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Back to the Real Economy: The Effects of Risk Mispricing on the Term Premium and Bank Lending*

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Abstract

Bond markets can plummet or rally on the back of sentiment-driven reactions which are unrelated to fundamentals. Therefore, changes in bond prices can not only be interpreted as reflecting risk but also mispricing of long-term assets. These *perceived* risks can often feed back into the economy by affecting the supply of credit. We construct a DSGE model with heterogeneous banks, asset pricing rules that generate a time-varying term premium, and introduce bond risk mispricing shocks to study their effects on the real economy. A risk mispricing shock, in which agents overprice perceived risk, increases the term premium and lowers output by reducing the availability of credit, as banks increase rates and tighten lending standards. However, when investors underprice risk, a compressed term premium leads to a 'bad' credit boom that results in a more severe recession once the snapback occurs.

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1 Introduction

Bond markets are generally thought to be driven by fundamentals, with prices reflecting the expected future path of short-term assets adjusted for risk. However, markets often plummet or rally on the back of changes to *risk perceptions* that do not necessarily stem from underlying actual *risks* in the economy, suggesting an occasional mispricing of risks in asset markets (e.g. Pflueger et al., 2020; Lewis et al., 2021). If market participants overprice or underprice long-term risk, can these 'mispricing shocks' affect credit markets and feed back to the real economy, potentially threatening financial stability?

In this paper, we develop a Dynamic Stochastic General Equilibrium (DSGE) model with a rich financial sector and bank lending that allows us to investigate the transmission mechanisms through which bond mispricing leads banks to reassess their lending behavior and ultimately impact economic activity. To this end, we introduce risk mispricing shocks, which influence the compensation for bearing long-term risk (the term premium) but are unrelated to economic fundamentals. An overpricing of risk prompts banks to pass on the impact of the shock to the private sector in the form of higher loan rates and reduce the amount of loans to entrepreneurs, as banks protect their profitability and capital-asset ratios. We find that in addition to loans becoming more expensive, a risk mispricing shock also alters the conditions for bank credit approval, increasing the de-facto required collateral necessary to take out a loan, making access to credit yet more difficult. Even without any initial change in underlying economic fundamentals, bond mispricing can lead to a pronounced contraction in investment and economic activity, linking risk mispricing shocks to the real economy via the term premium, through changes in bank lending decisions.

Our contribution to the literature is three-fold: (1) We combine a banking sector that is subject to macroprudential regulation with asset pricing rules for long-term bonds in order to investigate how changes in financial markets affect credit allocation to the real economy. (2) The rich structure of our general equilibrium model and non-linear solution technique enable us to match both macroeconomic and term premium moments to the data. (3) By introducing a wedge to the stochastic discount factor of financial agents, we model a shock that mimics how financial markets and the real economy respond to the mispricing of long-term risk, differentiating our shock from a more standard preference shock.

Our framework allows us to investigate lending conditions by differentiating between a 'good' credit boom driven by economic fundamentals versus a 'bad' credit boom driven by agents underpricing risk in the economy. We find that a bad boom that leads to a compressed term premium despite unchanged fundamentals, induces excessive lending relative to a good boom driven by improvements in technology. Moreover, once agents correct their mistake and price risk accordingly, the term premium snaps back and output falls more sharply during the subsequent bust. A bad credit boom also has stronger effects on financial markets than a good credit boom. Nonetheless, while a bad credit boom still drives investment, it is less supportive of consumption with wages remaining constant, as fundamentals and therefore productivity, remain unchanged. We conclude by highlighting the implications of different macroprudential policies targeted at reducing risk-taking behavior and promoting financial stability and find that higher target capital-asset ratios help mitigate the effects of risk mispricing shocks.

The paper is structured as follows. Section 2 describes the main assumptions and equations of the DSGE model that links the real economy and the financial sector via the term premium. Section 3 outlines our solution method and calibration, compares the model-implied responses of risk mispricing in the bond markets relative to a standard preference shock, and explores the role of different channels in amplifying the effects of risk mispricing. It also includes a detailed robustness section to different parametrization and shock specification, and the responses to other classic macroeconomic and financial shocks in our model. Section 4 explores the richness of our setup by simulating different credit boom-bust scenarios to analyze the effectiveness of macroprudential policies in mitigating financial instability. Section 5 concludes.

