0.001
25	$LM(\tilde{\tau}^*)$	0.017	0.036	0.030	0.228	0.038	0.945	0.050	1.000
	$LM(\tilde{\tau})$	0.008	0.021	0.009	0.140	0.007	0.924	0.007	1.000
	LM no break	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	DF no break	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
50	$LM(\tilde{\tau}^*)$	0.016	0.037	0.020	0.353	0.039	0.999	0.036	1.000
	$LM(\tilde{\tau})$	0.004	0.017	0.005	0.161	0.002	0.998	0.002	1.000
	LM no break	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	DF no break	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
100	$LM(\tilde{\tau}^*)$	0.006	0.029	0.016	0.579	0.025	1.000	0.043	1.000
	$LM(\tilde{\tau})$	0.001	0.005	0.000	0.239	0.000	1.000	0.000	1.000
	LM no break	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000
	DF no break	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Notes: δ_4 = break coefficient (magnitude of the break); λ = break location; T = # of time periods; N = # of cross sections; $LM(\tilde{\tau}^*)$ = transformed panel LM test statistic allowing for level and trend breaks; $LM(\tilde{\tau})$ = untransformed panel LM test statistic allowing for level and trend breaks; "LM no break" = a conventional LM-type panel unit root test without allowing for breaks; "DF no break" = a conventional Dickey-Fuller-type panel unit root test without allowing for breaks.

TABLE 5, PANEL A: UNIVARIATE LM UNIT ROOT TESTS ON INFLATION

Country:	Univariate LM Stat	Optimal Lag	Break Locations
Australia	–6.371***	7	1972, 1991
Austria	–6.738***	8	1972, 1982
Belgium	–7.084***	1	1972, 1988
Canada	–4.680**	7	1982, 1991
Finland	–9.007***	8	1976, 1992
France	–0.972	8	1973, 1985
Germany	–5.655***	5	1981, 1990
Greece	–4.399*	7	1974, 1979
Italy	–8.119***	8	1972, 1984
Japan	–4.278	8	1973, 1977
Korea	–4.213	7	1981, 1987
Luxembourg	–7.082***	7	1972, 1984
Netherlands	–4.306	3	1973, 1988
New Zealand	–5.881***	8	1977, 1990
Norway	–7.202***	7	1983, 1990
Portugal	–5.778***	2	1976, 1992
South Africa	–5.999***	8	1972, 1992
Spain	–3.162	2	1975, 1986
Sweden	–7.370***	8	1985, 1990
Switzerland	–4.158	7	1975, 1996
United Kingdom	–6.889***	2	1973, 1984
United States	–7.531***	1	1976, 1983

