Limits of stress-test based bank regulation*

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Abstract

Supervisory risk assessment tools, such as stress-tests, provide complementary information about bank-specific risk exposures. Recent empirical evidence, however, underscores the potential inaccuracies inherent in such assessments. We develop a model to investigate the regulatory implications of these inaccuracies. In the absence of such tools, the regulator sets the same requirement across banks. Risk assessment tools provide a noisy signal about banks' types, and enable bank specific capital surcharges, which can improve welfare. Yet, a noisy assessment can distort banks' ex ante incentives and lead to riskier banks. The optimal surcharge is zero when assessment accuracy is below a certain threshold, and increases with accuracy otherwise.
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1 Introduction

The great financial crisis revealed that banks, especially large and complex banks, are opaque [Gorton, 2009]. As a result, regulators in many countries are now increasingly relying on supervisory risk assessment tools to learn about bank-specific risk exposures and capital adequacy. Such tools complement financial reporting and disclosures in informing regulators about banks' riskiness [Morgan et al., 2014] and allow them to better align the baseline capital requirements with individual banks' risk profiles.[1]

In principle, capital regulation based on supervisory risk assessment can improve welfare by making requirements bank specific. However, in practice, such welfare gains depend on the precision of risk assessment tools. Several studies have shown that these tools can provide noisy assessments of banks' risk exposures [Plosser and Santos, 2018; Acharya et al., 2014]. Despite empirical evidence of such imprecision, there is lack of a theoretical framework in the literature to study how capital requirements must be set when the risk assessment is inherently noisy. Our goal in this paper is to start filling this gap. We develop a tractable model to study how banks respond to capital requirements that are based on potentially noisy signals about banks' riskiness, and derive the policy and welfare implications. We use the model to characterise the optimal requirements, and to study the trade-offs a regulator faces in making such risk assessments more accurate or in disclosing the assessment results to investors.

The key players in our model are a banker and a regulator. The banker runs a bank that takes deposits and invests in a risky project. The return on the project can be high or low, depending on the bank's *type*, which in turn depends on the effort it exerts ex-ante. A mis-priced deposit insurance combined with limited liability induce the bank to over-

[1] In the U.S., capital surcharges (among other requirements) are determined on the basis of stress-test results. See https://www.govinfo.gov/content/pkg/FR-2020-03-18/pdf/2020-04838.pdf for more details. In the Euro Area, stress-tests conducted by the European Banking Authority (EBA) are a crucial input into the Supervisory Review and Evaluation Process (SREP) which entails capital planning, reporting, and governance requirements tailored to individual banks. See https://eba.europa.eu/eba-launches-2020-eu-wide-stress-test-exercise for more details.

borrow relative to the social optimal.[2] In turn, this rationalises a minimum capital ratio requirement in our model, and allows us to study the welfare implications of counterfactual policies.[3]

We assume that the regulator cannot observe the bank's type, which means it cannot impose bank-specific requirements. We then consider stress-tests as a regulatory (supervisory) tool that provides an imprecise signal about the bank's type.[4] The regulator imposes a capital surcharge on top of the baseline capital requirement based on its performance in the stress-test (which results in the bank being *deemed* as low or high type). In doing so, the regulator faces a trade-off. Risk-assessment helps overcome (some) information frictions and align regulatory requirements with individual banks' risk profiles. This improves welfare. Yet, inaccuracies can lead to inefficiently low or high requirements for some banks, and at the same time distort banks' ex-ante incentives (also discussed in Prescott [2004] in the context of supervisory audits), which lowers welfare. We use the model to assess this trade-off formally.

Our main contribution is to show that under information frictions higher capital requirements can induce banks to become more risky, and to derive the attendant relationship between optimal capital surcharge and stress-test accuracy. When the risk assessment is sufficiently noisy, it can lead to an inefficiently high or low capital requirement and lower welfare. Moreover, because stricter requirement imposes a higher opportunity cost to a high-type bank, it also adversely affects a bank's ex ante incentives to exert effort towards

[2]Typical reasons for a mis-priced deposit insurance include the inability of the insurer to observe banks' risk profiles or impose risk-sensitive premium payments. See Flannery et al. [2017] for elaboration.

[3]A large literature provides several rationales for capital-ratio requirements, such as fire-sale externalities [Kara and Ozsoy, 2020], implicit government guarantees [Nguyen, 2015], moral hazard issues [Christiano and Ikeda, 2016; Gertler and Kiyotaki, 2010], and household preference for safe and liquid assets [Begenau, 2020]. The approach in this paper is related to that of Kareken and Wallace [1978], Santos [2001], and Van den Heuvel [2008] who show that over-borrowing, led by mis-priced deposit insurance or otherwise, justifies capital regulation.

[4]Stress-tests are one of the several ways in which regulators can obtain a signal about specific characteristics of banks. There are, indeed, other micro-prudential and supervisory tools, such as onsite risk assessments, that may provide similar signals and thus be subject to similar trade-offs that we model in the context of stress-tests.

becoming a high-type bank. We show that the optimal surcharge in this case is zero. For intermediate levels of accuracy, we show that the optimal surcharge increases with accuracy, but is still smaller than what the full information benchmark would imply. In case of a sufficiently accurate stress-test, the surcharge has a strong disciplining effect in terms of eliciting greater ex-ante effort from banks, and accordingly the optimal surcharge is closer to the full information case.[5]

The trade-offs faced by the regulator become even more pronounced when the regulator jointly chooses the level of test accuracy and the optimal surcharge. We consider two cases. In the first case, the regulator can alter the test design by incurring a social cost (e.g. due to higher supervisory burden on both regulators and banks) and reduce one or both false positive and false negative rates, which together determine test accuracy. We show in this case that as the cost of improving test accuracy becomes smaller, the regulator optimally increases the surcharge.

In the second case, we assume that it is not possible to improve one error rate without worsening the other error rate. This is typically the case when the regulator cannot improve the design of the test (say because it is prohibitively costly to do so) and can only adjust the threshold below which a bank is considered to be a low type. In this case, we show that as the cost of reducing one error rate – say false positive – goes down, the regulator will optimally choose a lower false positive rate. However, unlike in the previous case, this may not support higher surcharges because a lower false positive rate is achieved at the expense of a higher false negative rate. As such, in this case, a dichotomy between test accuracy and the optimal surcharge arises.

In an extended version of the model, we study two additional policy trade-offs associated with stress-tests: disclosure of test results, and the role of failure costs associated

[5] Our paper formalises the intuition James Bullard (President of the Federal Reserve Bank of St. Louis) had in the context of quantitative easing: *while state-contingent policies are generally desirable, they work well when the states on which the policy is contingent are known.* See this article for a coverage of his remark. Relatedly, our paper supports the remarks made by Mark Zelmer (Deputy Superintendent, OSFI Canada) in 2013 in the context of risk-sensitivity of capital requirements.

with too-big-to-fail banks. We show that disclosure of stress-test results can worsen regulatory trade-offs outlined in the baseline model. When stress-tests are sufficiently accurate in identifying bank types, disclosures improve market discipline and facilitate the use of capital surcharges. Yet, when tests are less accurate, disclosures can amplify adverse incentives, induce greater risk-taking by banks, and thus place further limits on the optimal use of surcharges. The regulatory trade-offs are also aggravated when bank failures are more costly, such as in the case of too-big-to-fail banks. We show that in this case, not only is the optimal baseline capital requirement stricter, the optimal surcharge for a given level of accuracy is also higher.

To illustrate our analytical results, we calibrate the parameters of the model using data on twenty major economies. Numerical computations allow us to fully characterise the relationship between accuracy and optimal surcharge. Consistent with the theoretical predictions, numerical simulations show that the surcharge should be zero if the signal quality is below a threshold, and should increase non-linearly with accuracy otherwise.

We conclude our discussion by alluding to potentially noisy assessments in the 2020 Dodd-Frank Act Stress Test in the U.S. that was conducted right before the Covid-19 crisis. We show that the cross-sectional variance in changes in banks' CET1 ratios on the basis of the stress-testing exercise is much higher than the observed changes, and that the two do not correlate. This observation points to the potentially substantial noise in stress-tests as a signal of bank capital adequacy, and lends support to using a combination of risk-assessment approaches for regulatory purposes.[6]

[6] An ideal appraisal of stress-testing would be one where the hypothetical stress scenario and the realised crisis are *identical*, but bank outcomes are not. While the Covid-19 crisis is not *identical* to the severely adverse scenario of the stress-test, several of the key indicators that characterise a macroeconomic scenario (such as GDP, employment, and stock prices) experienced comparable declines in the 2020 US stress-test scenario and the Covid-19 crisis. As such, broad concordance in banks' relative performances in the test and in the current crisis is to be expected. See Section 5 for more details.

Related literature

Supervisory risk assessment is inherently noisy, and some inaccuracies are inevitable. Inaccuracies can stem from noisy bank-level inputs used in assessment models [Ong et al., 2010], limits of internal risk models of banks [Leitner and Yilmaz, 2019; Plosser and Santos, 2018; Behn et al., 2016; Wu and Zhao, 2016], or limits of econometric models used by the regulators to predict bank losses [Covas et al., 2014]. It could also be that these assessments do not fully take into account the endogenous reaction of banks to shocks [Braouezec and Wagalath, 2018].[7] For instance, Acharya et al. [2014] find that in the 2011 European stress-test, the assessment of banks' risk was not in line with their realized risk following the disclosure of test results.[8] Despite acknowledgement of the noise inherent in risk-assessment, the literature has not formally assessed the attendant welfare implications. Our paper contributes by developing a tractable theoretical framework to study these implications.

Specifically, we analyse what are the implications for the setting of capital requirements on the basis of risk-assessment signals that are noisy. This aspect has received less attention in a growing literature on other aspects of risk-assessment such as efficient information acquisition [Parlatore and Philippon, 2020], transparency [Leitner and Williams, 2020; Quigley and Walther, 2020], and disclosure [Goldstein and Sapra, 2014; Bouvard et al., 2015; Williams, 2017; Goldstein and Leitner, 2018; Orlov et al., 2018]. This is despite a recognition of the regulatory requirement aspect in policy discussions [Zelmer, 2013; Powell, 2019]. In this paper, we hope to make progress in filling this gap.

A paper that closely shares our pursuit is Ahnert et al. [2020] which shows that the

[7]Technical and computational glitches can also lead to noisy assessments. For example, in September 2020, the U.S. Federal Reserve Bank published corrections to its previously issued stress-test results [Fed, 2020].

[8]Philippon et al. [2017] find similar results for the 2014 stress-test conducted by the European Banking Authority. Relatedly, Frame et al. [2015] show that stress-tests conducted by the U.S. Office of Federal Housing Enterprise Oversight in the pre-GFC period failed to detect risks on the balance sheets of Fannie Mae and Freddie Mac. More generally, Berger et al. [2000] show that supervisory assessments are generally less accurate than market indicators in predicting banks' future performances.

sensitivity of regulation to banks' types must depend on the precision of the signal generated by the risk assessment tool. Specifically, the authors show that beyond a level of accuracy, risk sensitivity of capital regulation should decrease with signal precision. By contrast, we show that when risk assessment, an input to capital regulation, is more noisy, higher capital requirements can lead to adverse incentives. In turn, this rationalises a less tight regulation in our model. The difference in conclusions stems from the fact that in our analysis a bank can affect the probability that they face a capital surcharge – as a result, regulation directly affects banks' ex-ante incentives. Relatedly, Morrison and White [2005] show that when the regulator's screening ability (or audit reputation) is lower, capital regulation must be tighter to compensate. The difference from our paper is that in Morrison and White [2005] screening and regulation are substitutes, whereas in our analysis, screening complements (ie is an input for) regulation.

More broadly, studies on state-contingent regulation are related to our analysis of bank-type dependent regulation – this is because bank-type can be interpreted as a *state*. For instance, Marshall and Prescott [2001] show that state-contingent fines on banks can increase welfare, but assume that the states are observable, unlike in our analysis. Lohmann [1992] shows that when future states are not fully known, it is sub-optimal to commit to a state-contingent policy. By comparison, while we share this insight, we allow the policy maker to choose the *degree* of state-contingency, and we characterise its optimal value.

A key element of our analysis is the modeling of how banks respond to capital requirements. To be sure, several papers have analysed this question before. An early work is by Koehn and Santomero [1980] who show that tighter capital requirements can lead some banks (modeled as portfolio managers) to become even more risky. In follow-up research, Kim and Santomero [1988] as well as Rochet [1992] show that this result disappears when risk-weights used to compute capital requirements are consistent with asset quality. By comparison, our analysis is based on the idea that risk assessment is inherently noisy, as a result of which capital requirements can lead to adverse incentives. While our headline conclusion resonates with that of Prescott [2004] where poorly executed supervisory audits can create adverse incentives for banks because they can disclose information strategically, or Gale et al. [2010] where higher capital can force banks to take more risk to achieve the required rate of return, the underlying mechanism in our paper is distinct. We show that a moral hazard issue arises as the cost of tighter regulation is greater for a high-type bank – indeed, in the presence of misdirected requirements, this can diminish a bank's incentives to improve its risk-return profile.

2 Model

Our goal is to analyse the welfare and policy implications of regulatory assessment of banks' risks – which are inherently noisy – and attendant capital requirements. To this end, we develop a model with the following main elements. First is a general equilibrium setup that enables us to capture the welfare effect of regulation. Second is a dynamic setup that allows us to assess the effect of stress-test and regulation on banks' ex-ante behavior. Third is a rationale for capital regulation: an inefficiency that warrants regulatory intervention. Fourth is information frictions: the regulator does not observe a bank's type – which in turn justifies the use of risk-assessment tools.[9] Accordingly, we consider an economy that lasts three periods (0, 1, and 2), and consists of a representative household, a banker whose decisions are socially inefficient and whose type is stochastic, a regulator that cannot (fully) observe the bank's type, and a government that runs a deposit insurance program.

Household The household is representative, and receives an unconditional income endowment $\bar{Y}$ on dates 1 and 2. On date-1, it decides how much to consume, c_1, and how much to deposit, d, in the bank.[10] Deposits are risk-free, and pay a gross return of R on

[9] We do not distinguish between a regulator and a supervisor, and use the terms interchangeably.

[10] A time subscript is used only for those quantities that are relevant on multiple dates. For instance, since d is only chosen once, on date-1, a time subscript is omitted.

date-2.

