Demandable debt without liquidity insurance

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Abstract

Debt that is demandable, such as deposits, is a common funding source for financial intermediaries. This feature allows creditors to withdraw early, thus exposing the institution to run risk. A large literature justifies demandability with the need to provide liquidity insurance to investors that are risk averse and face uncertainty about their consumption needs. By contrast, we show that demandable debt can be optimal even when investors have no demand for liquidity (i.e., they are risk neutral and have no early consumption need) as part of a bank's strategy for maximizing its own profits. Our paper therefore shows that debt demandability may be a much more generally optimal contractual feature than has been commonly assumed.

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1 Introduction

Demandable debt has been at the center of the literature on financial stability since its inception. Starting with the seminal paper by Diamond and Dybvig (1983), a large body of literature has analyzed demandable debt as a way to provide consumption flexibility for risk-averse depositors. Specifically, since investors are uncertain concerning the timing of their future consumption demand, deposit accounts represent claims that can provide investors with funds at a moment's notice, usually with few, if any, penalties as a result of withdrawal. Thus, a standard rationale for demandable debt is that it allows banks to provide liquidity to investors and increase overall welfare, despite exposing the banks to the risk of disintermediation resulting from depositor runs (Goldstein and Pauzner, 2005).

Importantly, the results summarized above are derived in settings where depositors are risk averse and the provision of liquidity serves to insure individuals against consumption uncertainty. Moreover, banks are assumed to operate in a perfectly competitive market for deposits and thus to act in the interest of depositors, maximizing their utility through the design of the deposit contracts. But to what extent are these elements – risk aversion, consumption uncertainty, and depositor utility maximization – necessary for demandability to emerge as a key design feature of the contracts offered to depositors? While other justifications for the issuance of demandable debt have been offered in the literature (see our discussion below), the literature on financial stability has generally considered these three ingredients as important for rationalizing demandability of deposit contracts. In this paper, we show, using a standard model of financial stability, that demandable debt may optimally be used by financial institutions even if none of these ingredients are included, and that banks may even offer "liquidity" to their investors by leveraging the benefit of their capital (i.e., equity) when setting the terms of deposit contracts. In other words, our paper shows that the demandability of debt can emerge as an optimal feature for deposit contracts in more general frameworks, independently of early consumption needs of risk averse depositors as well as of the degree of competition in the banking sector.

Specifically, we develop a simple two-period model with a representative bank and numerous investors, each with one unit of endowment. At the initial date, the bank raises funds to invest in risky, long-term projects whose expected return at the final date depends on the fundamentals of the economy as well as on the bank's costly monitoring effort. The fundamentals of the economy are realized at the interim date and each depositor receives a private imperfect signal about their realization. Based on this signal and the terms of the deposit contract, depositors make their withdrawal decisions. Unlike canonical models (e.g., Diamond and Dybvig, 1983, and Goldstein and Pauzner, 2005), all parties are risk neutral and are not subject to any consumption shock. Also, the bank can decide to use internal capital as a funding tool in addition to debt in order to maximize expected profits.

We show that the bank always finds it optimal to offer debtholders a demandable debt contract, i.e., a contract that allows depositors to withdraw early, at the intermediate date, and obtain a strictly positive repayment when doing so. Moreover, any repayment that is made must be done through the early liquidation of the long-term project. As a consequence, depositors may decide to withdraw early after observing a low signal on the economy's fundamentals, thus triggering a run. The bank therefore exposes itself to the risk of large liquidity outflows despite the fact that offering only a long-term contract, with no possibility of early withdrawal, would satisfy depositors' participation constraint and hence would be feasible for the bank. The intuition for the use of demandable debt is that by offering a positive early repayment, the bank can reduce the repayment at the final date and thus increase its expected profit by both lowering the probability of being insolvent and by increasing its payoff when no run occurs.

We also show that the size of the early repayment offered to depositors depends on the level of bank capital (or leverage) and on the liquidation value of the bank's project. In particular, the bank would like to offer as a high an early repayment as possible, as long as it can be honored by liquidating the project prematurely and does not trigger coordination failures among depositors, thus avoiding panic runs. The amount of this early repayment is increasing in the level of capital the bank has as well as in the project's liquidation value. In fact, when the bank's capital is sufficiently large, the early repayment will entail a premium relative to the amount initially deposited (i.e., an early repayment greater than one). In other words, depositors may be offered a liquidity premium even though, given that they are risk neutral and face no consumption uncertainty, they have no actual demand for liquidity. Rather, it is the bank's desire to increase its profits at the final date, which are the residual returns on the project after repaying its depositors, that drives the provision of liquidity.

We present four main extensions to our model. First, since our results emphasize the role of bank capital in the provision of liquidity to depositors, we show that indeed banks have incentives to use capital in addition to deposits to fund their projects as long as the cost of capital is not too high. Moreover, we also show that the amount of capital employed can be sufficiently high, relative to the liquidation value of the bank's project, that indeed the bank finds it optimal to offer an early repayment greater than the amount initially deposited.

Second, we modify the baseline framework to allow for the possibility that the liquidation value is stochastic. We show that in this case, the bank still finds it optimal to offer demandable debt with a positive early repayment. However, since it faces the uncertainty on the resources it will have available to repay depositors who withdraw early, the optimal early repayment it chooses will be such that panic runs may arise with some positive probability.

Third, we allow for bankruptcy costs to exist whenever the bank defaults on its promised payment to depositors at the final date. In practice, it is well documented that banks incur costs when insolvent (e.g., James, 1991), so our extension to introduce such costs can be viewed as adding realism to the framework. We show that having some value be destroyed when the bank defaults does not change the type of contract the bank chooses to offer depositors, and demandable debt will still be used.

Fourth, we allow the bank itself to choose whether to liquidate the project at the interim date without having to rely on a run by depositors in order to do so. We show that even in this case the bank wants to offer depositors a strictly positive date 1 repayment. In fact, rather than pushing the promised date 1 repayment to be as low as possible, the bank will instead choose the same promised repayment as in the case where early liquidation is only triggered by depositors' runs. In other words, allowing the bank to liquidate early and cash out investors has no impact on the equilibrium date 1 interest rate offered on deposits.

Our paper contributes to the vast literature, originated with Diamond and Dybvig (1983), studying the optimality of demandable debt and liquidity provision despite the consequent risk of a bank run. In our framework, these two interrelated aspects arise as a result of the bank's incentive to maximize its long-term return, while still satisfying depositors' need to obtain at least some minimum return. In this sense, we provide a rationale for the assumption in Rochet and Vives (2004) and Vives (2014) of a positive face value of debt irrespective of the withdrawal date in the absence of liquidity shocks to investors and profit maximizing banks.

An additional novel aspect of our framework concerns the role of bank capital. As we show, the promised early repayment embedded in the deposit contract increases with the level of bank capital, and it entails no penalties when banks are sufficiently well capitalized. This role for bank capital in determining banks' exposure to depositor runs is reminiscent of Diamond and Rajan (2000), athough in our framework the bank avoids to be exposed to panic runs by setting the early repayment appropriately. Importantly, however, the optimality of demandable debt in our model does not depend on either the bank having equity financing, or on its monitoring effort being endogenous. In fact, a bank with zero capital and an exogenous success probability would still offer to depositors the possibility to redeem their debt at the intermediate date.

The optimality of demandable debt has been justified in the literature by the presence of asymmetric information problems in credit markets (see, e.g., Flannery, 1986; and Diamond, 1991), conflicts between bank managers and shareholders (see e.g., Calomiris and Kahn, 1991; Diamond and Rajan, 2001; and Eisenbach, 2017), idiosyncratic liquidity shocks to banks' depositors (e.g., Diamond and Dybvig, 1983, and Goldstein and Pauzner, 2005) and the need of providing liquidity on demand on the asset side through credit lines (e.g., Kashyap, Rajan, and Stein, 2002). While these are likely important rationales for the pervasive use of demandability in deposit contracts, our analysis emphasizes that demandability may be a much more generally optimal feature, and is consistent with bank profit maximization in an environment where there is uncertainty about future returns and information about these returns is learned in the interim.

2 The model

Consider a three date economy ($t = 0, 1, 2$) with one representative bank and a continuum of atomistic investors, each with a unitary endowment at date 0.[1] All agents are risk neutral. The bank has access to a risky project requiring one unit of investment, and must raise funds at date 0 to finance this project in the form of (equity) capital, k, and debt/deposits, $1 - k$.[2] Investors have an outside option returning $u \geq 1$ at date 2, while the outside option of capital is given by $\rho > u$. The variable u can be interpreted as a measure of competition in the deposit market. For instance, in a similar spirit as in Carletti and Leonello (2019), u could be viewed an inverse measure of the switching costs investors face when moving their funds across banks. When u is small, switching costs are high and the bank has all the bargaining power, with $u = 1$ representing

[1] The exact number of banks is immaterial as long as there are relatively more investors than banks supplying funds to the bank inelastically whenever their reservation utility is satisfied. This is consistent with the idea of banks having some degree of market power in the deposit market.

[2] We will use the terms debt and deposits as well as debtholders and depositors interchangeably throughout the paper. In addition, we will refer to $1 - k$ as the level of bank leverage.

a monopolistic market. The assumption that $\rho > u$, which is empirically supported (e.g., Schepens, 2016) is standard in the literature on bank capital (see e.g., Hellmann, Murdock and Stiglitz, 2000; Repullo, 2004; Allen, Carletti and Marquez, 2011) and emerges naturally as an equilibrium outcome when investors incur a disutility from participating in financial markets (see, e.g., Allen, Carletti, and Marquez, 2015, or Caletti, Marquez, and Petriconi, 2020).

The bank's available project (or also technology) yields a fixed return or liquidation value $L < 1$ if liquidated at date 1, while it yields a stochastic return $\widetilde{P}$ at date 2 equal to

$$\widetilde{P} = \begin{cases} R\theta & \text{w.p. } q \\ 0 & \text{w.p. } 1-q. \end{cases}$$

The date 2 return on the project depends on the fundamental of the economy θ, with $\theta \sim U[0,1]$, and on an "effort" choice q of the bank, with $q \in [0,1]$. The latter represents the effort exercised by the bank in reducing the riskiness of its investment through, for example, the monitoring of its loans. Choosing a higher probability of success q is costly, and we assume that the bank bears a private non-pecuniary cost of $c\frac{q^2}{2}$.

The bank offers the $1-k$ investors a debt/deposit contract with promised repayment r_2 at date 2 and, in addition, a promised repayment r_1 if instead repayment is made at date 1. For convenience, we write such a contract as $\{r_1, r_2\}$. We assume that a value of r_2 exists such that $\int_0^1 qr_2 d\theta \geq u$, for some q that satisfies $\int_0^1 q\,(R\theta - r_2)\,d\theta - c\frac{q^2}{2} > 0$. This implies that a contract with $r_1 = 0$ and $r_2 > 0$ is feasible for the bank. As we will show below, however, such a contract will never be optimal. In fact, we will show that the bank will always choose to offer a contract with $qr_2 \geq r_1 > 0$. We will define a contract to be *demandable* if r_1 is strictly positive and to be *demandable without penalty* if $r_1 \geq 1$, so that the promise to depositors is redeemable early at least at par.

The promised repayment is made as long as the bank has enough resources. If depositors choose to withdraw at date 1, the bank liquidates as much of its assets as needed to satisfy withdrawals, obtaining $L < 1$ per unit liquidated, and carrying to time 2 any remaining amount. If the bank has insufficient resources to meet depositors' demands at date 1, all its assets are liquidated and the $1-k$ depositors receive a pro-rata share of the liquidation value L. Similarly, if the bank has insufficient resources to meet depositors' demands at date 2, depositors receive a pro-rata share of the project proceeds $R\theta$.

The variable θ is realized at the beginning of date 1, but is publicly revealed only at date 2.

After θ is realized at date 1, each depositor receives a private signal s_i of the form

$$s_i = \theta + \varepsilon_i, \tag{1}$$

where ε_i are small error terms that are independently and uniformly distributed over $[-\varepsilon, +\varepsilon]$. After the signal is realized, depositors decide whether to withdraw at date 1 or wait until date 2.

