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Basel II: A Case for Recalibration

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by

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ABSTRACT

Objectives for Basel II include the promulgation of a sound standard for risk measurement and risk-based minimum capital regulation. The AIRB approach, which may be mandatory for large U.S. banks, will give rise to large reductions in regulatory capital. This paper assess whether the reductions in minimum capital are justified by improvements in the accuracy of risk measurement under Basel II. Review of credit loss data and analysis of the economics of capital allocation methods identify important shortcomings in the AIRB framework that lead to undercapitalization of bank credit risks.

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1. INTRODUCTION

Under the June 2004 Basel II agreements, national supervisory authorities may choose among three alternative frameworks to set minimum regulatory capital for their internationally active banks. The standardized approach links minimum capital requirements to third-party credit ratings. The Foundation and Advanced (AIRB) Internal Ratings Based approaches assign minimum capital using a regulatory model that uses bank estimates of an individual credit's probability of default (*PD*), loss given default (*LGD*), and expected exposure at default (*EAD*). The U.S. implementation of the Basel II will include a modified version of the AIRB framework that will be mandatory for the largest internationally active banks.[1]

In the June 2006 discussion of the Basel II framework, the Basel Committee on Banking Supervision (BCBS) outlines its objectives for the revised Capital Accord. These include [BCBS 2006b, pages 2-4]:

- Strengthen the soundness and stability of the international banking system
- Promote the adoption of stronger risk management practices

[1] See the September 25, 2006 Notice of Proposed Rulemaking [Federal Register Vol. 71, No. 185, September 25, 2006, pp. 55830-55958]. In the U.S., Basel II implementation will require only the largest banks, the so-called core banks, to adopt the AIRB approach, while other banks may petition supervisors for AIRB capital treatment (so-called opt-in banks). Core banks are defined as institutions with total consolidated assets (excluding insurance subsidiary assets) in excess of $250 billion or total on-balance-sheet foreign exposure of $10 billion or more. A revised version of the 1988 Basel Accord, so-called Basel 1A, has been proposed as an alternative regulatory standard for non-AIRB banks, but has yet to be finalized.

- Institute more risk-sensitive capital requirements that are conceptually sound
- Provide a detailed set of minimum requirements designed to ensure the integrity of bank internal risk assessments
- Broadly maintain the aggregate level of capital requirements
- Prevent capital adequacy regulation from becoming a significant source of competitive inequality among internationally active banks
- Create incentives for the adoption of the more advanced framework approaches.

This paper will review the available evidence and assess the degree to which the U.S. implementation of Basel II promises to meet the ambitious goals articulated by the international bank supervisory community. The assessment will focus on the goals of improving financial stability and promoting sound risk measurement practices. We begin with a discussion of the AIRB approach including the logic used to set minimum capital requirements, the mathematical foundations of the AIRB rule, and the calibrations that have been selected in the U.S. implementation. Following this discussion, we review the existing evidence on the likely capital implications of Basel II and contrast these results with the goal of financial stability. Section 3 analyzes the AIRB as a risk measurement standard. We consider the benefits the AIRB approach may engender as it functions as the minimum risk measurement standard for bank internal capital allocation systems. A final section concludes the paper.

2. A REVIEW OF THE AIRB CAPITAL FRAMEWORK

The introductory section of the US Basel II NPR explains the logic that underlies the Basel II AIRB minimum capital rules. To set minimum capital needs, the AIRB focuses on the probability distribution of *potential credit losses*. The Basel II "soundness standard" for participating institutions is defined as the percentage of potential losses that must be covered by bank capital. The soundness standard determines the minimum probability that a bank will remain solvent over the coming year (e.g., 99.9 percent) [US Basel II NPR, pp. 55832-55833].

To restate the logic of the Basel II AIRB minimum capital rule in statistical terms, let $\widetilde{L}$ represent a credit portfolio's random potential loss and $\Psi(L), L \in [0, L]$ represent the cumulative distribution function for potential credit losses. The AIRB capital rule sets minimum capital equal to $\Psi^{-1}(0.999)$, or the inverse of the cumulative portfolio credit loss distribution evaluated at the 99.9 percentile.

The AIRB framework uses a regulatory model to approximate a bank's credit loss distribution and estimate $\Psi^{-1}(0.999)$. The framework is a modified version of the single factor Gaussian credit loss model first proposed by Vasieck (1991). Using a restrictive set of assumptions, this model creates a synthetic probability distribution for the default rate on a perfectly diversified portfolio of credits. AIRB capital requirements are set using a tail value of this synthetic distribution.

The single factor Gaussian model of portfolio credit losses uses a latent random factor to model whether an individual credit defaults within an unspecified time frame called the capital allocation horizon. There is a unique latent factor for each credit with the properties,

$$\begin{aligned} &\widetilde{V}_i = \sqrt{\rho}\, \widetilde{e}_M + \sqrt{1-\rho}\, \widetilde{e}_i \\ &\widetilde{e}_M \sim \phi(e_M) \\ &e_i \sim \phi(e_i), \\ &E(\widetilde{e}_i\, \widetilde{e}_j) = E(\widetilde{e}_M\, \widetilde{e}_j) = 0\ \forall\, i, j. \end{aligned} \tag{1}$$

$\widetilde{V}_i$ is normally distributed with $E(\widetilde{V}_i)=0$, and $E(\widetilde{V}_i^2)=1$. $\widetilde{e}_M$ is a factor common to all credits' individual latent factors, and the correlation between individual latent factors is ρ.

Firm *i* is assumed to default when $\widetilde{V}_i < D_i$ implying an unconditional probability that firm *i* will default, $PD_i = \Phi(D_i)$. The loss incurred should firm *i* default, *LGD*, is exogenous to the model and not specific to an individual credit. Time does not play an independent role in this model but is implicitly recognized through the calibration of input values; PD_i, for example, will differ according to the capital allocation horizon.

The model calculates the portfolio default rate distribution for a portfolio of N credits, where *N* is a very large number, and each credit is identical regarding its default threshold, $D_i = D$, and its latent factor correlation, ρ. For such a portfolio, credit losses depend only on the default rate experienced by the portfolio. The capitalization rate required for a single credit added to this so-called "asymptotic" portfolio is identical to the capitalization rate for the entire portfolio because idiosyncratic risks have been fully diversified. The model calculates capital for a perfectly diversified portfolio and ignores capital needs generated by risk concentrations.

The probability distribution for the portfolio default rate is defined using an indicator function,

$$\widetilde{I}_i = \begin{cases} 1 & if \quad \widetilde{V}_i < D \\ 0 & otherwise \end{cases} \tag{2}$$

$\tilde{I}_i$ is binomially distributed with an expected value of $\Phi(D)$. Conditional on a specific value for e_M, default indicators are independent and identically distributed binomial random variables. The default rate on a portfolio of N credits is $\tilde{X} = \frac{\sum_{j=1}^{N} \tilde{I}_j}{N}$.

If $\tilde{I}_j \mid e_M$ is used to represent the distribution of $\tilde{I}_i$ conditioned on a realized value $\tilde{e}_M = e_M$, then as $N \to \infty$, the Strong Law of Large Number requires,

$$\lim_{n\to\infty}\left(\tilde{X} \mid e_M\right) = \lim_{n\to\infty} \left(\frac{\sum_{i=1}^{n}\left(\tilde{I}_i \mid e_M\right)}{n} \right) \xrightarrow[a.s.]{} E\left(\tilde{I}_i \mid e_M\right) = \Phi\left(\frac{D - \sqrt{\rho}\, e_M}{\sqrt{1-\rho}} \right) \tag{3}$$

Recall that under Basle II, minimum capital requirements are set using the inverse of this unconditional portfolio loss distribution function, $\Psi^{-1}(\alpha), \quad \alpha \in [0,1]$. Expression (3) defines the inverse of the cumulative distribution for the portfolio's default rate. The portfolio default rate determines the unconditional portfolio loss distribution under the single factor Gaussian assumptions. Substituting for the default barrier, $D = \Phi^{-1}(PD)$, and the identity, $\Phi^{-1}(\alpha) = -\Phi^{-1}(1-\alpha)$, the inverse of the unconditional cumulative distribution for the portfolio default rate is given by,

$$\Phi\left(\frac{\Phi^{-1}(PD) + \sqrt{\rho}\,\Phi^{-1}(\alpha)}{\sqrt{1-\rho}} \right), \quad \alpha \in [0,1] \tag{4}$$

Assuming an identical exposure (*EAD*) for each credit in the portfolio and an exogenous identical *LGD* per dollar of *EAD* for all portfolio credits, the inverse of the portfolio unconditional credit loss distribution is,

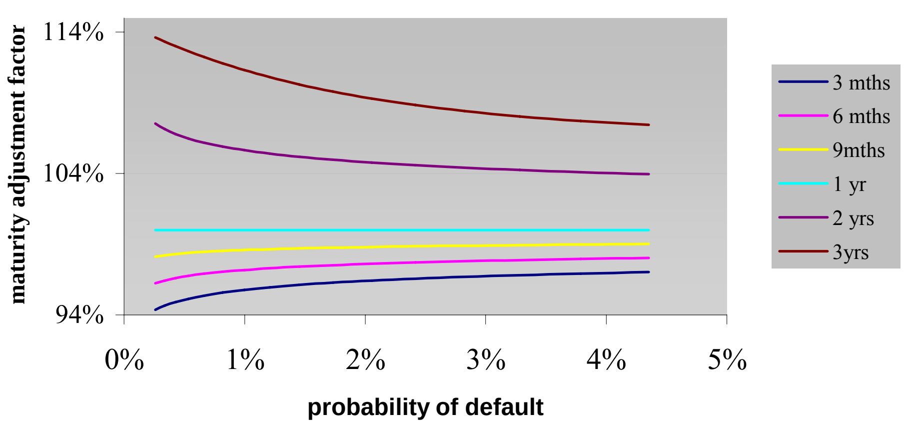
$$\Psi^{-1}(\alpha) = LGD \cdot EAD \cdot \Phi\left(\frac{\Phi^{-1}(PD) + \sqrt{\rho}\,\Phi^{-1}(\alpha)}{\sqrt{1-\rho}}\right), \quad \alpha \in [0,1] \tag{5}$$