2 A DSGE model with bank lending and risk mispricing

First, our model features patient households and impatient entrepreneurs to introduce lenders and borrowers with different sets of preferences. Households choose consumption, labor, and savings to maximize their recursive utility with Epstein and Zin (1989) preferences as in Rudebusch and Swanson (2012) and Van Binsbergen et al. (2012). Epstein-Zin preferences have the advantage that risk aversion can be modeled independently from the intertemporal elasticity of substitution, allowing for bond pricing dynamics to be amplified, while retaining the desirable features of standard preferences. As in Gambacorta and Signoretti (2014) and Bonciani and van Roye (2016), households own the banks that collect their deposits and the monopolistically competitive final-good firms (i.e., retail firms) for which they receive a profit. They also pay taxes to the government. Perfectly competitive entrepreneurs (that are credit constrained) need to borrow from the banks by providing capital as collateral in the spirit of Iacoviello (2005) in order to produce intermediate goods that are sold to retailers. This feature introduces a financial friction via the borrowing constraint referred to as the *collateral channel*, in which changes in the quantity and the value of capital impact consumption and investment decisions by entrepreneurs, generating the Bernanke and Gertler (1995) financial-accelerator effect on the economy. We follow Gerali et al. (2010) by allowing the borrowing constraint to depend on the LTV (loan-to-value) ratio modeled as an exogenous stochastic process. Therefore, independent of bank choices, the LTV ratio varies relative to a steady-state value set by the monetary policy authority and is subject to shocks that directly impact the entrepreneur's ability to borrow.

In addition, entrepreneurs own perfectly competitive capital producing firms who buy undepreciated capital stock and re-sell it to entrepreneurs at a new price taking into account a quadratic adjustment cost, which helps provide the equilibrium condition for the price of capital. As in Kiyotaki and Moore (1997), capital has many functions in this model and thus establishes an important feedback mechanism between the economy and the financial sector. Capital is used (i) in the production of intermediate goods, (ii) as collateral for entrepreneurs' loans, and (iii) as a source of funds for investment. Therefore capital, along with the LTV ratio and the bank lending rate, has a crucial impact on decisions made by entrepreneurs. Finally, monopolistically competitive retail firms owned by the households differentiate the intermediate goods and sell the final goods at a markup price subject to shocks, yielding a New Keynesian Phillips curve that incorporates the recursive preferences of households.

Second, we model the banking sector to intermediate credit in the economy. Similar to Gerali et al. (2010), Gambacorta and Signoretti (2014), and Bonciani and van Roye (2016), banks have branches that perform specific functions under different degrees of market power. As in Gambacorta and Signoretti (2014) and Bonciani and van Roye (2016), bank's demand for deposits is elastic, with the amount of deposits being determined by the households, such that the deposit rate is the same as the risk-free rate. On the other hand, the bank's wholesale branch manages its balance sheet by collecting the deposits from households and issuing wholesale loans under monopolistic competition, such that the interest rate on wholesale loans features a markup over the risk-free rate. The size of the markup, indicating the degree of market power, is governed by the bank's penalty cost for deviating from its target capital-asset ratio as in Gerali et al. (2010) and Gambacorta and Signoretti (2014). This represents the *bank-lending channel* through which the monetary policy pass-through to banks' funding costs becomes incomplete.

Similar to Gerali et al. (2010) and Gambacorta and Signoretti (2014), the bank's retail branch operates under monopolistic competition in the loan market for entrepreneurs. Market power is modeled with a Dixit and Stiglitz (1977) framework such that the loan rate is chosen relative to the wholesale-loan rate, amplified by a constant markup over the marginal cost. Different from their setup, is that our retail branch can also purchase long-term government bonds. The bank, however, takes the rate on long-term government bonds as given, which ultimately influences the loan rate set by the bank. In our model, government bond rates are subject to mispricing; therefore, our extension exposes bank loan rates to shocks that make access to credit more difficult by increasing the loan rate faced by entrepreneurs, a financial friction that we identify as the *loan-price channel*.