The timing of the model is as follows. At date 0, the bank, equipped with amount of capital k of internal capital, raises external funds with a deposit contract $\{r_1, r_2\}$, and then chooses how much effort to exert to reduce the riskiness of their portfolios q. At date 1, after receiving the private signal about the state of the fundamentals θ, depositors decide whether to withdraw early or wait until date 2. At date 2, the bank's project return is realized and depositors that chose to wait are repaid.

3 Equilibrium

We solve the model by backward induction, focusing first on depositors' withdrawal decisions, which occur at date 1. We then study the bank's choice of contract at date 0, as well as its monitoring decision. We treat bank capital structure as exogenous in this section and endogenize it in Section **4.1.**

3.1 Depositors' withdrawal decision

In this section, we analyze depositors' withdrawal decisions at date 1, taking the deposit contract $\{r_1, r_2\}$ and the riskiness of the portfolio q. The analysis relies on standard arguments in the global games literature (see, e.g., Goldstein and Pauzner, 2005) and allows to characterize the range of fundamentals where the bank faces a run by depositors. Differently from the literature on liquidity insurance á la Diamond and Dybvig (1983), investors are all patient in our model. This implies that their decision to withdraw early depends exclusively on their expectations about the date 2 returns and the possibility of receiving $r_1 > 0$ at date 1. We assume $r_1 > 0$ here and we will later show that the bank finds indeed it optimal to offer a positive date 1 repayment.

Since when receiving a high (low) signal a depositor expects a high (low) return of the bank's project, as well as that other depositors have also received a high (low) signal, he has low (high) incentives to withdraw at date 1. This suggests that depositors withdraw at date 1 when the signal is low enough, and wait until date 2 when the signal is sufficiently high. To show this formally, we first examine two regions of extremely bad and extremely good fundamentals, where each depositor's action is based on the realization of the fundamentals θ irrespective of his beliefs about other depositors' behavior. We start with the lower region.

Lower Dominance Region. The lower dominance region of θ corresponds to the range $[0, \underline{\theta})$ in which running is a dominant strategy. Upon receiving a signal in this region, a depositor is certain that the date 2 expected repayment $q \max\left\{r_2, \frac{R\theta}{1-k}\right\}$ is lower than the payment r_1 from withdrawing at date 1, even if no other depositor were to withdraw. Given that $qr_2 \geq r_1$ must hold in order for intermediation to be feasible,[3] we denote as $\underline{\theta}$ the cutoff value of fundamentals at which the minimum expected date 2 repayment, $\frac{R\theta}{1-k}$, equals r_1; that is, $\underline{\theta}$ solves:

$$r_1 = q\frac{R\theta}{1-k},$$

and is equal to

$$\underline{\theta} = \frac{(1-k)\, r_1}{qR}. \tag{2}$$

Upper Dominance Region. The upper dominance region of θ corresponds to the range $[\overline{\theta}, 1]$ in which fundamentals are so good that waiting to withdraw at date 2 is a dominant strategy. We make the same technological assumption as in Goldstein and Pauzner (2005) and construct this region by modifying the investment technology available to the bank. In particular, we assume that, for $[\overline{\theta}, 1]$, there is no inefficiency in liquidation, so that $L = R$, and the date 2 project fully pays off, so that $\widetilde{P} = R$. Given these assumptions, a bank needs to liquidate no more than 1 unit of its investment at date 1 for each withdrawing depositor and each depositor waiting until date 2 expects to receive $qr_2 \geq r_1$ for sure. As in Goldstein and Pauzner (2005), we consider the limit case where $\overline{\theta} \to 1$.

The Intermediate Region. When the signal indicates that θ is in the intermediate range, $[\underline{\theta}, \overline{\theta})$, a depositor's decision to withdraw early depends on the realization of θ as well as on his beliefs regarding other depositors' actions. To see how, we first calculate a depositor's utility differential between withdrawing at date 2 and at date 1. Using n to represent the fraction of depositors who choose to withdraw early, this differential is given by

$$v(\theta, n) = \begin{cases} qr_2 - r_1 & \text{if } 0 \leq n \leq \widehat{n}(\theta) \\ q\frac{R\theta\left(1 - \frac{n(1-k)r_1}{L}\right)}{(1-k)(1-n)} - r_1 & \text{if } \widehat{n}(\theta) \leq n \leq \overline{n} \\ 0 - \frac{L}{(1-k)n} & \text{if } \overline{n} \leq n \leq 1 \end{cases},$$

[3] If $qr_2 < r_1$, depositors would strictly prefer to withdraw early rather than waiting until date 2. This means that runs would occur for any θ, thus making unprofitable for the bank to operate.

where $\widehat{n}(\theta)$ solves

$$R\theta\left(1-\frac{\widehat{n}(1-k)r_1}{L}\right)-(1-\widehat{n})(1-k)r_2=0, \tag{3}$$

while $\overline{n}$ solves

$$L=\overline{n}(1-k)r_1.$$

The threshold $\widehat{n}(\theta)$ represents the proportion of depositors running at which the bank is no longer able to repay r_2 to those waiting until date 2, while $\overline{n}$ captures the proportion of withdrawing depositors at which a bank liquidates the entire portfolio at date 1.

Throughout, as is common in the literature on bank runs (e.g., Goldstein and Pauzner, 2005; Eisenbach, 2017; Allen et al., 2018), we focus our results on the limiting case where $\varepsilon \to 0$, so that the noise in depositors' information becomes vanishingly small. This implies that all depositors behave alike: they all either withdraw at date 1, thus originating a run, or wait until date 2. The following proposition characterizes depositors' withdrawal decisions and the threshold value of fundamentals θ below which runs occur.

Proposition 1 *The run risk depends on the level of bank capital as follows:*

a) When $(1-k)r_1 \leq L$, runs are triggered only by low realizations of θ and they occur when $\theta < \underline{\theta}(r_1, q)$ as given in (2).

b) When $(1-k)r_1 > L$, runs are driven also by panics and they occur when $\theta < \theta^*(r_1, r_2, q)$, which corresponds to the solution to

$$\int_0^{\widehat{n}(\theta)} qr_2 dn + \int_{\widehat{n}(\theta)}^{\overline{n}} q\frac{R\theta\left(1-\frac{n(1-k)r_1}{L}\right)}{(1-k)(1-n)}dn = \pi_1, \tag{4}$$

where $\pi_1 = \int_0^{\overline{n}} r_1 dn + \int_{\overline{n}}^1 \frac{L}{(1-k)n}dn$. Both thresholds $\underline{\theta}$ and θ^* increase with r_1 and decrease with q. Also, the threshold θ^* decreases with r_2.

The proposition shows that the terms of the deposit contract, interacting with the level of bank capital, determine whether runs are driven only by poor fundamentals or also by depositors' panics. For a given level of $r_1 > 0$, highly capitalized banks, those for which $(1-k)r_1 \leq L$, always have enough resources to repay r_1 by liquidating the project at date 1, even if all depositors were to withdraw. This implies that depositors do not have to worry about other depositors withdrawing early and so their decision to run is only driven by low fundamentals affecting the value of the expected date 2 repayment. Conversely, poorly capitalized banks, those for which $(1-k)r_1 > L$, cannot satisfy depositors' promised repayment even if they liquidate the entire loan prematurely in the event of large withdrawals. As a result, these banks are exposed to runs driven by depositors' fear of others running. In other words, poorly capitalized banks are subject to coordination failures among depositors for fundamentals in the range $[\underline{\theta}, \theta^*]$ and can therefore be subject to panics.

Note that, differently from the classic bank run models in Diamond and Dybvig (1983) and Goldstein and Pauzner (2005) where banks are financed exclusively through deposits and panics occur when $r_1 > L = 1$, here the condition for panics to potentially occur is $(1-k)\,r_1 > L$. As we will see below in the next section, the interest rate r_1 and the amount of capital k jointly determine, together with the size of L, the bank run risk.

As a final point, it is worth noting that the terms of the deposit contract affect the type of runs in addition to their probability. For a given level of bank capital, the bank can adjust its exposure to fundamental- and panic-driven runs by changing the promised repayments to depositors. In particular, both $\underline{\theta}$ and θ^* strictly increase with the date 1 promised repayment r_1, while θ^* decreases also with r_2. Finally, the bank's effort decision q affects run risk by reducing the probability of a run, whether based on fundamentals or as a result of a panic.

3.2 Bank's date 0 decisions

Having characterized depositors' withdrawal decisions, we now solve for the bank's effort q and the deposit contract $\{r_1, r_2\}$. The bank makes its date 0 decisions in order to maximize its expected profits, anticipating depositors' withdrawal decisions at date 1.

To solve the bank's maximization problem, we start by characterizing the bank's payoff. If no run occurs, the bank obtains positive profits only if it is able to repay the date 2 promised repayment r_2 to all $(1-k)$ investors. Thus, the bank is solvent if θ is above the cutoff θ^B, which is the solution to

$$R\theta - (1-k)\,r_2 = 0,$$

and is equal to

$$\theta^B = \frac{(1-k)\,r_2}{R}. \tag{5}$$

Since, as discussed above, $qr_2 \geq r_1$ must hold for intermediation to be feasible, it follows that $\theta^B \geq \underline{\theta}$ for any k, where $\underline{\theta}$ is the fundamental run threshold given in (2). The bank's payoffs thus depend crucially on the amount of capital. When $(1-k)\,r_1 < L$, if there is no run and the bank's monitoring is successful, the bank makes positive profits at date 2 only if $\theta > \theta^B$. If instead a run occurs, the bank makes expected profits at date 2 on any remainder after liquidating only what is needed to repay depositors at date 1. By contrast, when $(1-k)\,r_1 \geq L$, the bank's entire project is liquidated at date 1 when a run occurs. Thus, the bank can only obtain positive profits at date 2 when no run occurs, i.e., for $\theta > \max\left\{\theta^*, \theta^B\right\}$, and when its effort q has proven successful.

Denote now θ^R as the relevant run threshold, i.e., $\theta^R = \underline{\theta}$ when $(1-k)\,r_1 \leq L$ and $\theta^R = \theta^*$ when $(1-k)\,r_1 > L$. Given this, the maximization problem for the bank can be written as follows:

$$\max_{q,r_1,r_2} \Pi = \int_0^{\theta^R} q \max\left\{R\theta\left(1-\frac{(1-k)\,r_1}{L}\right), 0\right\} d\theta + \int_{\max\{\theta^R,\theta^B\}}^{1} q\left[R\theta-(1-k)\,r_2\right] d\theta - \frac{cq^2}{2} \quad (6)$$

subject to

$$\int_0^{\theta^R} \min\left\{\frac{L}{(1-k)}, r_1\right\} d\theta + \int_{\theta^R}^{\max\{\theta^R,\theta^B\}} q\frac{R\theta}{1-k} d\theta + \int_{\max\{\theta^R,\theta^B\}}^{1} qr_2 d\theta \geq u, \quad (7)$$

$$r_1 \leq qr_2, \quad (8)$$

and

$$\Pi \geq \rho k. \quad (9)$$

The first two terms in (6) capture the instances when the bank expects to accrue positive profits at date 2 if monitoring turns out to be successful, i.e., with probability q. The first term represents the case in which a run occurs for $\theta \leq \theta^R$ and the bank can still obtain positive profits at date 2 on the residual after repaying depositors, $\left(1-\frac{(1-k)r_1}{L}\right)$. This is possible only when $(1-k)\,r_1 < L$ so that $\theta^R = \underline{\theta}$. The second term captures the situation when there is both no run (for $\theta > \theta^R$) and the fundamentals are such that the bank remains profitable at date 2 (for $\theta \geq \theta^B$). Moving to costs, the last term in (6) represents the monitoring cost $\frac{cq^2}{2}$ the bank bears for the effort it exerts.