Basel II sets minimum capital equal to the 99.9 percentile level of this loss distribution. Adding the requirement that bank loan loss reserves (which count as regulatory capital) must be equal to (or greater than) expected portfolio loss, the bank minimum capital requirement in excess of loan loss reserves is,

$$K = EAD\left[LGD \cdot \Phi\left(\frac{\Phi^{-1}(PD) + \sqrt{\rho}\,\Phi^{-1}(.999)}{\sqrt{1-\rho}}\right) - PD \cdot LGD\right] \tag{6}$$

The Basel II AIRB capital rule is expression (6) with two additional modifications. Basel II assigns the correlation using a regulatory function that differs among regulatory exposure classes (wholesale, revolving retail, mortgages, and other retail). For wholesale exposures, expression (6) is also multiplied by a regulatory maturity adjustment function.

Figure 1: Maturity Adjustment Factors for Corporate, Bank and Sovereign Credits

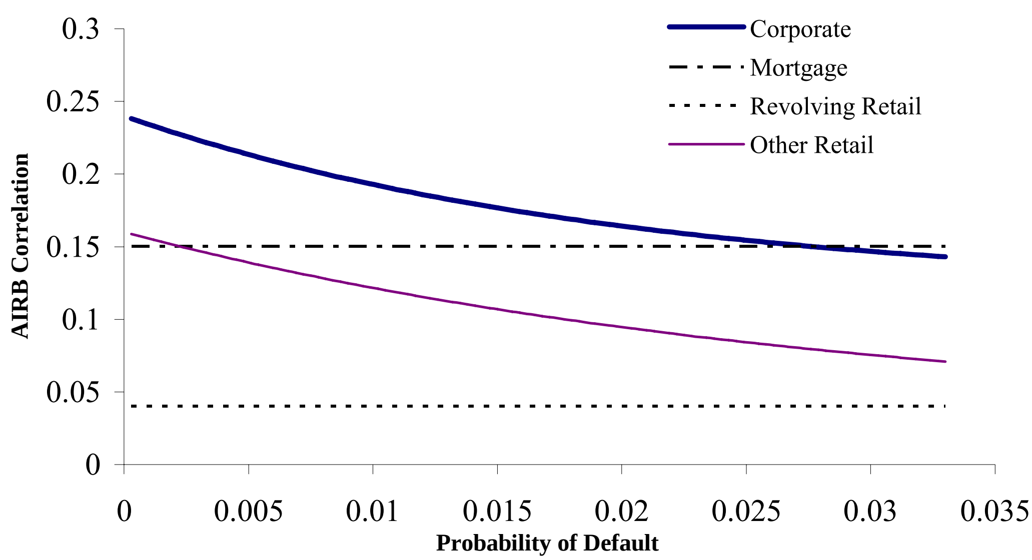


Source: Author's calculations using June 2006 AIRB maturity adjustments

The maturity factor for wholesale exposures (corporate, bank and sovereign credits) is plotted in Figure 1. There is no theoretical basis for the maturity correction factor as it was calibrated to make the AIRB rule mimic the capital allocation behavior of capital estimates calculated using KMV Portfolio Manager for different maturity and wholesale credit risk profiles [BCBS 2005, p.9]. The maturity adjustment factor is one for 1-year credits; it lowers capital for shorter-term credit and raises capital for longer term credits.

A regulatory function is used to specify the AIRB correlation parameter ρ . The correlation assignment depends on the type of credit (wholesale, residential mortgage, other retail, or qualifying revolving retail) and *PD*. The regulatory correlation is a constant for mortgages and revolving retail credits and a declining function of *PD* for wholesale, and other retail credits. AIRB correlation assumptions are plotted in Figure 2.

Figure 2: Basel II US AIRB Correlation Assumptions



Source: Author's calculations using June 2006 AIRB correlation assignment rules

The AIRB correlation functions were calibrated using data sets made available by G10 bank supervisors [see BCBS July 2005]. The BCBS interpretation of this data reportedly guided the calibration of the wholesale correlation curve. The data characteristics the BCBS reproduced include: (1) default correlation increases with firm size; and, (2) default correlations decrease as *PD* increases. Correlations mimic these features within a bound of 24 percent correlation for the lowest *PD*s, and 12 percent correlation for the highest *PD* wholesale exposures.

AIRB retail correlation assignments reportedly were “reverse engineered” from bank internal model data. Correlations were chosen so that, when used in conjunction with expression (6), they produced an AIRB capital requirement that was approximately equivalent to the capital requirement that was assigned by the internal capital allocation models of a group of large internationally active banks [See BCBS 2005, p. 14] .

Discussion

The AIRB is based on a very simple (and restrictive) model of portfolio credit risk in which potential credit losses are driven by the distribution of the proportion of portfolio credits that may default in a large and perfectly diversified portfolio. The model focuses entirely on a portfolio’s default rate and does not include other factors that may generate capital needs. The model, moreover, excludes interest earnings and thereby fails to measure the diversification benefits that arise from income that is generated when credits fully perform.

Among the more important risk factors that are omitted from the AIRB framework are systematic credit risks that are driven by random *LGD*s and, on portfolios of undrawn credit commitments, random *EAD*s . Depending on the characteristics of the *LGD* and *EAD* distributions, uncertainty in these factors may generate sources of risk that require additional capital. Appropriately measured, required capitalization rates may far exceed those calculated using the simple Vasicek approximation for a portfolio loss distribution.

Empirical evidence concerning *LGD*s finds significant time variability in realized *LGD*s. Default losses clearly increase in periods when default rates are elevated. Studies by Frye (2000), Schuermann (2004), Araten, Jacobs, and Varshney (2004), Altman, Brady, Resti and Sironi (2004), Hamilton, Varma, Ou and Cantor (2004), Carey and Gordy (2004), Emery, Cantor and Arnet (2004) and others show pronounced decreases in the recovery rates during recessions and periods of heightened defaults.

There is relatively little published evidence that characterizes the empirical characteristics of EADs for revolving exposures. The evidence that is available, including studies by Allen and Saunders (2003), Asarnow and Marker (1995), Araten and Jacobs (2001), and Jiménez, Lopez, and Saurina (2006) suggests that obligors draw on their lines of credit as their credit quality deteriorates. In other words, *EAD*s and *PD*s are positively correlated, suggesting that there is at least one common factor that simultaneously determines *EAD* and default realizations.

Basel II documents indicate that the BCBS is aware that the stochastic nature of *LGD* and *EAD* may affect minimum capital needs. The committee nonetheless did not decide to generalize the Vasicek model to account for these effects, and instead focused on including guidance that seeks to bolster the magnitude of bank *LGD* estimates.

The Basel II discussion defines ELGD as the simple average of historical *LGD* observations and requires that the *LGD* input into the AIRB capital rule equal *ELGD* plus an adjustment for the potential that losses might be elevated from *ELGD* should default occur during a recession. The framework excludes any formal method of adjustment or a technical standard to guide the estimation of so-called “downturn *LGD*. “ For revolving credits, Basel II requires that *EAD* estimates include recognition that obligors may draw on their credit lines, but again Basel II excludes any formal method, process, or standard for modeling *EAD*.

The calibration of the regulatory default correlation function raises a number of issues. For wholesale credits [corporate, bank and foreign sovereign exposures] and other retail credits [auto loans, boat loans, personal loans, etc], the BCBS specify a correlation parameter that declines as a credit’s *PD* increases. Low *PD* credits may have up to twice the default correlation of high *PD* exposures. Independent empirical evidence does not support this calibration.

In contrast to the BCBS characterization of the stylized facts [BCBS 2005, p. 12], studies including Allen, DeLong and Saunders (2004), Cowan and Cowan (2004), Dietsch and Petey (2004), and Das, Duffie, Kapadia and Saita (2004) find that default correlation increases as the credit quality of a portfolio declines (*PD* increases). The choice of the shape of the Basel II correlation curve is not consistent with empirical evidence, but likely was selected to attenuate fears that the AIRB might create “procyclicality,” or capital requirements that systematically vary with the business cycle.

Concerns about "procyclicality" are based on the idea that, during recessions, any given set of bank credits is more likely to be reclassified into lower-rated buckets.[2] In boom periods, the reverse will likely occur. If a portfolio of given credits migrates through various *PD* grades in response to changing economic conditions, AIRB minimum capital will rise during recessions and decline during booms. Such a cycle in minimum capital has the potential to discourage the extension of new bank credit during recessions and overly stimulate bank lending during boom periods and thereby unintentionally reinforce the bank lending cycle. It is seems likely that the BCBS intend to dampen the inherent procyclicality of the AIRB capital rule by specifying a correlation function that declines as *PD* increases. This calibration will reduce the minimum capital fluctuations that a credit may generate as it moves through an up-grade/down-grade cycle.