Third, we introduce long-term government bonds as a source of investment, similar to Gertler and Karadi (2013), Carlstrom et al. (2017), and Sims and Wu (2021). Government spending is modeled as an exogenous process subject to shocks that can be financed through taxes on households and by issuing government bonds. The supply of government bonds is also modeled exogenously as a stochastic process subject to shocks, but the rate of return on these bonds is determined in the financial markets via asset pricing conditions tied to the future state of the economy. An important extension of the model—and our key contribution to understanding macro-financial linkages—is the introduction of asset pricing rules where the stochastic discount factor of households is subject to risk mispricing shocks right before pricing long-term government bonds that endogenously elevate or compress the term premium, above and beyond economic fundamentals. We hence allow financial markets to misprice risk by introducing a wedge between the actual price of long-term government bonds determined by the level of risk in the economy versus the effective price that is driven by the mispricing of risk. A final and important point is that all debt in the economy is indexed to inflation in order to focus on the real effects of risk mispricing on the economy, rather than on the interaction between inflation risk and asset pricing.

To close the model, the monetary policy authority follows a conventional monetary policy rule that reacts to output and inflation deviations from their steady states similar to Bonciani and van Roye (2016). In addition, the real policy rate is subject to shocks, and the monetary policy authority determines the target capital-asset ratio for banks and sets the steady-state LTV ratio for entrepreneurs, which are then taken as given.

2.1 Households

Household i of unit mass chooses consumption $(c_{h,t})$, labor $(\ell_{h,t})$ and quantity of deposits (d_t) to maximize the recursive utility function

$$V_t(i) = U(c_{h,t}(i), \ell_{h,t}(i)) + \beta_h \left(\mathbb{E}_t V_{t+1}^{1-\xi}(i)\right)^{\frac{1}{1-\xi}}, \tag{1}$$

where β_h is the patience discount factor and ξ is the Epstein-Zin parameter that captures risk aversion. If $\xi = 0$, then $V_t(i)$ collapses to standard preferences. The intra-period utility function is given by

$$U(c_{h,t}(i), \ell_{h,t}(i)) = \frac{c_{h,t}^{1-\psi}(i)}{1-\psi} - \frac{\ell_{h,t}^{1+\phi}(i)}{1+\phi}, \tag{2}$$

with ϕ^{-1} representing the Frisch elasticity of labor and ψ^{-1}, the intertemporal elasticity of substitution (IES). Households receive w_t as (real) compensation for their labor, deposit their savings at the bank for which they receive a risk-free gross return $1 + r_t$ indexed to inflation, pay taxes to the government (T_t), and receive profits in real terms from the monopolistically competitive retail firms they own $(J_{R,t})$, such that their budget constraint is given by

$$c_{h,t}(i) + d_t(i) \leq w_t \ell_{h,t}(i) + (1 + r_{t-1}) d_{t-1}(i) - T_t(i) + J_{R,t}(i). \tag{3}$$

The first order conditions yield a standard labor supply schedule, $\ell_{h,t}^{\phi} = \frac{w_t}{c_{h,t}^{\psi}}$, and Euler equation,

$$1 \;=\; \mathbb{E}_t\left[m_{t,t+1}^h (1 + r_t)\right], \quad \text{for} \quad m_{t,t+1}^h = \beta_h \frac{c_{h,t}^{\psi}}{c_{h,t+1}^{\psi}} \left(\frac{\left(\mathbb{E}_t V_{t+1}^{1-\xi}\right)^{\frac{1}{1-\xi}}}{V_{t+1}}\right)^{\xi}, \tag{4}$$

where $m_{t,t+1}^h$ represents the real stochastic discount factor of the households. Rudebusch and Swanson (2012) show that with Epstein-Zin preferences, the consumption risk faced by households rises as the state of the economy worsens, increasing the model risk premium via the ξ parameter. Intuitively, these preferences imply that households are not just concerned with smoothing their consumption once sudden shocks are realized in the short term, but are also concerned with medium- and longer-term changes, allowing long-term risk to play a role in their decision-making process.