The condition in (7) represents depositors' participation constraint requiring that the expected repayments from depositing in a bank cannot be lower than investors' outside option $u \geq 1$. By depositing in a bank, depositors expect to receive the minimum between the pro-rata share $\frac{L}{1-k}$ and the promised repayment r_1 if there is a run at date 1 (i.e., when $\theta \leq \theta^R$), as represented in the first term of (7). If, instead, there is no run, then with probability q they receive the pro-rata share $\frac{R\theta}{1-k}$ of the bank's available resources at date 2 if the bank turns out to be insolvent (i.e., when $\theta \in (\theta^R, \max\left\{\theta^R, \theta^B\right\}])$, and the full repayment r_2 otherwise (i.e., when $\theta \in (\max\left\{\theta^R, \theta^B\right\}, 1])$. Note that the second term in (7) is positive only if $\theta^R < \theta^B$ as otherwise the bank would always be solvent at date 2 in case of no runs and thus depositors would always obtain the full promised repayment r_2.

Finally, the constraint (8) is an incentive compatibility constraint on the deposit contract, so that depositors expect to receive a higher repayment if they wait until the final date, and as a result don't have an incentive to always withdraw early. Similarly, the inequality in (9) is simply a non-negativity constraint on expected profits that requires that the bank can meet shareholders' required return in expectation.

3.2.1 Bank monitoring effort

Having characterized the bank's maximization problem, we now move on to solve it using backward induction. Hence, we start by computing the optimal monitoring level chosen by the bank for given repayments $\{r_1, r_2\}$ and bank capital k. We have the following result.

Proposition 2 *For any $k > 0$ and $r_2 > r_1 > 0$, the bank chooses $q \in (0, 1]$ as the solution to*

$$\int_0^{\underline{\theta}} R\theta \left(1 - \frac{(1-k)\, r_1}{L}\right) d\theta + \int_{\theta^B}^1 \left[R\theta - (1-k)\, r_2\right] d\theta + \frac{\partial \underline{\theta}}{\partial q} q R \underline{\theta} \left(1 - \frac{(1-k)\, r_1}{L}\right) - cq = 0, \quad (10)$$

if $(1-k)\, r_1 \leq L$ and to

$$\int_{\max\{\theta^*, \theta^B\}}^1 \left[R\theta - (1-k)\, r_2\right] d\theta - \frac{\partial \max\{\theta^*, \theta^B\}}{\partial q} q \left[R \max\{\theta^*, \theta^B\} - (1-k)\, r_2\right] - cq = 0, \quad (11)$$

if $(1-k)\, r_1 > L$.

As illustrated in the proposition, the bank chooses the monitoring effort q by trading off the marginal effort cost with its marginal benefits. Irrespective of the type of runs the bank is exposed to, a higher monitoring effort is associated with higher costs, as captured by the term $-cq$, and with higher profits as captured by the first two terms in (10) for the case where $(1-k)\, r_1 \leq L$, and the first one in (11) when $(1-k)\, r_1 > L$. In addition, an increase in the level of monitoring also reduces the bank's exposure to runs (i.e., $\frac{\partial \underline{\theta}}{\partial q} < 0$ and $\frac{\partial \theta^*}{\partial q} < 0$). This allows the bank to accrue more profits in the case when $(1-k)\, r_1 > L$, as captured by the term $-\frac{\partial \max\{\theta^*, \theta^B\}}{\partial q} q \left[R \max\{\theta^*, \theta^B\} - (1-k)\, r_2\right]$, while it shrinks the region where the bank remains solvent in the event of a run when $(1-k)\, r_1 \leq L$, as captured by the term $\frac{\partial \underline{\theta}}{\partial q} q R \underline{\theta} \left(1 - \frac{(1-k) r_1}{L}\right)$.

3.2.2 Deposit contract

Having characterized the bank's effort, we now move on to the choice of the deposit contract $\{r_1, r_2\}$. In doing so, we proceed in steps. First, we show that the bank always chooses the terms of the deposit contract in a way that panic runs never occur. Then, we characterize the choice of r_1 and r_2.

Lemma 1 *For any $k > 0$, the bank never finds it optimal to set $r_1 > \frac{L}{1-k}$. In equilibrium, therefore, there are no panic runs.*

The lemma establishes that, when all parties are risk neutral, the bank never offers a date 1 promised repayment r_1 so high that it would risk exposing itself to panic runs. Since panics arise only in the region $(1-k)\,r_1 > L$, the bank will never choose r_1 to be above the ratio between the liquidation value L and the amount of leverage $1-k$. In other words, r_1 must satisfy $r_1 \leq \frac{L}{1-k}$. This result is consistent with the literature studying bank runs and the provision of liquidity (e.g., Diamond and Dybvig, 1983, Jacklin and Bhattacharya, 1988, Allen and Gale, 2007, and Goldstein and Pauzner, 2005), where panic runs arise only when intermediaries insure risk-averse depositors against the possibilty of early consumption needs, but they do not occur when investors are either risk neutral or have no need for early consumption. In these latter cases, the bank sets r_1 equal to the liquidation value which, given the absence of capital, implies there are no coordination failures among depositors. The same result arises in our framework where investors are not exposed to any consumption shock and the bank maximizes its expected profit by choosing its deposit contract.

While Lemma 1 establishes that coordination failures among depositors – i.e., panics – will never arise in equilibrium, it does not pin down the equilibrium deposit contract $\{r_1, r_2\}$, nor does it suggest that the deposit contract must be demandable (i.e., $r_1 > 0$). In fact, common intuition would suggest that, given the absence of consumption shocks and depositors' risk aversion in our framework, the profit-maximizing bank could find it optimal to set $r_1 = 0$ and repay investors only at date 2 when the project succeeds. The following result shows instead that this is not the case.

Proposition 3 *The bank chooses the deposit contract $\{r_1, r_2\}$ as follows: $r_1 = min\left\{\frac{L}{1-k}, u\right\}$, $r_2 = \max\left\{\widetilde{r}_2, \frac{u}{q}\right\}$ where $\widetilde{r}_2$ is the solution to (7) holding with equality, that is*

$$\int_0^{\frac{L}{qR}} \frac{L}{1-k} d\theta + \int_{\frac{L}{qR}}^{\frac{(1-k)r_2}{R}} q\frac{R\theta}{1-k} d\theta + \int_{\frac{(1-k)r_2}{R}}^{1} q r_2 d\theta = u, \tag{12}$$

and q is given by the solution to (10).

The proposition shows that the bank finds it optimal to offer a demandable deposit contract, that is $r_1 > 0$, for any level of capital $k \in [0,1]$, even if investors are not subject to any consumption shock as in the traditional literature on bank runs, and despite the fact that the bank maximizes its own expected profits. The reason behind this result is that by choosing $r_1 > 0$, the bank can reduce the repayment to be made at date 2, r_2, and thus increase its profits when no run occurs. In fact, lowering r_2 benefits the bank in two ways. First, a lower r_2 implies a reduction of the threshold θ^B above which the bank makes positive profits at date 2. Second, a lower r_2 increases the net payoff of the bank at date 2 when it is solvent (i.e., $\theta > \theta^B$) and has exerted positive effort (i.e., with probability q). The key issue is that the tradeoff for the bank between increasing r_1 and reducing r_2 is different from that of depositors, who stand to benefit relatively more from an increase in r_1. In other words, the slope of depositors' iso-utility curve, $\frac{dr_2}{dr_1}\Big|_U$, is greater (in absolute magnitude) than the slope of the isoprofit curve for the bank, $\frac{dr_2}{dr_1}\Big|_\Pi$, and the bank exploits this difference by setting the date 1 deposit rate as high as possible in order to maximize its date 2 profits.

To understand the bank's choice of deposit contract in more detail, it is worth noting that the bank would like to set r_1 to be as high as possible without risking being exposed to panic runs, but is also subject to the incentive compatibility constraint $r_1 \leq qr_2$, as otherwise depositors would always strictly prefer to withdraw at date 1. The highest r_1 consistent with the bank not being exposed to panic runs is $r_1 = \frac{L}{1-k}$, which is increasing in k. Hence, we can denote as $\widehat{k} = 1 - \frac{L}{u}$ the level of capital for which $r_1 = u$. For any $k \geq \widehat{k}$, were the bank to choose $r_1 = \frac{L}{1-k} > u$, then depositors' participation constraint could be rewritten as

$$\int_0^{\frac{L}{qR}} \frac{L}{1-k} d\theta + \int_{\frac{L}{qR}}^{\frac{(1-k)r_2}{R}} q\frac{R\theta}{1-k} d\theta + \int_{\frac{(1-k)r_2}{R}}^{1} qr_2 = u,$$

thus showing that $qr_2 < r_1$ would be required for the constraint to hold with equality. As this is a violation of (8) and the bank always finds it optimal to have depositors' participation constraint bind, $qr_2 = r_1 = u$ follows. These considerations imply that $\{r_1, r_2\} = \left\{\frac{L}{1-k}, \widetilde{r}_2\right\}$ when the bank is highly leveraged and/or the deposit market is very competitive, i.e., for $k < \widehat{k}$. Conversely, when the bank's leverage is relatively low and/or competition in the deposit market is not intense (i.e., u is small, so that $k \geq \widehat{k}$), we have $\{r_1, r_2\} = \left\{u, \frac{u}{q}\right\}$.

Importantly, the maximization of the bank's expected profit does not imply eliminating runs. In fact, even though by assumption choosing $r_1 = 0$ and having no runs at all is feasible, the bank chooses to be exposed to fundamental-driven runs. The presence of capital allows the bank to offer $r_1 \geq L$ and yet not trigger coordination problems among investors.

So far we have shown that the bank finds it optimal to offer depositors a demandable deposit contract, that is, a contract whereby they can withdraw at date 1, even though depositors don't anticipate having any early consumption needs. Going further, it is straightforward to see, from Proposition 3, that the bank may even find it optimal to offer a return $r_1 > 1$ to early withdrawing depositors. Since $r_1 = min\left\{\frac{L}{1-k}, u\right\}$, (offering a demandable debt contract with no penalties, i.e., $r_1 > 1$, is optimal whenever $1 - k < L$.) This shows that whether r_1 is greater or less than 1 depends on value of the bank's project under liquidation, L, relative to the bank's leverage, $1 - k$, with a higher payment being promised the more capital the bank has and/or the higher is the liquidation value L. In other words, the bank provides "liquidity" to depositors even though there is no demand for liquidity from the investors and thus liquidity plays no insurance role. The intuition is that both a high level of capital k or a larger liquidation value L allows the bank to increase r_1 without exposing itself to panic runs.

Our analysis highlights a novel mechanism for why a bank may find it optimal to offer demandable debt without penalties, and highlights that neither risk-aversion for investors subject to consumption shocks, nor a competitive banking sector, are necessary ingredients for demandable debt to be optimal. In this respect, the result of Proposition 3 illustrates that the optimality of demandable debt (i.e., $r_1 > 0$) is more general than what is commonly discussed in the existing banking literature, and arises even in settings where banks enjoy substantial monopoly power.

4 Extensions

In this section, we present four extensions to the model that address issues of robustness and realism. First, we endogenize the bank's capital structure and characterize the bank's choice of k. Second, we consider the case of a stochastic liquidation value and ask whether the bank nevertheless finds it optimal to issue demandable debt despite the potential occurence of panic runs. Third, we analyze the situation where the bank incurs bankruptcy costs in the event of default at date 2. Such costs have been well-documented and are often regarded as substantial. Fourth, we study the case where, rather than passively investing at date 0 and holding its position until date 2 in the absence of runs, we allow the bank to itself choose to liquidate the project early.

4.1 Bank capital structure

In our analysis in Section 3, we treated k as a parameter for the bank, and showed that r_1 can be strictly greater than 1 if the bank is sufficiently capitalized. In this section, we prove that it is indeed optimal for the bank to finance itself with capital even if it is a more expensive source of funds than deposits. In other words, we endogenize the level of bank capital k, assuming that the bank takes such choice at date 0 before setting the terms of the deposit contract $\{r_1, r_2\}$ and its monitoring effort q. Hence, we solve the problem by backward induction, taking the choice of the optimal monitoring effort q and the deposit contract $\{r_1, r_2\}$ from the previous section.

As shown before, the bank chooses not to be exposed to panic runs, so the relevant thresholds are $\underline{\theta}$ and θ^B, as characterized in (2) and (5), respectively, with $\theta^B \geq \underline{\theta}$. We have the following result.

Proposition 4 *The bank chooses an amount of capital as follows:*

(1) When $\rho - u > 0$ is sufficiently small, the bank chooses $k > 0$;

(2) Fix the difference $\rho - u$ such that the result in (1) holds, i.e., that $k > 0$. Then, there is a value $\widehat{L} < 1$ such that, for $L > \widehat{L}$, $r_1(k) > 1$.