3. THE AIRB AND FINANCIAL STABILITY

Basel II will enhance financial stability if it improves upon the 1988 Basel Accord's ability to ensure that systemically important institutions retain adequate minimum capital to achieve social policy objectives. In a variety of published papers and public addresses, members of the BCBS have explained that the complexity of the AIRB is needed to ensure risk and minimum capital are properly aligned given the complexity of large international banking organizations and the need to foreclose opportunities for regulatory arbitrage that

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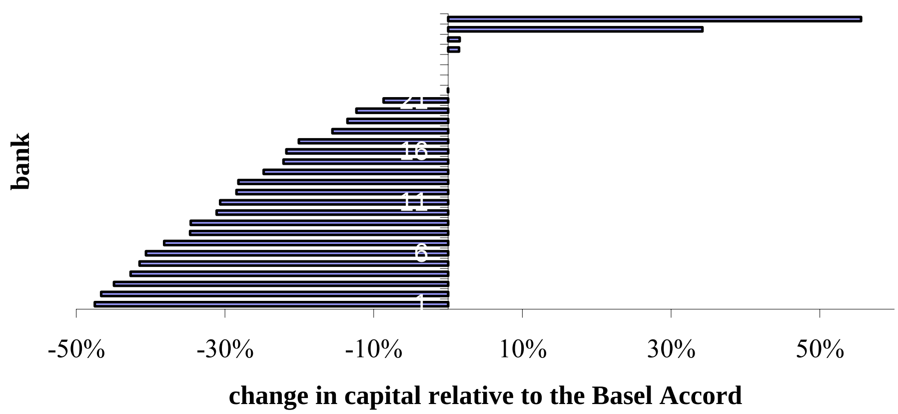
[2] See for example, Turner (2000), Lowe (2002), Allen and Saunders (2003), Kashyup and Stein (2004), or Gordy and Howells (2004).

exist under the 1988 Basel Accord.[3] Capital savings that arise under the AIRB are intended to offset costs associated with developing and operating AIRB systems. Reductions in capital also reflect a presumption that the AIRB approach will improve the accuracy of bank credit risk measures and thereby improve the assignment of minimum capital allocations within banks.

Figure 3: Estimates of Effective AIRB Changes in Minimum Required Capital of QIS4 Banks



Source: QIS 4 Interagency Analysis

The BCBS has conducted two Quantitative Impact Studies (QIS) following the June 2004 publication of the Basel II framework. QIS 4 included banks in the United States, Germany and South Africa. QIS 5 included banks in adopting countries in other nations. Both studies reported substantial declines in minimum capital requirements for AIRB banks relative to capital required under the 1988 Basel Accord. Figure 3 plots a histogram of estimates of the effective change in the levels of minimum capital that would be required

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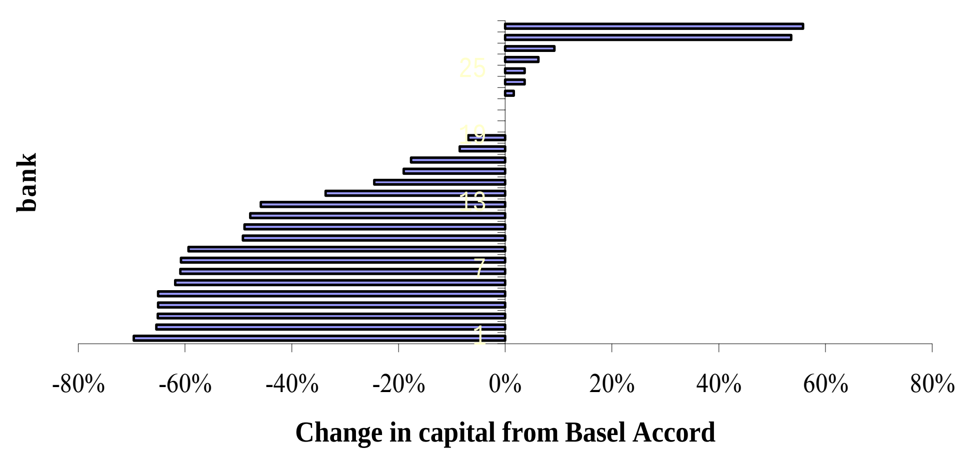
[3] See for example, Greenspan (1998), BCBS (1998, June 1999), Mingo (2000), Jones (2000), or Meyer (2001) or more recently, Bies (2005).

under the AIRB approach for banks participating in the QIS 4 exercise, relative to capital levels required under the U.S. implementation of the 1988 Basel Accord.

The QIS 4 study included 26 U.S. institutions, all of which reported using the AIRB approach.[4] The results show that, in aggregate, minimum regulatory capital for these institutions fell by 15.5 percent relative to existing capital requirements. Among these banks, the median reduction in capital was 26 percent and the median reduction in Tier I capital requirements was 31 percent. Of the few banks that experienced increases in minimum capital requirements under the AIRB, the increases were driven primarily by increases in capital for consumer retail portfolios and to a lesser extent by equity exposures.

Figure 4: QIS 4 Estimates of AIRB Change in Capital for Securitization Exposures



Source: Author’s calculations using QIS 4 Interagency data

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[4] See the Federal Reserve Board Press release, “Summary Findings of the Fourth Quantitative Impact Study,” available at www.federalreserve.gov

In addition to large declines in capital, QIS 4 results show a high degree of dispersion in reported estimates of minimum capital requirements. Banks reported widely divergent capital estimates for their constituent portfolios (corporate, mortgages, etc.). Although these differences could owe to differences in bank risk profiles that reflect differentiation among customer bases and business strategies, additional analysis conducted by the U.S. regulatory agencies using shared national credit data and a hypothetical mortgage portfolio indicated that banks reported widely divergent capital estimates for positions with substantially similar risk characteristics. The analysis suggested that a significant share of the variation in QIS 4 results may be attributed to differences in bank estimates of *PD*s and *LGD*s among credits with approximately equivalent risk characteristics. For the wholesale portfolio, for example, QIS 4 *LGD* estimates on non-defaulted credits varied from about 15 to 55 percent across banking institutions.

The minimum regulatory capital treatment of securitization exposures provides one indicator of the degree to which the AIRB approach meets Basel II objectives. Bank securitization activities have been specifically identified as the means through which Basel Accord minimum capital standards have been eroded [e.g., Jones (2000), Mingo (2000)]. The Basel AIRB approach includes a complex set of capital rules for measuring capital requirements on exposures related to securitized positions. Figure 4 plots the histogram of the changes in effective minimum capital required by the AIRB approach for QIS 4 participating banks. Changes are calculated relative to existing minimum capital requirements. In these estimates, AIRB rules that require deductions from capital are treated as a capital requirement of 100 percent. Figure 4 shows, for most banks, the AIRB will result in substantial reductions in required capital for exposures related to securitizations. Although a full analysis is not possible using QIS 4 data, a large part of the reductions likely owe to reductions in AIRB capital requirements for the assets that are included in these securitization structures.[5]

The QIS 5 study includes 382 banks in 32 countries outside of the U.S.[6] Of the banks that participated, the largest internationally active banks, so-called Group 1 banks, posted capital declines of 7.1 percent on average under the AIRB approach. Smaller banks, so-called Group 2 banks that are primarily nationally-focused institutions, experienced much larger declines in minimum regulatory capital [BCBC (2006a)]. Within Europe,[7] Group 1 banks posted average capital declines of 8.3 percent under the AIRB. For European Group 2 banks, capital declines averaged 26.6 percent under the AIRB. The QIS 5 analysis attributed the large declines in minimum regulatory capital requirements to bank concentrations in retail lending, especially residential mortgages.

The BCBS discussion of QIS 5 results does not provide detailed analysis of the dispersion of bank minimum capital estimates. The study does however report significant variation in AIRB input values. *LGD* estimates for wholesale credits, for example, range from 10.8 to 67.6 percent across reporting banks.

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[5] The Basel II capital rules for securitization exposures have a "look through" property, meaning that the minimum capital requirements that apply to the collateral in these structures in part determines the capital requirements for a bank's securitization position.

[6] See, BCBS (2006a). QIS 5 AIRB capital rules include a 1.06 scaling factor that was not included in the June 2004 calibration or the instructions that guided QIS 4. The inclusion of this scaling factor means the reported capital declines will appear less severe than those reported in the U.S.

[7] So-called CEBS (Committee of European Bank Supervisors) banks.

The results of the QIS 4 and QIS 5 studies show that, under the AIRB approach, most banks will face large reductions in their minimum required capital levels on their current portfolio positions. In practice, the AIRB will result in further capital reductions as banks optimize and adjust their positions to maximize the benefits available through new (unanticipated) regulatory arbitrage opportunities available under the AIRB approach.

Given the potential for large reductions in minimum bank capital that may materialize under the AIRB approach, it is important to assess whether or not these reductions are justified by improvements in risk measurement standards. There is a strong presumption in many Basel II-related documents and policy discussions that the AIRB approach represents a rigorous scientifically supported standard for measuring bank minimum capital needs. Unfortunately, this confidence is misplaced. A large body of evidence shows that the AIRB framework will undercapitalize credit risks.

