

Fine-Tuning a Corporate Hedging Portfolio – The Case of an Airline Company

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Fine-Tuning a Corporate Hedging Portfolio

– The Case of an Airline Company

Abstract

Non-financial companies typically face a multitude of risks potentially leading to significant fluctuations in the firm's cash-flows. This paper presents a case study approach demonstrating the hedging strategy employed by an international air carrier managing its jet-fuel price exposure. We contribute to the academic discussion on an optimal mix of linear and non-linear hedging instruments by providing an in-depth analysis of the firm's hedging policy and its derivative usage. Using the carrier's self-defined implicit objective function based on annual granularity, we show that the air carrier could fine-tune its current hedging portfolio by adding tailored exotic options. Specifically, we develop an *annual* average-price option and incorporate this derivative into the company's hedging portfolio, leading to a superior position in terms of hedging cost and the firm's overall gross exposure to jet-fuel price fluctuations.

Keywords: Corporate hedging policy, exotic derivatives, futures and options, commodity risk

JEL-Classification: F30, G30, G32

1 INTRODUCTION

Notwithstanding the work of Modigliani and Miller (1958), a significant body of literature on corporate risk management has addressed the issue of why a firm should- hedge its risk exposures. Smith and Stulz (1985) highlight three primary motives for hedging by value-maximizing corporations: (1) Asymmetric benefits of taxation, (2) Reduced costs of financial distress, (3) and Managerial risk aversion. In their seminal work, Froot, Scharfstein and Stein (1993) note a sound risk management framework mitigates the underinvestment problem. The authors develop a formal framework based on the assumption that internally generated funds are less-expensive than external financial resources, and that risk management activities are value-enhancing for all corporations regardless of the industry in which they operate. While there exists a large number of academic studies discussing the value-enhancing role of risk management, we note the fewer that provide guidance as to how a firm should hedge its risk exposure.

The corporate risk environment can commonly be divided into business, event and financial risks.¹ Accordingly, firms typically manage their financial exposure trading a wide range of derivative products to cover interest rate, exchange rate and commodity price risks. A number of academic papers cover the issues of establishing an optimal corporate hedging strategy applying linear and nonlinear contracts [Gay et al. (2003); Broll et al. (2001), Kolos and Ronn (2007)]. However, these studies are constrained by the confidentiality of hedging portfolios employed in corporate practice.

Applying a case study approach, this paper contributes to the existing academic literature by presenting an in-depth analysis of the corporate hedging strategy of an international air carrier managing its jet-fuel price exposure.² We compare the hedging policy actually employed by the firm with the academic discussion on an optimal mix of derivative instruments applying theoretical frameworks. In so doing, we present the airline's main strategic aspects – the firm's financial strength, its price and quantity correlations, and its risk profile – when establishing an optimal mix of derivatives. Moreover, we confirm the theoretical findings of Brown and Toft (2001) and Kolos and Ronn (2007) who show non-

¹ See Hommel (2005) for a further discussion on corporate hedging alternatives considering the three different risk categories.

² We follow the reasoning of Yin (1984, p. 23) who advocated the “case study research method as an empirical inquiry that investigates a contemporary phenomenon within its real-life context.”

financial firms are able to fine-tune their hedging portfolios by adding customized exotic options. Accordingly, we first develop a customized exotic derivative – an annual Asian option – while taking into account the air carrier’s specific hedging objectives. In a second step, we show the incorporation of such a derivative into the corporate portfolio leads to superior hedging position in terms of hedging expenditures as well as the firm’s gross exposure to jet-fuel price fluctuations.

The remainder of the paper is organized as follows. Section 2 gives an overview of the academic discussion on corporate hedging strategies and provides an in-depth insight into corporate practice by presenting and evaluating an airline’s hedging strategy to manage its jet-fuel price exposure. In Section 3 we introduce the mathematical derivation for the customized exotic derivative, an annual Asian option. In Section 4 we then value and incorporate the option developed into the carrier’s existing derivative portfolio to fine-tune its hedging strategy. Section 5 concludes.

2 A CASE STUDY APPROACH ON CORPORATE HEDGING

The first part of this section discusses the outcomes of academic studies on an optimal mix of derivative instruments managing financial risks. We present the hedging portfolio established by an international air carrier facing jet-fuel price risk. In so doing, we evaluate the strategic aspects from the firm’s perspective which ultimately lead to the derivative structures commonly applied by the company.

2.1 LITERATURE ON THE OPTIMAL HEDGING PORTFOLIO

Although the mean-variance framework is often used in academic studies, this approach provides significant shortcomings with respect to the performance evaluation of *non*-linear financial instruments in hedging portfolios. As is well-known, usage of options leads to a truncation of the price’s probability function and thus cannot be modeled with the mean-variance symmetric objective function which penalizes both downside risk and upside potential. Secondly, as argued by Lapan et al. (1991) the application of non-linear derivatives lead to a non-monotonic argument in the random variables of the applied utility function. Hence, normally-distributed returns imply linear instruments are superior to options contracts in hedging within a mean-variance framework, since such a framework neglects the asymmetric nature of risk exposures. In contrast, when examining the agricultural sector, other studies have considered quantity risk, multi-period frameworks, and skewed return

distributions. These extensions lead to the incorporation of options-type instruments when establishing an optimal hedging strategy.³

A second strand of literature predominantly focuses on the idea that a firm's main objective is to generate constant profits.⁴ As already pointed out by Froot et al. (1993), this argument follows the assumption that internal funds are more attractive from the firm's perspective as a financing source than externally raised capital. Gay et al. (2003) extend the standard set-up by accounting for financial distress costs. The authors primarily analyze the optimal mix of linear and non-linear derivatives with respect to a firm's specific price and quantity correlation. The researchers are able to show that a firm's optimal hedging portfolio depends critically on the level of correlation between (commodity) prices and realized firm output. Accordingly, a natural hedge exists if the price and the quantity are negatively correlated (with high prices and low output or vice versa), implying the firm's demand for linear hedging instruments declines.

Besides the discussion in respect to an optimal mix of linear and non-linear instruments, a few papers elaborate on the question whether exotic options are able to fine-tune a corporate hedging portfolio. Carr and Chou (1997) show that barrier options provide additional flexibility when applied in a firm's hedging approach. The authors point out that the exotic instruments allow the revision of a plain vanilla portfolio at no cost, once a specific critical price level is reached. A similar result is presented by Brown et al. (2001) who evaluate the benefits of incorporating barrier options into a corporate portfolio in case the firm has a certain market opinion regarding future price developments. Brown and Toft (2001) evaluate how an optimal portfolio of futures and options (plain vanilla and self-developed exotic options) can be constructed under the assumption that a value-maximizing firm faces distress costs. They show that by introducing customized exotic derivatives into a corporation's hedging portfolio, the firm is able to fine-tune its risk exposure due to the additional non-linear payoff component. They highlight the fact that tailor-made exotic options are commonly superior to their plain vanilla counterparts, especially when the

³ See for example the results of Sakong et al. (1993) discussing the role of quantity risk, Lence et al. (1994) applying a multi-period framework, and Vercammen (1995) considering skewed price distributions.

⁴ As already pointed out, the existence of constant profits allows for tax savings, the reduction of financial distress and the mitigation of the underinvestment problem.

correlations between prices and quantities are significantly large and positive.⁵ Similar to other studies, they note that with an increasing hedging horizon, a firm's hedging ratio will typically decrease, and non-linear instruments – especially exotic options - are preferable to their linear counterparts.

Finally, Kolos and Ronn (2007) analyze the problem of a corporation optimally managing its price-and-quantity risk exposure of long or short positions in a commodity using futures or options. Using a mean-value-at-risk objective function, the optimal hedges are derived by considering the trade-off between expected cash-flow maximization and risk minimization. Assuming hedging is costly in the sense the use of futures contracts reduces the *expected* cash flows of the corporation due a risk premium embedded in the futures contract price, they find managerial risk aversion increases the company's optimal hedge to initially acquiring options, then (as risk aversion increases) to replacing the options with futures contracts.

2.2 THE AIRLINE'S HEDGING STRATEGY

Airline companies compete in a market for homogenous goods. Consequently, the majority of market participants have to cope with two main types of financial exposures, exchange rates and commodity price risk. For simplicity, we henceforth ignore exchange rate risks and concentrate on the jet-fuel price risk in evaluating the firm's hedging strategy and its optimal combination of derivatives.

Now, to mitigate the impact of price fluctuations on the corporate cash flows, different sets of financial instruments are applicable. The need to hedge jet-fuel price risk on a corporate level is based on the homogeneity of the product offered by airlines and the highly competitive environment. Hence, passengers would immediately use the service of another airline if the air carrier were unique in passing commodity price increases to its customers. A practical difficulty in direct hedging the jet-fuel exposure is attributable to commodity exchanges not offering standard financial instruments on jet-fuel. In the course of the refining

⁵ Based on their assumptions, the strategy of a value-maximizing firm is superior compared to other hedging strategies such as the minimization of the variance of expected revenues, or the minimum variance framework.

process of crude oil, various products such as diesel fuel, heating oil and kerosene arise.⁶ Accordingly, the underlying for the vast majority of the derivatives utilized in the airline industry is crude oil and to a lesser extent heating oil. The root cause for this is the liquidity of financial contracts written on crude oil and heating oil at the major commodity exchanges (i.e., ICE or NYMEX). That said, the vanilla derivatives (mainly futures and plain vanilla options) bear the major shortcoming they constitute standardized, and thus relatively inflexible, products. Consequently, the company's basis risk potentially increases due to a mismatch in both the time horizon and the delivery location of the derivative. As a result, the discrepancy between a firm's desired hedging horizon and the maturities available for standardized derivative contracts results in a preference for OTC contracts. The high degree of flexibility inherent to OTC derivatives – esp. those which provide for payoffs on *average prices* during a specific delivery month – allows these companies to buy securities that match their financial needs and hedging objectives.

Due to the restrictive access to detailed information regarding derivative usage of non-financial companies, relatively few academic studies have evaluated the hedging portfolio of companies in general, and of airlines in particular.⁷ Consequently, the information demanded is commonly gathered from publicly available sources or through the usage of questionnaires. For example, Tufano (1996) and Bodnar et al. (1995) conducted extensive studies on corporate derivative usage. The first survey tests the notion risk management activities maximize shareholder value by providing an in-depth assessment of the hedging behavior of the North American gold industry. The latter study highlights that corporations commonly hedge their risk exposures, but frequently suffer from imperfect hedging.