TABLE 5, PANEL B: PANEL LM UNIT ROOT TESTS ON INFLATION

Panel LM Test Statistic = –9.807***

Notes: *Significant at 10%; **Significant at 5%; ***Significant at 1%

APPENDIX: MEANS AND VARIANCES FOR THE PANEL LM UNIT ROOT TEST USING PESARAN'S PROCEDURE

PANEL A: R = 0

		p=0		p=1		p=2		p=3		p=4		p=5		p=6		p=7		p=8	
N	*T*	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.
10	30	-2.14	0.86	-2.10	1.04	-1.96	1.17	-1.90	1.40	-1.75	1.57	-1.66	1.81	-1.48	2.07				
10	50	-2.15	0.75	-2.12	0.83	-2.05	0.91	-2.02	1.00	-1.94	1.09	-1.90	1.21	-1.81	1.29	-1.76	1.39	-1.67	1.46
10	100	-2.15	0.69	-2.14	0.72	-2.10	0.75	-2.09	0.80	-2.05	0.83	-2.03	0.87	-2.00	0.90	-1.98	0.94	-1.94	0.97
10	200	-2.15	0.66	-2.15	0.67	-2.13	0.67	-2.12	0.69	-2.11	0.70	-2.10	0.71	-2.08	0.73	-2.08	0.75	-2.06	0.76
20	30	-2.14	0.96	-2.09	1.23	-1.96	1.45	-1.89	1.75	-1.74	1.95	-1.65	2.20	-1.48	2.45				
20	50	-2.14	0.81	-2.12	0.93	-2.04	1.04	-2.01	1.19	-1.93	1.31	-1.89	1.47	-1.81	1.57	-1.76	1.70	-1.66	1.78
20	100	-2.15	0.73	-2.14	0.79	-2.10	0.84	-2.09	0.90	-2.05	0.94	-2.04	1.00	-2.00	1.05	-1.99	1.12	-1.95	1.17
20	200	-2.15	0.70	-2.15	0.71	-2.13	0.73	-2.12	0.74	-2.11	0.77	-2.10	0.79	-2.08	0.82	-2.08	0.85	-2.06	0.88
30	30	-2.14	1.09	-2.09	1.43	-1.96	1.74	-1.89	2.16	-1.74	2.37	-1.65	2.61	-1.48	2.80				
30	50	-2.14	0.90	-2.12	1.08	-2.05	1.22	-2.01	1.46	-1.93	1.64	-1.89	1.85	-1.81	1.98	-1.76	2.17	-1.66	2.20
30	100	-2.15	0.80	-2.14	0.87	-2.10	0.93	-2.09	1.01	-2.06	1.08	-2.04	1.17	-2.00	1.24	-1.99	1.33	-1.95	1.42
30	200	-2.15	0.77	-2.15	0.79	-2.13	0.81	-2.12	0.84	-2.11	0.87	-2.10	0.91	-2.08	0.95	-2.08	0.99	-2.06	1.02
50	30	-2.14	1.30	-2.09	1.85	-1.96	2.33	-1.89	2.95	-1.74	3.13	-1.65	3.50	-1.48	3.57				