There are many sources of bias in the AIRB capital rule. One source of undercapitalization arises because the AIRB underestimates the 99.9 percent loss value for banks' portfolio credit loss distributions. The AIRB approach synthesizes an estimate of a bank's 99.9 percent credit loss critical value using a model that ignores systematic risks in *LGD*s and the draw rates on revolving lines of credit. In addition, AIRB minimum capital requirements must be fortified to account for exposure concentrations that are assumed-away in the AIRB framework. A second source of bias is a flaw in the logic used to set AIRB minimum capital requirements. The AIRB capital rule ignores the need for a bank to pay interest on its own liabilities.

Some may argue that the weakness in the AIRB rule are known and market discipline and national supervisory discretion that may be exercised under pillar 2 will bolster bank capital and attenuate these weaknesses. Such claims are, however, untested. The Basel II prescription for pillar 2 powers does not ensure that national supervisors have the legal powers prescribed or that discretionary powers will be utilized. Claims of the veracity of market discipline or the ability to use pillar 2 supervisory powers to correct for AIRB shortcomings should not be a basis for codifying into regulation a seriously flawed risk measurement standard. The following sections discuss these issues in more detail.

4. ESTABLISHING A SOUND BENCHMARK FOR RISK MEASUREMENT PRACTICES

The Need for Capital for Bank Interest Expenses

Although the U.S. Basel II NPR discussion mirrors a textbook description of a credit VaR calculation, the procedure described will not set minimum capital requirements to ensure the 99.9 percent targeted soundness standard. An important flaw in credit VaR capital allocation method is its failure to recognize a bank's need to pay interest on its own liabilities. This oversight creates little bias when VaR measures are used to set capital over short horizons as they are for example, in the 1-day and 10-day horizons used in the market risk rule. Over longer horizons like the 1-year horizon used for Basel II, ignoring the need to pay interest will cause a substantial divergence between the intended and actual AIRB soundness standard. The magnitude of the deterioration in the intended safety margin will, moreover, depend on the level of interest rates. The omitted interest rate effect will magnify the procyclical nature of the AIRB capital rules.

Consider the problem of setting capital for a single credit. To avoid any questions about the magnitudes of the capital variations involved, we frame the example in terms of an exact pricing model for credit risk. We will use the Black and Scholes (1973) and Merton

(1974) model (hereafter BSM) to frame the analysis, but the qualitative result is true for any equilibrium asset pricing model.

Under simplifying assumptions, the BSM model establishes equilibrium pricing relationships that must hold for risky discount debt instruments. When the default-free term structure is not stochastic and flat at a rate, r_f , and a firms' assets have an initial value of A_0 and evolve in value following geometric Brownian motion with an instantaneous volatility of σ, the BSM model has shown that the equilibrium price, B_0 , of a one-year discount bond with a promised maturity value of Par and default risk is,

$$B_0 = e^{-r_f} \Phi\left(\frac{\ln(A_0) - \ln(Par) + \left(r_f - \frac{\sigma^2}{2} \right)}{\sigma} \right) - A_0 \, \Phi\left(\frac{\ln(Par) - \ln(A_0) - \left(r_f + \frac{\sigma^2}{2} \right)}{\sigma} \right) \qquad (7)$$

The value-at-risk measure for this bond is calculated using the physical probability distribution for the value of this bond at the end of one year, $\widetilde{B}_1$. Under the BSM model assumptions, $\widetilde{B}_1$ the physical probability distribution for the bond's value after one year is,

$$\widetilde{B}_1 = Min\left[A_0 \, e^{\left(\mu - \frac{\sigma^2}{2} \right) T + \sigma \sqrt{T} \, \widetilde{z}} , Par \right] \qquad (8)$$

where $\widetilde{z}$ is a standard normal variable, $\mu = r_f + \lambda\sigma$, and λ is the market price of risk.

The critical value of this distribution used to set a $VaR(\alpha)$ measure is,

$Min\left[A_0\, e^{\left(\mu-\frac{\sigma^2}{2}\right)T+\sigma\sqrt{T}\,\Phi^{-1}(1-\alpha)}, Par\right]$ which simplifies to $A_0\, e^{\left(\mu-\frac{\sigma^2}{2}\right)T+\sigma\sqrt{T}\,\Phi^{-1}(1-\alpha)}$ when the probability of default on the bond exceeds $(1-\alpha)$.

To determine the capital needed to fund this bond, note that any debt issue with a par value greater than $A_0\, e^{\left(\mu-\frac{\sigma^2}{2}\right)T+\sigma\sqrt{T}\,\Phi^{-1}(1-\alpha)}$ will default with a probability greater than $(1-\alpha)$ if $\tilde{B}_1$ is the only source of funds available to repay the funding debt. Thus $Par_F(\alpha) = A_0\, e^{\left(\mu-\frac{\sigma^2}{2}\right)T+\sigma\sqrt{T}\,\Phi^{-1}(1-\alpha)}$ is the maximum permissible par value for the funding debt. The cash flows from $\tilde{B}_1$ "pass through" the firm to payoff the funding debt issue, and so the BSM model can be used to price the bond issued by the bank. The difference between B_0 and the market value of the funding debt issue is the minimum equity capital needed to fund the risky bond. The minimum amount of capital needed to achieve a soundness standard of α is,

$$B_0 - Par_F(\alpha)e^{-r_f}\left[1-\Phi(d(\alpha))\right] - A_0\Phi(d(\alpha)+\sigma) \qquad (9)$$

where, $d(\alpha) = \Phi^{-1}(1-\alpha) + \dfrac{\mu - r_f}{\sigma}$.

The potential importance of the omission of bank funding costs from the Basel II AIRB capital calculations is illustrated in Figure 5 for a risky 1-year BSM discount bond. The bond has a par value of 70 and for the rights this claim, the bank lends \$66.14. The underlying assets of the borrower have an initial value of 100, and these assets evolve in value following geometric Brownian motion with an instantaneous drift rate of $\mu = .10$, and an instantaneous volatility $\sigma = .25$. One-year Treasury bonds pay a 5 percent rate.

The probability distribution of $\widetilde{B}_1$ is plotted in the top panel of Figure 1. In this example we consider a soundness standard of 99 percent which dictates that the bank's equity must be large enough to absorb 99 percent of all potential losses. The 99 percent critical value of the loss distribution is equivalent to the 1 percent critical value of the bond's future value distribution, or $59.82 in this example. Under the AIRB approach for setting capital, this bond requires $7.32 in capital ($66.14-$59.82) to cover both expected and unexpected losses. To fund the bond, the bank must sell debt that has an initial market value of $59.82.

The bottom panel of Figure 5 illustrates the potential outcome one year after the bond is purchased and funded according to an AIRB approach for setting minimum capital. If the bank raises $59.82 in debt finance to fund the bond, it owes bank debt holders $63.04 at the end of the year.[8] After accounting for the interest payments that are due on the bank debt, the true probability that the bank defaults on its debt is 1.7 percent.[9] The actual default rate is 70 percent higher than the default rate consistent with the minimum regulatory soundness standard.

[8] This value is calculated by inverting the BSM pricing model to find the par value of debt that would raise $59.82 when it is sold to investors. The bank's debt is risk so it must pay a rate higher than the one-year risk free rate.

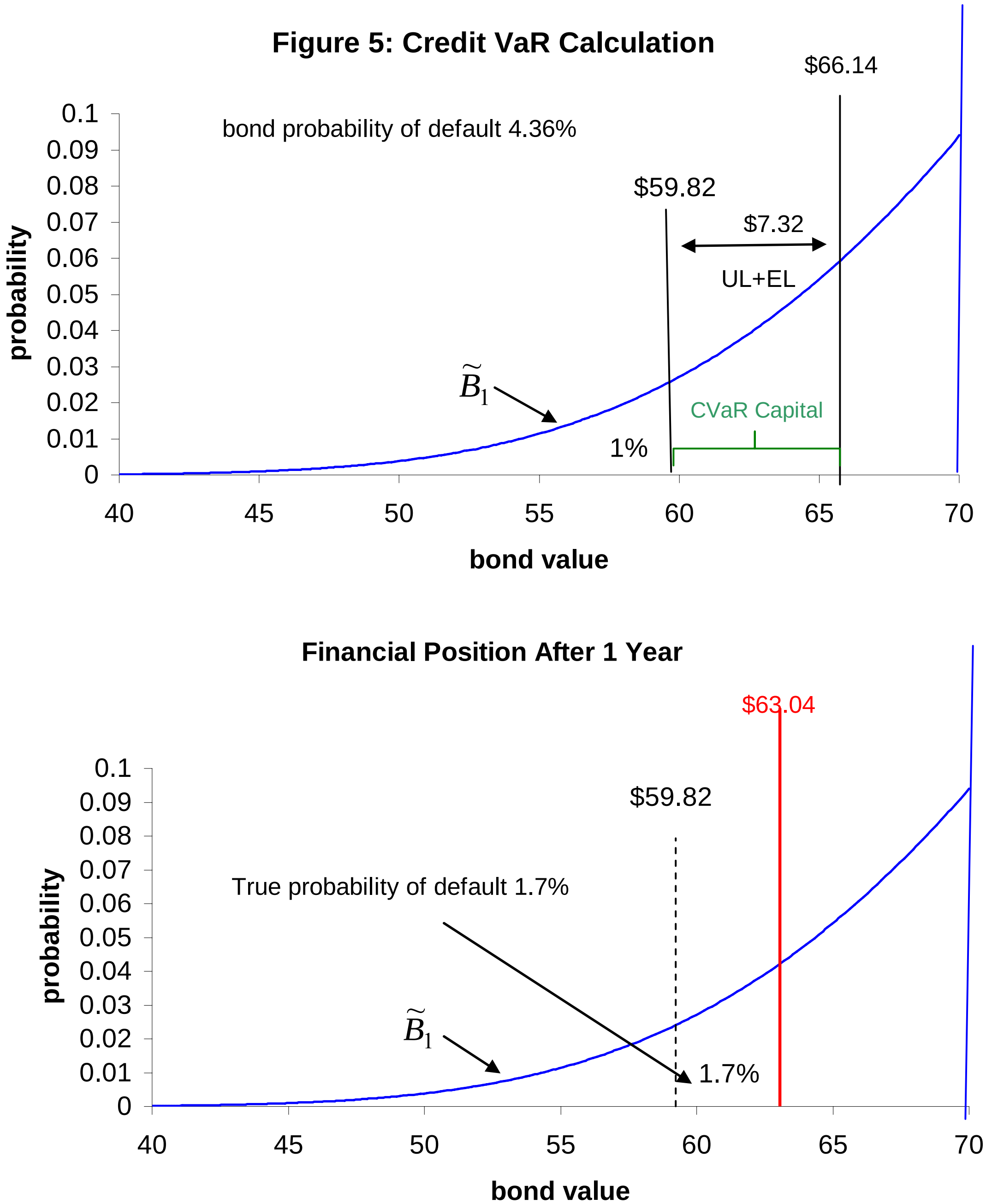
[9] The probability distribution for $\widetilde{B}_1$ includes the interest that is paid to the bank on the purchased risky bond.