For the most part, reports on hedging activities in the airline industry use information from publicly-available sources, such as 10-K reports. These studies mainly focus on the question of whether there is a causal relationship between the firm's hedging activities and firm value. Carter, Rogers and Simkins (2006a, 2006b) evaluate the hedging behavior of U.S. airlines between 1992 and 2003 and conclude jet-fuel hedging is positively related to market values. Their results are partly confirmed by Lin and Chang (2009), who examine a data

⁶ As all of the aforementioned products are based on the same natural resource, the prices of diesel fuel, heating oil and kerosene are highly correlated [Carter, Rogers and Simkins (2004) and Gjolberg and Johnson (1999)].

⁷ Detailed information regarding the actual hedging portfolios can potentially have a direct impact on the firm's stock price and is thus regarded as sensitive information.

sample of 69 airlines from 32 countries. The authors show U.S. airlines engaging in hedging activities increase their firm value by roughly 14%, while not finding evidence of differential valuation between hedging and non-hedging airlines outside the U.S.

Applying a case study approach, we now extend the academic literature by evaluating the hedging decisions of an international air carrier as it constructs its corporate hedging portfolio to manage jet-fuel price uncertainty.⁸ Four major aspects influence the airline's hedging decision: (i) The firm's financial strength and its current credit rating, (ii) its price and quantity correlations, (iii) fixed and variable transaction costs, and (iv) the firm's risk profile.

In discussions with the airline in question, we ascertained certain properties pertaining to the airline's hedging policies. The firm's credit rating is seen as the basis for any legal and contractual agreement the airline negotiates with its counterparties to trade derivatives at the OTC market. This rating thus defines both the scope for its derivative purchases and the amount of trading costs arising in the form of credit risk premiums. An excellent credit rating allows the firm to choose from a broad range of hedging instruments and apply derivative structures with a specific level of risk -- e.g., a collar containing a position in a short call (and a long put option). Secondly, the airline exhibits a positive price-quantity correlation: During times of strong economic conditions both the demand for flights and the prices for kerosene tend to increase. The reverse typically holds during a recession. Both the passenger and the cargo sectors face typical demand fluctuations pervading the airline industry. However, it is generally easier to pass an increase in jet-fuel expenditures through to the (mostly industrial) customers in the cargo sector than it is in the passenger division, due to the strong price sensitivity of this latter market segment. Accordingly, the airline first estimates the jet-fuel exposure for each business segment with respect to the competitive environment and then consolidates the single exposures to define a corporate-wide hedging portfolio. As a consequence, the hedging ratios may differ substantially between the cargo and passenger business segments.

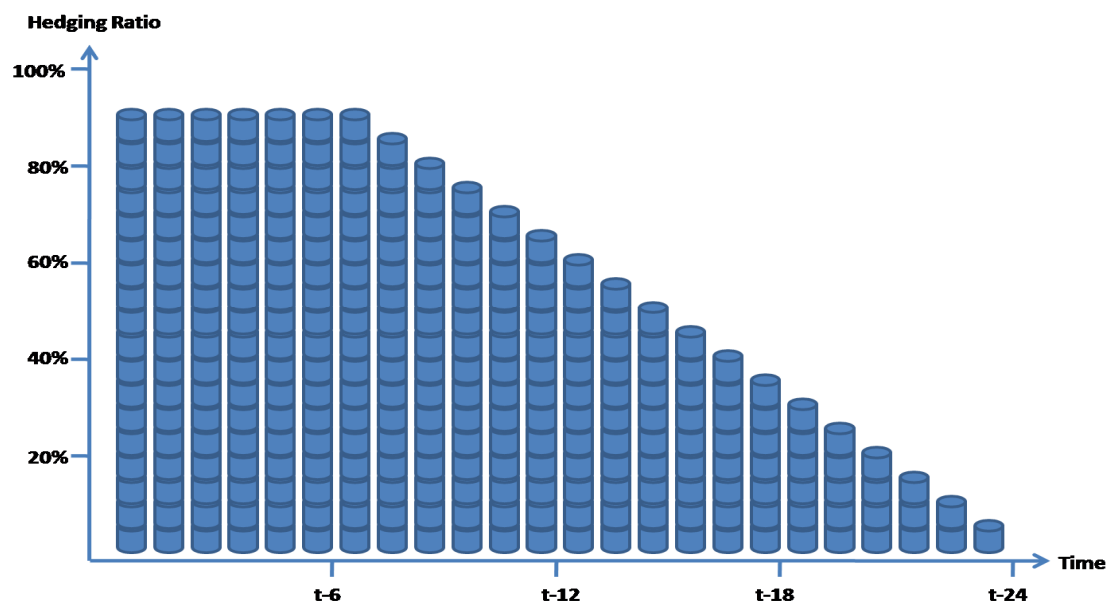
The firm's considerations with respect to fixed and variable transaction costs influence the airline's decision regarding derivative structures used to hedge its jet-fuel price risk. As seen in Section 2.3 below, the variable transaction costs play a key role when determining the firm's derivative usage.

⁸ We abstract from exchange-rate hedging activities.

Finally, the fourth major aspect from the airline's perspective represents its internal "risk profile." This factor is mainly driven by the firm's business strategy with respect to its airfare ticket sales and its flexibility in coping with new market conditions. As a large-scale international company the airline aims to implement a hedging approach which allows the firm to gradually adjust to changes in fuel costs – especially considering an increasing price environment – in order to define, decide and implement the necessary internal changes.

As a result of these considerations, the firm starts trading derivative contracts with a volume of 5% of the expected consumption 24 months before the actual date of usage. When moving closer to the consumption date, the airline continuously adds 5% per month up to a hedge ratio of about 85-90% six months prior to the actual consumption. This specific time horizon of 24 months period is mainly attributable to the flight schedule for the next two years being known with considerable certainty. In order to minimize the risk of overhedging, the firm limits its hedging activities to a net volume of 85%-90% of the expected kerosene consumption. See FIGURE 1.

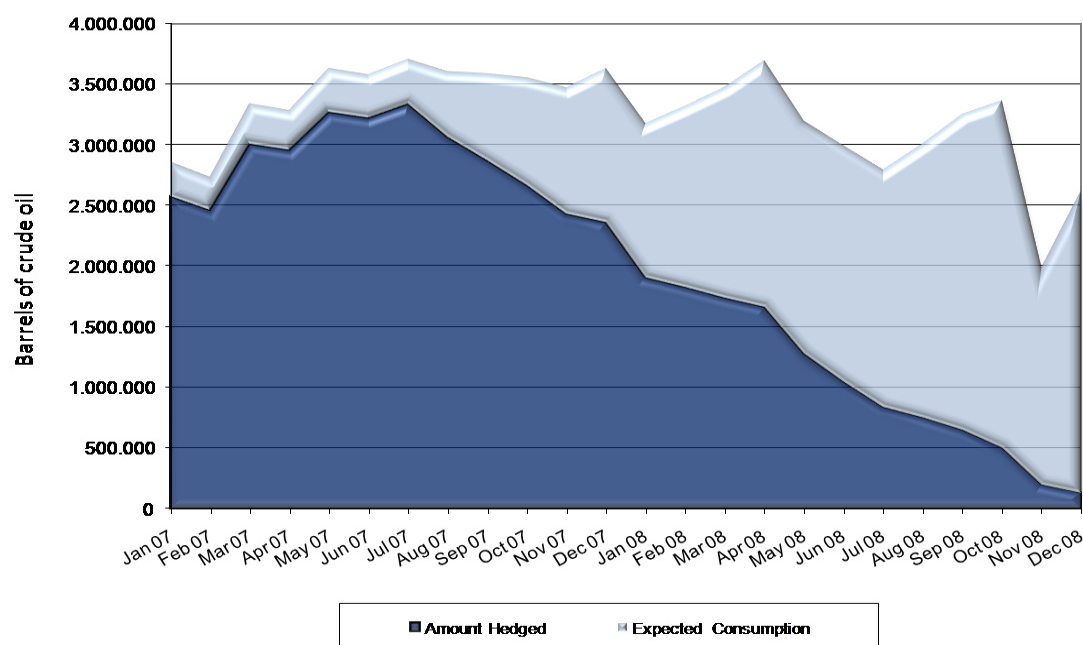
FIGURE 1: CONTINUOUS HEDGING APPROACH EXPRESSED AS A PERCENTAGE OF THE EXPECTED CRUDE OIL CONSUMPTION AT TIME T



This figure shows the layered hedging approach employed by the airline in relative terms. The firm starts trading derivative contracts with a volume of 5% of the expected consumption 24 months before the actual date of usage and then continually adds another 5% per month.

An additional critical aspect is the strong seasonal variation in demand, especially within the passenger transportation sector, with a peak during the summer months June, July and August. Consequently, hedging positions are seasonally adjusted to account for these fluctuations in monthly kerosene consumption. FIGURE 2 provides an outlook on the seasonality in the demand for hedging volume of the company as of 31st of December 2006, with hedging volume measured in barrels of crude oil. See also TABLE B.1.

FIGURE 2: HEDGING VOLUME (IN BARREL OF CRUDE OIL) FOR EACH MONTH BETWEEN 01/01/2007 AND 12/31/2008 (AS OF 12/31/2006)⁹



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This figure shows the layered hedging approach employed by the airline in absolute terms. Seasonal adjustments in hedging volume measured in barrels of crude oil become highlighted.

Comparing the hedge ratio in relative terms of the expected consumption (c.f. FIGURE 1) with the absolute hedging volume in barrels of crude oil, the airline's seasonal adjustment measured in barrels of oil hedged becomes obvious. A strong increase in traded contracts for the late spring and summer months of 2007 is due to the additional number of flights during the holiday season. The hedge ratio throughout the period January to July 2007 stays at 85-90% of the expected consumption; however, the absolute derivative purchases change due to the seasonal fluctuations in jet-fuel usage.

⁹ The results shown for the "Expected Consumption" are estimates based on the hedge ratio presented in Figure 1: I.e., the expected consumption in July 2007 is the product of the total amount hedged during this month multiplied by 1/hedge ratio. The ratio is in this case at roughly 90% at $t-7$ (July 2007) for time t (December 2006).

2.3 THE FIRM'S DERIVATIVE USAGE

Thus far we've presented the firm's strategic thoughts with respect to hedging horizon and the layered hedging approach. The four major aspects (i) – (iv) cannot easily be translated into quantitatively measurable rules. In fact, the company sees these factors more as pillars for the overall hedging strategy without a specific quantitative optimization framework. Management's hedging decisions predominately depend on experience and rules of thumb established over the years than on quantifiable calculations.