The proposition shows that it is optimal for the bank to be financed by a mix of debt and equity, despite equity being more costly to raise for the bank, when $\rho - u$ is not too large. The reason is that capital allows the bank to support its effort in reducing the riskiness of its investment project and also it reduces the run risk by lowering the run threshold (2). The positive level of bank capital pushes the interest rate on deposit r_1 above the liquidation value L and also implies that, for sufficiently large L, the bank finds it optimal to offer depositors demandable debt that can be redeemed early with a positive return, i.e., $r_1 > 1$.

The result in Proposition 4 shows again the importance of bank capital in our model. In particular, the bank has an incentive to raise a positive amount of capital because of the endogeneity of the bank's monitoring effort. In other words, as in other standard frameworks (e.g., Holmstrom and Tirole, 1997, and numerous subsequent papers such as Repullo, 2004, Allen, Carletti, and Marquez, 2011, Mehran and Thakor, 2011, Dell'Ariccia, Laeven and Marquez, 2014), here capital not only provides depositors with a buffer in case of losses, but it also provides the bank with an incentive to exert greater effort (see also Thakor, 2014, for a survey).

If instead q were to be exogenous, the bank would prefer to be entirely deposit financed given

$\rho - u > 0$ since there would be no commitment value associated with bank capital. This suggests that the demandability of the deposit contract in our framework does not hinge on its discipline role for the bank's incentives as in Calomiris and Kahn (1991), but rather exclusively on its role as a cost-minimizing tool. Indeed, even with no capital and a fixed q, in our framework the bank would still find it optimal to offer a deposit contract with $r_1 > 0$ as a way to reduce its funding costs.

4.2 Stochastic L and panic runs

So far we have shown that banks find it optimal to offer demandable debt with a date 1 promised repayment that exposes them to fundamental-driven runs but not to panic runs. This result was obtained in a framework with one single source of uncertainty related to the fundamentals of the economy. In this section, we study whether banks still find it optimal to offer demandable debt when there are other sources of uncertainty present, so that coordination failures among investors and panic runs may emerge.

There are various reasons why panics may arise even if banks take actions to minimize their likelihood. One simple reason may be the presence of uncertainty at date 0 about what the bank's project may be worth if liquidated at date 1. While in the analysis above we treat the liquidation value L as given and known at date 0, in reality this value may be uncertain at the time the bank is raising financing in order to invest.

We incorporate this type of uncertainty in the simplest way possible. Specifically, we modify the baseline model by assuming a binary stochastic liquidation value as given by

$$\widetilde{L} = \begin{cases} L_L & \text{w.p. } p \\ L_H & \text{w.p. } 1-p, \end{cases} \quad .$$

with $L_L < L_H$. The distribution of $\widetilde{L}$ is common knowledge at date 0, and uncertainty is resolved at the beginning of date 1.[4] Depositors observe the realization of $\widetilde{L}$ before they decide whether to withdraw at date 1. This implies that the run thresholds, as characterized in Proposition 1, continue to hold, with the only difference that in the expression (4) there will be L_i, with $i = L, H$. The model is again solved backward so that the bank anticipates depositors' withdrawal decisions as a function of the realized liquidation value when choosing the terms of the contract $\{r_1, r_2\}$.

[4] We assume that the distribution of $\widetilde{L}$ is uncorrelated with that of θ. This reflects the case when θ is not observable outside the bank at date 1 so that the liquidation value L can be viewed as independent of future realizations of θ. We make this assumption for tractability, but the analysis would go through in case $\widetilde{L}$ and θ were (positvely) correlated.

Since our only goal is to show that the bank will choose a strictly positive r_1, we simplify the analysis relative to the baseline model and assume that both q and k are exogenously positively given so that intermediation is feasible. In addition, we assume $u > \frac{L_L}{1-k}$ so that the optimal date 1 payment is a function of the project's liquidation value. We have the following preliminary result.

Lemma 2 *It is never optimal for the bank to choose* $r_1 < \frac{L_L}{1-k}$ *or* $r_1 > \frac{L_H}{1-k}$.

The result in Lemma 2 shows that it is never optimal for the bank to be always exposed to panic runs, as would be the case if the bank were to choose $r_1 > \frac{L_H}{1-k}$. Hence, the optimal r_1 must be in the interval $\left[\frac{L_L}{1-k}, \frac{L_H}{1-k}\right]$ and we can now state the following.

Proposition 5 *There is a value* $\underline{p} \in (0,1)$ *such that, for* $p \leq \underline{p}$, *the bank chooses* $r_1 > \frac{L_L}{1-k}$ *and is subject to panic runs whenever* $\widetilde{L} = L_L$.

The proposition shows that, differently from the case of a deterministic value of L, the bank may find it optimal now to increase r_1 to a level that exposes itself to panic runs. This occurs when the probability of the low realization of the liquidation value, L_L, is sufficiently low (i.e., $p \leq \underline{p}$) as this also implies a relatively low likelihood of panic runs. The equilibrium date 1 payment is again a function of the level of bank capital so that for large enough k, $r_1 > 1$ emerges in equilibrium.

4.3 The existence of bankruptcy costs

In this section we extend the model to include bankruptcy costs that may be incurred whenever the bank fails to meet its obligations to depositors and, hence, is insolvent. This is consistent with empirical evidence showing the existence of significant bankruptcy costs when banks enter into liquidation (e.g., James, 1991). To do so, we modify the model slightly as follows. The promised repayments $\{r_1, r_2\}$ are paid as long as the bank has enough resources, as above. However, if the bank fails to repay depositors r_2 at date 2, the bank enters a bankruptcy procedure and depositors experience losses as a result.[5] For simplicity, and to maximize the possible costs that may arise, we assume bankruptcy costs are 100%, so that depositors receive nothing upon insolvency of the bank at date 2. The bankruptcy costs may originate either from coordination failures among a bank's creditors, which makes it difficult and costly for them to seize the remaining value of the bank, or

[5] Following Goldstein and Pauzner (2005) and related papers, we assume that there are no bankruptcy costs at date 1 beyond the $1 - L$ units of resources that are lost due to the premature liquidation of banks' loans. Assuming full costs stemming from bankruptcy at date 1 does not qualitatively affect our results. Calculations can be provided to interested readers.

from the illiquidity of the bank's assets, where some value is lost when selling them to alternative users/lenders. The rest of the model is unchanged.

The analysis is similar to that above in that there is a run threshold below which it is optimal to run, and this threshold depends on whether $(1-k)\, r_1$ is greater or less than L. However, when $(1-k)\, r_1 \leq L$, given the presence of bankruptcy costs, each depositor knows that at date 2 he will receive $qr_2 \geq r_1$ only if the bank is solvent and thus able to make the promised repayment r_2 to all $1-k$ depositors, and 0 otherwise. Thus, his incentives to run depend on whether the bank is solvent or not, which boils down to how the fundamentals compare with the solution to

$$R\theta = (1-k)\, r_2, \tag{13}$$

which is the same as the threshold θ^B defined above in (5). Hence, for the case where $(1-k)\, r_1 \leq L$, withdrawing early is optimal with bankruptcy costs when $\theta < \theta^B$.

Similarly, applying the same arguments as in Proposition 1, one can show that for the case where $(1-k)\, r_1 > L$, a run threshold $\theta^* > \theta^B$ exists below which it is optimal for depositors to withdraw early. However, a minor extension of Proposition 1 establishes that, as in the case analyzed above without bankruptcy costs, it is never optimal for the bank to set r_1 such that $(1-k)\, r_1 > L$ and allow panic runs to arise in equilibrium. To keep the analysis concise we therefore leave out the construction of θ^* and restrict our focus to the case where $(1-k)\, r_1 \leq L$ and only fundamental runs are possible.

With this, we can now write the bank's profit as

$$\Pi = \int_0^{\theta^B} qR\theta \left(1 - \frac{(1-k)\, r_1}{L}\right) d\theta + \int_{\theta^B}^1 q\left(R\theta - (1-k)\, r_2\right) d\theta - \frac{cq^2}{2}. \tag{14}$$

The main difference between bank profits in (14) and those in (6) for the case where there are no bankruptcy costs is in the region of fundamental runs, where now runs occur for a larger parameter space than before (i.e., for $\theta < \theta^B$ instead of $\theta < \underline{\theta}$, with $\theta^B > \underline{\theta}$ as defined in **(5)).** In other words, early withdrawals occur more frequently simply because depositors recognize that, if θ is not sufficiently high that the bank will avoid default, waiting until date 2 will lead to losses arising from bankruptcy costs. Given that in default states depositors are effectively the residual claimants on any resources available at the bank, they also incur all of the losses that acrrue in bankruptcy and, hence, prefer to run for $\theta \in \left(\underline{\theta}, \theta^B\right)$ rather than wait until date 2.

For depositors, their participation constraint becomes

$$\int_0^{\theta^B} r_1 d\theta + \int_{\theta^B}^1 q r_2 d\theta \geq u, \tag{15}$$

where again the main difference from before stems from the larger region over which depositor runs occur. We can now state the following result, which is essentially equivalent to the main result presented above.

Proposition 6 *For large enough k, the bank sets the deposit contract $\{r_1, r_2\}$ as follows: $r_1 = \min\left\{\frac{L}{1-k}, u\right\}$ and $r_2 = \max\{u, \widetilde{r}_2\}$, where $\widetilde{r}_2$ corresponds to the value of r_2 that makes depositors' participation constraint (15) binding, with monitoring effort q chosen to maximize (14).*

The proposition shows that even in the case of bankruptcy costs at date 2, the bank still finds it optimal to offer a demandable deposit contract, even with a premium for large enough levels of capital k.

4.4 What if the bank can liquidate early?

So far, we have considered that the early liquidation of the project is only driven by depositors' decision to run and we have shown that the bank finds it optimal to offer investors a demandable debt contract. In doing this, we have ignored the possibility that the bank itself may prefer to liquidate the project prematurely rather than keeping it until the final date. Yet, when θ is sufficiently low, it may be more profitable for the bank to turn the project into cash at date 1 and repay depositors early. In this section, we analyze this possibility and show that offering a strictly positive repayment, and even sometimes a premium, at date 1 is still optimal. In other words, this means that the optimality of a debt contract that can be repaid early in our model is not driven by the bank's need to use depositors' runs as a way to achieve the premature projection liquidation. In what follows, we ignore bank runs and focus on the case when the bank can itself decide to interrupt the project at date 1.

We start by characterizing the banks' payoffs in the various instances. As in the baseline model, if the bank keeps the project until date 2, it accrues

$$R\theta - (1-k)\, r_2,$$

which is positive only for $\theta > \theta^B$ as given in (5). By contrast, when liquidating at date 1, the bank can decide to liquidate either the entire project or only a fraction of it, enough to repay r_1 to the

$(1-k)$ depositors. In the former case, the bank earns profits

$$L-(1-k)\,r_1,$$

while in the latter case, it earns instead

$$R\theta\left(1-\frac{1-k}{L}r_1\right)$$

with probability q. Thus, the bank prefers to liquidate the project entirely when $\theta<\theta^T$, where θ^T solves

$$L-(1-k)\,r_1=qR\theta\left(1-\frac{1-k}{L}r_1\right)$$

and is then equal to $\theta^T=\frac{L}{qR}$. For $\theta>\theta^T$, the bank instead would prefer to partially liquidate the project at date 1 rather than carrying it all over to date 2 as long as $\theta<\theta^L$, where θ^L solves

$$R\theta\left(1-\frac{1-k}{L}r_1\right)=R\theta-(1-k)\,r_2,$$

and is thus equal to $\theta^L=\frac{r_2}{R}\frac{L}{r_1}$.