There is nothing “staged” about this example. The AIRB approach for setting minimum regulatory capital requirements excludes any consideration of the need to compensate bank debt holders for the time value of money and credit risk. As a consequence the credit VaR based AIRB rule will always understate capital requirements. This is true in a portfolio context as well so long as the bank earns and pays competitive rates of return on its loans and liabilities. Kupiec [2006b] provides additional discussion including the portfolio generalization of this result.

Procyclicality of the AIRB Soundness Standard

The omission of bank interest expense in the AIRB capital rule engenders a soundness standard that varies over the business cycle. The soundness standard set by AIRB minimum capital requirements will decline (i.e., the probability of default will increase) when interest rates are high and the central bank is attempting to dampen economic activity and bank lending. Conversely, AIRB capital standards engender the strictest solvency standard when interest rates are low and the central bank is attempting to stimulate bank lending and economic activity. As a consequence, the potential safety net benefits to the banking system are increased during the boom phase of the economic cycle when banks compete on underwriting standards and stock up on the “bad loans” that default when a subsequent downturn materializes.

Figure 5: Credit VaR Calculation



Source: Author's calculations

The procyclicality of the soundness standard is illustrated in Figure 6. The top panel of the figure illustrates the credit VaR capital calculation for a bond identical to that analyzed in Figure 1. The only change in Figure 6 is a one-year Treasury rate of 10 percent instead of 5 percent. Since this new bond must satisfy equilibrium conditions, the higher default-free rate requires an increase in the instantaneous drift rate (μ = 15 percent) on the value of the underlying assets. Under these new equilibrium conditions, the credit VaR approach requires only $.18 for its minimum capital requirement, so the bond can be purchased for $63.07 and funded with $62.89 in debt.

The bottom panel of Figure 6 shows the possible outcomes one year later. After one year, the bank must pay its debt holders $69.79 to avoid default and retire its debt with accrued interest. The probability that the value $\tilde{B}_1$ is less than $69.79 is 2.8 percent. Thus the actual soundness standard set by the AIRB minimum capital rule is 97.20 percent and not the targeted 99.9 percent. The actual soundness standard set by the AIRB rule declined from 1.7 percent to 2.8 percent as risk free interest rates rose by 5 percentage points.[10]

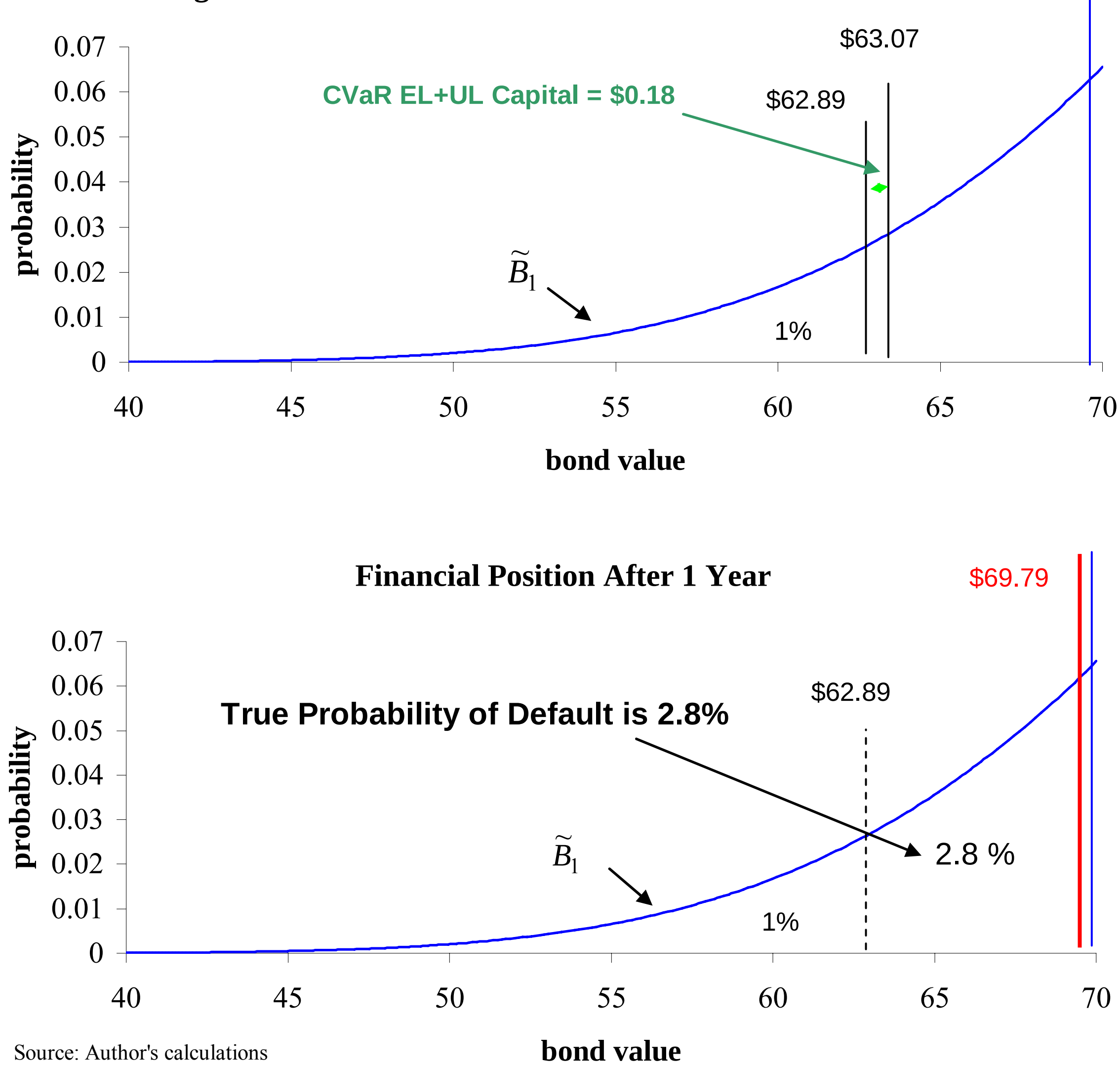
The omission of bank interest costs will induce procylicality in the AIRB regulatory soundness standard. To the extent that minimum regulatory capital requirements impose binding constraints on bank capital positions, this procyclicality may work to magnify the bank lending cycle. During the initial upturn phase of the business cycle, the demand for

---

[10] Notice that this increase in capital is for credit risk and not for interest rate risk as the one-year default free rate was changed *ceteris peribis* and not converted into a random variable.

credit is strong and banks may expand lending and grow without relaxing their underwriting standards or offering concessionary spreads.

Figure 6: Credit VaR Calculation with 10% Risk Free Rate



Source: Author's calculations

As the recovery phase matures toward the peak of the business cycle, growth opportunities wane, and banks compete aggressively to continue to grow. In this portion of the cycle, banks' risk of booking marginal quality credits increases. Concurrently, at this stage of the cycle, the central bank typically begins to increase interest rates in order to attenuate aggregate demand imbalances. Under the AIRB approach to setting capital, the increase in risk free interest rates will automatically reduce banks' minimum regulatory solvency standard.

When governments provide implicit or under-priced explicit guarantees on bank liabilities, banks debt is priced to reflect this guarantee. Because bank shareholders do not pay (or pay a fair price) for this guarantee, they profit from a government safety net subsidy. A reduction in a bank's soundness standard is equivalent to expanding the safety net subsidy enjoyed by banks. Banks may utilize the increased subsidy and continue to grow by adding marginal loans that otherwise might have been rejected under a stricter solvency standard. Reverse incentives will arise in a recession, as decreases in interest rates strengthen the regulatory solvency standard and discourage bank lending.