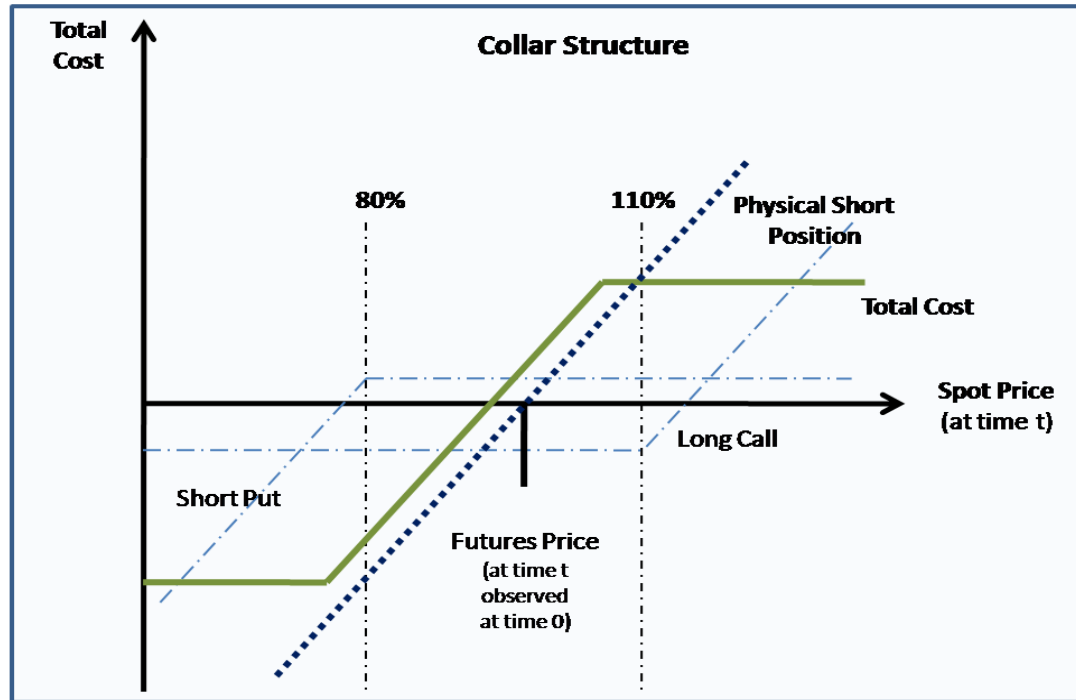
At the same time the airline's considerations of price and quantity correlations as well as hedging costs play also a significant role in the firm's choice of derivative contracts. The firm trades almost exclusively in derivatives on the liquid Brent crude oil commodity market. By purchasing these financial instruments, the firm faces two major disadvantages. First, the derivatives traded are standardized and thus do not perfectly fit the company needs. More importantly, the daily settlement required by the exchanges implies airlines have to maintain substantial cash reserves in order to meet margin calls. Thus, the firm considers such a procedure an unattractive alternative. Consequently, all financial contracts are traded OTC with the major investment banks as counterparties. On the other hand, the ability to trade OTC contracts depends on the firm's credit rating. In the case of a non-investment grade rating, the OTC market is usually inaccessible or can only be done at substantial credit costs which render such trades unattractive for most companies.¹⁰ Thus, the firm's financial strength significantly contributes to its ability to trade on the OTC market.

In contrast to the majority of non-financial institutions who commonly utilize linear instruments such as futures, forward or swap contracts, the airline considered here trades solely in option-type derivatives.¹¹ The payoff for these non-linear instruments depends on the monthly-averaged crude oil price (standard monthly Asian option). This derivative represents a very popular hedging instrument for industrial firms exposed to daily ratable consumption patterns. Due to the averaging effect of the commodity price the decrease in volatility leads to substantial cost savings when purchasing such exotic products.

¹⁰ Tsao and Liu (2008) examine in their paper the influence of credit risk arising in the OTC market on the price of an Asian Option traded accounting for the counterparty's credit risk. The paper of Hull and White (1995) present a model incorporating default risk to value derivative securities.

¹¹ An extensive survey about the usage of financial instruments of non-financial firms in the U.S. and Germany is provided by Bodnar and Gebhardt (1998).

FIGURE 3: PAYOFF STRUCTURE AND TOTAL COSTS OF THE FIRM'S STANDARD HEDGING STRATEGY (PREMIUM COLLAR)



This figure shows the Premium Collar structure most commonly applied by the firm. The costs of an unhedged physical short position (dotted dark blue line) is compared with the airline gross exposure (solid green line) trading a long call and a short put option.

The airline's standard combination of derivatives consists of a long call and short put position requiring net cash payment, or a so-called Premium Collar which is different from the Zero Cost Collar case. Thus, the airline is willing to spend a certain amount on option premiums to profit from a lower call option strike ultimately leading to higher pay-offs in times of increasing oil prices. Accordingly, the firm creates the collar with a long position in an out-of-the-money (OTM) call having a strike at roughly 110% of the respective date- t futures price and a short position in an OTM put with a strike of 80%. The resulting gross exposure (solid green line in FIGURE 3) allows the firm to limit the risk of increasing jet-fuel prices while at the same time participating to a certain extent from decreasing commodity price changes. The company pays the differential between the two option prices. Note the up-

front payment leads to the fact that the firm's total costs are higher compared to an unhedged physical short position until the long call option is in the money.

However, the simple premium collar is not taken as a given structure by the firm. In fact, the airline extends the standard hedging design based on its own price expectations with respect to the oil price. The firm seeks to lower the overall premiums paid by selling additional option contracts. Consequently, in addition to the simple premium collar, the company trades a few variations extending the standard version with either an additional short call (SC) – with a strike price commonly set at 125% of the date- t futures price – or with both an additional position in a short OTM call and a long OTM put (the latter with a strike price of 67%). The gross exposure of both alternatives – the “Collar + SC” and the “Collar + SC & LP” – exhibit unlimited risk in case of increasing kerosene prices. Thus, only price jumps up to roughly 125% of the futures price at time t are hedged with these types of constructs (see the respective payoff structure in Fig. A.2). The firm's decision regarding implementation of a simple collar structure or the extensions with a short calls or long puts respectively depends on both the current market prices for these out-of-the-money options (OTM) and the company's view of future price development of the crude oil price. The effective volatility smiles heavily influence the prices of OTM options. From the company's perspective the smiles in turn depend on the current supply and demand situation for these derivatives products and can be subject to trading decisions of major market players or other market forces. Consequently, in case the price for the short call seems to be high (or low for the long put) from the airline's perspective the firm commonly adds these instruments to its hedge portfolio. As a result, the firm's decision regarding the optimal collar structure strongly depends on the effective variable transaction costs and to a lesser extent on future price expectations.

Comparing the firm's hedging strategy employed (concentrating solely on non-linear instruments) with the results presented by Gay et al. (2003) we are able to highlight a few interesting outcomes. The authors conclude based on their theoretical framework that under “normal market and business operating conditions” (p. 226), firms should manage their exposures primarily with linear instruments. Moreover, they show that fixed transaction costs associated with initiating and maintaining a hedging program will not have a large effect on the optimal hedging position per se, but rather will have a stronger effect on the decision to hedge. The authors argue that variable transaction costs have a relatively larger effect on the

hedging positions, particularly with respect to the optimal non-linear hedging position.¹² We are able to support the latter statement that variable transaction costs represent a meaningful driver regarding the airline's decision to hedge with a simple premium collar or an extended non-linear structure. However, our findings suggest the decision to use linear versus non-linear instruments is conditioned on a number of strategic considerations made by the firm, rather than to the existence or absence of "normal" market conditions.

The abandonment of linear instruments is predominantly attributable to the highly competitive environment and positive price-quantity correlation. In this context, it may also be due in no small part to the contemporaneous existence of so-called "legacy" airlines, whose weak competitive structure precludes their hedging activities, and it is consequently incumbent on stronger market participants to be cognizant of the (price-unhedged) legacy airlines' stronger competitive position when crude-oil prices are low. The airline in question here prefers collar-type structures retaining upside potential (of lower prices) in times of economic downturns typically associated with lower oil prices. Thus, the firm is willing to accept the upfront hedging expenditures in terms of option premiums, in order to maintain a strong competitive position. Moreover, the current credit rating enables the firm to trade options managing its jet-fuel exposure, whereas corporations exhibiting low credit ratings may be unable to voluntarily choose between linear and non-linear instruments.

3 ANNUAL ASIAN OPTIONS

We now turn our focus to a theoretical discussion regarding the advantages of customized exotic options in corporate hedging portfolios. The two most relevant studies in this field are Kolos and Ronn (2007) and Brown and Toft (2001). In their seminal article the authors of the latter study show that by adding customized exotic derivatives into a corporation's hedging portfolio, the firm is able to fine-tune its risk exposure due to the non-linear payoff component. We aim to confirm their findings and present in this section a step-by-step procedure to develop such an exotic derivative – an "Annual Asian Option" – satisfying the specific needs of the air carrier. We price these derivatives using publicly available data on futures contracts and plain vanilla options. Thus, we present in this section the formal description of both a last-month-average option and an annual-average option.

¹² Option premiums can be seen as a part of variable hedging costs.

In comparison to the pay-off structure of a plain vanilla option, the payment of an Asian-type derivative depends on the average price of the underlying security during a specific time period. In the case of crude oil contracts, the determining period is commonly the last month of the contract (Hull 2003). Its popularity is based on the fact the option's payoff corresponds to the firm's physical commodity purchases. A second positive aspect is the lower option premium compared to its plain vanilla counterpart, due to the fact the monthly average price is less volatile than a price at a single point in time at maturity.

From a computational perspective two types of Asian options exist, namely the “geometric mean option” (GMO) and the “arithmetic mean option” (AMO). In a discrete-time world, the payoff function of a geometric mean option depends on the product of the underlying security (S) over a specific time horizon (with n being the number of periods):

$$\left(\prod_{i=1}^n S_i\right)^{1/n},$$

and the payoff to an AMO can be calculated as

$$\frac{1}{n}(\sum_{i=1}^n S_i)$$

The geometric option type is easier to price, since a closed-form solution can be obtained using the Black-Scholes framework (Black and Scholes, 1973), as shown by Kemna and Vorst (1990) and Angus (1999). However, these derivatives are relatively uncommon and only scarcely used in practical applications relative to their arithmetic counterparts (see Milevsky and Posner, 1997). Arithmetic mean options are less easy to price as no closed form solution can be derived.¹³ As a result, the academic literature has developed three alternatives to pricing an AMO.