50	50	-2.15	1.09	-2.12	1.34	-2.05	1.59	-2.02	1.92	-1.94	2.15	-1.90	2.44	-1.81	2.62	-1.76	2.86	-1.67	2.93
50	100	-2.15	0.91	-2.14	1.00	-2.10	1.09	-2.09	1.21	-2.05	1.32	-2.04	1.44	-2.00	1.55	-1.98	1.70	-1.94	1.82
50	200	-2.15	0.88	-2.15	0.93	-2.13	0.98	-2.12	1.03	-2.11	1.09	-2.10	1.14	-2.08	1.19	-2.08	1.25	-2.06	1.30
70	30	-2.14	1.58	-2.09	2.34	-1.96	3.05	-1.89	3.80	-1.74	4.09	-1.65	4.49	-1.48	4.45				
70	50	-2.15	1.25	-2.12	1.59	-2.05	1.95	-2.02	2.37	-1.94	2.74	-1.90	3.18	-1.81	3.37	-1.76	3.59	-1.67	3.69
70	100	-2.15	1.07	-2.14	1.20	-2.10	1.32	-2.09	1.48	-2.05	1.62	-2.04	1.81	-2.00	1.96	-1.98	2.14	-1.94	2.28
70	200	-2.15	0.99	-2.15	1.06	-2.13	1.12	-2.12	1.18	-2.11	1.25	-2.10	1.32	-2.08	1.39	-2.08	1.47	-2.06	1.54
100	30	-2.14	1.94	-2.09	3.00	-1.96	3.96	-1.89	5.13	-1.74	5.56	-1.65	5.87	-1.48	5.82				
100	50	-2.15	1.52	-2.12	1.99	-2.05	2.45	-2.01	3.05	-1.93	3.51	-1.90	4.02	-1.81	4.35	-1.76	4.79	-1.67	4.76
100	100	-2.15	1.28	-2.14	1.47	-2.10	1.65	-2.09	1.90	-2.05	2.09	-2.04	2.35	-2.00	2.57	-1.99	2.85	-1.95	3.06
100	200	-2.15	1.17	-2.15	1.25	-2.13	1.33	-2.12	1.42	-2.10	1.51	-2.10	1.61	-2.08	1.70	-2.07	1.80	-2.06	1.90
200	30	-2.14	3.25	-2.09	5.33	-1.96	7.16	-1.89	9.26	-1.74	9.86	-1.65	10.5	-1.48	9.82				
200	50	-2.15	2.42	-2.12	3.37	-2.05	4.27	-2.02	5.37	-1.94	6.27	-1.90	7.33	-1.81	7.90	-1.77	8.52	-1.67	8.52
200	100	-2.15	2.04	-2.14	2.40	-2.10	2.75	-2.09	3.18	-2.05	3.60	-2.04	4.07	-2.00	4.50	-1.98	4.97	-1.95	5.35
200	200	-2.15	1.82	-2.15	1.98	-2.13	2.13	-2.12	2.30	-2.11	2.48	-2.10	2.68	-2.08	2.87	-2.07	3.08	-2.06	3.28

Notes: R = # of breaks; p = order of autocorrelation; N = # of cross-section units; T = # of time periods