Incorporating Portfolio Interest Income

Quite apart from the need to recognize that bank capital requirements must be set to ensure that a bank can meet its interest expenses, well-formulated capital allocation estimates should also recognize the interest income received by a bank on fully performing credits. The AIRB framework calculates capital requirements using an approximation for the distribution of the default rate on a well-diversified portfolio. The model does not include any recognition of the loss diversification benefits that arise from the interest payments that are received on fully performing credits. Portfolio interest income can be recognized by formulating the model using an asymptotic approximation for the portfolio return distribution instead of the portfolio loss distribution [Kupiec (2006a)].

Consider the portfolio of identical credits analyzed in Section II. Let *YTM* represent the yield to maturity calculated using the initial market value of an individual credit and let *LGD* represent the loss from initial loan value should a loan default. All loans in a portfolio are assumed to have identical values for *YTM*, *PD,* and *LGD*.

Let $\tilde{R}_p$ represents the return on the portfolio of credits. The end-of-horizon conditional portfolio return is given by,

$$\tilde{R}_p = YTM - (YTM + LGD)\,\tilde{X} \tag{10}$$

where the distribution for $\tilde{X}$ follows from expression (3). Applying the same logic used in Section 2 to derive the Vasicek approximation for the portfolio's loss distribution, the unconditional cumulative return distribution for the portfolio, $\tilde{R}_P$ can be derived from the distribution for the portfolio default rate [expression (3)][11]. The critical value of the portfolio return distribution that is consistent with a regulatory soundness standard of 99.9 percent is,

$$\left(1 + YTM - \left(YTM + LGD\right)\Phi\left(\frac{\Phi^{-1}\left(PD\right) + \sqrt{\rho}\,\Phi^{-1}\left(.999\right)}{\sqrt{1-\rho}}\right)\right) \tag{11}$$

Assuming the bank earns and pays competitive rates on its assets and liabilities, *YTM* is a conservative estimate of the equilibrium required rate of return on the bank's funding debt when it is issued. Using this approximation, the minimum required portfolio (and individual credit) capitalization rate to ensure a 99.9 percent solvency standard is,

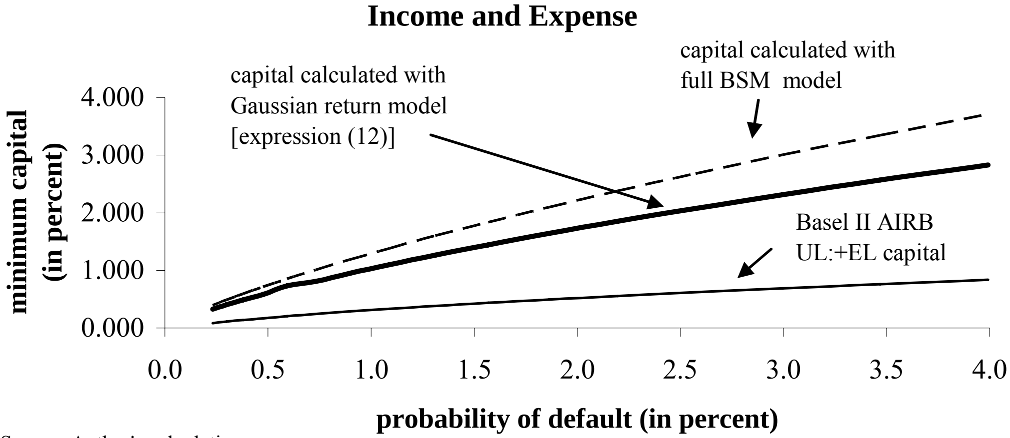
$$K\left(\alpha\right) \approx \frac{YTM + LGD}{1 + YTM}\,\Phi\left(\frac{\Phi^{-1}\left(PD\right) + \sqrt{\rho}\,\Phi^{-1}\left(.999\right)}{\sqrt{1-\rho}}\right) \tag{12}$$

[11] Kupiec (2006a) provides a full derivation.

Expression (12) is an approximation for the capital needed in a single common factor framework. It includes capital for both expected and unexpected loss as well as capital to cover bank interest expenses. Unlike the Basel II AIRB capital rule, it fully recognizes the capital reducing benefits of competitive rates of interest income earned by the fully performing credits in a portfolio. Capital requirements set according to expression (12) are uniformly larger than the capital requirements set by the Basel AIRB formula even when including capital for expected loss [expression (5)]. The relationship between the capital recommended by expression (12) and (5) is illustrated in Figure 7.

Figure 7 compares minimum capital requirements for a 99.9 percent soundness standard as set by the Basel AIRB rule for expected and unexpected loss [expression (5)] and expression (12). The minimum capital estimates are for hypothetical credit portfolios that are composed of credits that are priced to satisfy BSM equilibrium conditions [Kupiec (2006b) includes additional details].

Figure 3: Capital Requirements Recognizing Bank Interest Income and Expense



Source: Author's calculations

It is important to remember that the Basel AIRB rule and expression (12) are approximations for the true capital needed to satisfy a regulatory soundness standard. Both of these models are developed under a set of restrictive assumptions that allow the models to be parameterized in terms of *PD* and *LGD* and admit a closed form expression for capital. For reference, Figure 7 also includes the exact capital that is required to ensure the 99.9 percent soundness standard. These exact capital requirements are calculated using a full BSM capital allocation model developed in Kupiec [2004, 2006a, 2006b]. The full BSM model expression for capital is significantly more complex than expression (5) or expression (12), and it is not directly parameterized using common measures of credit risk (*PD, LGD*, default correlation) but instead is calibrated using a deeper set of model parameters (volatilities, drift rates, initial asset values, etc).

Capital for Systematic Risk in PD *and* LGD

Many studies have recognized that credit loss rate realizations may be tied to the business cycle. Recovery values tend to be depressed for defaults that occur when default rates are elevated. The Basel II AIRB model framework takes *LGD* as an exogenous parameter. Correlation between *PD* and *LGD* is not modeled, but must be accounted for through some ad hoc adjustment to expression (5). In the Basel II framework, this adjustment is made through requirements on how the *LGD* parameter must be estimated.

The U.S. Basel II NPR makes a distinction between two loss-given-default parameters. One parameter, expected loss given default, or *ELGD*, is the default-frequency weighted average default experience for an *LGD* grade. The second measure of loss given default, *LGD*, is the parameter that is to be used as the AIRB input. *LGD* is the greater of a bank's *ELGD* estimate for the exposure, or the loss per dollar of *EAD* that the bank would likely incur should the exposure default within a one-year horizon during an economic downturn [U.S. Basel II NPR, pp. 55847-8]. This regulatory definition of downturn *LGD* is not restrictive as to how *LGD* may be estimated. It is possible to formally incorporate random *LGD* into the AIRB model and to derive a rigorous statistical characterization of *LGD*.

Random Loss Given Default and "Downturn" LGD

Assume that a generic credit has a potential loss given default, $\widetilde{LGD}_i$, that is random. *LGD* uncertainty is driven by a latent Gaussian factor, $\widetilde{Y}_i$ with the following properties,

$$
\begin{aligned}
&\widetilde{Y}_i = \sqrt{\rho_Y}\,\widetilde{e}_M + \sqrt{1-\rho_Y}\,\widetilde{e}_{iY} \\
&\widetilde{e}_M \sim \phi(e_M) \\
&e_{iY} \sim \phi(e_{iY}), \\
&E(\widetilde{e}_{iY}\,\widetilde{e}_{jY}) = E(\widetilde{e}_M\,\widetilde{e}_{jY}) = E(\widetilde{e}_{iY}\,\widetilde{e}_j) = 0\ \forall\, i, j.
\end{aligned}
\tag{13}
$$

The common Gaussian factor, $\widetilde{e}_M$, in the latent factor $\widetilde{Y}_i$ is identical to the common Gaussian factor in expression (1), and so the latent default factor $\widetilde{V}_i$ and loss given default factor, $\widetilde{Y}_i$, are positively correlated provided $\sqrt{\rho_Y} > 0$.

The unconditional distribution for $\widetilde{LGD}_i$ can be approximated to any desired level of precision using a step function that is driven using the realized value of $\widetilde{Y}_i$. Without loss of generality, we assume that higher *LGD* realizations are associated with smaller realized values for $\widetilde{Y}_i$. For expositional simplicity, consider the following simple approximation,

$$
\widetilde{LGD}_i = \begin{cases} LGD_0 & for\ \tilde{Y}_i > B_{i1} \\ LGD_0 + \Delta LGD & for\ B_{i2} < \tilde{Y}_i < B_{i1} \\ LGD_0 + 2\,\Delta LGD & for\ B_{i3} < \tilde{Y}_i < B_{i2} \\ LGD_0 + 3\,\Delta LGD & for\ \tilde{Y}_i \le B_{i3} \end{cases} \tag{14}
$$

where $B_{i3} < B_{i2} < B_{i1}$. Let $\Omega(LGD_i)$ represent the cumulative distribution function for $\widetilde{LGD}_i$. Each level of the *LGD* step function approximation has an associated cumulative probability. This cumulative probability in turn defines the cumulative probability of the latent variable $\tilde{Y}_i$ crossing the threshold. This association is described in Table 1.