The first strand of literature uses numerical procedures estimating an approximation for the option value. Most commonly either the Laplace or the Fourier transformation is

¹³ An explanation for these difficulties is given by Milevsky and Posner (1997). The authors state that considering the Black-Scholes environment it is assumed that the option's underlying security is LogNormally distributed. A GMO is then “characterized by the correlated product of LogNormal distributed variables which, conventionally, is also LogNormally distributed” (p. 410). However, an AMO depends on the sum of correlated LogNormal distributed random variables, which is not itself LogNormal. Hence, a closed-form solution cannot be derived for AMOs.

applied to numerically estimate the density function (see e.g., Carverhill and Clewlow, 1990; Geman and Yor, 1993; Rogers and Shi, 1995). The second strand of literature concentrates on Monte-Carlo simulations applying variance reduction techniques with the most popular work presented by Kemma and Vorst (1990).

Finally, the third class of methods contains valuation approaches approximating the density distribution of the arithmetic average which then allows the application of a closed-form solution. Turnbull and Wakeman (1991) are able to show that by applying an Edgeworth Series expansion to the LogNormal distribution an estimate for the arithmetic average can be derived. Due to the fact we will be price our Annual Asian Option based on these authors' framework, we note their main assumptions: (1) the maturity of the average option is never shorter than the averaging period, (2) no transaction costs or taxes, (3) trading takes place continuously, (4) the interest rates' term structure is flat and non-stochastic, and (5) the underlying security is described by a LogNormal probability distribution such as

$$dS = S\alpha dt + S\delta dZ$$

where dZ is characterized by a standard Wiener process whose increments are uncorrelated, α represents the constant mean, and δ^2 is the constant variance of the instantaneous rate of return.

We follow the arguments of Levy (1992) that the usage of first two moments suffices to receive reliable results for the option value applying the approximation of Turnbull and Wakeman (1991). Consequently, an approximation adjusting the first two moments – the mean and the variance – is adequate to be consistent with the exact moments of the arithmetic average (see Appendix A for the standard valuation procedure of an Asian option shown by Haug, 1998, pp. 95-97).

2.3.1 THE LAST MONTH AVERAGE OPTION

In a first step, we present the analytics for the valuation of a “last-month-average option” (LMAO), the derivative instrument currently traded by the air carrier. We show the pricing procedure of an average-style option in which averaging takes place only in the final month prior to expiration. We assume the discrete time case and calculate the average volatility, Σ_T , for the LMAO in a two-step process based on publicly available data – using the plain vanilla options for Brent crude oil options traded at the Intercontinental Exchange (ICE) in London -- for implied plain vanilla volatilities (δ_T^2):

- (1) We first calculate the effective annualized volatility, δ_A^2 , during the option's last month which is lower than the implied plain vanilla one, δ_T^2 , due to the “volatility-dampening” effect of averaging:

$$\delta_A^2 = \frac{1}{t} \ln \left(\frac{2e^{\delta_T^2 t} - (1 + \delta_T^2 t)}{\delta_T^4 t^2} \right) \quad (1)$$

- (2) We then proceed to compute the effective “blended variance,” Σ_T^2 , which combines the variance of the non-averaging period and the variance during the averaged period (with $T \equiv (T - t) + t$):

$$\Sigma_T^2 = \frac{(T - t)\delta_T^2 + t\delta_A^2}{T}$$

The standard deviation can then be calculated as:

$$\Sigma_T = \sqrt{\frac{(T - t)\delta_T^2 + t\delta_A^2}{T}} \quad (2)$$

where T represents the length of the non-averaging period and t the one of the averaging period. By replacing δ_T^2 – the plain vanilla implied volatility -- with the derived “blended volatility,” Σ_T^2 , we are able to value a LMAO applying the standard Black formula for European style options on futures contracts (Black 1976):

$$Call_{LMAO} = e^{-rT} [FN(d) - KN(d - \Sigma_T \sqrt{T})]$$

$$Put_{LMAO} = e^{-rT} [KN(\Sigma_T \sqrt{T} - d) - FN(-d)]$$

where F denotes the current price of the futures contract, $N(.)$ stands for the cumulative normal distribution function, r denotes the risk free rate of interest, T is the time to option

expiration, Σ_T represents the “blended standard deviation”, and $d \equiv \frac{\log(\frac{F}{K})}{\Sigma_T \sqrt{T}} + \frac{1}{2} \Sigma_T \sqrt{T}$.

2.3.2 THE “ANNUAL ASIAN OPTION”

The popularity of standard monthly Asian options can mainly be explained by the averaging effect which naturally simulates the firm's commodity consumption during a calendar month and has the additional benefit of significant cost savings in option premiums. However, the

standard time period in terms of financial or accounting purposes commonly involves twelve months – the firm’s financial year. Hence, a natural extension would be the application of a financial hedging instrument averaging price effects over the whole financial year. The annualized averaging further reduces the high volatility commonly existing in commodity markets leading to an additional decrease in the option price. Consequently, managing price exposures with annual average options allows the firm to benefit from both decreasing hedging costs and protection against strong price variations.

Using publicly available data on futures prices and implied plain vanilla volatilities, the approximation for the annualized volatility is based on the procedure of Turnbull and Wakeman (1991). We consider in the following the case of annual Asian options with maturities ranging from one to twelve months (with $T = 1/12, 2/12, \dots, 1$). In doing so, for each specific futures contract prices are averaged only over the final month of expiration. It thus remains to convert the implied plain vanilla volatility, δ_T , to their last-month-averaged counterparts, Σ_T . As we do so, recall the computation of δ_A in Equation (1). We now add the subscript T to δ_A in order to denote the averaged volatility for month T , δ_{AT} , which can differ across the months due to $\delta_t \neq \delta_\tau$ for $t \neq \tau$. Hence, the only required inputs are the sequence of futures prices ($F_1, F_2, \dots, F_{12}$) and the corresponding implied volatilities of the plain vanilla options ($\delta_1, \delta_2, \dots, \delta_{12}$) which are both publicly available.

Applying then Equation (2) and using δ_{AT} , t , and T , the first two moments can then be calculated as following:

$$M_1 = \frac{1}{N} \sum_{T=1}^N F_T \quad (3)$$

$$M_2 = \frac{1}{N} \sum_{T=1}^N F_T^2 e^{\frac{\Sigma_T^2}{12}} + \frac{1}{N^2} \sum_T \sum_{t \neq T} F_T F_t e^{\left(\frac{\rho_T \Sigma_T \Sigma_t \min(T,t)}{12}\right)} \quad (4)$$

The requisite correlations ρ_{Tt} between the twelve futures contracts can be computed as following (Ronn, 2009):

$$\rho(\Delta t, t_{\min}) = A + (1 - A)e^{-\left(\frac{a}{t_{\min}}\right)} \quad (5)$$

where

$$A = e^{-b|\Delta t|}$$

a, b = two positive coefficients

Δt = time span between two futures contracts (in years)

$t_{\min}$ = time to maturity of the earlier of the two contracts

Finally, the one-year-averaged option volatility, denoted, Σ , is then obtained from the two moments M_1 and M_2 using $\tau = 1$:

$$\Sigma = \sqrt{\frac{1}{\tau} \left(\frac{M_2}{M_1^2} \right)} \quad (6)$$

The valuation of the Annual Asian Option is then computed according to Black (1976). For a given strike price K , and an interest rate r , the option can be valued with (1) the futures price F set to M_1 , (2) the time to expiration set to $\tau = 1$, and (3) the volatility set to Σ .

4 HEDGING FRAMEWORK & RESULTS

As noted in Section 2, the incorporation of exotic derivatives allows a firm to fine-tune its hedging portfolio. As noted in Brown and Toft (2001, p. 1286), “when correlation between price and quantity is positive, exotic derivatives offer additional gains over forwards or options alone, and these gains increase with greater quantity risk and less price risk.”

A positive price and quantity correlation can typically be observed in the airline industry due to demand-side shocks. Accordingly, customer demand in terms of flights and kerosene prices are positive correlated. Note, however, that the opposite relationship holds when considering supply-side disruptions. Hence, we demonstrate the incorporation of an exotic derivative leads to gains in hedging performance when considering a business environment with positive price-quantity correlation and the existence of quantity risk. This section implements an annual average-type option into an existing plain vanilla portfolio.

We first define a number of assumptions to reduce complexity and subsequently highlight the main effect of the exotic derivative. We examine the airline’s hedging portfolio as of December 31st 2007 for the subsequent twelve months, from January to December 2008. Hence, we limit ourselves to considering a time horizon of a financial year instead of the airline’s two-year hedging strategy. Moreover, we assume all option contracts are acquired on December 31st 2007, instead of the monthly-purchase policy currently exercised by the firm.

Furthermore, a constant expected crude oil consumption for each month in 2008 is considered (3,500,000 barrels of oil). The total monthly amount hedged is based on the firm's internal hedging database at the 31st of December 2007 (see TABLE 1). All of our assumptions are solely for the sake of simplicity and should not affect the validity of the results received.

TABLE 1: AIRLINE'S MONTHLY HEDGING POSITION
ASSUMING CONSTANT EXPECTED CONSUMPTION

Expected Consumption of Crude Oil & Monthly Hedged Amount (as of 31/12/2007)			
<i>Calendar Month</i>	<i>Expected Consumption</i>	<i>Total Amount Hedged</i>	<i>Hedge Ratio</i>
<i>2008</i>	<i>(in barrels)</i>	<i>(in barrels)</i>	
January	3,500,000	2,905,245	90 %
February	3,500,000	2,754,165	90 %
March	3,500,000	3,019,940	90 %
April	3,500,000	3,088,265	90 %
May	3,500,000	3,064,382	90 %
June	3,500,000	3,022,932	90 %
July	3,500,000	3,016,057	85 %
August	3,500,000	2,681,067	80 %
September	3,500,000	2,511,617	75 %
October	3,500,000	2,365,867	70 %
November	3,500,000	1,920,050	65 %
December 2	3,500,000	1,807,300	60 %
TOTAL 2008	42,000,000	32,156,890	

Moreover, since a premium collar position is typically applied by the air carrier as the standard hedging structure, we use this structure as the benchmark portfolio for our further analysis.¹⁴ The benchmark portfolio consists of a long monthly Asian OTM call with a strike at 110% of the underlying crude oil futures contract at time t , and a short monthly Asian OTM put with a strike price typically of 80%. In order to apply the pricing formulas derived for

¹⁴ We are aware the airline uses further derivative combinations (premium collar & short OTM call or premium collar & short OTM call + long OTM put). However, to reduce complexity we only focus on a simple collar structure.

both the “last-month-averaging option” and the “annual Asian option” we use publicly-available data on futures prices and implied volatilities for Brent crude oil contracts maturing on a monthly basis from January 2008 to December 2008. In other words, we assume for our research framework that all option contracts are being purchased at a single point in time, at the 31st of December 2007.