2.2 Entrepreneurs

Entrepreneurs operate under perfect competition to produce a homogeneous intermediate good, employ households, and consume, but also need to borrow from banks by providing capital as collateral. They form the link between the real economy and the banking sector and are thus important for generating a feedback loop between the financial and macroeconomic sides of the model. For simplicity, we assume that entrepreneurs have constant relative risk aversion, as they, unlike households, do not invest or price financial assets. Therefore, entrepreneur j of unit mass chooses consumption ($c_{e,t}$), labor demand ($\ell_{d,t}$), amount of bank loans ($b_{e,t}$), and capital stock (k_t) in order to maximize utility subject to budget and borrowing constraints,

$$\begin{aligned}
\max_{\{c_{e,t}(j),\ell_{d,t}(j),b_{e,t}(j),k_t(j)\}} \quad & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_e^t \log\left(c_{e,t}(j)\right) \qquad (5)\\
\text{s.t.} \quad & c_{e,t}(j) + (1+r_{b,t-1})b_{e,t-1}(j) + w_t \ell_{d,t}(j) + q_{k,t}k_t(j)\\
& \leq \frac{y_{e,t}(j)}{x_t} + b_{e,t}(j) + q_{k,t}(1-\delta_k)k_{t-1}(j)\\
& b_{e,t}(j) \leq \frac{\Omega_t \mathbb{E}_t\left[q_{k,t+1}k_t(j)(1-\delta_k)\right]}{1+r_{b,t}}.
\end{aligned}$$

β_e represents the entrepreneurs' patience discount factor applied to their stream of future utility. For small enough shocks, Iacoviello (2005) shows that $\beta_h > \beta_e$ makes entrepreneurs more impatient than households, ensuring that the borrowing constraint is binding around the steady state and credit is constrained in the economy. Therefore, households in this model are considered net lenders, while entrepreneurs are net borrowers. In the budget constraint, $1 + r_{b,t}$ is the gross interest rate on bank loans indexed to inflation; $q_{k,t}$ is the real price of capital, which depreciates at rate δ_k; and x_t represents the gross markup price of the final good over the intermediate good. $y_{e,t}(j)$ is the intermediate output produced by entrepreneur j, which follows a standard Cobb-Douglas form $y_{e,t}(j) = A_t k_{t-1}^{\alpha}(j)\, \ell_{d,t}^{1-\alpha}(j)$, where α denotes the capital share and A_t is an exogenous stochastic AR(1) process with persistence ρ_a, and i.i.d. technology shock $\varepsilon_{a,t}$ with variance σ_a^2.

The entrepreneurs are also subject to a borrowing constraint that limits how much can be borrowed from banks, depending on (i) the stock of physical capital they posses and can thus be offered as collateral, (ii) the expected price of capital stock, (iii) the lending rate set by the bank, and (iv) the LTV ratio Ω_t, modeled as an AR(1) process with persistence ρ_Ω, and i.i.d. shock $\varepsilon_{\Omega,t}$ with variance

σ^2_{Ω}. A high target LTV ratio, which can be interpreted as a high amount of loans relative to the future value of assets, implies that banks can lend more for the same amount of collateral in the steady state. The first order conditions yield the labor demand schedule $w_t = \frac{(1-\alpha)y_{e,t}}{\ell_{d,t}x_t}$ and the entrepreneurs' Euler equation

$$1 = \mathbb{E}_t\left[m^e_{t,t+1}\left(1+r_{b,t}\right)\right], \quad \text{for} \quad m^e_{t,t+1} = \beta_e\left(\frac{1}{c_{e,t}} - \lambda_{e,t}\right)^{-1}\mathbb{E}_t\frac{1}{c_{e,t+1}}, \tag{6}$$

where $m^e_{t,t+1}$ is the entrepreneur's real stochastic discount factor, in which the Lagrange multiplier on the borrowing constraint, $\lambda_{e,t}$, represents the discounted marginal value of one unit of additional borrowing.