Banker The banker has a capital endowment of k on date-1. It runs a bank that issues deposits d to invest $k + d$ in a risky project that pays $g(k + d)$ on date-2. $g(.)$ is a decreasing returns to scale (DRS) return function. is an investment shock whose density f_s depends on the banker's type s on date-1, which can be high (H) or low (L). Specifically, we assume that while both types face the same standard deviation of , namely σ, the high-type bank has a higher expected return, $\mu_H > \mu_L$, so that a high-type bank has a higher *risk-adjusted return.*The probability p with which the bank is of high-type depends on the effort e the banker exerts on date-0. The cost of exerting effort is $\zeta(e)$. The bank learns its type on date-1.

The bank's deposit liabilities on date-2 equal Rd, and thus the net cash-flow n equals $g(k + d) - Rd$. When is sufficiently high and the bank is solvent, the entire cash-flow is paid as dividends to the banker. However, when is low enough so that the cash-flow is negative, the bank fails and banker receives null. We assume that the banker only consumes on date-2, and that it has limited liability, so that it cannot be asked for additional capital to rescue a failing bank. Instead, the government takes the bank into receivership.

Government The government runs the deposit insurance scheme and ensures that depositors are fully protected against bank failure. When a bank fails, the government liquidates its assets, and covers any shortfall in the failed bank's liabilities. To fund the scheme, the government imposes a lumpsum tax T on the household. We assume that the insurance scheme is mis-priced – ie insensitive to the risks banks take – which, as we prove later, leads to a social inefficiency.[11] The government runs a balanced budget.

[11] The reason for introducing an inefficiency in our model is to rationalise capital requirements. A mis-priced deposit insurance is not the only way to do so, but it is a relatively simple method that helps keep our model tractable. Another paper to have taken this route is Van den Heuvel [2008].

Regulator The regulator is benevolent, i.e. it strives to maximise the joint welfare of the household and the banker. On date-0, it announces the minimum capital-ratio requirement χ that the bank must satisfy on date-1. However, we assume that the regulator cannot *observe* the bank's type on date-1.[12] As such, it must announce a requirement that does not depend on banks' type, i.e. applies universally to both types of banks on date-1. Next, on top this this baseline setup, we assume that the regulator possesses a tool that produces a noisy signal about the bank. For simplicity, we consider this tool to be a stress-test, but it could be other tools such as onsite examination. The signal is used to classify banks as high or low type depending on whether they pass or fail the test. The regulator then imposes a surcharge x on the bank deemed to be of the low type, effectively imposing a bank-type specific requirement $\chi_H = \chi$ and $\chi_L = \chi + x$.

Recursive formulation We now formally setup the problem statements of the agents in the economy. The household chooses d on date-1 to maximize its expected utility over dates 1 and 2, where   is the discount factor:

$$U = \max_{d} \quad c_1 + \quad \mathbb{E}c_2 \quad s.t. \; c_1 = \bar{Y} - d \; and \; c_2 = \bar{Y} + Rd - T. \tag{1}$$

The banker chooses e on date-0 which determines the probability of being an H-type on date-1:

$$[Date - 0]: \quad \max_{e} \quad -\zeta(e) + \quad \Big(p(e)V_H(\chi) + (1-p(e))V_L(\chi)\Big). \tag{2}$$

[12] In reality, regulators do have some knowledge about banks' characteristics (such as via regulatory filings). We assume that the observable characteristics are embedded in the return function $g(.)$ of the bank while *type* simply summarizes the unobservable characteristics. Furthermore, we assume that the bank cannot credibly communicate its type to the regulator, except via its performance in a stress-test.

where $V_s(\chi)$ is defined in equation (3). The bank of type $s \in \{H, L\}$ chooses d on date-1 to maximize the expected dividend it pays on date-2:

$$[Date - 1]: \quad V_s(\chi) = \max_{d} \int_{\frac{Rd}{g(k+d)}}^{\infty} \Big(\underbrace{g(k + d) - Rd}_{n} \Big) f_s(\psi) d\psi \quad s.t. \; \frac{k}{\chi} \geq d. \tag{3}$$

The lower limit on the integral is the cut-off – call it ψ_c – below which the bank fails (and no dividends are paid). χ is the minimum capital-ratio requirement.[13] The government's budget constraint is as follows:

$$T = \begin{cases} Rd - g(k + d) & \text{If the bank fails i.e. } \psi \leq \frac{Rd}{g(k+d)} \\ 0 & \text{Otherwise} \end{cases} \tag{4}$$

3 Qualitative Analysis

We begin by assessing the equilibrium conditions in the baseline economy. We then characterise – as a benchmark – the optimal regulation in the absence of stress tests. Finally, we analyse the optimal capital surcharge based on stress-test results, including when results are disclosed, and when bank failure is socially costly.

3.1 The baseline equilibrium

The first-order condition (FOC) of the bank's problem on date-0 shows that the effort the bank exerts depends on the *wedge*, say ω, between the value of being a high- as opposed to low-type on date-1:

$$-\zeta'(e) + p'(e) \underbrace{\Big(V_H(\chi) - V_L(\chi) \Big)}_{\omega} = 0 \tag{5}$$

[13] The requirement can vary across banks depending on their performance in the stress-test. This case is discussed later.

To see how the effort changes as the wedge increases, we take the total derivative of Equation 5 with respect to ω, from where it is straightforward to note Lemma 1:

$$-\zeta''(e)\frac{de}{d\omega} + p''(e)\omega\frac{de}{d\omega} + p'(e) = 0 \tag{6}$$

Lemma 1. *If $\zeta(.)$ is increasing and convex, and $p(.)$ is increasing and concave, then the bank exerts more effort when the difference in the value of being a high type compared to a low type increases, i.e. $de/d\omega > 0$.*[14]

Equation 5 provides the intuition for why effort would increase with the wedge ω. As the relative value of being a high-type bank increases, the marginal benefit of effort increases while the marginal cost is unaffected. Lemma 1 underscores that the minimum requirement (χ) affects the wedge ω by impacting the value of the bank on date-1. As such, the minimum requirement is a key factor in bank's effort choice on date-0, and will shape the regulator's choice of optimal ex ante capital requirement as we show later in Section 3.3.[15]

As regards the date-1 FOCs, we have the following:

$$\text{Bank:} \qquad \int_{\frac{Rd}{g(k+d)}}^{\infty} \Big(g\psi(k+d) - R \Big) f_s(\psi)\, d\psi - \Lambda_s = 0 \tag{7}$$

$$\text{Household:} \qquad R = 1/\beta \tag{8}$$

Note in the bank's FOC that Λ_s is the Lagrange multiplier on the regulatory constraint, and that two of the three terms which arise from a routine application of the Leibniz rule are equal to zero. The system of FOCs (5), (7), (8) and the government's budget constraint (4) together characterise the competitive equilibrium of the model economy for a given set

[14] The result does not depend on the concavity of the cost function; a linear $\zeta(.)$ would suffice. Yet, a decreasing marginal effect of effort is a realistic assumption to have.

[15] Lemma 1 is related to a similar result proven in Christiano and Ikeda [2016], but the channel through which regulation has an impact on the banker's effort is different.

of minimum capital-ratio requirements (χ_H, χ_L).

3.2 Optimal ex-post regulation

We now assess the efficiency of the equilibrium, and discuss the role that regulation could play in improving welfare. In this section, we focus on the date-1 economy, and turn to the date-0 economy (and the discussion of stress tests) in the next subsection.

Inefficiency of the equilibrium We compare allocations in an unregulated date-1 economy with a benevolent social planner's allocations. As such, we ignore for now the banker's date-0 problem, ie it's effort choice (and return to this consideration later). Without loss of generality, refer to a banker of type s, where s could be either high or low.

We consider a constrained social planner who maximizes the date-1 and date-2 equally weighted welfare of the household and the banker by choosing the level of deposit funding on behalf of the banker, taking as given the household's first order condition:

$$\max_{d} \quad c_1 + \quad \mathbb{E}(c_2 + n) \quad s.t. \; R = 1/\beta \quad c_1 = \bar{Y} - d; \quad c_2 = \bar{Y} + Rd - T$$

Recall that the banker does not consume on date-1, and note that $c_2 + n$ denotes the combined consumption of the household and the banker on date-2. Since the planner internalises the effect of choosing d on n and T we can solve for $c_2 + n$ using expressions for n and T from equations (3) and (4) respectively:

$$c_2 + n = \bar{Y} + \quad g(k + d) \tag{9}$$

Next, we rewrite the planner's objective after plugging in the expressions for c_1, c_2, n, rearranging terms using the household's FOC, and segregating the expectation (i.e. the integral on $c_2 + n$) at the cutoff for failure of the bank:

$$\max_{d} \quad (1+ \ )\bar{Y}\psi + \underbrace{\int_{\frac{Rd}{g(k+d)}}^{\infty} \Big(g(k\psi + d) - Rd\Big) f_s(\ ) d\psi}_{\text{Banker's date-1 objective}} + \int_{0}^{\frac{Rd}{g(k+d)}} \Big(\ g(k\psi + d) - Rd\Big) f_s(\ ) d\psi . \psi \tag{10}$$

By segregating the integral into two parts, the first part matches the bank's objective function, and thus facilitates a comparison of bank's and planner's FOCs, as shown below:

$$\int_{\frac{Rd}{g(k+d)}}^{\infty} \Big(g'(k\psi + d) - R\Big) f_s(\ ) d\psi + \underbrace{\int_{0}^{\frac{Rd}{g(k+d)}} \Big(g'(k\psi + d) - R\Big) f_s(\ ) d\psi}_{\text{Bank-failure inefficiency}} = 0 . \psi \tag{11}$$

Equation (11) uncovers a wedge between the planner's FOC and the bank's FOC in the unregulated economy (i.e. equation (7) with $\Lambda_s = 0$). This wedge stems from limited liability and a mis-priced deposit insurance. Because of limited liability, the bank does not internalise the losses corresponding to the left tail of the distribution of – the part that corresponds to bank failure. And because of deposit insurance, the depositors do not charge a premium for risk of non-repayment of deposit proceeds post bank failure. The bank thus over-borrows. The planner, on the contrary, chooses the level of deposits taking into account the entire distribution of . We refer to this wedge as the bank-failure inefficiency, which the following lemma characterises.

Lemma 2. *The bank's capital ratio, defined as k/d, is smaller in the competitive equilibrium as compared to that in the constrained planner's problem, i.e. the second best.*

Proof. Assume that the inefficiency term is positive. Then, $\int_{\frac{Rd}{g(k+d)}}^{\infty} \Big(\ g'(k+d) - R\Big) f_s(\ ) d$ must also be positive (since integral is increasing in). But this is a contradiction since the overall expression for the planner's FOC must equal zero (or both terms must be zero, which is trivial). As such, the inefficiency term must be negative. In turn, this implies that $\int_{\frac{Rd}{g(k+d)}}^{\infty} \Big(\ g'(k\psi + d) - R\Big) f_s(\ ) d\psi > 0$. We know that d^{CE} (the level of deposits in the competitive equilibrium) satisfies $\int_{\frac{Rd}{g(k+d)}}^{\infty} \Big(\ g'(k\psi + d) - R\Big) f_s(\ ) d\psi = 0$. But since $g(.)$ is concave, it must be that $d^{CE} > d^*$ where d^* solves the constrained planner's problem.[16] ■

Implementability of the constrained efficient allocation That the competitive equilibrium exhibits an inefficiency implies that $W^{CE}_\psi \leq W^*_\psi$ where W^{CE} is the welfare in the competitive equilibrium and W^* is the second-best welfare. The question that follows is whether a regulatory intervention can help implement or approach the second best.

To this end, we consider a benevolent regulator who sets a minimum capital-ratio requirement $k/d_\psi \geq \chi^\psi_s$ on the bank in order to maximize welfare. In choosing χ^ψ_s, the regulator faces the following trade-off. A higher χ^ψ_s forces the bank to reduce deposit-based funding and accordingly its failure probability, which has a welfare improving effect due to a smaller bank-failure inefficiency. Yet, a higher χ^ψ_s depresses expected output, which has a welfare reducing effect.

In effect, the regulator's decision problem is very similar to that of a constrained planner. This is because choosing deposits on behalf of the bank to maximise welfare is equivalent to imposing a minimum capital-ratio requirement with the same objective when capital is fixed and the requirement is binding. This is formally seen by comparing equations (7) and (11). Indeed, the first terms are identical. And to the extent the Lagrange multiplier Λ_s on (i.e. the shadow cost of) the regulatory constraint in (7) is equal to the absolute value of the bank-failure inefficiency term in (11), the solution to the two equations must be identical. We note this result in the lemma below, and denote the optimal regulation for an s-type bank by χ^o_s.

Lemma 3. *The solution to the constrained planner's problem can be implemented via a minimum capital-ratio requirement.*[17]

[16] The finding that the bank takes more leverage than what is socially optimal is not unique to this paper, nor is it our main contribution. Several other studies have related findings, such as Van den Heuvel [2008] and Christiano and Ikeda [2016], for instance. Our approach is to develop a relatively parsimonious model that has the mechanisms needed to study the welfare effects of stress-test based capital requirements.

[17] A capital-ratio requirement is not the only regulatory tool that can implement the second best. A

Before turning to the date-0 problem, we document a result that will be useful later. It compares the optimal date-1 regulation for high- and low-type banks. Assume that the regulator can perfectly observe bank type.

Lemma 4. *The regulator optimally sets a higher ex-post requirement on the low-type bank as compared to a high-type bank, i.e.* $\chi_L^o > \chi_H^o$.

Proof. Consider the non dis-aggregated version of the planner's date-1 FOC – i.e. equation (11) – for both high- and low-type banks. This characterises the optimal level of deposits in each case.

$$0 = \int_0^\infty \Big(\psi g'(k + d) - R \Big) f_s(\psi) d\psi = \mu_s g'(k + d) - R \quad s \in \{H, L\} \tag{12}$$

The total derivative of d with respect μ_s implies:

$$g'(k + d) + \mu_s g''(k + d) \frac{\partial d}{\partial \mu_s} = 0 \implies \frac{\partial d}{\partial \mu_s} > 0 \quad s \in \{H, L\} \tag{13}$$

This immediately implies that the optimal d is higher, or equivalently, the optimal χ^o is lower for a high-type bank. ■

Intuitively, for a given level of deposits, a low-type bank not only generates lower expected output, but is also more likely to fail. This underpins the stricter regulation for the low-type bank.