TABLE 2: ESTIMATED STRIKE PRICES OF MONTHLY ASIAN CALL AND PUT OPTIONS BASED ON FUTURES PRICES AT 31ST OF DECEMBER 2007

Brent Oil – Futures Price Curve and Option Strikes (in \$)			
Data are shown as of 31/12/2007			
<i>Month</i>	<i>Futures Price - F_t</i>	<i>Call Strike - K_t^{Call}</i>	<i>Put Strike - K_t^{Put}</i>
	(100%)	(110%)	(80%)
January 2008	\$ 93.95	\$ 103.35	\$ 75.16
February 2008	\$ 93.66	\$ 103.03	\$ 74.93
March 2008	\$ 93.48	\$ 102.83	\$ 74.78
April 2008	\$ 93.22	\$ 102.54	\$ 74.58
May 2008	\$ 92.91	\$ 102.20	\$ 74.33
June 2008	\$ 92.58	\$ 101.84	\$ 74.06
July 2008	\$ 92.25	\$ 101.48	\$ 73.80
August 2008	\$ 91.92	\$ 101.11	\$ 73.54
September 2008	\$ 91.62	\$ 100.78	\$ 73.30
October 2008	\$ 91.28	\$ 100.41	\$ 73.02
November 2008	\$ 90.95	\$ 100.05	\$ 72.76
December 2008	\$ 90.65	\$ 99.72	\$ 72.52

TABLE 2 shows the futures price curve and the calculated strike prices for both the call and put options in each month. The prices for the strip of LMAO call and put options can now easily be calculated applying the standard Black framework. To do so, the following information for each month, t , is used: (i) Futures prices on Brent crude oil (F_t), (ii) strike prices for the call and put options (K_t^{Call}, K_t^{Put}), (iii) an interest rate of 3% p.a., and (iv) the “blended volatility” Σ_T , calculated based on Equation (2) using the implied volatilities for plain vanilla options. Comparing the plain vanilla with the monthly averaged volatilities for the calendar year 2008,

a strong decrease due to the dampening effect can be observed, with the most significant reduction obtained for January, which in turn leads to a significant cost reduction in terms of option premiums (see TABLE 3).

TABLE 3: IMPLIED VANILLA VOLATILITIES AND OPTION PRICES
WITH LAST-MONTH-AVERAGED OPTIONS FOR 2008

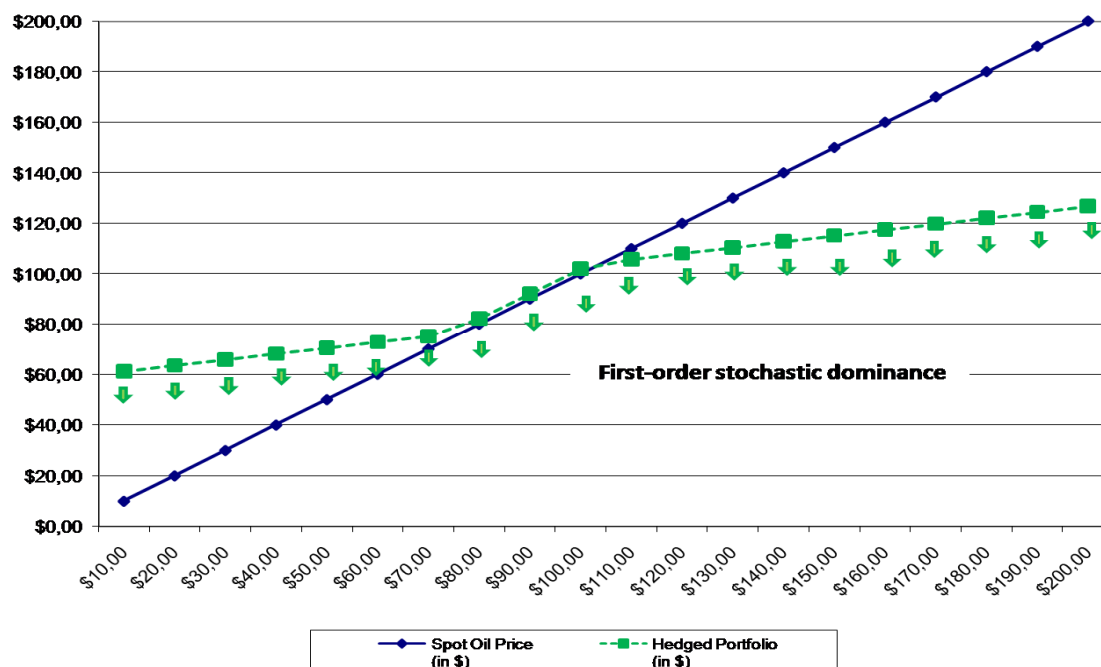
Volatilities and Option Prices (as of 12/31/2007)						
	<i>Volatilities (in %)</i>		<i>Option Prices (in \$)</i>			
	<i>Implied Plain</i>	<i>LMAO</i>	<i>Plain</i>	<i>LMAO</i>	<i>Plain</i>	<i>LMAO</i>
	<i>Vanilla</i>	<i>Volatility</i>	<i>Vanilla Call</i>	<i>Call</i>	<i>Vanilla Put</i>	<i>Put</i>
<i>Jan 2008</i>	32.72%	18.90%	\$ 0.76	\$ 0.66	\$ 0.02	\$ 0.01
<i>Feb 2008</i>	31.43%	25.66%	\$ 1.66	\$ 1.40	\$ 0.17	\$ 0.12
<i>Mar 2008</i>	28.96%	25.54%	\$ 2.12	\$ 1.87	\$ 0.32	\$ 0.23
<i>Apr 2008</i>	26.89%	24.55%	\$ 2.47	\$ 2.22	\$ 0.43	\$ 0.34
<i>May 2008</i>	25.67%	23.90%	\$ 2.79	\$ 2.49	\$ 0.60	\$ 0.44
<i>Jun 2008</i>	25.19%	23.75%	\$ 3.18	\$ 2.73	\$ 0.73	\$ 0.54
<i>Jul 2008</i>	24.69%	23.48%	\$ 3.50	\$ 3.00	\$ 0.88	\$ 0.66
<i>Aug 2008</i>	24.31%	23.28%	\$ 3.81	\$ 3.30	\$ 1.04	\$ 0.80
<i>Sep 2008</i>	24.04%	23.13%	\$ 4.11	\$ 3.60	\$ 1.20	\$ 0.94
<i>Oct 2008</i>	23.70%	22.89%	\$ 4.36	\$ 3.88	\$ 1.34	\$ 1.09
<i>Nov 2008</i>	23.08%	22.37%	\$ 4.48	\$ 4.13	\$ 1.42	\$ 1.23
<i>Dec 2008</i>	21.61%	21.00%	\$ 4.29	\$ 4.09	\$ 1.32	\$ 1.35

Now, the total cost of the benchmark hedging portfolio for calendar-year 2008 applying the premium collar structure can be computed. The airline spends roughly \$85 million on call option premiums and receives about \$20 million by selling the same amount of OTM put options. In summary, the firm spends approximately \$2.02 on variable hedging costs for each barrel of oil consumed (see TABLE B.3). The significant up-front investment can be explained by the fact the price of the call is considerably higher than the put on the same underlying and identical expiration date.

To this point, we have considered the variable cost of establishing the hedging strategy. We now turn to evaluating the firm's hedging portfolio. Academic research developed a number

of formalized frameworks (e.g., mean-variance approach). In this paper we concentrate on the airline's objective function in order to evaluate the effectiveness of alternative hedging structures. The firm gross exposure to jet-fuel price risk is assessed based on a range of oil price scenarios for the calendar year 2008. The company considers 20 different scenarios assuming a constant oil price ranging from \$10 to \$200 per barrel of oil.

FIGURE 4: FIRM'S EXPOSURE APPLYING THE "BENCHMARK PORTFOLIO" (PREMIUM COLLAR)



This figure shows the airline crude oil exposure – in \$ per barrel of oil consumed – applying the “Benchmark Portfolio” (dotted green line) compared to an unhedged position (solid blue line) assuming twenty different oil price scenarios in 2008 (x-axis).

FIGURE 4 illustrates the firm's exposure function applying the “Benchmark Portfolio” structure for each of these oil price scenarios (x-axis) inclusive of option premium costs.¹⁵ Accordingly, the premium collar portfolio (“S”-curve) with the monthly hedge ratio as mentioned in TABLE 1 is compared with a completely unhedged position (the linear line)

¹⁵ The firm's annual exposure for each oil price scenario considering the standard collar position is calculated as following: The airline's total consumption in 2008 sums up to 42,000,000 barrels. The firm hedges 32,156,890 barrels with its option structure. Hence, the airline total exposure is the payoff of the hedge portfolio plus the unhedged amount of 9,843,110 barrels. Consequently, the unhedged volume depends on the oil price scenario considered – ranging from a spot price \$10 to \$200.

showing the case where the firm needs to purchase its total consumption in 2008 on the spot market.

In the following, the “S”-curve serves as our benchmark reflecting the firm’s objective function when assessing the effects of alternative hedging portfolios. That is, we take the firm’s own “S”-curve depiction of risk and return, which uses the implicit assumption a specific variable price (from \$10 to \$200) as prevailing throughout the year. While one might take issue with this objective function, we take what is used as given and then aim to optimize and fine-tune the exposure to create combinations of derivatives which are first-order stochastically dominant for every oil price scenario from \$10 up to \$200. In other words, the goal of our study is to extend the current mix of instruments with the annual Asian option, so that the firm’s exposure – measured in dollars per barrel of Brent oil – is lower for each oil price scenario within 2008.