Finally, the investment Euler equation equalizes the marginal benefit to the marginal cost of saving capital. Since capital serves as collateral, the equation also depends on the discounted Lagrange multiplier of the entrepreneur and the stochastic LTV ratio,

$$\frac{q_{k,t}}{c_{e,t}} = \mathbb{E}_t\beta_e\frac{1}{c_{e,t+1}}\left(r_{k,t+1} + q_{k,t+1}(1-\delta_k)\right) + \lambda_{e,t}\left(\frac{\Omega_t\mathbb{E}_t\left[q_{k,t+1}(1-\delta_k)\right]}{1+r_{b,t}}\right), \tag{7}$$

where $r_{k,t}$ is the return to capital defined as $r_{k,t} \equiv \alpha\frac{A_t k^{\alpha-1}_{t-1}\ell^{1-\alpha}_{d,t}}{x_t}$.

2.3 Capital good producers

Capital good producers are perfectly competitive and their main task is to transform the old, undepreciated capital from entrepreneurs to new capital without any additional costs. In addition, capital producers 'invest' in the final goods bought from retailers, which are not consumed by households, and also transform these into new capital. They then resell the new capital to entrepreneurs in the next period. Taking the entrepreneurs' stochastic discount factor from Eq. (6) as given, capital producers maximize their expected discounted profits accounting for the law of motion of physical capital, $k_t = (1-\delta_k)k_{t-1} + \left(1 - \frac{\kappa_i}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right)I_t$, such that

$$1 = q_{k,t}\left(1 - \frac{\kappa_i}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 - \kappa_i\left(\frac{I_t}{I_{t-1}} - 1\right)\frac{I_t}{I_{t-1}}\right) + \beta_e\mathbb{E}_t\left[\frac{c_{e,t}}{c_{e,t+1}}q_{k,t+1}\kappa_i\left(\frac{I_{t+1}}{I_t} - 1\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right], \tag{8}$$

where I_t represents aggregate investment and κ_i governs the importance of the adjustment cost associated with the production of the final good.

2.4 Retailers

Let $y_t(v)$ be the final quantity of output sold by retailer v of unit mass and $P_t(v)$ the associated price. Given the final-good production technology: $y_t = \left(\int_0^1 y_t(v)^{\frac{\epsilon_{y,t}-1}{\epsilon_{y,t}}} dv\right)^{\frac{\epsilon_{y,t}}{\epsilon_{y,t}-1}}$, where $\epsilon_{y,t}$ represents the elasticity of substitution between differentiated final goods, the retailer's demand for consumption goods is $y_t(v) = \left(\frac{P_t(v)}{P_t}\right)^{-\epsilon_{y,t}} y_t$, with aggregate price level $P_t = \left(\int_0^1 P_t(v)^{1-\epsilon_{y,t}} dv\right)^{\frac{1}{1-\epsilon_{y,t}}}$. The monopolistically competitive retailers differentiate the intermediate goods from entrepreneurs sold at wholesale price $P_{W,t}$ at no cost and sell them with a markup in order to maximize the expected discount value of profits, subject to the demand for consumption goods:

$$\begin{aligned} \max_{\{P_t(v)\}} \quad J_{R,t}(v) = \quad & \mathbb{E}_{t-1} \sum_{t=1}^{\infty} m^h_{t-1,t} \left[P_t(v) y_t(v) - P_{W,t} y_t(v) - \frac{\kappa_\pi}{2} \left(\frac{P_t(v)}{P_{t-1}(v)} - 1 \right)^2 P_t y_t \right] \qquad (9) \\ \text{s.t.} \qquad & y_t(v) = \left(\frac{P_t(v)}{P_t} \right)^{-\epsilon_{y,t}} y_t. \end{aligned}$$

Since retail firms are owned by the households, who keep their profits, retailers take their stochastic discount factor as given and face quadratic price adjustment costs parametrized by κ_π, which cause prices to be sticky (Rotemberg, 1982), and yield the New Keynesian Phillips curve,

$$0 = 1 - \frac{\mu_{y,t}}{\mu_{y,t}-1} + \frac{\mu_{y,t}}{\mu_{y,t}-1} \frac{1}{x_t} - \kappa_\pi (\pi_t + 1) \pi_t + \mathbb{E}_t \left[\frac{m^h_{t,t+1}}{m^h_{t-1,t}} \kappa_\pi (\pi_{t+1} + 1) \pi_{t+1} \frac{y_{t+1}}{y_t} \right], \qquad (10)$$