PANEL B: *R* = 1

		p=0		*p*=1		*p*=2		*p*=3		*p*=4		*p*=5		*p*=6		*p*=7		*p*=8	
N	*T*	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.
10	32	-2.76	0.69	-2.72	0.77	-2.55	0.79	-2.45	0.91	-2.24	1.09	-2.10	1.43	-1.86	1.93				
10	50	-2.76	0.60	-2.74	0.64	-2.65	0.66	-2.61	0.70	-2.50	0.73	-2.44	0.80	-2.31	0.86	-2.23	1.00	-2.09	1.14
10	100	-2.76	0.54	-2.75	0.56	-2.71	0.57	-2.70	0.59	-2.65	0.60	-2.64	0.62	-2.59	0.62	-2.57	0.64	-2.52	0.65
10	200	-2.75	0.53	-2.75	0.53	-2.73	0.53	-2.73	0.54	-2.71	0.54	-2.71	0.54	-2.69	0.55	-2.68	0.55	-2.66	0.55
20	32	-2.76	0.70	-2.72	0.82	-2.55	0.87	-2.45	0.97	-2.24	1.16	-2.10	1.52	-1.85	2.08				
20	50	-2.76	0.61	-2.74	0.67	-2.65	0.70	-2.61	0.75	-2.50	0.78	-2.44	0.84	-2.31	0.91	-2.23	1.05	-2.09	1.23
20	100	-2.76	0.54	-2.75	0.56	-2.71	0.57	-2.70	0.58	-2.66	0.60	-2.64	0.62	-2.59	0.63	-2.57	0.66	-2.52	0.69
20	200	-2.76	0.51	-2.75	0.52	-2.74	0.52	-2.73	0.53	-2.71	0.54	-2.71	0.55	-2.69	0.56	-2.68	0.58	-2.66	0.59
30	32	-2.76	0.74	-2.72	0.89	-2.55	0.93	-2.45	1.03	-2.24	1.21	-2.10	1.62	-1.86	2.24				
30	50	-2.76	0.61	-2.74	0.67	-2.65	0.72	-2.61	0.80	-2.50	0.83	-2.44	0.90	-2.31	0.97	-2.23	1.13	-2.09	1.29
30	100	-2.76	0.55	-2.75	0.59	-2.71	0.60	-2.70	0.64	-2.66	0.66	-2.64	0.69	-2.59	0.70	-2.57	0.72	-2.52	0.75
30	200	-2.76	0.52	-2.75	0.53	-2.74	0.54	-2.73	0.55	-2.71	0.57	-2.71	0.58	-2.69	0.59	-2.68	0.60	-2.66	0.61
50	32	-2.76	0.85	-2.72	1.08	-2.55	1.11	-2.45	1.17	-2.24	1.37	-2.10	1.85	-1.85	2.56				
50	50	-2.76	0.70	-2.74	0.81	-2.65	0.88	-2.61	0.95	-2.50	0.98	-2.44	1.03	-2.31	1.10	-2.23	1.31	-2.10	1.48
50	100	-2.76	0.59	-2.75	0.64	-2.71	0.67	-2.70	0.73	-2.66	0.76	-2.64	0.80	-2.59	0.83	-2.57	0.85	-2.52	0.85
50	200	-2.76	0.53	-2.75	0.55	-2.73	0.56	-2.73	0.58	-2.71	0.59	-2.70	0.62	-2.69	0.64	-2.68	0.66	-2.66	0.67
70	32	-2.76	0.94	-2.72	1.22	-2.55	1.26	-2.45	1.35	-2.24	1.53	-2.10	2.09	-1.86	2.98				
70	50	-2.76	0.75	-2.74	0.89	-2.65	0.99	-2.61	1.08	-2.50	1.12	-2.44	1.20	-2.31	1.27	-2.23	1.49	-2.09	1.71
70	100	-2.76	0.62	-2.75	0.68	-2.71	0.74	-2.70	0.81	-2.66	0.85	-2.64	0.92	-2.59	0.95	-2.57	0.97	-2.52	0.98
70	200	-2.76	0.58	-2.75	0.60	-2.73	0.62	-2.73	0.65	-2.71	0.66	-2.71	0.69	-2.69	0.72	-2.68	0.74	-2.66	0.76
100	32	-2.76	1.07	-2.72	1.46	-2.55	1.50	-2.45	1.60	-2.24	1.82	-2.10	2.46	-1.86	3.47				
100	50	-2.76	0.83	-2.74	1.04	-2.65	1.19	-2.61	1.29	-2.50	1.31	-2.44	1.39	-2.31	1.51	-2.23	1.77	-2.09	2.01
100	100	-2.76	0.68	-2.75	0.76	-2.71	0.82	-2.70	0.91	-2.66	0.97	-2.64	1.03	-2.59	1.06	-2.57	1.10	-2.52	1.11
100	200	-2.76	0.62	-2.75	0.65	-2.73	0.68	-2.73	0.73	-2.71	0.76	-2.70	0.80	-2.68	0.84	-2.68	0.87	-2.66	0.90
200	32	-2.76	1.55	-2.72	2.27	-2.55	2.35	-2.45	2.38	-2.24	2.73	-2.10	3.73	-1.86	5.28				
200	50	-2.76	1.14	-2.74	1.57	-2.65	1.85	-2.61	2.03	-2.50	2.01	-2.44	2.09	-2.31	2.24	-2.23	2.66	-2.10	3.08
200	100	-2.76	0.88	-2.75	1.04	-2.71	1.18	-2.70	1.35	-2.66	1.49	-2.64	1.63	-2.59	1.69	-2.57	1.75	-2.52	1.76
200	200	-2.76	0.78	-2.75	0.85	-2.73	0.92	-2.73	0.99	-2.71	1.06	-2.71	1.13	-2.69	1.20	-2.68	1.28	-2.66	1.33