4.1 THE INCORPORATION OF ANNUAL ASIAN OPTIONS

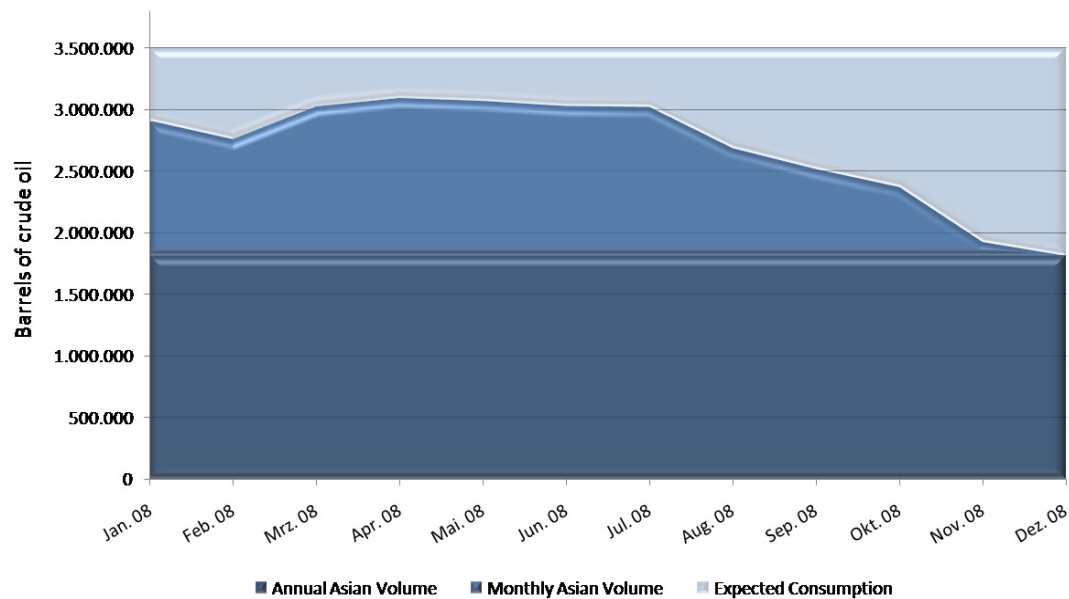
We begin the implementation of AAOs by replacing a certain number of monthly Asian option contracts with annual Asian ones. The arithmetic annual average of the crude oil volatility for 2008 is calculated using Equation (6). This estimate is then used as an input for Black’s formula to compute the value for both the Annual Asian call and put option. Employing the implied volatilities published at the 31st of December 2007, we derive an annual averaged volatility of 14.07% (see TABLE 3) which is significantly lower than the individual monthly averages (compare TABLE 4).

TABLE 4: INPUT VALUES FOR BLACK (1976) FRAMEWORK TO CALCULATE THE ANNUAL CALL AND PUT OPTION PRICES

Input Values for Annual Asian Option			
<i>Futures Price</i> <i>(100%)</i>	<i>Call Strike</i> <i>(110%)</i>	<i>Put Strike</i> <i>(80%)</i>	<i>Arithmetic Volatility</i>
\$ 92.37	\$ 101.76	\$ 74.01	14.07%

The other input values required to calculate the annual Asian option values are easily accessible from publicly available sources assuming the effective date of 31/12/2007: (1) The annual averaged futures price is the arithmetic average of the twelve contracts from January to December 2008 leading to \$92.37 and (2) the strikes for the annual Asian call and put options is set at 110% and 80% resulting in \$101.76 and \$74.01 respectively. The firm's hedging strategy currently established leads to a hedge ratio of 65% of the expected consumption twelve months from today, so that the overall volume hedged for December 2008 adds up to 1,807,000 barrels of oil (compare TABLE B.1). In order to define a reasonable hedging volume for replacing monthly with annual Asian options, we need to avoid the risk of being overhedged. Using a hedge ratio of 65% of the expected consumption for the annual average price options, we replace the December 2008 hedging volume of monthly Asian options with the same amount of annual derivatives for each single month during the calendar year 2008 (more precisely the amount of long call and short monthly put positions, see TABLE B.2). The resulting "Alternative Portfolio" then includes a mix of both monthly and annual Asian call and put options (see FIGURE 2.5).

FIGURE 5: VOLUME HEDGED WITH MONTHLY AND ANNUAL ASIAN OPTION CONTRACTS ON A MONTHLY BASIS APPLYING THE “ALTERNATIVE PORTFOLIO”

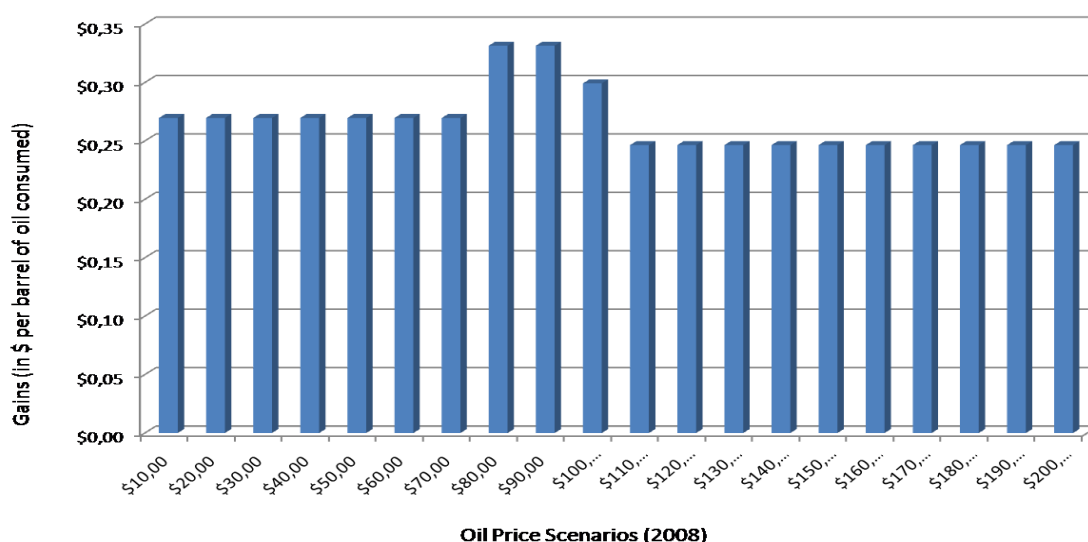


Th

This figure shows the mix of derivative instruments in each month in 2008 applying the “Alternative Portfolio” measured in barrels of crude oil (y-axis). Thereby, two different options are considered: a constant layer of annual Asian options (dark blue area) and a fluctuation layer of monthly Asian option (blue area). The remaining unhedged volume (light blue area) is assumed to be purchased at the spot market.

In the following we compare the “Alternative Portfolio” structure with the “Benchmark Portfolio” in two regards. First, we highlight the aggregated hedging costs in terms of option premiums for each strategy for calendar year 2008. TABLE B.3 present an overview of both the hedging amount (in barrels of oil) and the actual expenditures or earnings for each short and long position on an aggregated basis. The incorporation of annual Asian options leads to a reduced cost of 30 cents per barrel of oil hedged. As a result, the firm needs to spend \$1.72 per barrel on option premiums which is a cost reduction of roughly 15% to the benchmark case. The discount is attributable to the lower volatility ultimately leading to significant savings for the purchase of the long call option contracts.

FIGURE 6: HEDGING EXPOSURE ADVANTAGE OF THE “ALTERNATIVE PORTFOLIO” IN COMPARISON TO THE “BENCHMARK” FOR EACH OIL PRICE SCENARIO IN 2008



This figure shows the cost savings (in \$ per barrel of oil consumed by the airline) applying the “Alternative Portfolio” in comparison to the currently established “Benchmark” considering twenty different oil price scenarios in 2008 (x-axis).

Secondly, we evaluate the firm’s gross exposure on oil price scenarios. Noting we aim to develop alternative hedging strategies which are first-order stochastically dominant for each oil price ranging from \$10 to \$200, FIGURE 6 shows the advantage in gross exposure on a barrel of oil basis of the Alternative Portfolio for each price scenario considered. We are able to demonstrate the decrease in hedging costs is perfectly correlated with an improvement in portfolio value regardless of the oil price in 2008. Consequently, the firm is in the position to purchase every barrel of oil consumed for at least \$0.25 less compared to the Benchmark approach currently established by the firm.

4.2 OPTION STRIKE OPTIMIZATION

We now turn our focus to a further scalar potential. To this point, we accepted the airline’s assumptions of setting the strike prices for the call and put options at 110% and 80%, respectively, of the futures price. In the following we aim to implement an optimization procedure to identify the optimal strikes for both the Annual call and put options assuming the firm’s exposure still needs to be first-order dominant.

Accordingly, the sum of the firm's annual oil price exposure (in \$), E^{opt} , and the aggregated option premiums (in \$), P , is minimized considering each oil price scenario ranging from \$10 to \$200. Moreover, we ensure that the optimized strike portfolio is first order stochastic dominant irrespective of the crude oil price scenario i considered in 2008 – $E_i^{opt} + P^{opt} \leq E_i^{bench} + P^{bench}$. Furthermore, we develop two optimized portfolios. First, we optimize the call option strike – $\$0.01 \leq Call_{Strike}^{annual} \leq \200 – keeping the strike of the put at the predefined price level ($Put_{Strike}^{annual} = \73.94). Secondly, we then optimize the put strike – $\$0.01 \leq Put_{Strike}^{annual} \leq \200 – and hold the call strike fixed ($Call_{Strike}^{annual} = \101.67). Consequently, we can define the following two optimization problems to derive optimized strike price portfolios:¹⁶

$$\min \sum_{i=1}^n E_i^{opt} + P_i \quad (8)$$

$$s.t.: \quad E_i^{opt} + P^{opt} \leq E_i^{bench} + P^{bench}$$

where i represents the oil price scenarios ranging from \$10 ($i=1$) to \$200 ($i=20$), E^{opt} provides the net oil price exposure (in \$) of the optimized portfolio given a specific oil price scenario, E^{bench} the net exposure of the standard collar portfolio, and P^{opt} and P^{bench} are the aggregated \$-expenditures of the option premiums paid for the optimized and the benchmark portfolio, respectively. The following additional restrictions apply considering the optimized call strike portfolio:

$$Put_{Strike}^{annual} = \$73.94, \quad (9)$$

$$\$0.01 \leq Call_{Strike}^{annual} \leq \$200. \quad (10)$$

When developing the optimized put strike portfolio, the Restrictions (9) and (10) are replaced with:

$$Call_{Strike}^{annual} = \$101.67, \quad (11)$$

$$\$0.01 \leq Put_{Strike}^{annual} \leq \$200. \quad (12)$$

Note we do not utilize any specific projections regarding the expected oil price in 2008, as we account for each scenario from \$10 to \$200 equally. We solely aim to optimize the strike

¹⁶ We use the 'Excel Solver' applying the 'Generalized Reduced Gradient' (GRG2) Algorithm for optimizing nonlinear problems developed by Lasdon et al. (1978).

prices of the annual options developing a first-order stochastic dominant hedging portfolio. The optimization of the strike prices is done separately for both the call and the put option strike defined by the Restrictions (9) and (10) for the call strike, and (11) and (12) for the put strike. Thus, in the first case the annual Asian call option strike – currently set at \$101.76 – is optimized keeping the put option strike fixed at \$74.01. In the second optimization problem, the same procedure is applied to the put optimization holding the call strike fixed at \$101.76. In the following we compare the performance of the two hedging alternatives with the current strategy by evaluating both the variable hedging costs and the firm's exposure with respect to the different oil scenarios.