where $\frac{P_t}{P_{t-1}} = \pi_t + 1$ is the gross inflation rate, $\frac{1}{x_t} = \frac{P_{W,t}}{P_t}$, and $\epsilon_{y,t} = \frac{\mu_{y,t}}{\mu_{y,t}-1}$. The firm's price markup, $\mu_{y,t}$, is assumed to follow a stochastic AR(1) process with an autocorrelation coefficient $\rho_{\mu y}$ and an i.i.d. price markup shock, $\varepsilon_{\mu y,t}$, with variance $\sigma^2_{\mu y}$.

2.5 Banks

The banking sector consists of a wholesale and a retail branch. The wholesale branch manages the overall balance sheet of the bank, such that bank loans are financed through deposits and bank capital. This branch therefore chooses the amount of deposits and total bank lending (b_t) in order to maximize the discounted sum of cashflows subject to a standard balance sheet constraint:

$$\max_{\{b_t\}} \quad \mathbb{E}_{t-1} \sum_{t=1}^{\infty} m_{t-1,t}^h \left[(1+r_{w,t})\, b_t - b_{t+1} + d_{t+1} + \Delta k_{b,t+1} - (1+r_t)\, d_t - \frac{\theta}{2} \left(\frac{k_{b,t}}{b_t} - \nu \right)^2 k_{b,t} \right] \tag{11}$$
$$\text{s.t.} \quad b_t = d_t + k_{b,t},$$

where $1 + r_{w,t}$ is the gross interest rate on total bank lending (or wholesale loans) and $k_{b,t}$ is bank capital, which accumulates out of reinvested profits. The bank's optimal choices are subject to a quadratic adjustment cost of deviating from a target capital-asset ratio, ν, which is exogenously set by the monetary policy authority and is parametrized by θ. This setup yields the following condition for the net interest rate on wholesale loans,

$$r_{w,t} = r_t - \theta \left(\frac{k_{b,t}}{b_t} - \nu \right) \left(\frac{k_{b,t}}{b_t} \right)^2, \tag{12}$$

that relates $r_{w,t}$ to the deposit rate and the capital position of the bank relative to its target. When $\frac{k_{b,t}}{b_t} = \nu$, the wholesale rate equals the deposit rate, as in the case of no market power. Therefore, the difference between the wholesale and the risk-free rates widens the more over- or under-capitalized a bank is, with θ regulating the magnitude of the level shift relative to the risk-free rate; the *bank-lending channel.*

The bank's retail branch buys wholesale loans from its wholesale branch at the given rate $r_{w,t}$, differentiates them at no cost, and either lends to entrepreneurs to earn $r_{b,t}$ interest rate or purchases $b_{l,t}$ government bonds to earn $r_{l,t}$ interest rate. While the bank chooses the interest rate on loans subject to the entrepreneurs' Dixit-Stiglitz demand for debt, the interest rate on long-term government bonds is determined by asset pricing relationships and thus taken as given. The retail branch therefore sets the loan rate that maximizes profits for bank n subject to the total assets condition, i.e., bank assets are comprised of loans to entrepreneurs and government bonds, and the demand for loans to entrepreneurs,

$$\max_{\{r_{b,t}(n)\}} \mathbb{E}_{t-1} \sum_{t=1}^{\infty} m_{t-1,t}^h \left[r_{b,t}(n)\, b_{e,t}(n) + r_{l,t} b_{l,t}(n) - r_{w,t} b_t(n) \right] \tag{13}$$
$$\text{s.t.} \quad b_t(n) = b_{e,t}(n) + b_{l,t}(n)$$
$$b_{e,t}(n) = \left(\frac{r_{b,t}(n)}{r_{b,t}} \right)^{-\epsilon_b} b_{e,t},$$

where $b_{e,t}(n)$ depends on overall loan volumes and on the interest rates charged by bank n relative to the average rate prevailing in the economy, with constant elasticity of substitution equal to $\epsilon_b > 1$. The equilibrium condition for the bank leads to the net loan rate depending on the wholesale-loan rate, the interest rate on long-term government bonds, and a constant markup, after imposing a symmetric equilibrium condition:

$$r_{b,t} = (r_{l,t} + r_{w,t}) \frac{\epsilon_b}{\epsilon_b - 1}. \tag{14}$$

Notice that, everything else held constant, a shock that increases the long-term government bond rate would also increase the loan rate charged by retail banks, making credit more expensive for entrepreneurs. This is effectively the *loan-price channel* through which increases in long-term government bond rates via risk mispricing make borrowing less attractive.