3.3 Optimal ex-ante regulation

The bank forms expectations and chooses its date-0 decisions based on date-1 requirements announced by the regulator on date-0.[18] However, because the bank's type on date-1 is

tax (or a deposit insurance premium) that is a function of the balance sheet choice of the bank may also achieve the same objective.

[18]We abstract away from time-inconsistency issues, and assume that regulatory announcements are credible.

private information, the regulator must adopt a uniform capital requirement – say χ – which is applicable on date-1 irrespective of the bank's type. To characterize the optimal χ, we begin with the following result.

Lemma 5. *Assume that regulation χ binds for both bank types on date-1.*[19] *Then the effort the bank chooses to exert on date-0 decreases as χ rises.*

Proof. As shown in Lemma 1, the bank's date-0 effort e depends on $\omega = V_H(\chi) - V_L(\chi)$, i.e. the wedge between the value of being a high- versus low-type on date-1. The key then to proving this lemma is to characterise how regulation impacts ω.

$$\omega = \int_{\frac{Rd}{g(k+d)}}^{\infty} \left(g(k + d) - Rd \right) f_H(\ ) d\psi - \int_{\frac{Rd}{g(k+d)}}^{\infty} \left(g(k + d) - Rd \right) f_L(\ ) d\psi$$

where $d = k/\chi$. The derivative of ω with respect to χ gives:

$$\frac{\partial\omega\psi}{\partial\chi} = -\frac{k\psi}{\chi^2} \left(\underbrace{\int_{\frac{Rd}{g(k+d)}}^{\infty} \left(g'(k + d) - R \right) f_H(\ ) d\psi}_{\Lambda_H} - \underbrace{\int_{\frac{Rd}{g(k+d)}}^{\infty} \left(g'(k + d) - R \right) f_L(\ ) d\psi}_{\Lambda_L} \right) \tag{14}$$

where Λ_s is the Lagrange multiplier on the regulatory constraint in the bank's problem.

To sign this expression, we proceed as follows. First note that since the (binding) regulatory requirement is the same for both types of bank, their deposit choices and thus the failure cutoffs $_c$ are also the same. Then let $\hat{F}_H$ and $\hat{F}_L$ be the distribution functions of for high- and low-type banks, truncated below at $_c$. Since $\mu_H > \mu_L$ (while the variances are the same), $\hat{F}_H$ $FOSD$ $\hat{F}_L$, that is $\hat{F}_H(\ ) \leq \hat{F}_L(\ )\ \forall$. Finally, since $(\ g'(k + d) - R)$ is an increasing function of , it follows that:

[19] A concave yet sufficiently close to linear asset return function $g(.)$ would ensure that regulation always binds. In practice, banks hold a so-called management buffer beyond the minimum requirement, but this can be included in our framework as a constant on top of the minimum requirement.

$$\int \left(\left(g(k + d) - R\right)\right) d\hat{F}_H(\ ) - \int \left(\left(g(k + d) - R\right)\right) d\hat{F}_L(\ ) = \Lambda_H - \Lambda_L > 0.$$ [20]

In turn, this implies that $\frac{\partial \omega}{\partial \chi} < 0$. Then from Lemma 1 we know that $\frac{\partial e}{\partial \omega} > 0$, which completes the proof since:

$$\frac{\partial e}{\partial \chi} = \frac{\partial e}{\partial \omega}\frac{\partial \omega}{\partial \chi} < 0.$$

■

Lemma 5 captures a key insight of this paper. Because a high-type bank's assets are more profitable, the opportunity cost of stricter capital requirements is greater for this bank. As such, an increase from a given level of requirement leads to a greater decline in the expected value of the high-type bank than the low type bank. This, in turn, lowers the returns to exerting more effort. In contrast to the conventional wisdom that more skin-in-the-game (via higher capital requirement) can induce banks to become safer, our finding is that under information frictions banks might respond to stricter regulation by becoming riskier.

This insight points to an important trade-off the regulator faces while setting χ. Compared to no regulation ($\chi = 0$), a higher χ can improve welfare *ex-post* by mitigating some of the inefficiency associated with the bank's choices, especially in case of a low-type bank. Yet, a higher χ can reduce welfare due to its adverse impact on effort exerted *ex-ante*.

Before characterising the optimal ex-ante regulation, we note that the assumption that regulation binds for both bank types is not critical for Lemma 5. The case where regulation binds for only one bank – which has to be the high type bank since it chooses a lower capital

[20] To prove this formally, consider continuous distribution functions G and H such that $\forall x, H(x) \leq G(x)$, and define $y(x) = H^{-1}(G(x))$. Then for any increasing function $w(x)$, $\int w(y(x))dH(y(x)) = \int w(y(x))dG(x)$. Next, note that $y(x) = H^{-1}(G(x)) \implies y(x) \geq x$ since $\forall x, H(x) \leq G(x)$. In turn, since $w(.)$ is an increasing function, $w(y(x)) \geq w(x)$. Thus, $\int w(y(x))dG(x) \geq \int w(x)dG(x)$. Indeed, intuitively, the *shadow cost* of the minimum capital-ratio constraint should be greater for a bank whose assets are *ceteris paribus* more profitable.

ratio in the unregulated economy – leads to the same result because in that case $\Lambda_L = 0$. The case where regulation does not bind for any bank is not relevant nor interesting because we already showed that an inefficiency rationalises *some* regulation.

Proposition 1. *The optimal ex-ante requirement χ^o in the case where the regulator cannot observe the bank's type, lies between by the optimal ex-post requirement for low- and high-type banks, ie $\chi^o_L \geq \chi^o \geq \chi^o_H$.*

Proof. The problem of a benevolent regulator on date-0 when it cannot impose bank-specific requirements, is as follows:

$$\max_{\chi} \quad p(e)U_H(\chi) + \quad (1 - p(e))U_L(\chi) - \zeta(e)$$

Here U_s is the household's and banker's combined expected lifetime consumption utilities when the banker turns out to be of type s, while $\zeta(e)$ accounts for the banker's effort on date-0. We will prove the proposition via the method of contradiction. Let χ^o solve the above problem. Then, if $\chi^o > \chi^o_L > \chi^o_H$, it means that the requirement is more strict than the optimal requirement for both bank types, and thus a lower χ^o would improve welfare in case of each bank type, as well as the total expected welfare. Similarly, if $\chi^o_L > \chi^o_H > \chi^o$, it means that the requirement is more liberal than the optimal requirement for both bank types, and thus a higher χ^o would improve total welfare. ■

Intuitively, this proposition shows that when there is information asymmetry, the regulator chooses a *middle-ground* relative to the optimal bank-type specific requirements.

3.4 Mitigating information frictions via supervisory assessment

Risk assessments produce information about banks' types. Mitigating some information frictions allows capital requirements to be better aligned to the banks' types. This is desirable as it can improve welfare.

We model risk assessment as a tool that produces a noisy signal to the regulator about the bank's type. Based on the test outcome, the regulator deems the bank to be of the low or high type (see Figure 1 for the timeline). We assume that the probability that a high-type (low-type) bank is deemed high is q_H (q_L).[21]

The accuracy of the stress-test is fully captured by the tuple (q_H, q_L). Any test can thus be represented by a point in the set $[0, 1] \times [0, 1]$, as shown in Figure 2. In this format, $(1 - q_H)$ denotes the 'false positive' or Type-I error rate (high-type bank fails the test), while q_L is the 'false negative' or Type-II error rate (low-type bank passes the test). A convenient benchmark, which is equivalent to the full-information case, is when $q_H = 1$ and $q_L = 0$, i.e. a *perfect* signal that exactly identifies the type of the bank. In all other cases, we refer to the test as *imperfect* because an H-type bank can fail the test ($q_H < 1$) or an L-type bank can pass the test ($q_L > 0$).

The regulator uses the outcome of the stress-test to adjust the baseline capital requirement χ^o. We assume that a bank that passes the stress test is deemed high-type and is allowed to operate at χ^o.[22] A failed bank is deemed to be of the low-type, and the regulator strives to align the capital-ratio requirement towards $\chi_L^o \geq \chi^o$ by imposing a surcharge $x \geq 0$.

For banks that pass the test, in line with stress-testing in practice, we do not allow a negative surcharge ie no capital relief.[23] Even if we were to allow for negative surcharges,

[21]The pass probabilities are rationalised as follows. We assume that the signal distribution, say Q_H, of high-type banks dominates (in the first order stochastic (FOSD) sense) the signal distribution Q_L of low-type banks. Depending on its preferences for true- and false- positive and negative rates, the supervisor uses a signal cutoff η^c above (below) which the bank is considered pass (fail) and is deemed to be of the high- (low-) type. Thus the probability that a high-type bank passes the test is given as $q_H = 1 - Q_H(\eta^c)$, and the same for a low-type bank is given as $q_L = 1 - Q_L(\eta^c)$. Moreover, $Q_H \succcurlyeq_{FOSD} Q_L \implies q_H > q_L$. Note that we do not model the regulator's preferences for Type-I and Type-II error rates that determine the signal cutoff η^c and thus the pass probabilities. This mapping is standard in the literature, and is typically based on the receiver operating characteristics (ROC) curve. In section 3.5 we assess the implications of a change in the signal cutoff for the optimal policy, and also the case where the regulator can incur a cost and improve the overall accuracy of the stress-test – i.e. increase the area under the ROC curve.

[22]Note that in practice, most jurisdictions no longer assign an official pass or fail grade to the banks. Yet, whether a bank has done well or not in absolute (i.e. pass or fail) terms can still be inferred from its capital ratio under stress and the minimum capital-ratio requirement in the economy.

[23]In the U.S., for instance, banks that perform poorly in the test (in terms of their losses under stress)

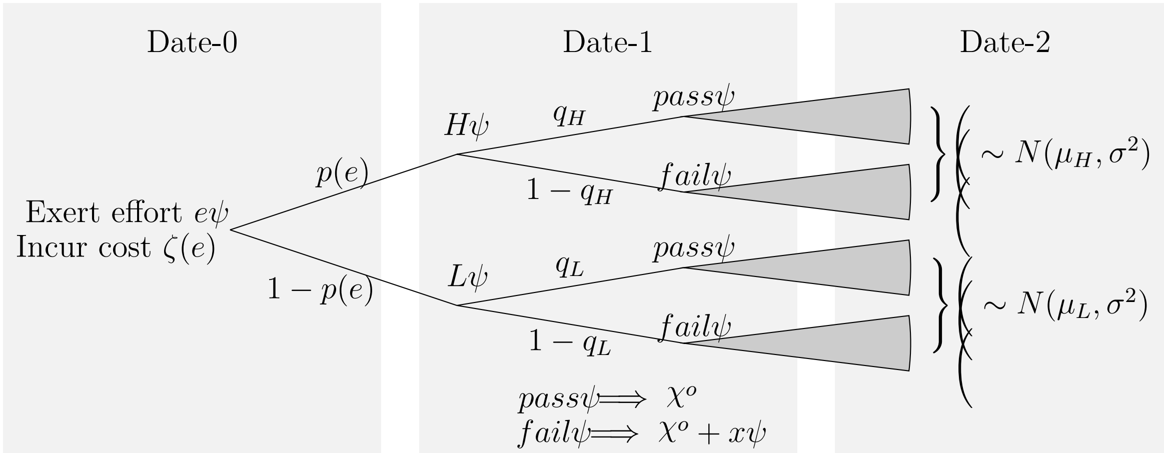


Figure 1: The timeline of events when there is information asymmetry about the bank's type, and stress tests serves as a tool to (partially) mitigate information frictions.

we do not expect our qualitative findings to change because negative surcharges would be subject to the same regulatory trade-offs as the positive surcharges as long as $q_H < 1$.[24]

The core question of interest then is as follows: what is the welfare maximising level of surcharge x that the regulator must announce on date-0. The choice of x is non-trivial, and is subject to a three-way trade-off.

1. In case of the low-type bank, the surcharge (upon failing the test) **increases** welfare *ceteris paribus* as long as $x \leq \chi_L^o - \chi^o$. This is because the surcharge brings the requirement $(\chi^o + x)$ closer to the optimal (χ_L^o).

2. In case of the high-type bank, the surcharge (upon failing the test) **decreases** welfare *ceteris paribus*. This is because $\chi^o + x > \chi^o \geq \chi_H^o$, as a result of which the surcharge takes the effective requirement away from the optimal.

3. The surcharge affects the wedge between the expected value of being high- versus low-type on date-1, and thus impacts the bank's behaviour on date-0. Depending on

must satisfy a higher capital surcharge, while those that do well are not given any relief and must continue to satisfy the baseline requirements.

[24] Alternatively, if χ^o was set to be equal to χ_H^o, the regulator would optimally never choose a negative surcharge, and yet face the same qualitative trade-offs as in our original setup. That is, Lemma 6 and Propositions 2 and 3, which together form the main results of the paper, would continue to hold.

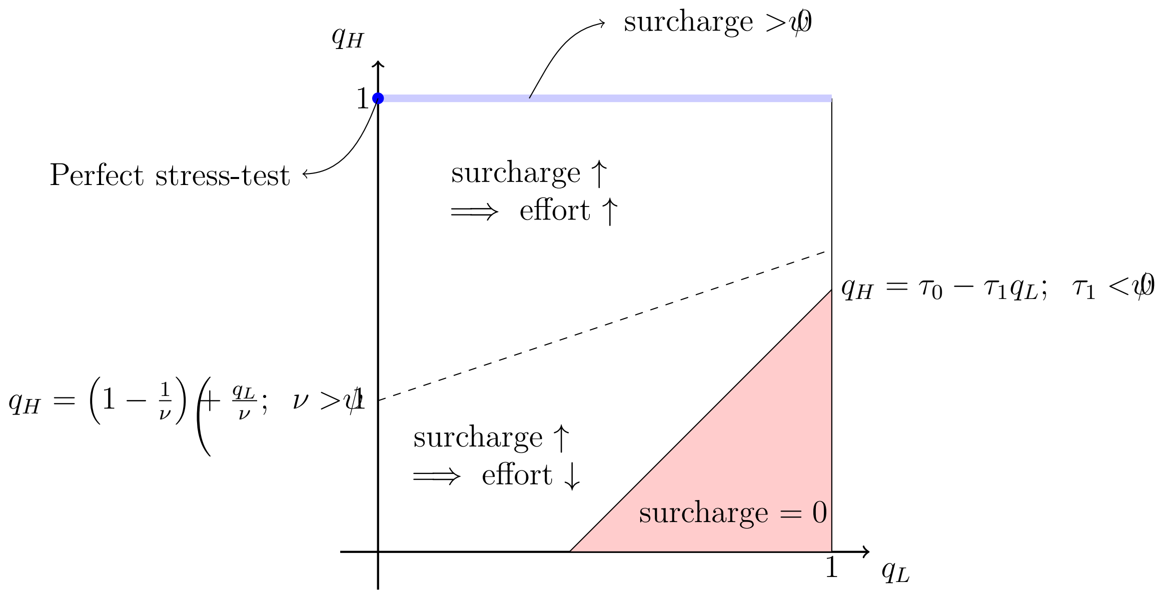


Figure 2: **Stress-test accuracy, effect on ex-ante effort, and optimal penalties:** Each point on the unit square characterises the accuracy of a stress-test. A higher q_H and a lower q_L indicate a more accurate test. The optimal surcharge is zero if accuracy is below the solid diagonal line (i.e. in the shaded area), and positive if $q_H = 1$. Effort increases with the surcharge above the dotted diagonal line, and decreases otherwise.

the accuracy of the stress test, this can lead to an increase or decrease in the bank's effort. We prove this result in Lemma 6 below. Accordingly, *ceteris paribus*, a higher surcharge can **increase or decrease** welfare through its effect on effort.