Notes: R = # of breaks; p = order of autocorrelation; N = # of cross-section units; T = # of time periods

PANEL C: *R* = 2

		p=0		*p*=1		*p*=2		*p*=3		*p*=4		*p*=5		*p*=6		*p*=7		*p*=8	
N	*T*	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.
10	35	-3.30	0.63	-3.29	0.66	-3.12	0.66	-3.02	0.86	-2.79	1.12	-2.56	1.51	-2.22	2.03				
10	50	-3.29	0.57	-3.29	0.58	-3.20	0.56	-3.15	0.59	-3.02	0.67	-2.95	0.82	-2.78	0.96	-2.64	1.15	-2.42	1.36
10	100	-3.27	0.50	-3.28	0.50	-3.24	0.50	-3.23	0.50	-3.19	0.50	-3.17	0.51	-3.12	0.51	-3.10	0.53	-3.04	0.56
10	200	-3.25	0.49	-3.26	0.48	-3.24	0.47	-3.24	0.47	-3.23	0.46	-3.23	0.46	-3.21	0.46	-3.21	0.47	-3.19	0.46
20	35	-3.30	0.65	-3.29	0.68	-3.12	0.67	-3.02	0.86	-2.79	1.21	-2.56	1.70	-2.22	2.25				
20	50	-3.28	0.55	-3.29	0.57	-3.19	0.55	-3.15	0.59	-3.03	0.66	-2.95	0.81	-2.78	1.00	-2.64	1.25	-2.41	1.49
20	100	-3.26	0.48	-3.27	0.49	-3.24	0.49	-3.23	0.49	-3.19	0.50	-3.17	0.51	-3.12	0.50	-3.10	0.54	-3.04	0.58
20	200	-3.26	0.46	-3.26	0.46	-3.25	0.46	-3.25	0.45	-3.23	0.46	-3.23	0.46	-3.21	0.46	-3.21	0.47	-3.19	0.47
30	35	-3.30	0.65	-3.29	0.67	-3.13	0.68	-3.03	0.91	-2.79	1.26	-2.56	1.81	-2.22	2.40				
30	50	-3.28	0.57	-3.29	0.60	-3.20	0.59	-3.15	0.62	-3.03	0.69	-2.95	0.88	-2.79	1.09	-2.64	1.40	-2.42	1.66
30	100	-3.27	0.48	-3.27	0.49	-3.24	0.49	-3.23	0.51	-3.19	0.50	-3.18	0.52	-3.12	0.53	-3.10	0.56	-3.04	0.59
30	200	-3.26	0.45	-3.26	0.45	-3.25	0.45	-3.25	0.45	-3.23	0.45	-3.23	0.45	-3.21	0.46	-3.21	0.46	-3.19	0.46
50	35	-3.30	0.71	-3.29	0.77	-3.13	0.74	-3.03	1.02	-2.79	1.44	-2.56	2.10	-2.22	2.75				
50	50	-3.28	0.61	-3.29	0.66	-3.19	0.64	-3.15	0.64	-3.03	0.74	-2.95	0.93	-2.78	1.19	-2.64	1.54	-2.42	1.95
50	100	-3.27	0.50	-3.27	0.52	-3.24	0.53	-3.23	0.55	-3.19	0.54	-3.17	0.56	-3.12	0.57	-3.10	0.60	-3.04	0.63
50	200	-3.26	0.47	-3.26	0.48	-3.25	0.49	-3.25	0.49	-3.23	0.48	-3.23	0.49	-3.21	0.50	-3.21	0.51	-3.19	0.51
70	35	-3.30	0.76	-3.29	0.83	-3.12	0.78	-3.02	1.07	-2.79	1.59	-2.56	2.37	-2.22	3.30				
70	50	-3.28	0.64	-3.29	0.70	-3.20	0.69	-3.15	0.70	-3.03	0.78	-2.95	1.01	-2.78	1.34	-2.64	1.82	-2.42	2.34
70	100	-3.27	0.52	-3.27	0.55	-3.24	0.56	-3.23	0.58	-3.19	0.59	-3.17	0.60	-3.12	0.61	-3.10	0.63	-3.04	0.67
70	200	-3.26	0.49	-3.26	0.50	-3.25	0.51	-3.25	0.51	-3.23	0.52	-3.23	0.54	-3.21	0.55	-3.21	0.55	-3.19	0.55
100	35	-3.30	0.86	-3.29	0.96	-3.12	0.86	-3.03	1.19	-2.79	1.84	-2.56	2.85	-2.22	3.98				
100	50	-3.28	0.68	-3.29	0.78	-3.20	0.78	-3.15	0.79	-3.03	0.90	-2.95	1.19	-2.78	1.63	-2.64	2.21	-2.42	2.78
100	100	-3.27	0.54	-3.27	0.58	-3.24	0.62	-3.23	0.66	-3.19	0.68	-3.18	0.69	-3.12	0.69	-3.10	0.72	-3.04	0.77
100	200	-3.26	0.49	-3.26	0.51	-3.25	0.52	-3.25	0.54	-3.23	0.55	-3.23	0.58	-3.21	0.59	-3.21	0.60	-3.19	0.60
200	35	-3.30	1.12	-3.29	1.33	-3.12	1.18	-3.02	1.64	-2.79	2.60	-2.56	4.26	-2.22	6.22				
200	50	-3.28	0.84	-3.29	1.07	-3.19	1.06	-3.15	1.05	-3.03	1.17	-2.95	1.64	-2.78	2.30	-2.64	3.27	-2.42	4.36
200	100	-3.27	0.64	-3.27	0.74	-3.24	0.80	-3.23	0.85	-3.19	0.87	-3.17	0.90	-3.12	0.91	-3.10	0.95	-3.04	1.02
200	200	-3.26	0.56	-3.26	0.61	-3.25	0.65	-3.25	0.69	-3.23	0.73	-3.23	0.76	-3.21	0.78	-3.21	0.80	-3.19	0.80