Table 1: Probability Distribution Approximation for *LGD*

Loss Step Function Increment	LGD Level	Cumulative Probability of LGD Level	Cumulative Probability for Latent Variable $\tilde{Y}_i$
0	LGD_{i0}	$\Omega(LGD_{i0})$	$1-\Phi(B_{i1})$
ΔLGD	$LGD_{i0} + \Delta LGD$	$\Omega(LGD_{i0} + \Delta LGD)$	$1-\Phi(B_{i2})$
$2\,\Delta LGD$	$LGD_{i0} + 2\,\Delta LGD$	$\Omega(LGD_{i0} + 2\Delta LGD)$	$1-\Phi(B_{i3})$
$3\,\Delta LGD$	$LGD_{i0} + 3\,\Delta LGD$	$\Omega(LGD_{i0} + 3\Delta LGD)$	$\Phi(B_{i3})$

In this example, the loss distribution for an individual account can be defined using four indicator functions, one for default status and three to represent the realized *LGD*,

$$
\tilde{I}_i = \begin{cases} 1 & if\ \ \tilde{V}_i < D_i \\ 0 & otherwise \end{cases}, \quad \tilde{H}_{ij} = \begin{cases} 1 & if\ \ \tilde{Y}_i < B_{ij} \\ 0 & otherwise \end{cases}, \quad for\ \ j = 1,2,3. \tag{15}
$$

Each indicator variable has a binomial distribution with a mean equal to the cumulative standard normal distribution evaluated at the indicator functions threshold value. For example, $\widetilde{I}_i$ has a binomial distribution with an expected value of $\Phi(D_i)$; similarly, $\widetilde{H}_{i1}$ is binomial with an expected value of $\Phi(B_{i1})$, and so on for the remaining indictors.

The loss rate for account *i* measured relative to EAD_i, can be written

$$L\widetilde{R}_i = \widetilde{I}_i \left(LGD_{i0} + \Delta LGD \sum_{k=1}^{3} \widetilde{H}_{k1} \right) \tag{16}$$

Define $\widetilde{I}_i \mid e_M$ and $\widetilde{H}_{ik} \mid e_M$ as the distributions of the default indicator functions conditional on a realized value for e_M for ($k = 1, 2, 3$). The conditional indicator functions are independent binomial random variables with the properties,

$$E\left(\widetilde{I}_i \mid e_M\right) = \Phi\left(\frac{D - \sqrt{\rho_d}\; e_M}{\sqrt{1-\rho_d}}\right), \quad E\left(\widetilde{H}_{il} \mid e_M\right) = \Phi\left(\frac{B_{il} - \sqrt{\rho_Y}\; e_M}{\sqrt{1-\rho_Y}}\right) \tag{17}$$

Using the conditional indicator function notation, the conditional loss rate for an individual credit can be written,

$$L\widetilde{R}_i \mid e_M = \left(\widetilde{I}_i \mid e_M\right)\left(LGD_{i0} + \Delta LGD \sum_{k=1}^{3} \left(\widetilde{H}_{k1} \mid e_M\right) \right) \tag{18}$$

Asymptotic Portfolio Loss Distribution

Consider a portfolio composed of *N* accounts with identical latent-factor correlations, $\{\rho, \rho_Y\}$, default thresholds, $D_i = D$, and unconditional loss given default distributions, $LG\widetilde{D}_i = LG\widetilde{D}$. Individual credit *LGD*s are drawn from a common distribution defined by expression (14) with parameters: $B_{i1} = B_1, B_{i2} = B_{2,}$ and $B_{i3} = B_3$. Under these assumptions, $\tilde{I}_i \mid e_M$ and $\tilde{H}_{ik} \mid e_M$ are independent and identically distributed across individual credits i in the portfolio, $\tilde{I}_i \mid e_M \sim \tilde{I}_j \mid e_M, \forall i, j,$ and $\tilde{H}_{ik} \mid e_M \sim \tilde{H}_{jk} \mid e_M, \forall i, j, k$.

Define $L\tilde{R}_P \mid e_M$ as the loss rate on the portfolio of accounts conditional on a realization of e_M, $L\tilde{R}_P \mid e_M = \left(\frac{\sum_{i=1}^{N} \left(L\tilde{R}_i \mid e_M \right)}{N} \right)$. Because $\left(L\tilde{R}_i \mid e_M \right)$ is independent of $\left(L\tilde{R}_j \mid e_M \right)$ for all $i \neq j$, and these conditional losses are identically distributed, the Strong Law of Large Numbers requires, for all e_M,

$$\lim_{N \to \infty} \left(L\tilde{R}_P \mid e_M \right) = \lim_{N \to \infty} \left(\frac{\sum_{i=1}^{N} \left(L\tilde{R}_i \mid e_M \right)}{N} \right) \xrightarrow[a.s.]{} E\left(L\tilde{R}_i \mid e_M \right) \tag{19}$$

Independence of the conditional indicator functions for a single credit implies,

$$E\left(\tilde{I}_i \mid e_M \cdot \tilde{H}_{ik} \mid e_M \right) = E\left(\tilde{I}_i \mid e_M \right) \cdot E\left(\tilde{H}_{ik} \mid e_M \right) \quad \forall k, i. \tag{20}$$

and so the asymptotic portfolio return distribution converges almost surely to,

$$\lim_{N \to \infty} \left(L\tilde{R}_P \mid e_M \right) = \lim_{N \to \infty} \left(\frac{\sum_{i=1}^{N} \left(L\tilde{R}_i \mid e_M \right)}{N} \right) \xrightarrow[a.s.]{} E\left(\tilde{I} \mid e_M \right) \cdot \left(LGD_0 + \Delta LGD \sum_{k=1}^{3} E\left(H_k \mid e_M \right) \right) \tag{21}$$

The i subscript has been dropped on the indicator functions in the final term of expression (21) as they are no longer necessary.

The number of steps that may be included in the approximations for the *LGD* unconditional density functions is not restricted. If the number of steps in the approximations is M, after substituting the binomial expressions for the conditional indicators' expected values, the conditional portfolio loss distribution converges almost surely to,

$$\lim_{N\to\infty}\left(L\widetilde{R}_P \mid e_M\right)\xrightarrow[a.s.]{}\Phi\left(\frac{D-\sqrt{\rho_d}\ e_M}{\sqrt{1-\rho_d}}\right)\cdot\left(LGD_0+\Delta LGD\sum_{k=1}^{M}\Phi\left(\frac{B_k-\sqrt{\rho_Y}\ e_M}{\sqrt{1-\rho_Y}}\right)\right) \tag{22}$$

The inverse of the unconditional distribution function for the portfolio loss rate can be derived using expression (22) and the density function for $\widetilde{e}_M$. For a soundness standard (α), the critical value of $\widetilde{e}_M$ is $\Phi^{-1}(1-\alpha)$. Latent factor threshold values can be defined using the characteristics of the individual account's unconditional *PDs* and their unconditional *LGD* probability distribution. These threshold values are defined in Table 2.

Table 2: Latent Factor Model Parameters

Default process	LGD Process
$D=\Phi^{-1}(PD)$	$B_1=\Phi^{-1}(1-\Omega(LGD_0))$
	$B_2=\Phi^{-1}(1-\Omega(LGD_0+\Delta LGD))$
	$\vdots$
	$B_M=\Phi^{-1}(LGD_0+(M-1)\Delta LGD)$

Making use of the identity $\Phi^{-1}(1-\alpha)=-\Phi^{-1}(\alpha)$, the inverse of the unconditional cumulative distribution function for the asymptotic portfolio loss rate can be written,

$$LR_p(\alpha)=\Phi\left(\frac{\Phi^{-1}(PD)+\sqrt{\rho_d}\ \Phi^{-1}(\alpha)}{\sqrt{1-\rho_d}}\right)\cdot\left(LGD_0+\Delta LGDB(\alpha)\right),\quad \text{for } \alpha\in[0,1], \tag{23}$$

where,

$$B(\alpha)=\sum_{j=1}^{M}\Phi\left(\frac{\Phi^{-1}\left(1-\Omega\left(LGD_0+(j-1)\Delta LGD\right)\right)+\sqrt{\rho_Y}\;\Phi^{-1}(\alpha)}{\sqrt{1-\rho_Y}}\right) \tag{24}$$

The first term in expression (23) is the inverse of the cumulative distribution function of the Vasicek portfolio loss rate model, the standard Gaussian model in which *LGD* is an exogenous constant. The second term in the expression adjusts the distribution to account for random *LGD*.

When $\rho_Y \to 0$, it is straight forward to show $\left(LGD_0+\Delta LGD\, B(\alpha)\right)\to E\left(LG\widetilde{D}\right)$. So when *LGD* is random, but uncertainty is completely idiosyncratic, expression (23) becomes,

$$LR_p(\alpha)=\Phi\left(\frac{\Phi^{-1}(PD)+\sqrt{\rho_d}\;\Phi^{-1}(\alpha)}{\sqrt{1-\rho_d}}\right)\cdot E\left(LG\widetilde{D}\right) \tag{25}$$

When $\rho_Y \neq 0$, The function $B(\alpha)$ can be interpreted as a function that shifts the probability distribution for $LG\widetilde{D}$. When $\rho_Y > 0$, the $B(\alpha)$ function shifts probability mass into the right tail of the unconditional *LGD* distribution and, in effect, forms a new "stress *LGD*" distribution.[12] A numerical example that follows will help to clarify the transformation.

As an example, we consider the capital calculation for a portfolio of credits that have unconditional *LGD* distributions consistent with the distribution in Table 3. In this distributions one-third of all loss rates are 33.3 percent, one third are 66.7 percent, and the final one third are 100 percent. In step function form, the distribution can be parameterized

[12] Should $\rho_Y < 0$, $B(\alpha)$ would shift weight towards the left tail of the *LGD* distribution.

with $LGD_0 = .333$, and $\Delta LGD = .333$. The cumulative probability associated with the first threshold value is .333; the second threshold has a cumulative probability of .667. The expected value of the unconditional *LGD* distribution is 66.70 percent.