In sum, there are two regions where early repayment of depositors occurs: (1) for $\theta\in\left[0,\theta^T\right]$, the bank liquidates fully and gets $L-(1-k)\,r_1$; (2) for $\theta\in\left(\theta^T,\theta^L\right)$, the bank only partially liquidates and gets $qR\theta\left(1-\frac{1-k}{L}r_1\right)$. Having characterized the liquidation thresholds θ^L and θ^T, we can write bank's expected profits and depositors' participation constraint, respectively, as follows:

$$\Pi=\int_0^{\theta^T}\left(L-(1-k)\,r_1\right)d\theta+q\int_{\theta^T}^{\theta^L}R\theta\left(1-\frac{(1-k)\,r_1}{L}\right)d\theta+q\int_{\theta^L}^{1}\left(R\theta-(1-k)\,r_2\right)d\theta,$$

and

$$U=\int_0^{\theta^L}r_1d\theta+\int_{\theta^L}^{1}qr_2d\theta\geq u. \tag{16}$$

We can now establish the following result.

Proposition 7 *The bank chooses to set* $r_1=\min\left\{\frac{L}{1-k},u\right\}$ *and* $r_2=\min\left\{\widetilde{r}_2,\frac{u}{q}\right\}$, *where* $\widetilde{r}_2$ *solves (16) with equality.*

The proposition shows that the bank finds it optimal to offer a positive early repayment also when it is free to liquidate the project prematurely. Again, this result arises in a more general framework as a way for the bank to optimze its profits without the need of assuming risk averse depositors and uncertainty about their consumption preferences.

5 Conclusion

In this paper, we have shown that demandable debt can arise as the optimal contract arrangement in a model with risk neutral investors and profit maximizing banks. The key intuition is that banks may prefer to allow depositors to obtain an early repayment as this allows the bank to reduce the interest rate in the final date, and thus increase its overall return. The promised early repayment is set to be equal to the maximum amount compatible with having only fundamental-based runs, and its level depends on the bank's capital: when capital is high enough, the bank offers demandable debt with a premium, i.e., depositors are promised to receive more than the amount deposited initially when withdrawing prematurely. In other words, the bank provides liquidity to depositors when it is sufficiently capitalized despite depositors not demanding such liquidity since they have no insurance needs. These results are robust to modifications of the baseline model. In particular, they emerge also when banks choose their capital structure, when they can be subject to panic runs due to stochastic liquidation values, when there are bankruptcy costs in the final date, and when the bank can unilateraly decide to prematurely terminate the project.

6 References

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7 Appendix

Proof of Proposition 1: The proof follows closely that in Carletti, Leonello, Marquez (2022). Their arguments establish that depositors run if and only if the signal they receive is below a common signal $s^*(r_1)$ and, at the limit when $\varepsilon \to 0$, this threshold converges to the upper-bound of the lower dominance region $\underline{\theta}(r_1)$ for any $(1-k)\, r_1 \leq L$, and to $\theta^*(r_1, r_2, q) > \underline{\theta}(r_1, q)$ for any $(1-k)\, r_1 > L$. The key difference relative to their framework is that that there are no bankruptcy costs and so depositors acquire a pro-rata share of bank's available resources in the event the bank is unable to repay r_2 to depositors at date 2.

Depositors' withdrawal decisions are characterized by the pair $\{s^*, \theta^*\}$, as given by the solution of the following system of equations:

$$R\theta\left(1-\frac{n\left(\theta,s^*\right)\left(1-k\right)r_1}{L}\right)-\left(1-n\left(\theta,s^*\right)\right)\left(1-k\right)r_2=0, \tag{17}$$

and

$$\begin{aligned} &qr_2\Pr\left(\theta>\theta^*|\,s^*\right)+q\max\left\{\frac{R\theta\left(1-\frac{n(\theta,s^*)(1-k)r_1}{L}\right)}{\left(1-n\left(\theta,s^*\right)\right)\left(1-k\right)},0\right\}\Pr\left(\theta<\theta^*|\,s^*\right) \\ &=r_1\Pr\left(\theta>\theta_n|\,s^*\right)+\frac{L}{\left(1-k\right)n\left(\theta,s^*\right)}\Pr\left(\theta<\theta_n|\,s^*\right), \end{aligned} \tag{18}$$

where $n(\theta, s^*)$ is the fraction of depositors withdrawing early and $\theta_n = s^* + \varepsilon - 2\varepsilon\frac{L}{(1-k)r_1}$ represents the level of θ for which the bank liquidates the entire portfolio at date 1. The former is equal to the probability of receiving a signal below s^* and so equal to

$$n\left(\theta,s^*\right)=\begin{cases} 1 & \text{if } \theta\leq s^*-\varepsilon \\ \frac{s^*-\theta+\varepsilon}{2\varepsilon} & \text{if } s^*-\varepsilon<\theta\leq s^*+\varepsilon \\ 0 & \text{if } \theta>s^*+\varepsilon \end{cases},$$

while the latter is equal to the solution to

$$n\left(\theta,s^*\right)\left(1-k\right)r_1=L.$$

Condition (17) identifies the level of fundamentals, θ^*, at which the bank is at the brink of insolvency at date 2 when $n(\theta^*, s^*) > 0$ depositors run, for given s^*. When no depositor is expected to run, i.e., $n(\theta^*, s^*) = 0$, (17) simplifies to

$$R\theta-\left(1-k\right)r_2=0,$$

and we denote as θ^B the level of fundamental solving the condition above, i.e.,

$$\theta^B=\frac{\left(1-k\right)r_2}{R}.$$

Condition (18) is depositors' indifference condition: the LHS represents a depositor's expected utility from withdrawing at date 2, while the RHS represents the expected utility from withdrawing at date 1. This condition pins down s^* given $\theta^*(s^*)$ from (17), so that together the two equations characterize the equilibrium withdrawal decisions $\{s^*, \theta^*\}$.

Rearranging (17) as follows:

$$R\theta - (1-k)\,r_2 - n\,(.)\left[R\theta\frac{(1-k)\,r_1}{L} - (1-k)\,r_2\right],$$

it is easy to see that when $\theta < \theta^B$, the bank can never repay the promised repayment r_2 at date 2 for any n, so that depositors expect to receive:

$$q\max\left\{0, \frac{R\theta\left(1-\frac{n(.)(1-k)r_1}{L}\right)}{(1-n\,(.))\,(1-k)}\right\}$$

For any $n\,(.) > 0$, $\frac{R\theta\left(1-\frac{n(.)(1-k)r_1}{L}\right)}{(1-n(.))(1-k)} < \frac{R\theta}{1-k}$. Hence, at $\theta = \underline{\theta}\,(r_1, q)$, as defined in (2), depositors strictly prefer to run. This implies that, at the limit, when $\varepsilon \to 0$ and $s^* \to \theta^*$, $\theta^* > \underline{\theta}$. Symmetrically, when $\theta > \overline{\theta}$, (17) is always slack so that depositors expect to receive qr_2 and so have never an incentive to run. This implies that when $\varepsilon \to 0$ and $s^* \to \theta^*$, $\theta^* < \overline{\theta}$.

Differentiating (17) with respect to θ and n, we obtain, respectively,

$$R\left(1-\frac{n\,(\theta,s^*)\,(1-k)\,r_1}{L}\right) - \frac{\partial n\,(\theta,s^*)}{\partial\theta}\left[R\theta\frac{(1-k)\,r_1}{L} - (1-k)\,r_2\right] > 0,$$

and

$$-R\theta\frac{(1-k)\,r_1}{L} + (1-k)\,r_2 < 0,$$

as long as (17) is not negative and for any $\theta > \theta^B$ since $(1-k)\,r_1 > L$ and $\frac{\partial n(\theta,s^*)}{\partial\theta} < 0$. Since $n\,(\theta, s^*)$ is a decreasing function of θ, it follows that the LHS in (17) strictly increases in θ and so does the expected utility at date 2. This also implies that a depositor's expected utility differential between withdrawing at date 2 and date 1, which corresponds to the difference between the LHS and RHS in (18). Hence, it follows that a unique threshold s^* exists at which a depositor is indifferent between withdrawing at date 2 and at date 1.

To obtain the expression for $\theta^*\,(r_1, r_2, q)$ as in the proposition, we perform a change of variable by defining $\theta^*\,(n) = s^* + \varepsilon\,(1-2n)$ and evaluate (18) at the limit when $\varepsilon \to 0$, $\theta^*\,(n) \to s^*$.

Having characterized the panic run threshold, we now move on to show that the relevant run threshold is $\underline{\theta}\,(r_1, q)$ when $(1-k)\,r_1 \le L$ and $\theta^*\,(r_1, r_2, q)$ when $(1-k)\,r_1 > L$. To do this, it is useful to rearrange the expression in (4) as follows:

$$\int_0^{\overline{n}} q\min\left\{r_2, \frac{R\theta\left(1-\frac{n(1-k)r_1}{L}\right)}{(1-n)\,(1-k)}\right\}dn = \pi_1, \tag{19}$$

with $\pi_1 = \int_0^{\overline{n}} r_1 dn + \int_{\overline{n}}^1 \frac{L}{(1-k)n} dn$ as in the proposition.

Consider first the case in which $(1-k)\, r_1 \leq L$. When $(1-k)\, r_1 = L$, the RHS in (19) simplifies to r_1 since $\overline{n} = 1$ when $(1-k)\, r_1 = L$. The LHS (19) simplifies to

$$\int_0^1 \left(q \min\left\{ r_2, \frac{R\theta}{1-k} \right\} \right) dn$$

whose sign depends on the level of θ. When $\theta > \theta^B$, $\frac{R\theta}{1-k} > r_2$, so that the indifference condition in (19) can be written as

$$qr_2 = r_1.$$

This means that depositors expect to receive the same repayment at date 2 and date 1, in which case, they choose not to run. When $\theta < \theta^B$, $\frac{R\theta}{1-k} < r_2$, thus depositors expect to receive a pro-rata share of bank's available resources $q\frac{R\theta}{(1-k)}$ and the indifference condition simplifies to

$$q\frac{R\theta}{1-k} = r_1.$$

This is the same as the condition pinning down the fundamental run threshold $\underline{\theta}\,(r_1, q)$, as defined in (2). Then, it follows that running is optimal when $\theta \leq \underline{\theta}\,(r_1, q)$ that is the relevant run threshold is $\underline{\theta}\,(r_1, q)$ when $(1-k)\, r_1 = L$. When $(1-k)\, r_1$ falls below L, the expression for the pro-rata share in the LHS of condition (19) increases for any n when $(1-k)\, r_1$ falls below L. This implies that (19) is still satisfied for $\theta = \underline{\theta}\,(r_1, q)$, that is $\underline{\theta}\,(r_1, q)$ is still the relevant run threshold when $(1-k)\, r_1 < L$.

Consider now the case where $(1-k)\, r_1 > L$. Since (17) is negative when $\theta = \underline{\theta}\,(r_1, q)$, it follows that $\theta^*\,(r_1, r_2) > \underline{\theta}\,(r_1)$ when $(1-k)\, r_1 > L$.