Lemma 6. *The bank's effort may increase or decrease with a surcharge, depending on the accuracy of the stress test.*

Proof. The date-0 problem of the bank is:

$$\max_e \quad -\zeta(e) + \quad p(e)\underbrace{\left(q_H V_H(\chi^o) + (1-q_H)V_H(\chi^o + x)\right)}_{\mathbb{E}V_H} + (1-p(e))\underbrace{\left(q_L V_L(\chi^o) + (1-q_L)V_L(\chi^o + x)\right)}_{\mathbb{E}V_L} \tag{15}$$

We begin by noting that similar to the case without stress testing, the effort the bank exerts increases with the *expected* value function wedge $\omega = \mathbb{E}V_H - \mathbb{E}V_L$. Taking the derivative of ω with respect to x at $x = 0$ gives:

$$\frac{\partial \omega}{\partial x}\bigg|_{x=0} = (1-q_H)V'_H(\mathcal{X}^o) - (1-q_L)V'_L(\mathcal{X}^o)$$

where V' indicates the derivative of the value function. To determine the sign of this expression, divide everything by $V'_L(\mathcal{X}^o)$:[25]

$$sgn\left(\frac{\partial \omega}{\partial x}\bigg|_{x=0}\right) = -sgn\left((1-q_H)\underbrace{\frac{V'_H(\mathcal{X}^o)}{V'_L(\mathcal{X}^o)}}_{\nu} - (1-q_L)\right)$$

Next, recall from the proof of Lemma (5) that $V'_H(\mathcal{X}^o) - V'_L(\mathcal{X}^o) < 0$, which implies that $\nu > 1$ since $V'_H(\mathcal{X}^o) < 0$ and $V'_L(\mathcal{X}^o) < 0$. Thus, the effect of surcharge on the bank's effort choice depends on the accuracy of the test as follows:

$$(1-q_L) - (1-q_H)\nu \begin{cases} > 0 & \Longrightarrow \quad \text{efforts increases with surcharge} \\ = 0 & \Longrightarrow \quad \text{efforts does not change with surcharge} \\ < 0 & \Longrightarrow \quad \text{effort decreases with surcharge} \end{cases}$$

■

Intuitively, ν captures the relative shadow cost of tightening regulation for the high- and low-type banks. *Ceteris paribus*, a higher ν makes imposing a surcharge less desirable by making it more likely that the bank reduces effort. Similarly, for a given ν, a higher Type-I (i.e. lower q_H) or Type-II error rate (higher q_L) would make $(1-q_L)-(1-q_H)\nu$ more negative and cause the bank to reduce effort following a higher surcharge. Indeed, if a high-type bank is sufficiently likely to fail the stress-test and the low-type bank is sufficiently likely to pass, then the high-type bank will often face a surcharge while the low-type bank

[25] Since the value of a more regulated bank is lower, $V'_L(\mathcal{X}^o) < 0$. As such, we add a minus sign to the RHS expression.

will not, thereby reducing the relative benefit to being a high-type bank. This will induce the bank to exert less effort towards becoming high-type in the first place. Relatedly, it is clear from Lemma 6 that with a perfect stress test, i.e. when $(q_H = 1, q_L = 0)$, effort increases with surcharge, while when $q_H = q_L = 0.5$, effort decreases with surcharge. We indicate these insights qualitatively (i.e., not to scale) in Figure 2.[26]

Next we assess the relationship between accuracy of the stress-test and the optimal surcharge.

Proposition 2. *No surcharge must be imposed if the error rate of stress testing as measured by an (endogenously-defined) linear combination of the Type-1 and Type-II error rates is higher than a cutoff.*[27]

Proof. Welfare as a function of the surcharge x can be written based on the regulator's problem as follows (note that e also depends on x in this expression):

$$\max_{x} \quad W(x) = \quad p(e)\Big(q_H U_H(\mathcal{X}^o) + (1-q_H)U_H(\mathcal{X}^o + x)\Big) +$$

$$(1-p(e))\Big(q_L U_L(\mathcal{X}^o) + (1-q_L)U_L(\mathcal{X}^o + x)\Big) - \zeta(e)$$

Our goal is to identify 'a' non-trivial set of (q_H, q_L) where $W(0) > W(x)\ \forall\ x > 0$, i.e. a zero surcharge is optimal.[28] A sufficient condition for this to be the case is $W'(x) < 0\ \forall\ x > 0$. To this end, we consider the first-order condition of the regulator's problem:

$$\frac{dW}{dx} = p'(e)e'(x)\Big(q_H U_H(\mathcal{X}^o) + (1-q_H)U_H(\mathcal{X}^o + x)\Big) + p(e)(1-q_H)U_H'(\mathcal{X}^o + x) -$$

$$p'(e)e'(x)\Big(q_L U_L(\mathcal{X}^o) + (1-q_L)U_L(\mathcal{X}^o + x)\Big) + (1-p(e))(1-q_L)U_L'(\mathcal{X}^o + x) - \zeta'(e)e'(x)$$

[26] In case the test has non-trivial discriminatory power, the set of parameters of interest is $q_H > q_L$.

[27] That a linear combination of the two error rates is a *measure* of the accuracy of the stress-test is not imposed; it is an endogenous by-product of the regulator's preferences over the overall welfare in the economy.

[28] Our goal is to not fully characterise the set of (q_H, q_L) for which the optimal surcharge is zero. We only wish to show that with low-enough accuracy, imposing a surcharge is sub-optimal.

To characterise the sign of this expression, we make a few assumptions, again with the goal to find *sufficient* conditions under which the optimal surcharge is zero.

– First we assume that $x \in [0, \chi_L^o - \chi^o]$. The upper bound corresponds to a surcharge amount that results in a requirement for the low-type banks that is equal to the ex-post optimal requirement χ_L^o. In principle, the optimal surcharge could be higher (due to its effect on improving ex-ante effort), but that would entail a welfare decreasing effect in case of both high- and low-type banks.

– Second, we assume that (q_H, q_L) are such that the effort exerted by the bank decreases as surcharge increases (as per Lemma 6).

Next, since $U_s(\chi^o + x), s \in \{L, H\}$ is a concave function of x, $\chi_L^o \geq \chi^o \geq \chi_H^o$ implies the following: (i) $U_H(\chi^o) \geq U_H(\chi^o + x)$; (ii) $U_H'(\chi^o + x) \leq 0$; (iii) $U_L(\chi^o) \leq U_L(\chi^o + x)$; and (iv) $U_L'(\chi^o + x) \geq 0$; $\forall\, x \in [0, \chi_L^o - \chi^o]$. It then follows that:

$$\frac{dW}{dx} \leq p'(e)e'(x)U_H(\chi^o) + p(e)(1 - q_H)U_H'(\chi^o + x) - p'(e)e'(x)U_L(\chi^o) +$$

$$(1 - p(e))(1 - q_L)U_L'(\chi^o) - \zeta'(e)e'(x)$$

Finally, we re-arrange and set the right-hand-side expression to zero:

$$p(e)U_H'(\chi^o + x) + (1 - p(e))U_L'(\chi^o) - p(e)q_H U_H'(\chi^o + x) - (1 - p(e))q_L U_L'(\chi^o) +$$

$$\underbrace{p'(e)e'(x)\Big(U_H(\chi^o) - U_L(\chi^o)\Big)}_{A<0} - \zeta'(e)e'(x) = 0$$

$$\implies \underbrace{\frac{A}{p(e)U_H'(\chi^o + x)} + 1 + \frac{(1 - p(e))U_L'(\chi^o)}{p(e)U_H'(\chi^o + x)} - \zeta'(e)e'(x)}_{\tau_0 <> 0} - q_L \underbrace{\frac{(1 - p(e))U_L'(\chi^o)}{p(e)U_H'(\chi^o + x)}}_{\tau_1 < 0} = q_H$$

$$\implies q_H = \tau_0 - \tau_1 q_L \tag{16}$$

In equation (16), while the slope is positive, the intercept can be positive or negative, depending on the underlying parameters. The equation implies that when $q_H < \tau_0 - \tau_1 q_L$ the surcharge should be zero, as also indicated in Figure 2. ■

Intuitively, the proposition shows that when q_H is low and/or q_L is high – both of which reflect a relatively less accurate stress-test – the surcharge must be zero. Next we explore conditions under which the optimal surcharge can be strictly positive.

Consider a stress-test that is accurate in identifying high-type banks i.e. $q_H = 1$, but is possibly inaccurate in identifying low-type banks i.e. $1 > q_L \geq 0$. In this case a higher x does not affect $\mathbb{E}V_H$, but decreases $\mathbb{E}V_L$ (recall equation (15)). As a result, the banker increases effort as surcharge increases. Second, consider the regulator's problem:

$$\max_x \quad p(e)U_H(\chi^o) + \quad (1 - p(e))\Big(q_L U_L(\chi^o) + (1 - q_L)U_L(\chi^o + x)\Big) - \zeta(e)$$

A higher x does not affect welfare when the bank passes the test, but increases welfare when it fails the test as long as $x \leq \chi_L^o - \chi^o$ (recall from Proposition 4 that beyond this threshold, the effective requirement on the low-type bank is higher than the optimal requirement χ_L^o.)

Combining the effect of a surcharge on effort e and $U_L(\chi^o + x)$, both of which increase as x increases, and given that $U_H(\chi^o) > U_L(\chi^o)$, it is clear that welfare, ignoring the effect of the surcharge on the cost of effort, must increase as x rises above zero. Thus, if the cost of effort is sufficiently small, the optimal surcharge must be strictly positive. Together with proposition 2, this insight points to a material shift in the relation between optimal surcharge and stress-test accuracy, with the optimal surcharge being zero (positive) if the level of accuracy of the stress tests is sufficiently low (high).

In what follows, we formalise this insight using a simpler version of the model where the probability that a bank is of a given type is fixed.[29]

[29] A similar result cannot be proven analytically in the fully specified model. Although numerical simulations show that the result also holds in the fully specified model.

Proposition 3. *The optimal surcharge increases with stress-test accuracy.*

Proof. The regulator's problem in this case is as follows:

$$\max_{x} \quad p\Big(\Big(q_H U_H(\mathcal{X}^o) + (1-q_H)U_H(\mathcal{X}^o + x)\Big) + \quad (1-p)\Big(q_L U_L(\mathcal{X}^o) + (1-q_L)U_L(\mathcal{X}^o + x)\Big)\Big($$

The first order condition is:

$$[x] \quad 0 = p(1-q_H)U_H'(\mathcal{X}^o + x) + (1-p)(1-q_L)U_L'(\mathcal{X}^o + x)$$

Next consider an increase in accuracy via a higher q_H (the proof in case of a lower q_L is similar):

$$0 = -pU_H'(\mathcal{X}^o + x) + p(1-q_H)U_H''(\mathcal{X}^o + x)\frac{\partial x\psi}{\partial q_H} + (1-p)(1-q_L)U_L''(\mathcal{X}^o + x)\frac{\partial x\psi}{\partial q_H}$$

Since $U\psi$ is concave, and $U_H'(\mathcal{X}^o + x)$ is negative (because $\mathcal{X}^o$ is higher than the optimal requirement for the high-type bank), $\frac{\partial x}{\partial q_H} > \psi 0$. ■

3.5 Endogenous accuracy

Thus far, we have considered the accuracy of the stress test – as summarised by (q_H, q_L) – to be given exogenously. In reality, regulators may be able to influence or even choose the level of accuracy, and may prefer to increase it given the welfare gains it entails. Yet, they may be constrained by various factors. In this section, we discuss two cases summarizing the trade-offs regulators face in choosing a higher level of test accuracy.

Case I: Improving accuracy along one or both dimensions possible Consider the case where the regulator is able to improve accuracy along one or both dimensions i.e. increase area under the receiver operating characteristic (ROC) curve (recall discussion in Section 3.4). This could be achieved, for instance, by making the test harder – e.g.

by using a more severe crisis scenario – to lower the false negative rate, and at the same time exercising greater caution in assessing test results to mitigate any increase in the false positive rate as a result of a harder test. Designing such a test is likely to be more costly not just for the regulator, but also for the banks. Indeed, a more extensive review of the banks' balance sheet and its risk models would not only absorb additional supervisory force, but also more bank resources.

To model these trade-offs, we consider the problem of a regulator that jointly chooses surcharge x and a test-design parameter $y \geq 0$ that maps to the pass probability of the high-type bank: $q_H(y) \uparrow 1$ as $y \to \infty$, while keeping q_L fixed.[30] In addition, we assume that adjusting the design of the test y to improve accuracy entails a social cost $C(y) = {}_c y$. This setup leads to the following result, which we prove in Appendix A.

Proposition 4. *The regulator increases stress-test accuracy q_H as well as the surcharge for failing banks as the cost of accuracy decreases.*

Intuitively, higher accuracy along one or both dimensions of the stress-test reduces the likelihood that a high-type bank is penalised, and this in turn mitigates the adverse incentives that a capital surcharge can generate. As such, a higher surcharge is optimal.