Optimizing our objective function for the call strike leads to a slight reduction from \$101.76 to \$99.93. The hedging costs of the “Optimized Call Portfolio” add \$2.02 per barrel of oil hedged. Consequently, the decrease in the call strike results in a significant increase in premiums for the call options (totaling \$9,507,620). So the firm needs to spend almost the same aggregated amount of hedging costs as compared to the “Benchmark Portfolio”. The optimization of the put strike – holding the call strike fixed at \$101.76 – leads to an optimal value of \$0.01 (which is the lowest value possible in our optimization framework). This implies the firm would find it optimal to waive the capital inflow of premiums received by selling the put options. The advantage of profiting from low crude oil prices below \$74.01 (the initial strike price of the annual put option) is superior for the firm compared to the additional premium income of roughly \$6,000,000 (see TABLE B.3 for comparison of the capital inflow due to the sold put options). The overall spending on option premiums in 2008 is then roughly \$61 million for average hedging costs of \$1.91 representing a reduction of 5.5%.

Evaluating the hedging performance in terms of oil price exposure for the two optimized portfolios, the results for both the “Optimized Call Portfolio” and the “Optimized Put Portfolio” are positive. The slight decrease of the call strike price leads to an average profit of \$0.87 per barrel of oil consumed in 2008 with respect to any oil price scenario between \$110 and \$200. Considering oil prices below \$110 for the calendar year, the firm's risk exposure is almost equal to the benchmark case (see FIGURE B.2). The abandonment of shorting any annual Asian put options (based on the proposed strike price of \$0.01 within our optimization framework) allows the airline to exhibit a superior hedging position for oil-price scenarios of \$ 74.01 and lower. This can be explained by the fact that the premiums collected from shorting puts do not provide an equivalent advantage to offset the exposure in times of

decreasing oil prices. In other words, the firm's ability to purchase its crude oil quantities at the spot market in low-price scenarios (instead of delivering the put option at a price of \$74.01) lead to substantial gains per barrel of oil consumed. Moreover, these gains significantly increase in low spot price scenario allowing the company to establish a strong competitive position in such price environments.

In summary, by moving from monthly average to annual average options, we are able to reduce the cost of the long call but also reduce the cost of the short put. However, the benefit is not as large as it would be with purely long options. Thus, when we turn to long positions in annual average call options, whilst eliminating the short put options, the portfolio does give rise to substantial benefit. As a result, establishing the alternative "Optimized Put Portfolio" allows the firm to purchase low-cost call protection and at the same time waive the premiums earned by selling put contracts¹⁷ and profit from low price oil scenarios (see TABLE B.3). This hedging strategy perfectly meets the airlines objectives and at the same time fine-tunes the firm's kerosene exposure.

5 CONCLUSION

Our work primarily adds two aspects to current research on corporate hedging and the ongoing academic discussion on the optimal mix of hedging instruments for non-financial firms.

First, we present a case-study approach evaluating the hedging strategy employed by a major player in the airline industry facing jet-fuel price risk. We provide an in-depth insight into the strategic considerations of the firm and a detailed overview of the instruments and derivate structures applied. In so doing, we present the major aspects from the firm's perspective leading to its derivative portfolio mix presented: (1) the firm's financial strength and its current credit rating, (2) its price and quantity correlations, (3) its fixed and variable transaction costs, and (4) the hedging activities of its major competitors. We show these aspects naturally lead to the firm's objective function of reducing price risk and retaining "upside capture" due to potentially decreasing prices.

Secondly, we extend the findings of Brown and Toft (2001) showing that non-financial firms are able to fine-tune its hedging portfolio by adding tailor-made exotic options.

¹⁷ The airline still sells the volume of monthly Asian put options above the amount of annual puts it waives.

We develop a customized exotic derivative, an annual averaged price option, satisfying the airline's financing and hedging objectives by averaging price fluctuation throughout the financial year. While we have made simplifying assumptions regarding the depth of oil-market liquidity and abstracted from any intra-year liquidity issues, we show the incorporation of such exotic derivatives provides a superior hedging position compared to the mix of financial instruments currently utilized by the airline. In addition, we are able to provide an analytical framework optimizing the annual option strikes to further fine-tune the airlines gross exposure. We show that sale of annual Asian put options is suboptimal considering the objective function defined. Therefore, we propose an optimal hedging mix of long annual and monthly Asian call options and short monthly Asian put options (our so-called "Optimized Put Portfolio").

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APPENDIX A – Approximation of an Arithmetic Average Option based on Turnbull and Wakeman (1991)

The approximation adjusts the mean and variance so that they are consistent with the exact moments of the arithmetic average. The adjusted mean, b_a , and variance, δ_A^2 , are then used as input in the generalized Black-Scholes formula (1976):

$$Call_{Asian} \approx Se^{(b_A - r)T_2} N(d_1) - Xe^{-rT_2} N(d_2)$$

and

$$Put_{Asian} \approx Xe^{-rT_2} N(d_2) - Se^{(b_A - r)T_2} N(d_1)$$

with

$$d_1 = \frac{\ln(\frac{S}{X}) + (b_A + \delta_A^2/2)T_2}{\delta_A \sqrt{T_2}}$$

$$d_2 = d_1 - \delta_A \sqrt{T_2}$$

$$\delta_A = \sqrt{\frac{\ln(M_2)}{T}} - 2b_A$$

$$b_A = \frac{\ln(M_1)}{T}$$

T_2 as the remaining time to maturity of the Asian Option

The calculation of the exact first and second moments of the arithmetic average is as following:

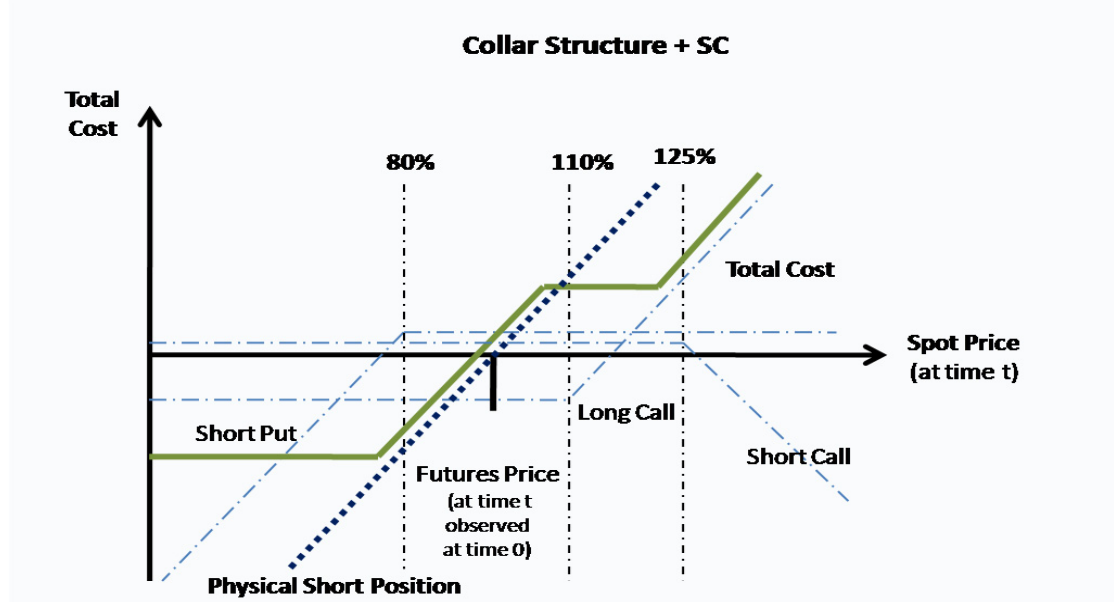
$$M_1 = \frac{e^{bT} - e^{b\tau}}{b(T - \tau)}$$

$$M_2 = \frac{2e^{(2b + \delta^2)T}}{b(T + \delta^2)(2b + \delta^2)(T - \tau)^2} + \frac{2e^{(2b + \delta^2)\tau}}{b(T - \tau)^2} \left[\frac{1}{2b + \delta^2} - \frac{e^{b(T - \tau)}}{b + \delta^2} \right]$$

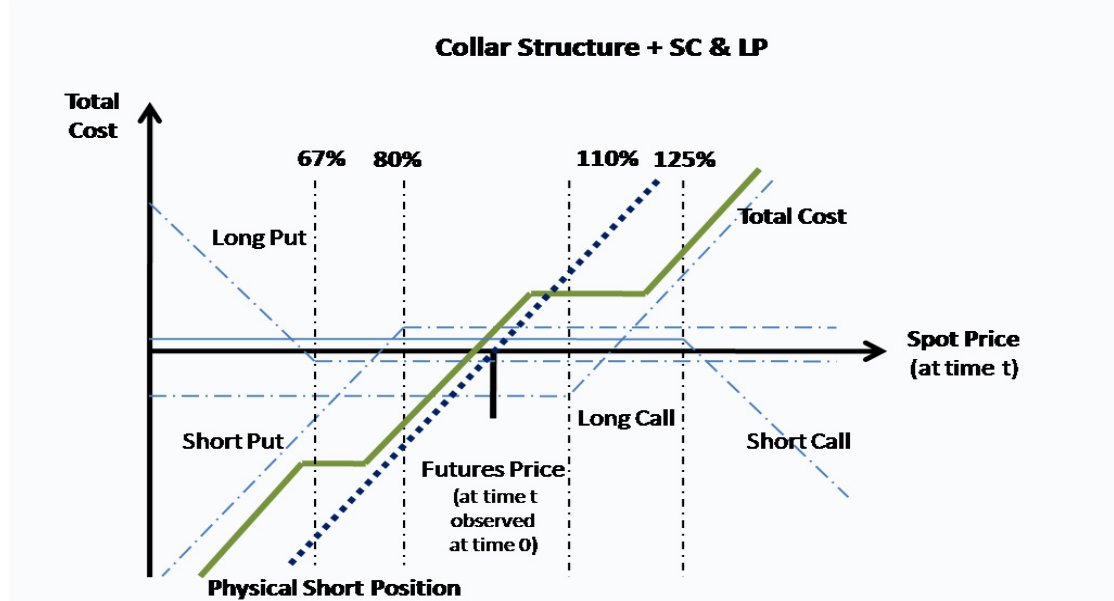
APPENDIX B

FIGURE B.1: MODIFIED COLLAR STRUCTURES COMMONLY TRADED BY THE FIRM

Panel I: Premium Collar plus a short call (SC)