To complete the proof, we characterize the effect that r_1, r_2, q and k have on the run thresholds. Consider first the case where $(1-k)\, r_1 \leq L$, so that the relevant run threshold is $\underline{\theta}$. It is easy to see that

$$\frac{\partial \underline{\theta}}{\partial r_1} = \frac{(1-k)}{qR} = \frac{\underline{\theta}}{r_1} > 0,$$

and

$$\frac{\partial \underline{\theta}}{\partial q} = -\frac{(1-k)\, r_1}{q^2 R} = -\frac{\underline{\theta}}{q} < 0.$$

Consider now the case where $(1-k)\, r_1 > L$, so that the relevant run threshold is θ^* as given by the solution to (4). We use the implicit function theorem to compute $\frac{\partial\theta^*}{\partial r_1}$, $\frac{\partial\theta^*}{\partial r_2}$ and $\frac{\partial\theta^*}{\partial q}$ and obtain:

$$\frac{\partial\theta^*}{\partial r_1} = -\frac{-\int_{\widehat{n}(\theta^*)}^{\overline{n}} \frac{qR\theta^* n\frac{(1-k)}{L}}{(1-k)(1-n)}dn - \frac{\partial\pi_1}{\partial r_1}}{\int_{\widehat{n}(\theta^*)}^{\overline{n}} q\frac{R\left(1-\frac{n(1-k)r_1}{L}\right)}{(1-k)(1-n)}dn} = \frac{\int_{\widehat{n}(\theta^*)}^{\overline{n}} \frac{qR\theta^* n\frac{(1-k)}{L}}{(1-k)(1-n)}dn + \frac{\partial\pi_1}{\partial r_1}}{\int_{\widehat{n}(\theta^*)}^{\overline{n}} q\frac{R\left(1-\frac{n(1-k)r_1}{L}\right)}{(1-k)(1-n)}dn} > 0,$$

$$\frac{\partial\theta^*}{\partial r_2} = -\frac{\int_0^{\widehat{n}(\theta^*)} qdn}{\int_{\widehat{n}(\theta^*)}^{\overline{n}} q\frac{R\left(1-\frac{n(1-k)r_1}{L}\right)}{(1-k)(1-n)}dn} < 0,$$

and

$$\frac{\partial\theta^*}{\partial q} = -\frac{\int_0^{\widehat{n}(\theta^*)} r_2 d\theta + \int_{\widehat{n}(\theta^*)}^{\overline{n}} \frac{R\theta^*\left(1-n\frac{(1-k)r_1}{L}\right)}{(1-k)(1-n)}dn}{\int_{\widehat{n}(\theta^*)}^{\overline{n}} q\frac{R\left(1-\frac{n(1-k)r_1}{L}\right)}{(1-k)(1-n)}dn} < 0,$$

since the derivatives of the extreme of the integrals cancel out and $\frac{\partial\pi_1}{\partial r_1} = \int_0^{\overline{n}} dn > 0$. Hence, the proposition follows. □

Proof of Proposition 2: The proof is straightforward. The two expressions in the proposition are obtained by differentiating (6) with respect to q after evaluating it for the case $(1-k)\,r_1 \leq L$ and $(1-k)\,r_1 > L$, respectively. □

Proof of Proposition 1: The proof proceeds in steps. We first show that for a fixed q, it is optimal for a bank to choose r_1 not to be exposed to panic runs. Then, we move on to the case where q is endogenous. Suppose $(1-k)\,r_1 > L$, so that the bank's payoff is

$$\int_0^{\max\{\theta^*,\theta^B\}} 0d\theta + q\int_{\max\{\theta^*,\theta^B\}}^{1} \left[R\theta - (1-k)\,r_2\right]d\theta - \frac{cq^2}{2} - \rho k$$

and for depositors either

$$U_1 = \int_0^{\theta^*} \frac{L}{1-k}d\theta + \int_{\theta^*}^{\theta^B} q\frac{R\theta}{(1-k)}d\theta + \int_{\theta^B}^{1} qr_2 d\theta - u$$

when $\theta^* < \theta^B$ or

$$U_2 = \int_0^{\theta^*} \frac{L}{1-k}d\theta + \int_{\theta^*}^{1} qr_2 d\theta - u,$$

when $\theta^* > \theta^B$.

Recall that $\frac{\partial\theta^*}{\partial r_1} > 0$, while θ^B does not depend on r_1. Then, it is easy to see that bank's profits are weakly decreasing in r_1, which implies that the bank always has an incentive to reduce r_1 below the value that triggers panic runs, i.e., $(1-k)\, r_1 = L$.

Let's now move on to see how a change in r_1 affects depositors. We start with the case where $\theta^* > \theta^B$. Since $(1-k)\, r_1 > L \Leftrightarrow r_1 > \frac{L}{1-k}$, and $qr_2 \geq r_1 > \frac{L}{1-k}$ must hold for the deposit contract to be incentive compatible and $\theta^* < \overline{\theta}$, then U_2 will increase if r_1 decreases since less weight will be put on the term with $\frac{L}{1-k}$, and more weight on the term with qr_2. Therefore, depositors are better off with lower r_1.

Consider now the opposite case where $\theta^* < \theta^B$. For this case, we compare $\frac{L}{1-k}$ to $q\frac{R\theta}{(1-k)}$, or more simply L to $qR\theta$. Since $\theta^* \geq \underline{\theta} = \frac{(1-k)r_1}{qR}$, and we are in the region where $\theta \geq \theta^*$, this immediately gives us that $L < qR\theta$ for $\theta \geq \theta^*$. Hence, reducing r_1, which reduces θ^*, increases U_1, and is hence again good for depositors. As a result, if $(1-k)\, r_1 > L$ so that there are panic runs, depositors would always be better off if r_1 were to be reduced. Since this is also beneficial for the bank, when q is exogenous, the bank will never choose r_1 such that there could be panic runs.

Consider now the case where q is endogenous and is given by the solution to (10) and (11), depending on whether $(1-k)\, r_1 \gtreqless L$. Suppose that $r_1^P > \frac{L}{1-k}$, and that at the bank's optimal choice of q, $q^*\left(r_1^P|k\right)$, there exists an $r_2^P \geq r_1^P$ that satisfies depositors' participation constraint, i.e., the deposit contract $\left\{r_1^P, r_2^P\right\}$ is a feasible contract. Define also the contract $r^M = \left\{r_1^M, r_2^M\right\}$ where $r_1^M = \frac{L}{1-k}$ and r_2^M is obtained from satisfying depositors' participation constraint with equality, for $q = q^*\left(r_1^M|k\right)$. We can interpret this as the "maximal" contract that allows only fundamental runs, and note that $r_1^M < r_1^P$.

Suppose now that the bank offers the contract $\left\{r_1^M, r_2^M\right\}$ rather than $\left\{r_1^P, r_2^P\right\}$, which implies it will choose $q^*\left(r_1^M|k\right)$ rather than $q^*\left(r_1^P|k\right)$ and depositors' participation constraint will be satisfied. We want to show that $\Pi^M = \Pi\left(r_1^M, r_2^M, q^*\left(r_1^M|k\right)|k\right) > \Pi\left(r_1^P, r_2^P, q^*\left(r_1^P|k\right)|k\right) = \Pi^P$. To see why, consider an arbitrary q, and notice that $\Pi\left(r_1^M, r_2^M, q\right) > \Pi\left(r_1^P, r_2^P, q\right)$ by the argument above for fixed q (for ease of notation, we leave out the conditioning on k in the bank's profits). Now let $q = q^*\left(r_1^P|k\right)$, and note that as a result $\Pi\left(r_1^M, r_2^M, q^*\left(r_1^P|k\right)\right) > \Pi\left(r_1^P, r_2^P, q^*\left(r_1^P|k\right)\right)$. But since $\Pi\left(r_1^M, r_2^M, q^*\left(r_1^M|k\right)\right) \geq \Pi\left(r_1^M, r_2^M, q^*\left(r_1^P|k\right)\right)$ by optimality of q^*, this immediately implies $\Pi^M > \Pi^P$, as desired. Hence, the bank never wants to offer a contract where $r_1 > \frac{L}{1-k}$, or in other words where depositor panics may arise. $\square$

Proof of Proposition 3: Based on the result of Proposition 1, the bank is only exposed to fundamental runs so that its maximization problem simplifies to

$$\max_{r_1,r_2} \Pi = \int_0^{\underline{\theta}} qR\theta \left(1 - \frac{(1-k)\, r_1}{L}\right) d\theta + \int_{\theta^B}^{1} q\left[R\theta - (1-k)\, r_2\right] d\theta - \frac{cq^2}{2} \tag{20}$$

subject to

$$\int_0^{\underline{\theta}} r_1 d\theta + \int_{\underline{\theta}}^{\theta^B} q\frac{R\theta}{1-k} d\theta + \int_{\theta^B}^{1} q r_2 d\theta \geq u. \tag{21}$$

The choice of r_2 is straightforward since profits in (20) are strictly decreasing in r_2. This implies that the bank chooses the lowest r_2 consistent with the depositors participating. In other words, r_2 corresponds to the solution to the binding depositors' participation constraint.

Consider now the choice of r_1. From (21) holding with equality, we can obtain an expression for $-\int_{\theta^B}^{1} q\,(1-k)\, r_2 d\theta$ and substitute it into bank's profits, thus obtaining:

$$\max_{r_1} \Pi = \int_0^{\underline{\theta}} qR\theta \left(1 - \frac{(1-k)\, r_1}{L}\right) d\theta + \int_0^{\underline{\theta}} (1-k)\, r_1 d\theta + \int_{\underline{\theta}}^{1} qR\theta d\theta - (1-k)\, u - k\rho - c\frac{q^2}{2}, \tag{22}$$

with q being the solution to (10). Differentiating (22) with respect to r_1, we obtain:

$$\begin{aligned}
& -\int_0^{\underline{\theta}} qR\theta \frac{(1-k)}{L} d\theta + \int_0^{\underline{\theta}} (1-k)\, d\theta + \frac{\underline{\theta}}{r_1}\left[qR\underline{\theta}\left(1 - \frac{(1-k)\, r_1}{L}\right) + (1-k)\, r_1 - qR\underline{\theta}\right] \\
& + \frac{dq}{dr_1}\left[\int_0^{\underline{\theta}} R\theta\left(1 - \frac{(1-k)\, r_1}{L}\right) d\theta + \int_0^{\underline{\theta}} (1-k)\, r_1 d\theta + \int_{\underline{\theta}}^{1} R\theta d\theta \right. \\
& \left. + \frac{\partial\underline{\theta}}{\partial q} qR\underline{\theta}\left(1 - \frac{(1-k)\, r_1}{L}\right) + \frac{\partial\underline{\theta}}{\partial q}(1-k)\, r_1 - \frac{\partial\underline{\theta}}{\partial q} qR\underline{\theta} - cq\right] = 0.
\end{aligned} \tag{23}$$

Evaluating the (23) at r_2 being the solution to the depositors' participation constraint, we obtain:

$$\begin{aligned}
& -\int_0^{\underline{\theta}} qR\theta \frac{(1-k)}{L} d\theta + \int_0^{\underline{\theta}} (1-k)\, d\theta + \underline{\theta}\,(1-k)\left[-\frac{qR\underline{\theta}}{L} + 1\right] \\
& + \frac{dq}{dr_1}\left[\frac{\partial\underline{\theta}}{\partial q}(1-k)\, r_1 - \frac{\partial\underline{\theta}}{\partial q} qR\underline{\theta} + \frac{u\,(1-k)}{q} + \int_0^{\underline{\theta}} \frac{(1-k)\, r_1}{q} d\theta\right] \\
& \Longleftrightarrow \\
& -\frac{1}{2} qR\left(\underline{\theta}\right)^2 \frac{(1-k)}{L} + \underline{\theta}\,(1-k) + \underline{\theta}\frac{(1-k)}{L}\left[-(1-k)\, r_1 + L\right] \\
& + \frac{dq}{dr_1}\left[-\frac{\underline{\theta}}{q}(1-k)\, r_1 - \frac{\partial\underline{\theta}}{\partial q} qR\underline{\theta} + \frac{u\,(1-k)}{q} + \frac{\underline{\theta}\,(1-k)\, r_1}{q}\right] \\
& \Longleftrightarrow \frac{\underline{\theta}\,(1-k)}{2}\left[-\frac{(1-k) r_1}{L} + 1\right] + \underline{\theta}\frac{(1-k)}{L}\left[-(1-k)\, r_1 + L\right] + \frac{dq}{dr_1}\left[R\left(\underline{\theta}\right)^2 + \frac{u\,(1-k)}{q}\right]
\end{aligned}$$

It is easy to see that since $L > (1-k)\, r_1$, all terms in the expression above are positive besides $\frac{dq}{dr_1}\left[R\left(\underline{\theta}\right)^2 + \frac{u(1-k)}{q}\right]$, whose sign we are going to characterize below. The bracket $\left[R\left(\underline{\theta}\right)^2 + \frac{u(1-k)}{q}\right] > 0$, so we only need to establish the sign of $\frac{dq}{dr_1}$.