Case II: Improving accuracy along both dimensions not possible In practice, it may not be possible to improve accuracy along one dimension without worsening it along the other dimension, ie a statistical tension. For instance, a prohibitively high cost of making the test more comprehensive may leave the regulator in a situation where reducing one error rate invariably increases the other error rate. Or there may be fundamental constraints to improving accuracy given that predicting bank performance in a hypothetical scenario rests on a number of assumptions, and is an inherently hard endeavor.[31] In

[30] This is without loss of generality: the other case where q_L is adjusted can be handled similarly and leads to similar conclusions.

[31] See Parlatore and Philippon [2020] for a discussion of the underlying technical constraints.

practical terms, this situation implies that the regulator cannot increase the area under the ROC curve, and can only move *along* it, i.e. vary the signal cutoff for failures.

To obtain the regulatory implications in this case, we assume without loss of generality that by reducing false positive rate (i.e. increasing q_H), the regulator also ends up increasing the false negative rate (i.e. higher q_L). In this case, we find that a lower cost of improving accuracy along the q_H dimension induces the regulator to improve the accuracy along that dimension but does not necessarily enable the bank to choose a higher x, thus creating a dichotomy between accuracy and surcharge (see proof in Appendix B).

Overall, our analysis suggests that stress-test design and the subsequent capital surcharge decisions are intricately linked, and must inform each other. This is especially given our finding that higher accuracy along one dimension does not necessarily imply room to impose a higher surcharge on banks.

3.6 Additional policy trade-offs

Disclosure policy A contrasting aspect of stress-testing compared to other forms of micro-prudential supervision and regulation is that the testing methodology and test results are disclosed to the wider public in some detail. Disclosure of results can have an additional impact on banks via market discipline, for instance via the surprise element in test results, i.e. the difference between how investors perceive a bank and its stress-test performance. Depending on the direction of surprise in test results, investors may seek a higher or lower return for providing funding to banks. This can impact how banks respond to stress-tests, and have implications not only for stress-test disclosure policy, as discussed in [Goldstein and Sapra, 2014; Goldstein and Leitner, 2018; Leitner and Williams, 2020], but also for how test-based capital requirements must be set.[32]

[32] Other studies in this literature include Corona et al. [2019] who assess how bailout regime and disclosure policy interact, Orlov et al. [2018] who characterise the optimal disclosure policy for high- and low-risk banks, and Bouvard et al. [2015] who show that the optimal disclosure policy must vary along the business cycle.

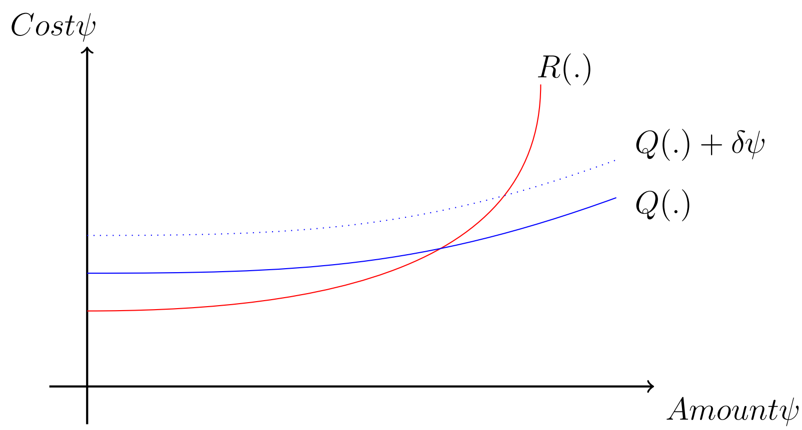


Figure 3: Cost of insured deposits and uninsured investor funding

To assess this latter aspect, we extend our model to include a role for uninsured investors that react to stress-test results. To create an incentive for the bank to pursue the two types of funding, we assume that deposit based funding is not easily scalable, and thus the unit cost of deposit funding $R(d)$ increases with the funding amount. At the same time, investor funding w, even though more costly for smaller amounts, is easily scalable, and is the relatively cheaper source of financing for larger amounts (see Figure 3). Yet, when a bank fails the stress-test, while insured depositors do not seek a higher return, uninsured investors raise their required return $Q(w)$ by, say, δ.[33] The date-1 problem of the bank in this case is as follows:

$$V_s(\chi_s) = \max_{d,w} \int_{\frac{R(d)d+Q(w)w}{g(k+d+w)}}^{\infty} \left(\left(g(k + d + w) - R(d)d - Q(w)w \right) \right) f_s(\quad) d$$
$$s.t. \ \frac{k}{\chi_s} \geq (d + w). \tag{17}$$

Assuming that both forms of financing are used in equilibrium, we assess the implications for banks and for the regulator. We first note that failure in the test is now more costly for the bank – not only does it need to satisfy a higher capital ratio, its unit cost of funding is higher compared to the case where disclosures have no material impact (i.e.

[33] Relatedly, Chen et al. [2020] provide empirical evidence of the fact that uninsured deposit flows are more sensitive to information about bank performance.

$\delta = 0$). Formally, the FOCs of the bank's problem imply that d and w are determined in the case of passing and failing banks as follows, respectively:

$$\frac{k}{\chi} = d + w; \quad R'(d)d + R(d) = Q'(w)w + Q(w)$$

$$\frac{k}{\chi + x} = d + w; \quad R'(d)d + R(d) = Q'(w)w + Q(w) + \delta$$

To make analytical progress, we assume simple forms of the cost functions: $R(d) = R_0 + R_1 d$ and $Q(w) = Q$ such that they continue to reflect the underlying intuition that investor funding is more elastic than deposit funding. Solving the FOCs explicitly leads to:

$$d_{pass} = \frac{Q - R_0}{2R_1}; \quad w_{pass} = \frac{k}{\chi} - \frac{Q - R_0}{2R_1};$$

$$d_{fail} = \frac{Q + \delta - R_0}{2R_1}; \quad w_{fail} = \frac{k}{\chi + x} - \frac{Q + \delta - R_0}{2R_1};$$

That is, upon failure in the test, the bank reduces its overall balance sheet and funding, and tilts its funding composition towards deposits. At the same time, the total funding cost (TC) of a failing bank is increasing in δ.[34] In turn, the value of a failing bank is decreasing in δ.

To derive the implications for the effort the bank exerts on date-0, we assess the impact of δ on the expected value function wedge (recall equation 6). The value of a high- or low-type bank that passes the test – ie $V_H(\chi^o)$ and $V_L(\chi^o)$ – remains unaffected by δ. However, δ leads to a larger decline in the value of a high-type bank that fails the test. To see this formally, consider the resolved value function of the s-type bank, where we have already

[34] To see this, consider the total funding cost (TC) of a failing bank as a function of δ : $TC(\delta) = R(d)d + (Q + \delta)(k/(\chi + x) - d)$ where $d = (Q + \delta - R_0)/(2R_1)$. Taking the derivative of the above expression with respect to δ immediately leads to the above result: $TC'(\delta) > 0$.

solved for the d, w decisions as a function of δ:

$$V_s(\chi^o + x; \delta) = \int_{\frac{TC(\delta)}{g(k+d+w)}}^{\infty} \Big(g(k + d + w) - TC(\delta) \Big) f_s(\psi) d\psi$$

The derivative of the value function with respect to δ implies:[35]

$$\frac{dV_s(\chi^o + x; \delta)}{d\delta} = - TC'(\delta) \int_{\frac{TC(\delta)}{g(k+d+w)}}^{\infty} f_s(\psi) d\psi$$

The above expression proves that the value function of each type of bank is decreasing in δ since $TC'(\delta) > 0$. Moreover, since the failure cutoff and $TC(.)$ are independent of bank type, the decline in value is greater in case of the high-type bank. Intuitively, the probability that the high-type bank is solvent is higher, which means that it is more likely to incur the higher funding cost. Therefore, it follows that $V_H(\chi^o + x; \delta = 0) - V_H(\chi^o + x; \delta > 0) > V_L(\chi^o + x; \delta = 0) - V_L(\chi^o + x; \delta > 0)$. As such, *ceteris paribus,* a higher δ depresses the expected value function wedge.

The above analysis shows that depending on the accuracy of stress-tests, disclosing the results of the test can strengthen or worsen the bank's ex-ante incentives. When the test is sufficiently accurate, the disclosure can help improve market discipline and increase the effort banks' exert ex-ante. Yet, when the test is less accurate (ie moving south-east from the north-west corner in Figure 2), disclosure can worsen ex-ante incentives, and place further constraints on using stress-test results for imposing bank-specific capital surcharges.

Failure costs Failure of a bank can impose a social cost. This cost can stem from, for instance, forced sale of a failed bank's assets, as well as due to resolution related expenses. It can be a major cost in the case of large banks (due to contagion/knock-on effects), when the resolution framework is not well functioning, or during a crisis when many banks are

[35]Note that $d + w$ is independent of δ and that only one of the terms following an application of the Leibniz rule is non-zero.

in insolvency at the same time.

Failure costs exacerbate the trade-off for regulators. A higher surcharge (compared to the case without failure costs) may be justified on the grounds that it lowers the expected failure rate and attendant social costs. Yet, to the extent the stress test is not sufficiently accurate, a higher surcharge would not only lower welfare in the case of a high type bank, but also would lower the ex-ante e ort exerted by the bank. As such, it is not obvious as to whether the surcharge must be adjusted upwards or downwards in the presence of higher failure costs.

To formally assess the effect of failure cost on optimal regulation, we adapt the model as follows. We assume that once a bank fails, the recovery value of its assets is less that a hundred percent. This cost – denoted Δ – is borne by the deposit insurance program and is funded via taxes:

$$T(\psi) = \begin{cases} Rd - \psi g(k + d)(1 - \Delta) & \text{If the bank fails i.e. } \psi \leq \frac{Rd}{g(k+d)} \\ 0 & \text{Otherwise} \end{cases}$$

In what follows, we prove that the failure cost exacerbates the inefficiency banks pose, and rationalises a higher ex-post requirement χ^o and also a higher ex-ante surcharge $x\psi$ associated with failing the stress test.[36]

We begin by assessing the ex-post requirement, while abstracting away from bank-type as before. Household and banker consumption on date-2 in this case is given as:

$$c_2 + n = \bar{Y} + \left(\psi g(k + d) - \Delta \psi g(k + d) \mathbb{1} \left(\psi \leq \frac{Rd}{g(k + d)} \right) \right)$$

[36]We assume that the cost of failure is fixed. In practice, implicit guarantees for too-big-to-fail banks may imply that Δ is smaller for larger banks, which in turn can induce banks to pursue leverage. Yet, we abstract away from such considerations to keep the model focused.

Accordingly, the planner's problem is:

$$\max_{d} \quad (1+\psi)\bar{Y} + \int_{\frac{Rd}{g(k+d)}}^{\infty} \Big(\psi g(k+d) - Rd\Big) f(\psi)\, d\psi + \int_{0}^{\frac{Rd}{g(k+d)}} \Big(\psi g(k+d) - Rd - \Delta \psi g(k+d)\Big) f(\psi)\, d\psi,$$

while the attendant first-order-condition is:

$$\begin{aligned} 0 \;=\; & \int_{\frac{Rd}{g(k+d)}}^{\infty} \Big(\psi g'(k+d) - R\Big) f(\psi)\, d\psi \;+ \\ & \underbrace{\int_{0}^{\frac{Rd}{g(k+d)}} \Big(\psi g'(k+d)(1-\Delta) - R\Big) f(\psi)\, d\psi - \Delta g(k+d) \frac{\partial \frac{Rd}{g(k+d)}}{\partial d} f\Big(\frac{Rd}{g(k+d)}\Big)}_{\text{Bank-failure inefficiency}} \end{aligned} \tag{18}$$

We know from the discussion of equation (11) that the inefficiency term in that equation, namely $\int_0^{\frac{Rd}{g(k+d)}} \Big(\psi g'(k+d) - R\Big) f(\psi)\, d\psi$, is negative. This means that the integral in the second row of equation (18) is also negative, and lower in value compared to the second integral in equation (11). At the same time, since $g(.)$ is concave:

$$\frac{\partial \frac{Rd}{g(k+d)}}{\partial d} = \frac{R\Big(g(k+d) - d g'(k+d)\Big)}{g(k+d)^2} > 0.$$

As such, the inefficiency term in equation (18) is negative and larger in magnitude relative to the inefficiency term in equation (11). Thus failure cost amplifies the bank-failure inefficiency. In turn, as shown in Lemma 3, greater inefficiency rationalises a higher minimum capital-ratio requirement. We note this result in Lemma 7.

Lemma 7. *The regulator must optimally impose a higher ex-post minimum capital-ratio requirement on a bank that, all else equal, exhibits a higher failure cost.*

Next we examine how the optimal surcharge must change as failure cost increases. Unfortunately, it is not possible to characterise the change generally. For analytical tractability, we assume (as in the previous subsection) that the probability of being a high-type (or equivalently low-type) bank is given and that there is no effort choice involved. We find that irrespective of the accuracy of the test (and thus the attendant adverse incentives it generates), the optimal surcharge is higher when failure is more costly (see proposition below and proof in Appendix C).

Proposition 5. *Assuming $p(e) \equiv p$, the optimal surcharge must increase as Δ increases.*

Relaxing the assumption that $p(e) = p$ does not lead to a general result, that is, $\frac{dx}{d\Delta}$ cannot be signed unless the specific values of the parameters of the model are known. As such, we pursue this more general case in the quantitative analysis. Nonetheless, the above proposition suggests that if the stress test is sufficiently accurate so that effort e and thus the probability of being a high-type bank increase as the surcharge increases, then it is likely that the surcharge must be optimally adjusted upwards as the failure cost increases.

4 Numerical illustration

We now calibrate the model parameters. Our goal is not to draw empirical predictions, but to provide a relevant numerical illustration of our analytical results. To this end, we set the parameters such that model generated moments are equal to the corresponding data moments (see Table 1). As an example, we use data on G20 member countries. Unless otherwise mentioned, for each data moment, we consider the average across these countries. We focus on data from 2019 to abstract away from any Covid-19 crisis related aberrations in the data.