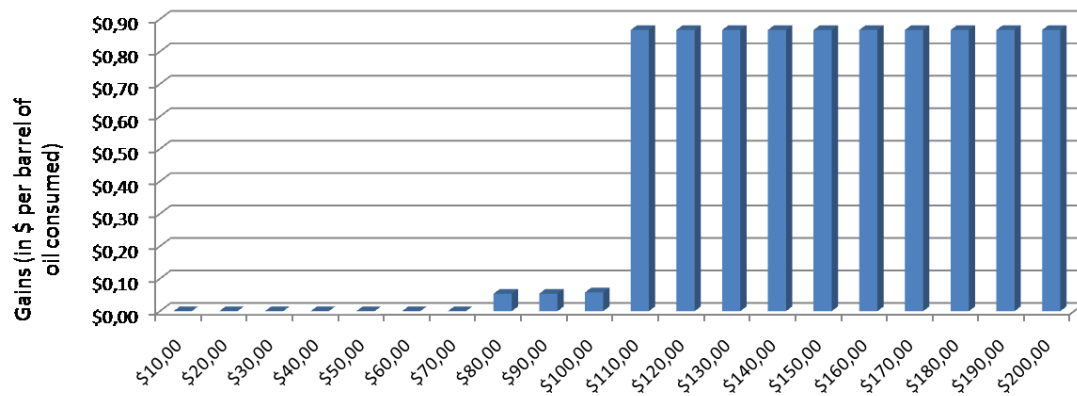
Panel II: Premium Collar plus a short call (SC) and a long put (LP)



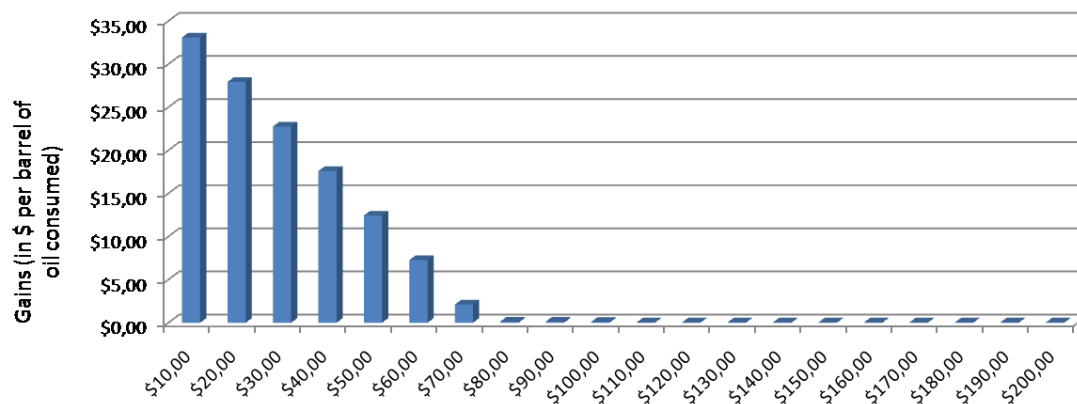
This figure shows the two modified Premium Collar structures commonly applied by the airline. The costs of an unhedged physical short position (dotted blue line) is compared with the airline gross exposure (solid green line) trading (1) a premium collar plus a short OTM call option (Panel I), and (2) a premium collar plus a short OTM call and a long OTM out option (Panel II).

FIGURE B.2: HEDGING EXPOSURE ADVANTAGE OF THE “OPTIMIZED CALL” AND “OPTIMIZED PUT” IN COMPARISON TO THE “BENCHMARK” PORTFOLIO FOR EACH OIL PRICE SCENARIO IN 2008

Panel I: “Optimized Call Portfolio” versus “Benchmark Portfolio”



Panel II: “Optimized Put Portfolio” versus “Benchmark Portfolio”



This figure shows the cost savings (in \$ per barrel of oil consumed by the airline) applying the “Optimized Call Portfolio” (Panel I) and the “Optimized Put Portfolio” (Panel II) in comparison to the currently established “Benchmark Portfolio” considering twenty different oil price scenarios in 2008 (x-axis).

TABLE B.1: VOLUME OF BARRELS OF OIL HEDGED FOR EACH MONTH (AS OF 31/12/2006)

Expected Consumption, Hedged & Unhedged Volume (as of 31/12/2006)				
<i>Month</i>	<i>Expected Consumption¹⁸</i>	<i>Hedged Volume</i>	<i>Spot Market Purchases</i>	<i>Hedge Ratio</i>
<i>Jan 07</i>	2,858 ¹⁹	2,572	286	~ 85-90 %
<i>Feb 07</i>	2,732	2,459	273	~ 85-90 %
<i>Mar 07</i>	3,337	3,003	334	~ 85-90 %
<i>Apr 07</i>	3,281	2,953	328	~ 85-90 %
<i>May 07</i>	3,628	3,265	363	~ 85-90 %
<i>Jun 07</i>	3,575	3,217	358	~ 85-90 %
<i>Jul 07</i>	3,703	3,333	370	~ 85-90 %
<i>Aug 07</i>	3,601	3,060	541	~ 80-85 %
<i>Sept 07</i>	3,584	2,867	717	~ 75-80 %
<i>Oct 07</i>	3,551	2,663	888	~ 70-75 %
<i>Nov 07</i>	3,468	2,428	1,040	~ 65-70 %
<i>Dec 07</i>	3,627	2,358	1,269	~ 60-65 %
<i>Jan 08</i>	3,173	1,904	1,269	~ 55-60 %
<i>Feb 08</i>	3,317	1,824	1,493	~ 50-55 %
<i>Mar 08</i>	3,472	1,736	1,736	~ 45-50 %
<i>Apr 08</i>	3,692	1,661	2,031	~ 40-45 %
<i>May 08</i>	3,194	1,277	1,917	~ 35-40 %
<i>Jun 08</i>	2,990	1,046	1,944	~ 30-35 %
<i>Jul 08</i>	2,792	837	1,955	~ 25-30 %

¹⁸ Expected consumption is being calculated based on the values of total amount hedged times the standard hedge ratio of the airline; so for January it would be 2.572K barrels x 90% = 2.858K barrels.

¹⁹ In thousands of barrels of crude oil.

TABLE B.1 (CONTINUED): VOLUME OF BARRELS OF OIL HEDGED FOR EACH MONTH

<i>Aug 08</i>	3,008	752	2,256	~ 20-25 %
<i>Sept 08</i>	3,250	650	2,600	~ 15-20 %
<i>Oct 08</i>	3,362	504	2,858	~ 10-15 %
<i>Nov 08</i>	1,990	199	1,791	~ 5-10 %
<i>Dec 08</i>	2,625	131	2,494	~ 0-5 %

TABLE B.2. VOLUME OF BARRELS OIL HEDGED UNDER “ALTERNATIVE HEDGING PORTFOLIO”

Expected consumption of crude oil & amount hedged per month				
(as of 31/12/2007)				
<i>Month</i>	<i>Expected consumption</i>	<i>Annual Asian Hedged</i>	<i>Monthly Asian Hedged</i>	<i>Unhedged Volume Spot Purchases</i>
<i>Jan 08</i>	3,500	1,807	1,097	594
<i>Feb 08</i>	3,500	1,807	946	745
<i>Mar 08</i>	3,500	1,807	1,212	480
<i>Apr 08</i>	3,500	1,807	1,280	411
<i>May 08</i>	3,500	1,807	1,257	435
<i>Jun 08</i>	3,500	1,807	1,215	477
<i>Jul 08</i>	3,500	1,807	1,208	483
<i>Aug 08</i>	3,500	1,807	873	818
<i>Sept 08</i>	3,500	1,807	704	988
<i>Oct 08</i>	3,500	1,807	558	1,134
<i>Nov 08</i>	3,500	1,807	112	1,579
<i>Dec 08</i>	3,500	1,807	0	1,692

TABLE B.3: AGGREGATED HEDGING COSTS IN TERMS OF OPTION PREMIUMS AND GROSS EXPOSURE PER BARREL OF OIL CONSUMED

	Benchmark Premium Collar		Alternative Annual Portfolio		Optimized Call Portfolio		Optimized Put Portfolio	
Hedging Instruments	Aggregated Premiums (in Mil. \$)	Volume Hedged (Mil. barrels)	Aggregated Premiums (in Mil. \$)	Volume Hedged (Mil. barrels)	Aggregated Premiums (in Mil. \$)	Volume Hedged (Mil. barrels)	Aggregated Premiums (in Mil. \$)	Volume Hedged (Mil. barrels)
Monthly Call (long)	\$84.549	32.156	\$ 24.640	10.469	\$ 24.640	10.469	\$24.640.096	10.469.290
Monthly Put (short)	- \$19.502	32.156	-\$ 5.125	10.469	-\$ 5.125	10.469	-\$5.125.488	10.469.290
Annual Call (long)	-	-	\$ 41.817	21.687	\$51.229	21.687	\$41.829.091	21.687.600
Annual Put (short)	-	-	-\$ 6.025	21.687	-\$5.930	21.687	-	-
<i>Total cost</i>	\$65.047		\$55.306		\$64.814		\$61.343	
<i>Gross Exposure /barrel consumed</i>	\$ 2,02		\$ 1,72		\$ 2,01		\$ 1,91	
<i>Cost reduction²⁰</i>	-		14.9 %		~ 0 %		5.4 %	

This table shows the aggregated option premiums of the “Benchmark Portfolio” currently established by the airline and the three alternative portfolio including annual Asian options. Moreover, the gross exposure (total cost) in \$ per barrel of oil consumed is shown based on the derivatives’ payoff traded and the remaining spot market purchases.

²⁰ Compared to the “Benchmark Portfolio” (in %)