To compute $\frac{dq}{dr_1}$, we start from the expression in (10), which can be simplified it to

$$\frac{1}{2}R\left(\underline{\theta}\right)^2\left(1-\frac{(1-k)\,r_1}{L}\right)+\int_{\theta_B}^{1}\left[R\theta-(1-k)\,r_2\right]d\theta-cq-R\left(\underline{\theta}\right)^2\left(1-\frac{(1-k)\,r_1}{L}\right)=0.$$

Furthermore, using the fact that r_2 comes from the participation constraint holding with equality, we can rearrange the expression above as follows:

$$-\frac{1}{2}R\left(\underline{\theta}\right)^2\left(1-\frac{(1-k)\,r_1}{L}\right)+\int_{\underline{\theta}}^{1}R\theta d\theta-cq+\underline{\theta}\frac{(1-k)\,r_1}{q}-\frac{u\,(1-k)}{q}=0. \tag{24}$$

Denoting the LHS in (24) as FOC_q, we use the implicit function theorem to compute

$$\frac{dq}{dr_1}=-\frac{\frac{\partial FOC_q}{\partial r_1}}{\frac{\partial FOC_q}{\partial q}}.$$

The denominator is negative when q is an interior solution. Hence, the sign of $\frac{dq}{dr_1}$ is equal to the sign of $\frac{\partial FOC_q}{\partial r_1}$, which is given by

$$\begin{aligned}
&-\frac{\partial\underline{\theta}}{\partial r_1}R\underline{\theta}\left(1-\frac{(1-k)\,r_1}{L}\right)+\frac{1}{2}R\left(\underline{\theta}\right)^2\frac{(1-k)}{L}-\frac{\partial\underline{\theta}}{\partial r_1}R\underline{\theta}+\frac{\partial\underline{\theta}}{\partial r_1}\frac{(1-k)\,r_1}{q}+\underline{\theta}\frac{(1-k)}{q}\\
&=-R\frac{\left(\underline{\theta}\right)^2}{r_1}\left(1-\frac{(1-k)\,r_1}{L}\right)+\frac{1}{2}R\left(\underline{\theta}\right)^2\frac{(1-k)}{L}-R\frac{\left(\underline{\theta}\right)^2}{r_1}+\underline{\theta}\frac{(1-k)}{q}+\underline{\theta}\frac{(1-k)}{q}\\
&=-R\frac{\left(\underline{\theta}\right)^2}{r_1}+R\left(\underline{\theta}\right)^2\frac{(1-k)}{L}+\frac{1}{2}R\left(\underline{\theta}\right)^2\frac{(1-k)}{L}-R\frac{\left(\underline{\theta}\right)^2}{r_1}+\underline{\theta}\frac{(1-k)}{q}\\
&=-R\frac{\left(\underline{\theta}\right)^2}{r_1}+\underline{\theta}\frac{(1-k)}{q}+\frac{3}{2}R\left(\underline{\theta}\right)^2\frac{(1-k)}{L}=-\underline{\theta}\frac{(1-k)}{q}+\underline{\theta}\frac{(1-k)}{q}+\frac{3}{2}R\left(\underline{\theta}\right)^2\frac{(1-k)}{L}\\
&=\frac{3}{2}R\left(\underline{\theta}\right)^2\frac{(1-k)}{L}>0.
\end{aligned}$$

Hence, it follows that (23) is positive and the bank would like to set r_1 as high as possible conditional with avoiding panic runs, i.e., $r_1=\frac{L}{1-k}$.

It is easy to see that r_1 increases with k, with $\lim_{k\to1}r_1=+\infty$, while $r_1=L<1$ when $k=0$. This means that there is a threshold level of k, which we denote as $\widehat{k}$, such that $r_1=\frac{L}{1-k}=u$ at $k=\widehat{k}$, i.e., $\widehat{k}=1-\frac{L}{u}$. When $r_1\geq u$, depositors' participation constraint can only hold with equality if $qr_2\leq u$. However, $qr_2<u$ violates $qr_2>r_1$. It follows that for any $k\geq\widehat{k}$, $r_1=qr_2=u$. The expression (12) in the proposition is obtained evaluating (21) at $r_1=\frac{L}{1-k}$. $\square$

Proof of Proposition 4: We consider separately the case in which $k<\widehat{k}$ so that $r_1=\frac{L}{1-k}$ and $k\geq\widehat{k}$, which implies that $r_1=qr_2=u$. We start from the former. Based on the result of

Proposition 3, we can rewrite bank's expected profits as

$$\Pi = \int_{0}^{\frac{L}{Rq}} L d\theta + \int_{\frac{L}{Rq}}^{1} qR\theta d\theta - (1-k)\, u - k\rho. \tag{25}$$

Differentiating (25) with respect to k, we obtain:

$$\left(\int_{\frac{L}{Rq(k)}}^{\theta^B} R\theta d\theta + \int_{\theta^B}^{1} R\theta d\theta - cq\left(k\right)\right) \frac{dq\left(k\right)}{dk} + u - \rho. \tag{26}$$

Using the FOC for q, as given in (10), evaluated at $r_1 = \frac{L}{1-k}$, we have

$$\int_{\theta^B}^{1} R\theta d\theta - cq = \int_{\theta^B}^{1} \left(1-k\right) r_2 d\theta. \tag{27}$$

Substituting the expression above in (26), we obtain the expression

$$\left(\int_{\frac{L}{Rq(k)}}^{\theta^B} R\theta d\theta\right) \frac{dq\left(k\right)}{dk} + u - \rho = 0.$$

It is easy to see that the term in the bracket is positive for any $k \in [0, \widehat{k})$. Hence, to show that $k > 0$, we need to show that $\frac{dq(k)}{dk} > 0$ for all k.

Denote as FOC_q the expression in (27), where r_2 corresponds to the solution to the depositor's participation constraint and it is, thus, a function of q. Using the implicit function theorem, we have that

$$\frac{dq\left(k\right)}{dk} = -\frac{\frac{\partial FOC_q}{\partial k}}{\frac{\partial FOC_q}{\partial q}}.$$

Since the solution for q is an interior and so $\frac{\partial FOC_q}{\partial q} < 0$, the sign of $\frac{dq}{dk}$ is equal to the sign of $\frac{dFOC_q}{dk}$, which corresponds to the following expression

$$\begin{aligned}
&-\left[\frac{\partial\theta^B}{\partial k} + \frac{\partial\theta^B}{\partial r_2}\frac{dr_2}{dk}\right] R\theta^B + \left[\frac{\partial\theta^B}{\partial k} + \frac{\partial\theta^B}{\partial r_2}\frac{dr_2}{dk}\right]\left(1-k\right) r_2 + \int_{\theta^B}^{1}\left[r_2 + \frac{dr_2}{dk}\left(1-k\right)\right] d\theta \\
&= -\left[\frac{\partial\theta^B}{\partial k} + \frac{\partial\theta^B}{\partial r_2}\frac{dr_2}{dk}\right]\left[R\theta^B - \left(1-k\right) r_2\right] + \int_{\theta^B}^{1}\left[r_2 + \frac{dr_2}{dk}\left(1-k\right)\right] d\theta \\
&= \int_{\theta^B}^{1}\left[r_2 + \frac{dr_2}{dk}\left(1-k\right)\right] d\theta,
\end{aligned}$$

given that, from the definition of θ^B, we know that $R\theta^B - \left(1-k\right) r_2 = 0$. To establish the sign of the expression above, we need to establish the sign of the bracket $\left[r_2 + \frac{dr_2}{dk}\left(1-k\right)\right]$. Using the implicit function theorem on depositor's participation constraint, we can compute

$$\frac{dr_2}{dk} = -\frac{\frac{\partial \underline{\theta}}{\partial k}\left[r_1 - q\frac{R\underline{\theta}}{1-k}\right] + \frac{\partial \theta^B}{\partial k} q \left[\frac{R\theta^B}{1-k} - r_2\right] + \int_{\underline{\theta}}^{\theta^B} q\frac{R\theta}{(1-k)^2}d\theta}{\frac{\partial \theta^B}{\partial r_2} q\left[\frac{R\theta^B}{1-k} - r_2\right] + \int_{\theta^B}^{1} q d\theta}$$

$$= -\frac{\int_{\underline{\theta}}^{\theta^B} q\frac{R\theta}{(1-k)^2}d\theta}{\int_{\theta^B}^{1} q d\theta} < 0.$$

Hence,

$$r_2 + \frac{dr_2}{dk}(1-k) = r_2 - \frac{\int_{\underline{\theta}}^{\theta^B} q\frac{R\theta}{(1-k)}d\theta}{\int_{\theta^B}^{1} q d\theta} = \int_{\theta^B}^{1} q r_2 d\theta - \int_{\underline{\theta}}^{\theta^B} q\frac{R\theta}{(1-k)}d\theta$$

$$= q\left[\int_{\theta^B}^{1}(1-k) r_2 d\theta - \int_{\underline{\theta}}^{\theta^B} R\theta d\theta\right] > 0,$$

which implies that $\frac{dq(k)}{dk} > 0$ for all k including $k = 0$ and, as a result, $k > 0$ follows.

Consider now the interval $k \in [\widehat{k}, 1]$. In this range, $r_1 = u = qr_2$. We follows the same steps as above. Bank's expected profits are then equal to:

$$\Pi = \int_0^{\frac{(1-k)u}{qR}} qR\theta\left(1 - \frac{(1-k)u}{L}\right)d\theta + \int_{\frac{(1-k)u}{qR}}^{1} qR\theta d\theta - (1-k)\int_{\frac{(1-k)u}{qR}}^{1} u d\theta - \frac{cq(k)^2}{2} - \rho k,$$

since when $r_1 = qr_2 = u$, $\theta^B = \underline{\theta}$.

Differentiating the expression above with respect to k, we obtain:

$$\int_0^{\frac{(1-k)u}{qR}} qR\theta\frac{u}{L}d\theta + \int_{\frac{(1-k)u}{qR}}^{1} u d\theta - \frac{u}{qR}\left[qR\frac{(1-k)u}{qR}\left(1 - \frac{(1-k)u}{L}\right) - qR\frac{(1-k)u}{qR} + (1-k)u\right] \tag{28}$$

$$+\left[\int_0^{\frac{(1-k)u}{qR}} R\theta\left(1 - \frac{(1-k)u}{L}\right)d\theta + \int_{\frac{(1-k)u}{qR}}^{1} R\theta d\theta - cq(k)\right.$$
$$\left.-\frac{(1-k)u}{q^2R}qR\frac{(1-k)u}{qR}\left(1 - \frac{(1-k)u}{L}\right) - qR\frac{(1-k)u}{qR} + (1-k)u\right]\frac{dq(k)}{dk} - \rho.$$

If we evaluate it at $k = 1$, the expression above simplify to

$$\left(\int_0^1 R\theta d\theta - cq(k)\right)\frac{dq(k)}{dk} = -\rho + u, \tag{29}$$

with $\frac{dq(k)}{dk}$ being equal to

$$\frac{dq(k)}{dk} = -\frac{-\frac{\partial \underline{\theta}}{\partial k}(1-k)u\left[\frac{1}{L} - 1\right] + \int_0^{\underline{\theta}} R\theta\frac{u}{L}d\theta + \int_{\underline{\theta}}^{1}\frac{u}{q}d\theta}{\frac{\partial \underline{\theta}}{\partial q}R\underline{\theta}\left(\frac{(1-k)u}{L}\right) + \frac{\partial \underline{\theta}}{\partial q}(1-k)u - c}$$

$$= \frac{\frac{u^2}{qR}(1-k)\left[\frac{1}{L} - 1\right] + \int_0^{\underline{\theta}} R\theta\frac{u}{L}d\theta + \int_{\underline{\theta}}^{1}\frac{u}{q}d\theta}{c - \frac{\partial \underline{\theta}}{\partial q}(1-k)u - \frac{\partial \underline{\theta}}{\partial q}R\underline{\theta}\left(\frac{(1-k)u}{L}\right)} > 0$$

for any c sufficiently large to insure that q is an interior solution.

Evaluating the expression for FOC for q in (10) at $k = 1$, we obtain

$$\int_0^1 R\theta d\theta = cq,$$

which implies that (29) simplifies to

$$-\rho + u < 0.$$

Hence, for any $\rho > u$, $k = 1$ is not a solution and the optimal level of capital k chosen by the bank falls in the range $(0, 1)$.