We consider the following moments as targets. First is the gross return on assets, which takes into account interest as well as non-interest income. Second is the equity capital to assets ratio. Third is a typical regulatory or bank-management-imposed value-at-risk threshold of 1%. Together, these three moments capture aspects of a bank's financials that are key for their response to regulation. The fourth moment is the household savings rate, approximated as the domestic savings to GDP ratio. Fifth is the deposit interest rate. Finally, Δ is set in line with the losses associated with bank failures in the US in

Parameter	Description	Value	Target moments	Value
	Payoff exponent: (k+d)	0.9668	Gross return on assets	5.84%
$\mu\psi$	Mean of	1.1501	Equity capital to assets ratio	8.17%
$\sigma\psi$	Standard-deviation of	0.0549	Value-at-risk threshold	1%
$\bar{Y}\psi$	Household income	43.701	Household savings rate	25.72%
	Discount factor	0.9760	Deposit interest-rate	2.46%
Δ	Failure cost	0.22	US bank failure losses	22%

Table 1: Parameter values and target moments. The mean gross return on assets and capital ratio are computed using data on the top 3 or more banks in each G20 country, and is sourced from Fitch. The household savings rate is calculated as the average domestic savings to GDP ratio from the World Bank. Deposit rate is also sourced the World Bank, and data on bank failure losses are from the FDIC. Note that the last two parameters and target moments have a one-to-one mapping (i.e. they need not be estimated jointly), and that without loss of generality $k\psi$is normalised to unity. The value of the moments in data exactly match those implied by the model.

the period after the Great Financial Crisis.[37] According to the Federal Deposit Insurance Commission (FDIC), there have been 367 bank failures during this period, and the median estimated loss is about 21% of the failed bank's assets, while the inter-quartile range is 13% to 30%. Our target moment is the mean, which is 22%.

As regards the functional forms, we assume the cost of exerting effort by the bank on date-0 as $\zeta(e) = {}_{e}e^2$, and the related probability of the bank becoming a high-type on date-1 as $p(e) = 1-1/(1+e)$. The exact functional forms do not matter for our qualitative results as long as $\zeta(.)$ is (weakly) convex and $p(.)$ is concave. We choose $_{e}$ so that the bank is high- or low-type with equal probability. As regards μ_H and μ_L, we assume a symmetric perturbation of 25 basis points around μ. Finally, we treat q_H and q_L as free parameters that we conduct comparative statics with respect to.

Optimal ex-post regulation We begin by analyzing the impact of a minimum capital-ratio requirement on the bank's behavior and overall welfare on date-1. Without loss of generality, we consider a high-type bank. Starting from the unregulated economy, a higher minimum capital-ratio requirement forces the bank to deleverage (first panel in Figure 4).

[37]Data on bank failure related losses is not available systematically for all the countries in our sample; hence we only use the US data for this moment.

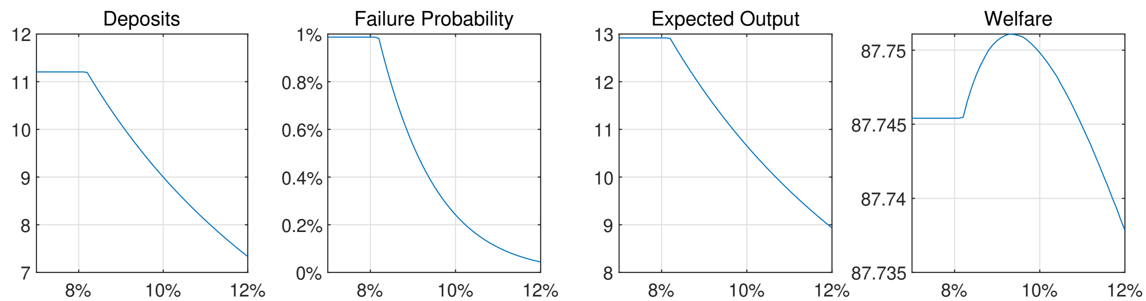


Figure 4: The effect of minimum capital-ratio requirement (x-axis) on the high-type bank and on overall welfare.

This reduces the failure probability (second panel), but also lowers expected output (third panel). The overall effect – one that weighs welfare gains from lower bank failure against the welfare loss from lower expected output – is an inverted U-shaped welfare profile as a function of $\mathcal{X}$. This finding is consistent with Lemmas 2 and 3 where we showed that the unregulated equilibrium is sub-optimal and that a minimum capital-ratio requirement can improve welfare, and also with the broader literature (e.g. Begenau [2020], Christiano and Ikeda [2016]).[38]

Relatedly, as bank failure costs increase, not only is the optimal requirement higher (as proven in Lemma 7), the welfare gain from regulation is also higher (see left-hand panel in Figure 5).

Finally, we compare the optimal ex-post requirement for low- and high-type banks. Consistent with Lemma 4, we find that the requirement is higher for the low-type bank (see right-hand panel of Figure 5, dotted lines).

Optimal ex-ante regulation When the regulator cannot observe banks' types ex-post, the optimal ex-ante requirement announced on date-0 cannot be bank-type specific. Consistent with Proposition 1, we find that it is saddled by the ex-post optimal requirements (see solid line in the right-hand panel of Figure 5).

[38] Given the stylized nature of our model, the simulations are not meant to pin down the optimal level of capital requirements in the real world, but rather to illustrate the comparative statics of the model with respect to calibrated parameters.

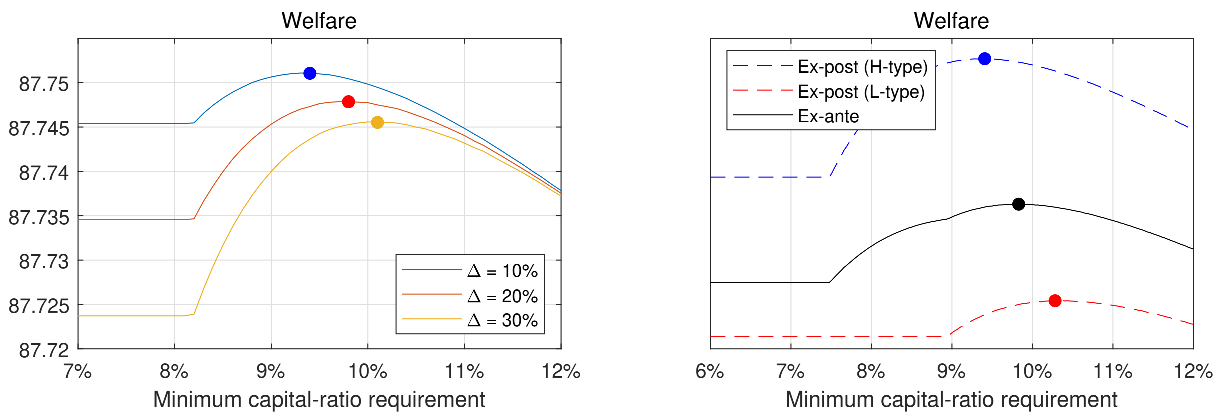


Figure 5: *Left-hand panel*: The welfare maximizing regulation for varying levels of bank failure costs. *Right-hand panel*: Optimal ex-post requirement depending on bank type, and the optimal ex-ante requirement in the absence of stress tests.

Next we assess how a stress-test led surcharge affects bank's behavior. A higher surcharge decreases the value of both high- and low-type banks (left-hand panel of Figure 6). The decrease is starker for a high-type bank – indeed the opportunity cost of not being able to use its balance sheet capacity is higher for a bank whose assets have a higher return. And as long as the stress test is not fully perfect, both $\mathbb{E}V_H$ and $\mathbb{E}V_L$ decrease as $x\psi$ increases.

The difference between $\mathbb{E}V_H$ and $\mathbb{E}V_L$, namely $\omega\psi$ – as we showed in Lemma 6 – can increase or decrease depending on the accuracy of the test (see centre panel of Figure 6). This immediately means that the effort banks exert can also increase or decrease as the surcharge is raised (recall that $e\psi$ depends on ω; see the proof of Lemma 5). This is a key insight of the paper – a higher surcharge may not necessarily act as a disciplining device if the basis on which the surcharge is imposed is not sufficiently accurate.

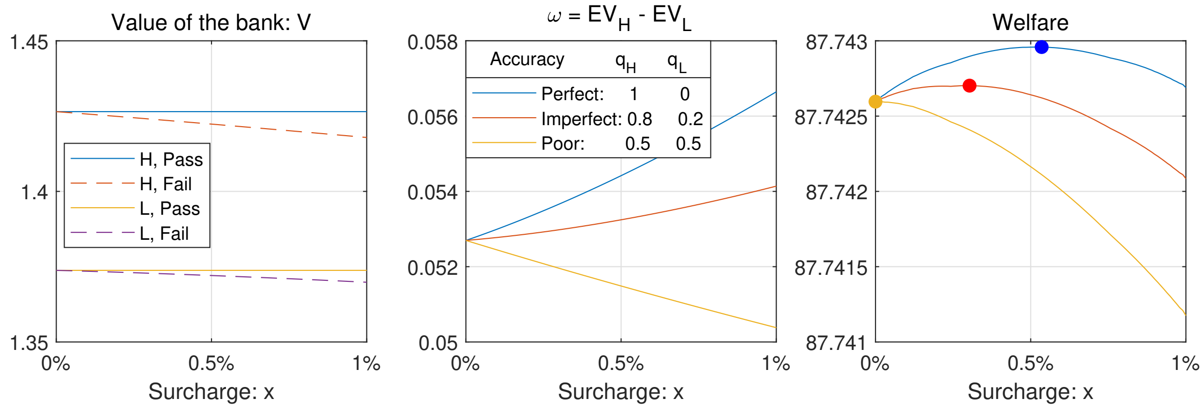


Figure 6: *Left-hand panel:* The value $V\psi$ of the bank in various cases as a function of the surcharge. *Centre panel:* The expected value wedge changes in response to the surcharge for different levels of accuracy of the stress test. *Right-hand panel:* Optimal surcharge.

Overall, the optimal surcharge depends on the following trade-off. Higher capital charges for banks that fail the assessment improve welfare to the extent that low-type banks are penalised, but it reduces welfare by asking some high-type banks to raise unnecessary capital. As such, when the test is sufficiently noisy, capital surcharges may not increase expected welfare. In particular, they may induce banks to reduce their effort. We confirm this insight quantitatively. For a poor level of accuracy, consistent with proposition 2, the optimal surcharge is zero (right-hand panel of Figure 6). For higher levels of accuracy, including the case of a perfect stress test, the optimal surcharge is higher (recall Proposition 3).

We illustrate the optimal surcharge for each accuracy level of the stress-test in the left-hand panel of Figure 7, thus confirming the broad indications sketched in Figure 2. Indeed, a phase shift is evident: for sufficiently low levels of accuracy, the optimal surcharge is zero. Moving closer to a perfect stress test ($q_H = 1, q_L = 0$) increases the size of the optimal surcharge.

Next we elaborate upon the result proven in Proposition 5, and show that as the failure cost increases, the optimal surcharge also increases (see the right-hand panel of Figure 7).

Finally we illustrate the implications of endogensing accuracy (recall the setup in sub-

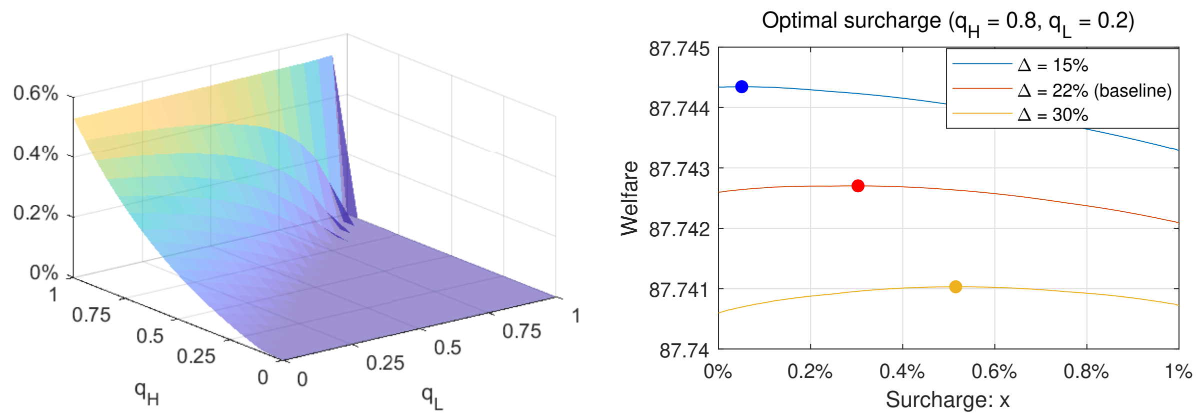


Figure 7: *Left-hand panel*: Optimal surcharge as a function of the accuracy of the stress-test. *Right-hand panel*: Change in optimal surcharge as the cost of failure increases.

section 3.5). We assume that $C(y) = \ _c y$, $q_H(y) = 1 - \ _q/(1+y)$, and $q_L(y) = 1 - q_H(y)$ (see left-hand panel of Figure 8 for an example). Consistent with the analytical result in proposition 4, we find that as the cost of accuracy decreases ($\ _c \downarrow$), the regulator must optimally work with more accurate stress tests ($y \Uparrow$), and at the same time, revise upwards the surcharge it imposes on failings banks ($x \Uparrow$) (see right-hand panel of Figure 8).

5 Covid-19 crisis: A test of risk-assessment tools?

In this section, we use the supervisory stress-tests in the U.S. as an illustration of the noise that is inherent in this type of risk-assessment exercises. In particular, we compare the assessments conducted in 2020, right before the Covid-19 crisis, with the impact of the crisis on banks' balance sheets.[39]

Stress-tests evaluate whether banks have sufficient capital to absorb losses resulting from adverse economic conditions.[40] In the US, the Federal Reserve imposes a capital surcharge on banks based on their performance in the test, as measured by the projected decline in their Common Equity Tier 1 (CET1) capital ratios in the severely adverse

[39] We acknowledge that the comparison is somewhat "unfair" because stress-tests are not designed to predict crises, and only test banks' capital adequacy in a single hypothetical stress scenario.

[40] This involves projecting revenues, expenses, losses, and, crucially, the capital ratios of the participating banks in a recession. The projections use a standard set of capital action assumptions.