Notes: *R* = # of breaks; *p* = order of autocorrelation; *N* = # of cross-section units; *T* = # of time periods

PANEL D: *R* = 3

		p=0		*p*=1		*p*=2		*p*=3		*p*=4		*p*=5		*p*=6		*p*=7		*p*=8	
N	*T*	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.
10	38	-3.79	0.61	-3.84	0.61	-3.69	0.66	-3.60	1.03	-3.27	1.35	-2.95	1.77	-2.50	2.16				
10	50	-3.76	0.56	-3.80	0.54	-3.72	0.52	-3.68	0.61	-3.53	0.82	-3.40	1.05	-3.13	1.24	-2.90	1.53	-2.59	1.73
10	100	-3.72	0.48	-3.75	0.47	-3.72	0.45	-3.73	0.44	-3.69	0.44	-3.67	0.46	-3.62	0.49	-3.59	0.53	-3.52	0.60
10	200	-3.70	0.46	-3.71	0.45	-3.70	0.43	-3.71	0.43	-3.70	0.42	-3.70	0.42	-3.69	0.42	-3.69	0.41	-3.67	0.40
20	38	-3.79	0.61	-3.84	0.59	-3.69	0.66	-3.60	1.03	-3.27	1.44	-2.95	1.93	-2.51	2.38				
20	50	-3.76	0.55	-3.80	0.53	-3.72	0.50	-3.68	0.60	-3.53	0.81	-3.39	1.10	-3.13	1.34	-2.90	1.62	-2.59	1.87
20	100	-3.72	0.46	-3.74	0.45	-3.72	0.43	-3.73	0.42	-3.69	0.42	-3.68	0.44	-3.62	0.48	-3.59	0.54	-3.52	0.61
20	200	-3.70	0.44	-3.71	0.43	-3.70	0.42	-3.71	0.41	-3.70	0.41	-3.70	0.40	-3.69	0.40	-3.69	0.39	-3.67	0.40
30	38	-3.79	0.63	-3.83	0.60	-3.69	0.67	-3.60	1.07	-3.27	1.46	-2.95	2.05	-2.51	2.61				
30	50	-3.76	0.55	-3.80	0.54	-3.72	0.49	-3.68	0.62	-3.53	0.83	-3.39	1.14	-3.13	1.43	-2.90	1.78	-2.59	2.08
30	100	-3.72	0.46	-3.74	0.46	-3.72	0.44	-3.72	0.45	-3.69	0.44	-3.68	0.46	-3.62	0.47	-3.59	0.53	-3.52	0.60
30	200	-3.70	0.42	-3.71	0.42	-3.70	0.41	-3.71	0.40	-3.70	0.40	-3.70	0.40	-3.69	0.40	-3.69	0.40	-3.67	0.40
50	38	-3.79	0.66	-3.84	0.64	-3.69	0.66	-3.60	1.16	-3.27	1.64	-2.95	2.36	-2.51	2.98				
50	50	-3.76	0.58	-3.80	0.57	-3.72	0.54	-3.68	0.65	-3.53	0.88	-3.39	1.25	-3.12	1.60	-2.89	2.01	-2.58	2.41
50	100	-3.72	0.47	-3.74	0.47	-3.72	0.46	-3.73	0.46	-3.69	0.46	-3.68	0.48	-3.62	0.51	-3.59	0.58	-3.52	0.67
50	200	-3.70	0.43	-3.71	0.43	-3.70	0.43	-3.71	0.43	-3.70	0.42	-3.70	0.42	-3.69	0.42	-3.69	0.43	-3.67	0.43
70	38	-3.79	0.68	-3.84	0.66	-3.69	0.70	-3.60	1.24	-3.27	1.85	-2.95	2.73	-2.51	3.54				
70	50	-3.76	0.59	-3.80	0.60	-3.72	0.56	-3.68	0.68	-3.53	0.96	-3.39	1.41	-3.12	1.83	-2.89	2.34	-2.58	2.80
70	100	-3.72	0.49	-3.74	0.49	-3.72	0.48	-3.73	0.47	-3.69	0.47	-3.67	0.50	-3.62	0.55	-3.59	0.62	-3.52	0.71
70	200	-3.70	0.43	-3.71	0.44	-3.70	0.44	-3.71	0.44	-3.70	0.44	-3.70	0.44	-3.69	0.44	-3.69	0.43	-3.67	0.43
100	38	-3.79	0.74	-3.84	0.70	-3.69	0.75	-3.60	1.42	-3.27	2.18	-2.95	3.22	-2.51	4.15				
100	50	-3.76	0.64	-3.80	0.65	-3.72	0.61	-3.68	0.72	-3.53	1.07	-3.39	1.60	-3.12	2.16	-2.89	2.79	-2.59	3.34
100	100	-3.72	0.50	-3.74	0.51	-3.72	0.51	-3.73	0.52	-3.69	0.51	-3.68	0.53	-3.62	0.56	-3.59	0.66	-3.52	0.78
100	200	-3.70	0.44	-3.71	0.44	-3.70	0.44	-3.71	0.45	-3.70	0.45	-3.70	0.46	-3.69	0.46	-3.69	0.46	-3.67	0.46
200	38	-3.79	0.93	-3.84	0.90	-3.69	0.93	-3.60	1.85	-3.27	3.08	-2.95	4.86	-2.51	6.40				
200	50	-3.76	0.76	-3.80	0.83	-3.72	0.73	-3.68	0.92	-3.53	1.45	-3.39	2.31	-3.13	3.15	-2.89	4.21	-2.59	5.27
200	100	-3.72	0.56	-3.74	0.63	-3.72	0.64	-3.72	0.64	-3.69	0.62	-3.68	0.64	-3.62	0.71	-3.59	0.84	-3.52	1.04
200	200	-3.70	0.49	-3.71	0.51	-3.70	0.53	-3.71	0.56	-3.70	0.57	-3.70	0.57	-3.69	0.57	-3.69	0.58	-3.67	0.57

Notes: *R* = # of breaks; *p* = order of autocorrelation; *N* = # of cross-section units; *T* = # of time periods