From the result in Proposition 3, we know that $r_1 = \min\left\{u, \frac{L}{1-k}\right\}$. Hence, it follows that for $k \in (0, 1)$, there exists a value of the liquidation value $\widehat{L} = (1-k) < 1$, such that for any $L \geq \widehat{L}$, $r_1 \geq 1$ and the proposition follows. □

Proof of Lemma 2: When $r_1 < \frac{L_L}{1-k}$, the bank has enough resources to fully repay depositors at date 1 so that panic runs do not occur and $\theta^R = \underline{\theta}$. As shown in Proposition 3, in this case the bank would find it optimal to set $r_1 = \min\left\{\frac{L_L}{1-k}, u\right\}$. Hence, given that $u > \frac{L_L}{1-k}$, $r_1 < \frac{L_L}{1-k}$ is not optimal. Consider now the case when $r_1 > \frac{L_H}{1-k}$. In this case, $r_1 > \frac{L_L}{1-k}$ also holds, which implies that the bank faces panic runs irrespective of the realization of $\widetilde{L}$. A minor extension of the result in Proposition 1 can be used to establish that a contract with $r_1 > \frac{L_H}{1-k}$ would be dominated by offering $r_1 = \frac{L_H}{1-k}$ instead. Hence, $r_1 > \frac{L_H}{1-k}$ cannot hold and the lemma follows. □

Proof of Proposition 5: The proof hinges on the result of Proposition 1 and Proposition 3 that establish that, for a given L, the bank always finds it optimal not to be exposed to panic runs and sets $r_1 = \frac{L}{1-k}$.

Given Lemma 2, the bank's maximization problem can be written as

$$\max_{r_1, r_2} p\left\{\int_0^{\max\{\theta^*(L_L),\theta^B\}} 0 d\theta + q\int_{\max\{\theta^*(L_L),\theta^B\}}^1 \left[R\theta - (1-k)\, r_2\right] d\theta\right\} \tag{30}$$
$$+ (1-p)\left\{q\int_0^{\underline{\theta}} R\theta\left(1 - \frac{r_1 (1-k)}{L_H}\right) d\theta + q\int_{\theta^B}^1 \left[R\theta - (1-k)\, r_2\right] d\theta\right\} - \frac{cq^2}{2} - \rho k$$

subject to

$$p\left\{\int_0^{\theta^*(L_L)} \frac{L_L}{1-k} d\theta + \int_{\theta^*(L_L)}^{\theta^B} q\frac{R\theta}{(1-k)} d\theta + \int_{\theta^B}^1 q r_2 d\theta\right\} + (1-p)\left\{\int_0^{\underline{\theta}} r_1 d\theta + \int_{\underline{\theta}}^{\theta^B} q\frac{R\theta}{(1-k)} d\theta + \int_{\theta^B}^1 q r_2 d\theta\right\}$$
$$-u \geq 0 \tag{31}$$

when $\theta^*(L_L) < \theta^B$ or

$$p\left\{\int_0^{\theta^*(L_L)} \frac{L_L}{1-k} d\theta + \int_{\theta^*(L_L)}^1 q r_2 d\theta\right\} + (1-p)\left\{\int_0^{\underline{\theta}} r_1 d\theta + \int_{\underline{\theta}}^{\theta^B} q \frac{R\theta}{(1-k)} d\theta + \int_{\theta^B}^1 q r_2 d\theta\right\} - u \geq 0 \tag{32}$$

when $\theta^*(L_L) > \theta^B$.

When $p = 0$, $\widetilde{L} = L_H$ with certainty and the bank chooses $r_1 = \min\left\{\frac{L_H}{1-k}, u\right\}$. When $p = 1$, $\widetilde{L} = L_L$ with certainty and so the bank chooses $r_1 = \min\left\{\frac{L_L}{1-k}, u\right\}$. Since bank profits in (30) are continuous in p, it is straightforward to see that for p sufficiently close to but strictly greater than 0, choosing $r_1 = \frac{L_H}{1-k}$ dominates choosing $r_1 = \frac{L_L}{1-k}$ for the bank, even if there may be some other intermediate value of r_1 strictly less than $\frac{L_H}{1-k}$ that does even better. As a result, for p low enough, the bank chooses $r_1 > \frac{L_L}{1-k}$ and is subject to panic runs with probability p. Hence, the proposition follows. □

Proof of Proposition 6: Using (14) and (15), we can compute:

$$\theta^B = \frac{(1-k)\, r_2}{R},$$

$$\left.\frac{dr_2}{dr_1}\right|_U = -\frac{\frac{\partial U}{\partial r_1}}{\frac{\partial U}{\partial r_2}} = -\frac{\int_0^{\theta^B} d\theta}{\int_{\theta^B}^1 q d\theta + \frac{\partial \theta^B}{\partial r_2}(r_1 - q r_2)} = -\frac{\int_0^{\theta^B} d\theta}{\int_{\theta^B}^1 q d\theta + \frac{\partial \theta^B}{\partial r_2}(r_1 - q r_2)}$$

and

$$\left.\frac{dr_2}{dr_1}\right|_\Pi = -\frac{\frac{\partial \Pi}{\partial r_1}}{\frac{\partial \Pi}{\partial r_2}} = -\frac{-\int_0^{\theta^B} qR\theta \frac{(1-k)}{L} d\theta}{\frac{\partial \theta^B}{\partial r_2} qR\theta^B\left(1 - \frac{(1-k)r_1}{L}\right) - \frac{\partial \theta^B}{\partial r_2} q\left(R\theta^B - (1-k)\, r_2\right) - \int_{\theta^B}^1 q(1-k)\, d\theta}.$$

Since $\theta^B = \frac{(1-k)r_2}{R}$, the second term in the denominator is zero, and this expression reduces to

$$\left.\frac{dr_2}{dr_1}\right|_\Pi = -\frac{\int_0^{\theta^B} R\theta \frac{(1-k)}{L} d\theta}{\int_{\theta^B}^1 (1-k)\, d\theta - \frac{\partial \theta^B}{\partial r_2} R\theta^B\left(1 - \frac{(1-k)r_1}{L}\right)}.$$

We can compare $\left.\frac{dr_2}{dr_1}\right|_U$ to $\left.\frac{dr_2}{dr_1}\right|_\Pi$, which gives:

$$\frac{\left(\theta^B\right)^2 R}{2L\left(1-\theta^B\right) - 2L\theta^B + 2\theta^B(1-k)\, r_1} < \frac{\theta^B R}{qR\left(1-\theta^B\right) - qR\theta^B + (1-k)r_1}$$

$$\Longleftrightarrow$$

$$qR\theta^B\left(1-\theta^B\right) - qR\left(\theta^B\right)^2 + \theta^B(1-k)r_1 < 2L\left(1-\theta^B\right) - 2L\theta^B + 2\theta^B(1-k)\, r_1$$

$$\Longleftrightarrow$$

$$\left[qR\theta^B - L\right]\left(1 - 2\theta^B\right) - \theta^B(1-k)\, r_1 < 0,$$

As $k \to 1$, $\theta^B \to 0$, thus giving $-L < 0$. Therefore, for large enough k, $\frac{dr_2}{dr_1}_U < \frac{dr_2}{dr_1}_\Pi < 0$, which establishes the result: The bank finds it optimal to increase r_1 as much as possible, while reducing r_2, but without crossing into the region where panic runs are possible. This maximum value is $r_1 = \frac{L}{1-k}$ as long as $\frac{L}{1-k} < u$ and becomes $r_1 = u$ otherwise using the same argument as in the proof of Proposition 3. Hence, the proposition follows. □

Proof of Proposition 7: The proof relies on the same arguments as the proof of Proposition 6 and compares the slope of the iso-profit and iso-utility curves, as given by $\frac{dr_2}{dr_1}_\Pi$ and $\frac{dr_2}{dr_1}_U$. We compute $\frac{dr_2}{dr_1}_\Pi$ and $\frac{dr_2}{dr_1}_U$ to obtain:

$$\frac{dr_2}{dr_1}_\Pi = -\frac{\int_0^{\theta^T}(1-k)\,d\theta + q\int_{\theta^T}^{\theta^L} R\theta\frac{(1-k)}{L}d\theta}{q\int_{\theta^L}^1 (1-k)\,d\theta} = -\frac{\int_0^{\theta^T} d\theta + q\int_{\theta^T}^{\theta^L} R\theta\frac{1}{L}d\theta}{q\int_{\theta^L}^1 d\theta} < 0,$$

and

$$\frac{dr_2}{dr_1}_U = -\frac{\int_0^{\theta^L} d\theta - \frac{\partial\theta^L}{\partial r_1}[qr_2 - r_1]}{\int_{\theta^L}^1 qd\theta - \frac{\partial\theta^L}{\partial r_2}[qr_2 - r_1]},$$

where $\frac{\partial\theta^L}{\partial r_1} = -\frac{r_2}{R}\frac{L}{r_1^2} = -\frac{\theta^L}{r_1} < 0$ and $\frac{\partial\theta^L}{\partial r_2} = \frac{1}{R}\frac{L}{r_1} = \frac{\theta^L}{r_2} > 0$.

We would like to show that $\frac{dr_2}{dr_1}_U < \frac{dr_2}{dr_1}_\Pi < 0$. To do that, note that the denominator of $\frac{dr_2}{dr_1}_U$, $\int_{\theta^L}^1 qd\theta - \frac{\partial\theta^L}{\partial r_2}[qr_2 - r_1]$, is not larger in absolute value than the denominator of $\frac{dr_2}{dr_1}_\Pi$, $\int_{\theta^L}^1 qd\theta$, because $\frac{\partial\theta^L}{\partial r_2}[qr_2 - r_1]$ is non-negative given $\frac{\partial\theta^L}{\partial r_2} > 0$ and $qr_2 \geq r_1$. Looking at the numerators, for $\frac{dr_2}{dr_1}_U$ we have that

$$\int_0^{\theta^L} d\theta - \frac{\partial\theta^L}{\partial r_1}[qr_2 - r_1] = \theta^L + \frac{\theta^L}{r_1}qr_2 - \theta^L = \frac{\theta^L}{r_1}qr_2 = \frac{r_2}{R}\frac{L}{r_1}\frac{qr_2}{r_1}.$$

For the bank, the numerator of $\frac{dr_2}{dr_1}_\Pi$ can be expressed as

$$\begin{aligned}\int_0^{\theta^T} d\theta + q\int_{\theta^T}^{\theta^L} R\theta\frac{1}{L}d\theta &= \theta^T + \frac{1}{2L}Rq\left(\left(\theta^L\right)^2 - \left(\theta^T\right)^2\right)\\ &= \frac{L}{qR} + \frac{1}{2L}Rq\left(\left(\frac{r_2}{R}\frac{L}{r_1}\right)^2 - \left(\frac{L}{qR}\right)^2\right) = \frac{1}{2}L\frac{q^2r_2^2 + r_1^2}{Rqr_1^2}.\end{aligned}$$

Comparing these two, we have that

$$\frac{r_2}{R}\frac{L}{r_1}\frac{qr_2}{r_1} > \frac{1}{2}L\frac{q^2r_2^2 + r_1^2}{Rqr_1^2} \Leftrightarrow \frac{qr_2^2}{r_1^2} > \frac{1}{2}\frac{q^2r_2^2 + r_1^2}{qr_1^2}.$$

Rewrite the right hand side as $\frac{1}{2}\frac{qr_2^2}{r_1^2} + \frac{1}{2q}$, so that the condition holds as long as

$$\frac{qr_2^2}{r_1^2} > \frac{1}{2}\frac{qr_2^2}{r_1^2} + \frac{1}{2q} \Leftrightarrow \left(\frac{qr_2}{r_1}\right)^2 \geq 1,$$

which is always true since $qr_2 \geq r_1$. Therefore, since the numerator of $\frac{dr_2}{dr_1}_U$ is greater or equal (when $qr_2 = r_1$) in absolute value than the numerator of $\frac{dr_2}{dr_1}_\Pi$, and the denominator is smaller or equal (when $qr_2 = r_1$) in absolute value, we have that $\frac{dr_2}{dr_1}_U < \frac{dr_2}{dr_1}_\Pi < 0$, as desired. The rest of the proof follows from the fact that bank profits are decreasing in r_2, so that r_2 comes from the solution of the binding participation constraint and by the fact that when $\frac{L}{1-k} > u$, $r_1 = qr_2 = u$ needs to hold for the depositors' participation constraint (16) and the incentives compatibility constraint $qr_2 \geq r_1$ to be satisfied. Hence, the proposition follows. $\square$