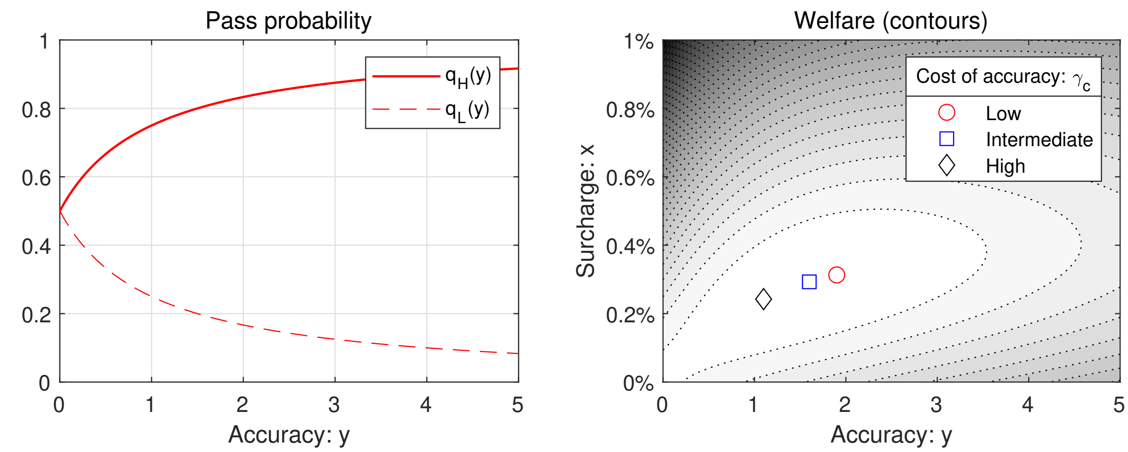


Figure 8: *Left-hand panel:* Pass probabilities for high and low-type banks as a function of accuracy. *Right-hand panel:* The jointly optimal accuracy and surcharge for different levels of cost of accuracy $_c$. Welfare contours correspond to the intermediate level of $_c$.

scenario. Banks that perform poorly face a higher Stressed Capital Buffer (SCB).

The hypothetical scenario in the 2020 stress-test in the US comprised of a peak unemployment rate of 10 percent, a decline in real GDP of 8.5 percent, and a drop in equity prices of 50 percent through the end of 2020, among other macroeconomic developments.[41] Thirty-three entities participated in the test and the results were published in June 2020. The average decline in CET1 ratio was 2.7 percentage points (see Figure 9). The figure also shows that stress-tests and the attendant capital surcharges have a strong and positive relationship beyond the minimum SCB of 2.5%.

A comparison of banks' CET1 ratios in the 2020 US stress-test with their actual ratios in the Covid-19 crisis can help us learn something about the inherent noise in stress testing. Several factors make the Covid-19 crisis a useful natural experiment, but there are caveats too.[42]

For one, several key macroeconomic indicators (such as GDP, employment, and stock

[41] The severely adverse scenario was designed in late 2019 and was published in February 2020. While the 2020 DFAST did not adapt the severely stress scenario to incorporate the Covid-19 crisis, it disclosed additional information about predicted aggregate losses in the banking sector based on a sensitivity analysis viz-a-viz the Covid-19 crisis. Bank-level results from this exercise were not disclosed.

[42] Older stress-tests cannot be appraised against banks' actual performances in the Covid-19 crisis as banks are likely to act on the test results and evolve materially in the meantime.

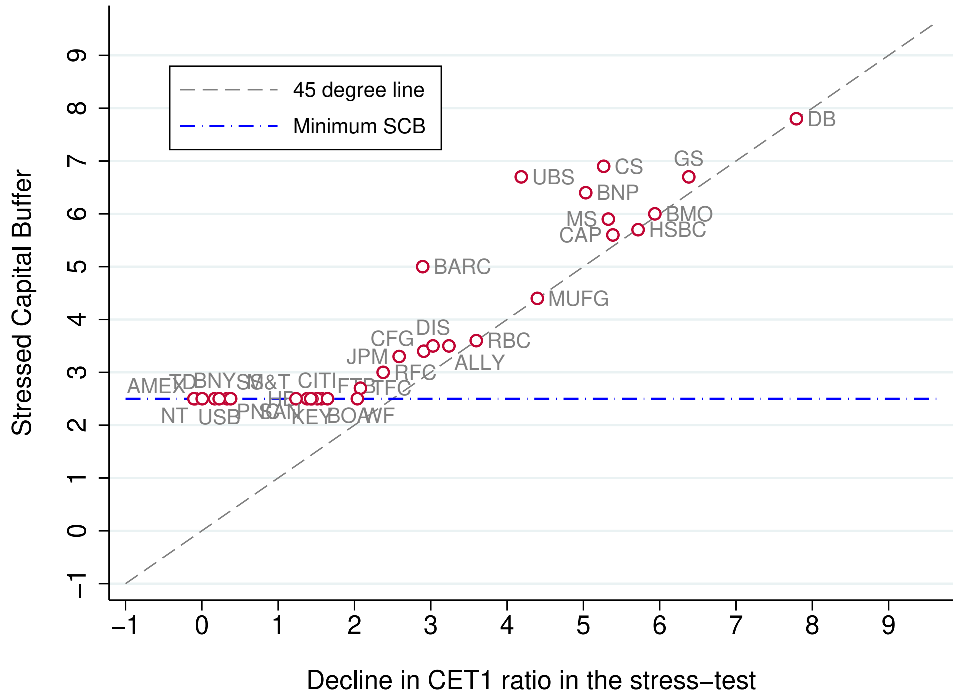


Figure 9: A comparison of the decline in CET1 ratio in the 2020 DFAST and the Stressed Capital Buffer (SCB) imposed on banks. Unit of both axes is percentage points.

prices) in the Covid-19 crisis line up with the 2020 stress-test scenario.[43] Yet, some indicators such as the house price index do not. While the differences can render the comparison of 'a' bank's performance less meaningful, broad concordance in banks' relative performances in the test and in the current crisis is to be expected (also see Acharya et al. [2014], for instance).

Second, the Covid-19 shock was completely unexpected, like in the case of stress-tests where the hypothetical scenarios are not known to banks in advance.

Third, that risk-weighted assets and loan loss provisions (LLPs) are forward looking, and that banks front-loaded their response to the crisis by increasing LLPs substantially in Q2 2020, means that the Q2 capital ratios likely reflect how banks would eventually perform in the crisis.[44] This helps address, at least partially, claims that the CET1 ratio of

[43]The U.S. economy contracted by close to 30% (YoY) in Q2 2020; the peak unemployment rate was 15%; and the Dow Jones Index plunged by close to 30% in March 2020.

[44]Our conclusions below are robust to using Q3 or Q4 2020 data.

banks may not yet reflect fully the impact of the Covid-19 crisis. Although concerns that extraordinary support was provided to banks and to the broader economy have dented the impact of the crisis on banks (so far) may still apply.

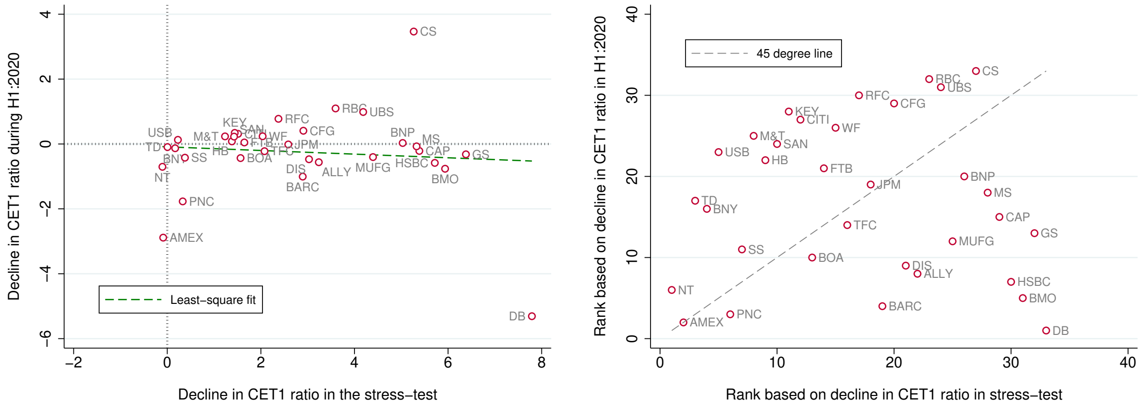


Figure 10: (Left-hand panel: A comparison of the decline in CET1 ratio in the stress-test and the actual decline observed between Q4:2019 and Q2:2020. Unit of both axes is percentage points. Right-hand panel: A comparison of the rank based on decline in CET1 ratio in the stress-test and rank based on the actual decline observed between Q4:2019 and Q2:2020. A lower rank (number) indicates a smaller decline in the ratio.

All in all, our assessment is that the Covid-19 crisis can shed light on the noise inherent in supervisory risk assessments. To this end we compare the test-implied and Q2 2020 CET1 ratios of banks (see Figure 10). We document that the cross-sectional variance in test-driven changes in CET1 ratios is much higher than the observed changes, and that the two do not correlate. In fact, while the ratios declined for almost all banks in the test, it rose for many in reality. In the case of Deutsche Bank USA, for instance, the CET1 ratio declined by 8 percentage points (pp) in the test, while during H1 2020, the same ratio rose by 5 pp. We observe similar degree of discordance in the case of cross-sectional ranking of banks' CET1 ratios, changes in CDS spreads during the crisis – a market indicator of banks' resilience – and loan loss provisions – an indicator of banks' self assessment of expected credit losses (see Figure 11).[45] The higher variance in test results and its lack of

[45]Credit losses are a major component of the CET1 ratio projections in the stress-test. This makes loan loss provisions during the Covid-19 crisis a useful indicator to compare banks' stress-test performance with.

correlation with observed outcomes is suggestive of the noise inherent in risk-assessments.

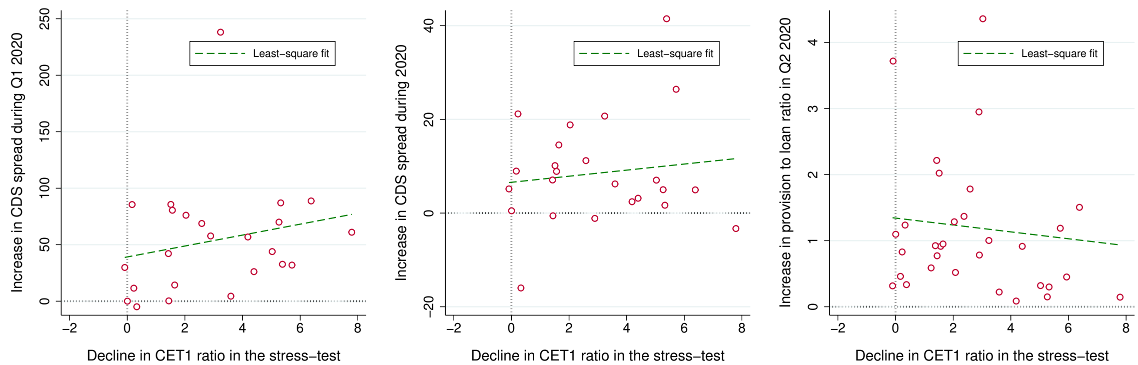


Figure 11: A comparison of the decline in CET1 ratio in the stress-test and the increase in CDS spreads in Q1 2020 (left hand panel), during the full year 2020 (center panel), and loan loss provisions to loans ratio in Q2 2020 (right-hand panel). CDS data is not available for all banks in the sample.

6 Conclusion

Use of supervisory assessments of bank risk to determine bank-specific regulation has become an important tool for policymakers. They have helped regulators in gauging banks' idiosyncratic risks and in bolstering financial stability. They continue to evolve and improve based on lessons learnt over the years. Despite these enhancements, such assessments continue to be noisy, not least due to fundamental difficulties inherent in identifying risks. Given that such assessments underpin banks' capital requirements, noisy assessment can lead to misdirected requirements and can have a large impact on banks' capital costs, on their operational incentives, and on overall economic welfare.

To assess the implications, we build a model that incorporates noisy risk-assessment of banks, and show that noise not only reduces overall welfare directly, but can also create adverse ex-ante incentives for banks. Going against the conventional wisdom, we thus show that in the presence of information frictions, higher capital requirements may lead to more risky banks. As such, the graduation of capital requirements on the basis of noisy assessments should be inversely related to the noise. In the extreme case of very low reliability, capital requirements should be insensitive to the signal.

The parsimony and tractability of our model makes it amenable to extensions of interest. For instance, some regulators have discussed maintaining a surprise element in stress-tests on the grounds that it can help avoid pre-positioning or complacency by banks.[46] The welfare effects of surprise in stress tests is not obvious because while it can limit the scope for gaming by banks, higher regulatory uncertainty can weaken the link between the effort banks exert and their performance in the stress-test. This can make banks exert less effort towards improving their risk-return profile. Our model can be extended to study this trade-off by allowing for uncertainty around type-specific capital requirements.

References

Acharya, V., R. Engle, and D. Pierret (2014). Testing macroprudential stress tests: The risk of regulatory risk weights. *Journal of Monetary Economics 65*, 36–53.

Ahnert, T., J. Chapman, and C. Wilkins (2020). Should bank capital regulation be risk sensitive? *Journal of Financial Intermediation*.

Begenau, J. (2020). Capital requirements, risk choice, and liquidity provision in a business-cycle model. *Journal of Financial Economics 136*.

Behn, M., R. F. Haselmann, and V. Vig (2016). The limits of model-based regulation. *ECB working paper*.

Berger, A. N., S. M. Davies, and M. J. Flannery (2000). Comparing market and supervisory assessments of bank performance: Who knows what when? *Journal of Money, Credit and Banking*, 641–667.

[46]See, for instance, the remarks by Mr Jerome H Powell, Chair of US Federal Reserve System, at the research conference titled ”Stress Testing: A Discussion and Review” on 9 July 2019. In fact, the continuous evolution of the stress-test regime may be motivated by this pursuit.

Bouvard, M., P. Chaigneau, and A. D. Motta (2015). Transparency in the financial system: Rollover risk and crises. *The Journal of Finance 70*(4), 1805–1837.

Braouezec, Y. and L. Wagalath (2018). Risk-based capital requirements and optimal liquidation in a stress scenario. *Review of Finance 22*(2), 747–782.

Chen, Q., I. Goldstein, Z. Huang, and R. Vashishtha (2020). Bank transparency and deposit flows. *SSRN working paper (No 3212873).*

Christiano, L. and D. Ikeda (2016). Bank leverage and social welfare. *American Economic Review 106*(5), 560–64.

Corona, C., L. Nan, and G. Zhang (2019). The coordination role of stress tests in bank risk-taking. *Journal of Accounting Research 57*(5), 1161–1200.

Covas, F. B., B. Rump, and E. Zakrajšek (2014). Stress-testing US bank holding companies: A dynamic panel quantile regression approach. *International Journal of Forecasting 30*(3), 691–713.