Table 3: Step Function Approximation for the Corporate LGD Distribution

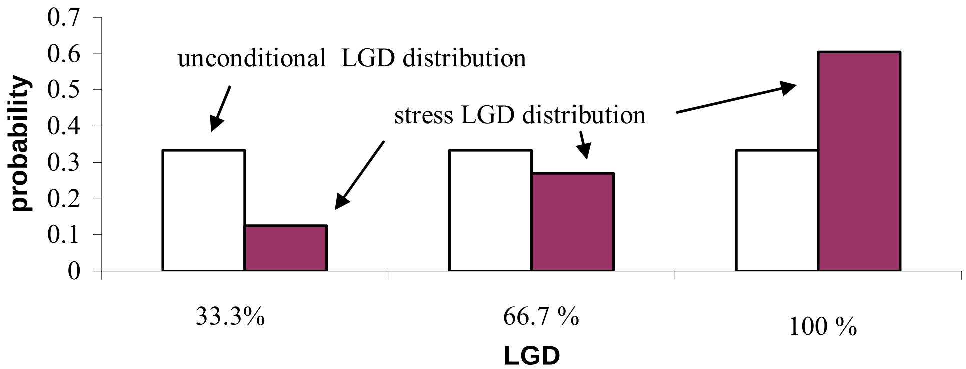
LGD rate thresholds	cumulative probability of LGD level	cumulative probability of LGD increment	threshold for $\tilde{Y}$
33%	33%	33.3%	0.432
67%	67%	33.3%	-0.432
100%	100%	33.3%	
mean	66.70%		

In this example, we assume the correlation among *LGD*s is positive and take $\rho_Y = .05$. The threshold values for the latent variable $\tilde{Y}_i$ are set as $B_1 = \Phi^{-1}(1 - \Omega(LGD_0)) = 0.431644$, and $B_2 = \Phi^{-1}(1 - \Omega(LGD_0 + \Delta LGD)) = -0.431644$. Using these thresholds in the $B(\alpha)$ function, the loss given default term in expression (23) is, $LGD_0 + \Delta LGD\, B(\alpha) = .333 + .333(.8753 + .6049) = 0.827$.

The final value for the loss given default term, 0.827, is equivalent to the expected value of a new shifted *LGD* distribution, where probability mass in the unconditional *LGD* distribution has been shifted to higher *LGD* realizations. We call this new modified *LGD* distribution the stress *LGD* distribution.

Figure 8 plots the unconditional and stress *LGD* distributions for $\alpha = 99.9$ percent and $\rho_Y = .05$. The amount of probability mass that is shifted under the stress measure depends on α, the cumulative probability at which the portfolio loss rate is being evaluated, and on the latent *LGD* factor correlation ρ_Y.

Figure 8: Unconditional and Stress LGD Distributions



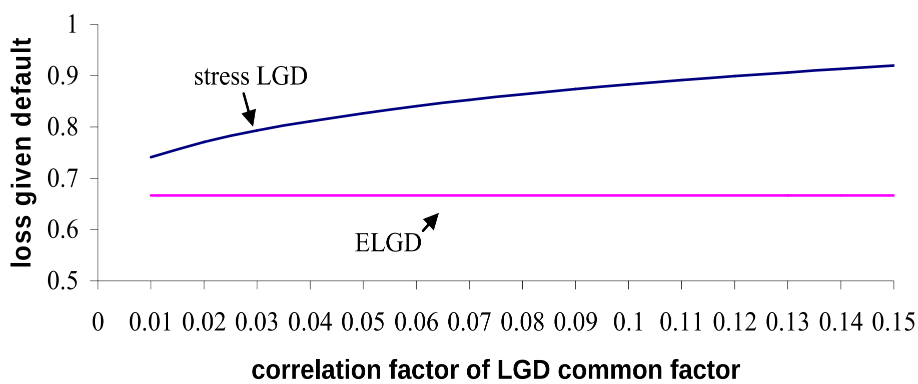
Source: Author's calculations

If stress *LGD* is defined to be the expected value of the stress *LGD* distribution, $E\left(\widetilde{LGD}^S\right) = \left(LGD_0 + \Delta LGD\, B(\alpha)\right)$, upon substitution using the expression for capital that recognizes interest payments and expense (expression (12)), the approximate minimum capital requirement necessary to ensure a soundness standard of 99.9 percent can be written as ,

$$K(\alpha) \approx \frac{YTM + E\left(\widetilde{LGD}^S\right)}{1 + YTM} \Phi\left(\frac{\Phi^{-1}(PD) + \sqrt{\rho}\, \Phi^{-1}(.999)}{\sqrt{1-\rho}}\right) \tag{26}$$

To provide a sense of the potential importance of a positive correlation between *PD* and *LGD*, consider an example in which credits' unconditional *LGD* is consistent with the unconditional *LGD* distribution in Table 3. Figure 9 plots the expected value of the unconditional *LGD* distribution and the corresponding stressed *LGD* measure that is appropriate for use in setting capital for an asymptotic portfolio of credits. Small increases in the correlations between exposures' potential *LGD*s can lead to large changes in minimum capital requirements. For example, an increase in ρ_Y from 0 to 10 percent will increase required capital by 28.2 percent when capital is calculated using expression (26) using the unconditional *LGD* distribution in Table 3.[13]

Figure 9: Correlation and Stress LGD



Source: Author's calculations

Random Exposures at Default (EAD)

The AIRB framework treats *EAD* an exogenous parameter. For revolving exposures, banks using the AIRB are required to estimate *EAD*, but Basel II rules give very little guidance as to how *EAD* should be estimated. For example, the guidance suggests that banks must have methods for estimating *EAD* but the only quantitative standard imposed is that an *EAD* estimate must be at least as large as an obligor's current exposure. As discussed in Section 2, there is a growing body of evidence that suggests that credit facility draw rates are

[13] If one uses the AIRB rule for setting capital [expression (5)], the increase in capital necessary to account for random *LGD* is nearly 33 percent.

higher for low quality credits and credits nearing default implying a positive correlation between *PD* and *EAD*.

Similar to the case of random *LGD*, if the random exposure realizations of the credits in a portfolio are positively correlated, then the ability to reduce credit risk using portfolio diversification is limited. Kupiec [2006c] includes a random *EAD* into the Vasicek framework and shows that, similar to the case of correlated *LGD*s, an expression for minimum capital can be defined in terms of "stressed *EAD*." Correlation among *EAD*s will lead to the need for substantially higher minimum capital requirements.

The Basle II AIRB capital rule will underestimate capital needs for revolving credit portfolios unless banks somehow compensate and input *EAD* rates that are significantly elevated relative to their average facility *EAD*s. The Basel AIRB standard is underdeveloped relative to the treatment of revolving credit exposures. Further model development and recalibration can deliver substantial improvements in accuracy even in the context of the simple single factor Gaussian approximation for measuring portfolio credit risks.

8. CONCLUSIONS

Basel II objectives include the enhancement of financial stability and the promotion of sound risk measurement standards. Unless Basel II fortifies the minimum bank capital requirements for any given set of exposures, it is unclear how it will lead to enhanced stability in the banking sector. Quantitative Impact Studies (QIS) show that large internationally active banks will benefit from large capital reductions under Basel II, especially under the AIRB approach. Once banks are allowed to optimize under the AIRB approach, capital levels will be further eroded.

The results of the QIS studies call into question whether the Basel AIRB approach in its current form should even be considered a minimum regulatory capital standard. The idea of a standard implies that positions with identical risks are subject to identical minimum capital requirements. QIS studies show that AIRB estimates of minimum capital requirements for positions with similar risks vary by wide margins across banks. These results suggest that the AIRB rule and its associated guidance for implementation standards have been vaguely formulated and allow substantial capital differences or subjective interpretations. It is difficult to envision that supervisors around the globe will use pillar 2 powers and impose national implementation standards that ensure equal capital for equal risk. With wide latitude to interpret the input values for the AIRB capital rule, the AIRB approach cannot be viewed as a well-formulated standard.

Concerns about reductions in required capital under the AIRB approach are amplified when the economic foundations of the AIRB rule are examined. The current AIRB capital rule cannot accurately measure the credit risks taken in large complex banking institutions. The AIRB framework does not formally model capital needs that arise because *EAD* and *LGD* are themselves random factors with systematic components. The stochastic properties of *EAD* and *LGD* create potentially large unexpected credit losses that are not modeled in the AIRB framework. The current framework, moreover, is without a sound economic foundation. It ignores the capital needed to satisfy bank interest expenses. This oversight leads to a large understatement in AIRB capital requirements. The AIRB also omits any measure of the capital benefits that are generated by bank interest earnings on its credit portfolio. The adequacy of banks' pricing of credit risk is a primary factor of importance in measuring portfolio credits risk and assigning minimum capital needs

This analysis in this paper suggests that it is improbable that the AIRB approach will either enhance financial stability or serve as a sound standard against which bank credit risk measurement processes are evaluated. Although the list of apparent weakness in the AIRB approach discussed here may seem long, there are still other serious shortcomings that have not been discussed. This paper's analysis has not addressed issues attendant to the AIRB approach not setting capital surcharges for credit risk concentrations which undoubtedly are an important source of risk in many banking institutions. The analysis has also been silent on issues regarding the accuracy of AIRB operational risk measurement standard. Analysis of these and other issues are left for future research.

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