Fed (2020). Corrected stress test results stemming from an error in projected trading losses and as a result, revised the capital requirements for two banks. *Press release.*

Flannery, M., B. Hirtle, and A. Kovner (2017). Evaluating the information in the Federal Reserve stress tests. *Journal of Financial Intermediation 29*, 1–18.

Frame, W. S., K. Gerardi, and P. Willen (2015). The failure of supervisory stress testing: Fannie Mae, Freddie Mac, and the Office of the Federal Housing Enterprise Oversight. *Federal Reserve Bank of Atlanta Working Paper*.

Gale, D. et al. (2010). The effects of bank capital on lending: What do we know, and what does it mean? *International Journal of Central Banking 6*(34), 187–204.

Gertler, M. and N. Kiyotaki (2010). Financial intermediation and credit policy in business cycle analysis. *Handbook of Monetary Economics 3*, 547.

Goldstein, I. and Y. Leitner (2018). Stress tests and information disclosure. *Journal of Economic Theory 177*, 34–69.

Goldstein, I. and H. Sapra (2014). Should banks' stress test results be disclosed? An analysis of the costs and benefits. *Foundations and Trends in Finance*.

Gorton, G. (2009). Information, liquidity, and the (ongoing) panic of 2007. *American Economic Review 99*(2), 567–72.

Kara, G. I. and S. M. Ozsoy (2020). Bank regulation under fire sale externalities. *The Review of Financial Studies 33*(6), 2554–2584.

Kareken, J. H. and N. Wallace (1978). Deposit insurance and bank regulation: A partial-equilibrium exposition. *Journal of Business*, 413–438.

Kim, D. and A. M. Santomero (1988). Risk in banking and capital regulation. *The Journal of Finance 43*(5), 1219–1233.

Koehn, M. and A. M. Santomero (1980). Regulation of bank capital and portfolio risk. *The Journal of Finance 35*(5), 1235–1244.

Leitner, Y. and B. Williams (2020). Model secrecy and stress tests. *SSRN working paper*.

Leitner, Y. and B. Yilmaz (2019). Regulating a model. *Journal of Financial Economics 131*(2), 251–268.

Lohmann, S. (1992). Optimal commitment in monetary policy: credibility versus flexibility. *The American Economic Review 82*(1), 273–286.

Marshall, D. A. and E. S. Prescott (2001). Bank capital regulation with and without state-contingent penalties. *Carnegie-Rochester Conference Series on Public Policy 54*.

Morgan, D. P., S. Peristiani, and V. Savino (2014). The information value of the stress test. *Journal of Money, Credit and Banking 46*(7), 1479–1500.

Morrison, A. D. and L. White (2005). Crises and capital requirements in banking. *American Economic Review 95*(5), 1548–1572.

Nguyen, T. T. (2015). Bank capital requirements: A quantitative analysis. *SSRN Working Paper*.

Ong, L. L., R. Maino, and N. Duma (2010). Into the great unknown: Stress testing with weak data. *IMF Working Paper*.

Orlov, D., P. Zryumov, and A. Skrzypacz (2018). Design of macro-prudential stress tests. *Stanford University Working Paper 3548*.

Parlatore, C. and T. Philippon (2020). Designing stress scenarios. *Working paper*.

Philippon, T., P. Pessarossi, and B. Camara (2017). Backtesting European stress tests. *NBER Working Paper 23083*.

Plosser, M. C. and J. A. Santos (2018). Banks' incentives and inconsistent risk models. *The Review of Financial Studies 31*(6), 2080–2112.

Powell, J. H. (2019). Welcoming remarks: Stress testing: A discussion and review. *Speech: Research conference at the Federal Reserve Bank of Boston.*

Prescott, E. S. (2004). Auditing and bank capital regulation. *Economic Quarterly, Federal Reserve Bank at Richmond 90*(4), 47–63.

Quigley, D. and A. Walther (2020). Inside and outside information: Reverse unraveling and stress test design. *Working Paper*.

Rochet, J.-C. (1992). Capital requirements and the behaviour of commercial banks. *European Economic Review 36*(5), 1137–1170.

Santos, J. A. C. (2001). Bank capital regulation in contemporary banking theory: A review of the literature. *Financial Markets, Institutions & Instruments 10*(2), 41–84.

Van den Heuvel, S. J. (2008). The welfare cost of bank capital requirements. *Journal of Monetary Economics 55*(2), 298–320.

Williams, B. (2017). Stress tests and bank portfolio choice. *Working Paper, New York University*.

Wu, H.-M. and Y. Zhao (2016). Optimal leverage ratio and capital requirements with limited regulatory power. *Review of Finance 20*(6), 2125–2150.

Zelmer, M. (2013). Simplicity and supervision: Rules are not enough. *Remarks to the RISK USA Conference.*

Appendix

A Endogenous accuracy: Case I

The regulator's problem in this case is as follows:

$$\max_{x,y} \quad p\Big(\Big(q_H(y)U_H(\mathcal{X}^o)+(1-q_H(y))U_H(\mathcal{X}^o+x)\Big)+ \ (1-p)\Big(q_L U_L(\mathcal{X}^o)+(1-q_L)U_L(\mathcal{X}^o+x)\Big)\Big(- \ {}_c y\psi$$

The first order conditions are:

$$[x] \quad 0 = p(1-q_H(y))U_H'(\mathcal{X}^o+x)+(1-p)(1-q_L)U_L'(\mathcal{X}^o+x)$$

$$[y] \quad 0 = \ p q_H'(y)(U_H(\mathcal{X}^o)-U_H(\mathcal{X}^o+x)) - \ {}_c$$

Next consider an increase in the cost of accuracy $_c$. A total derivative of the FOCs leads to:

$$[x]: \quad 0 = p\Big((1-q_H(y))U''_H(\chi^o+x)\dot{x} - q'_H(y)\dot{y}U'_H(\chi^o+x)\Big) + (1-p)\Big((1-q_L)U''_L(\chi^o+x)\dot{x}\Big)$$

$$[y]: \quad 0 = p\Big(q''_H(y)\dot{y}(U_H(\chi^o)-U_H(\chi^o+x)) - q'_H(y)U'_H(\chi^o+x)\dot{x}\Big) - 1$$

where $\dot{y} = \frac{\partial y}{\partial _c}$ and $\dot{x} = \frac{\partial x}{\partial _c}$. The first total derivative implies that $\dot{x}$ and $\dot{y}$ are of the same sign since U is concave, $U'_H < 0$, and $q'_H > 0$. This means that accuracy and surcharge go hand in hand. To assess the direction of change, consider the second total derivative, and replace $\dot{x}$ using the first total derivative. This results in the following expression:

$$\dot{y}\Bigg(pq''_H(y)(U_H(\chi^o)-U_H(\chi^o+x)) - \frac{(pq'_H(y)U'_H(\chi^o+x))^2}{p(1-q_H(y))U''_H(\chi^o+x)+(1-p)(1-q_L)U''_L(\chi^o+x)}\Bigg) = 1$$

The coefficient on $\dot{y}$ is negative following the second-order sufficiency optimality condition (i.e. negative-definite Hessian matrix). This implies that $\dot{y} < 0$, and from the above discussion, that $\dot{x} < 0$.

B Endogenous accuracy: Case II

The regulator's problem in this case is as follows, where both q_H and q_L increase as y, the underlying accuracy of the test, increases:

$$\max_{x,y} \quad p\Big(q_H(y)U_H(\chi^o) + (1-q_H(y))U_H(\chi^o+x)\Big) + (1-p)\Big(q_L(y)U_L(\chi^o) + (1-q_L(y))U_L(\chi^o+x)\Big) - {}_c y$$

The first order conditions are:

$$[x] \quad 0 = p(1-q_H(y))U_H'(\mathcal{X}^o + x) + (1-p)(1-q_L(y))U_L'(\mathcal{X}^o + x)$$

$$[y] \quad 0 = \; pq_H'(y)(U_H(\mathcal{X}^o) - U_H(\mathcal{X}^o + x)) + \; (1-p)q_L'(y)(U_L(\mathcal{X}^o) - U_L(\mathcal{X}^o + x)) - \; _c$$

Next consider an increase in the cost of accuracy $_c$. A total derivative of the FOCs leads to:

$$[x]: \quad 0 = p\Big((1-q_H(y))U_H''(\mathcal{X}^o + x)\dot{x} - q_H'(y)\dot{y}U_H'(\mathcal{X}^o + x)\Big) +$$
$$(1-p)\Big((1-q_L(y))U_L''(\mathcal{X}^o + x)\dot{x} - q_L'(y)\dot{y}U_L'(\mathcal{X}^o + x)\Big)$$
$$[y]: \quad 0 = \; p\Big(q_H''(y)\dot{y}(U_H(\mathcal{X}^o) - U_H(\mathcal{X}^o + x)) - q_H'(y)U_H'(\mathcal{X}^o + x)\dot{x}\Big) +$$
$$(1-p)\Big(q_L''(y)\dot{y}(U_L(\mathcal{X}^o) - U_L(\mathcal{X}^o + x)) - q_L'(y)U_L'(\mathcal{X}^o + x)\dot{x}\Big) - 1$$

where $\dot{y} = \frac{\partial y}{\partial \; _c}$ and $\dot{x} = \frac{\partial x}{\partial \; _c}$. The first total derivative no longer (compare to Proposition 4) implies that $\dot{x}$ and $\dot{y}$ are of the same sign given that U is concave, $U_H' < 0$, $q_H' > 0$, $U_L' > 0$, and $q_L' > 0$. This means that accuracy and surcharge do not go hand in hand. To assess the direction of change in y, consider the second total derivative, and replace $\dot{x}$ using the first total derivative. Like before, the second-order sufficiency condition for optimality (ie negative-definite Hessian matrix) implies that $\dot{y} < 0$.

C Optimal surcharge with failure costs

The regulator's problem is as follows:

$$\max_x \quad W(x) = \; p\Big(q_H U_H(\mathcal{X}^o, \Delta) + (1-q_H)U_H(\mathcal{X}^o + x, \Delta)\Big)$$
$$(1-p)\Big(q_L U_L(\mathcal{X}^o, \Delta) + (1-q_L)U_L(\mathcal{X}^o + x, \Delta)\Big)$$

Here Δ in the utility function formally expresses the dependence of welfare on failure costs. The attendant first-order condition is as follows, where the D_i operator indicates the derivative with respect to the i^{th} argument of U:

$$p(1-q_H)D_1U_H(\chi^o+x,\Delta)+(1-p)(1-q_L)D_1U_L(\chi^o+x,\Delta)=0$$

Next, we take the total derivative of this expression with respect to Δ:

$$p(1-q_H)\left(D_{11}U_H(\chi^o+x,\Delta)\frac{dx}{d\Delta}+D_{12}U_H(\chi^o+x,\Delta)\right)+$$

$$(1-p)(1-q_L)\left(D_{11}U_L(\chi^o+x,\Delta)\frac{dx}{d\Delta}+D_{12}U_L(\chi^o+x,\Delta)\right)=0$$

$$\implies -\underbrace{\left(p(1-q_H)D_{11}U_H(\chi^o+x,\Delta)+(1-p)(1-q_L)D_{11}U_L(\chi^o+x,\Delta)\right)}_{A}\frac{dx}{d\Delta}=$$

$$p(1-q_H)D_{12}U_H(\chi^o+x,\Delta)+(1-p)(1-q_L)D_{12}U_L(\chi^o+x,\Delta)$$

Since both U_H and U_L are concave functions of x, $A<0$. To sign the RHS, consider $U_s, s=\{H,L\}$:

$$U_s(\chi^o+x,\Delta)=\bar{Y}-d-g(k+d)\left(\mu_s-\Delta\int_0^{\frac{Rd}{g(k+d)}}f_s(\ )d\ \right)\quad where\ d=\frac{k}{\chi^o+x}$$

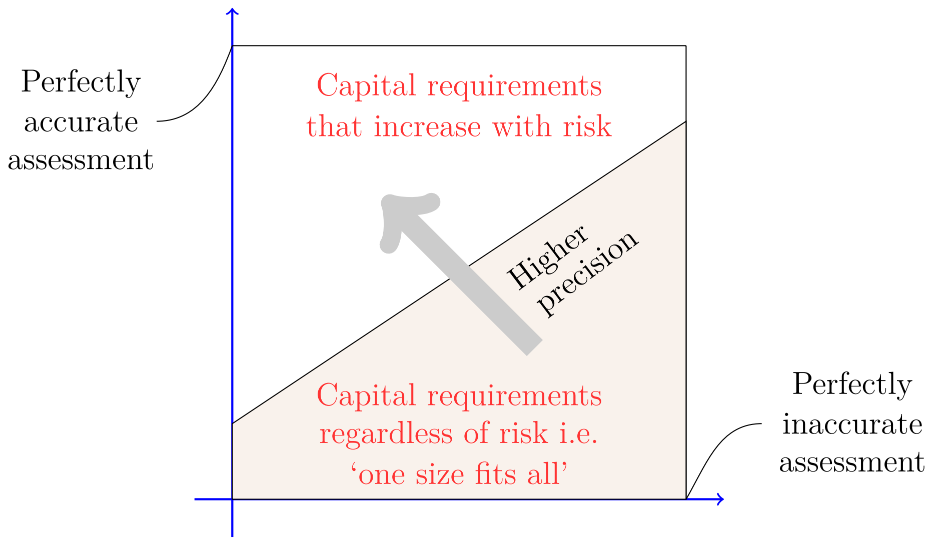
$$\implies D_2U_s(\chi^o+x,\Delta)=-g(k+d)\int_0^{\frac{Rd}{g(k+d)}}f_s(\ )d\ $$

$$\implies D_{21}U_s(\chi^o+x,\Delta)=-\left(g'(k+d)\frac{dd}{dx}\int_0^{\frac{Rd}{g(k+d)}}f_s(\ )d\ +\right.$$

$$\left.g(k+d)\frac{d}{dd}\left[\frac{Rd}{g(k+d)}\right]f_s\left(\frac{Rd}{g(k+d)}\right)\frac{dd}{dx}\right)$$

As x increases, d decreases i.e. $\frac{dd}{dx}<0$. Also, as d increases, the upper limit on the integral is increases (recall $g(.)$ is concave), which means that by application of Leibniz rule, $D_{21}U_s(\chi^o + x, \Delta) > 0$. Since U is a continuous function in both its arguments, $D_{21}U_s(\chi^o + x, \Delta) = D_{12}U_s(\chi^o + x, \Delta) > 0$ for both $s = H, L$.

Optimal capital surcharges depending on risk assessment accuracy



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