Finally, bank profits $(J_{b,t})$ from both branches,

$$J_{b,t} = r_{b,t}b_{e,t} + r_{l,t}b_{l,t} - r_t d_t - \frac{\theta}{2}\left(\frac{k_{b,t}}{b_t} - \nu\right)^2 k_{b,t}, \tag{15}$$

get reinvested as bank capital every period, which is used by the bank at rate δ_b under the following accumulation equation:

$$k_{b,t} = (1 - \delta_b) k_{b,t-1} + J_{b,t-1}. \tag{16}$$

2.6 Government sector

For simplicity, we assume that government spending, G_t, follows a stationary AR(1) process

$$G_t = (1 - \rho_g)\overline{G} + \rho_g G_{t-1} + \varepsilon_{g,t}, \tag{17}$$

where $\overline{G}$ is the steady state value of government spending, ρ_g captures the degree of persistence in fiscal policy, and $\varepsilon_{g,t}$ is a government spending i.i.d. shock with variance σ_g^2. Government spending is financed through taxes on households and through debt held by banks, modeled as an exogenous AR(1) process around the bond supply steady state $\left(\overline{b_l}\right)$ with autocorrelation coefficient ρ_{bl},

$$b_{l,t} = (1 - \rho_{bl})\overline{b_l} + \rho_{bl} b_{l,t-1} + \varepsilon_{bl,t}, \tag{18}$$

which is subject to i.i.d. shocks $\varepsilon_{bl,t}$ with variance σ_{bl}^2. Finally, government spending is subject to the standard budget constraint:

$$G_t + (1 + r_{l,t-1})\, b_{l,t-1} = T_t + b_{l,t}. \tag{19}$$

2.7 Asset pricing and risk mispricing

Long-term government bonds are default-free securities issued by the fiscal authority that pay a geometrically declining coupon every period in perpetuity, normalized to 1.[1] Let $p_{l,t}$ be the price of the long-term government bond at time t, where δ_c is the coupon decay rate that controls the duration of the bond, discounted by the household's stochastic discount factor subject to risk mispricing:

$$p_{l,t} = \mathbb{E}_t \left[\left(m_{t,t+1}^h + \epsilon_{risk,t} \right) (1 + \delta_c p_{l,t+1}) \right], \tag{20}$$

where risk mispricing, $\epsilon_{risk,t}$, is an exogenous AR(1) process with autocorrelation coefficient ρ_{risk} and i.i.d. shock $\varepsilon_{risk,t}$ with variance σ_{risk}^2,

$$\epsilon_{risk,t} = \rho_{risk}\epsilon_{risk,t-1} - \varepsilon_{risk,t}. \tag{21}$$

Notice that our risk mispricing shock is not a preference shock, as it only changes the level of the stochastic discount factor when pricing bonds, acting more like a wedge when it comes to asset prices rather than a fundamental change in agents' beliefs.[2] This setup is intuitive in that one can imagine how events or news that trigger a higher perception of risk might cause investors to change their portfolio allocations immediately yet not alter their consumption patterns on impact.

The rate that equates the price of the long-term bond today to the present discounted value of cashflows on perpetuity is given by

$$r_{l,t} = ln \left(\frac{\delta_c p_{l,t} + 1}{p_{l,t}} \right), \tag{22}$$

[1] This is equivalent to assuming that government bonds are infinitely-lived *consol-style* bonds as in Chin et al. (2015). The purpose of this assumption is to reduce the pricing relationship to just one recursive equation in the model, rather than having to solve for each maturity level. As shown in Rudebusch and Swanson (2012), this simplification still generates equivalent results to using ten-year zero-coupon bonds, while significantly reducing the computational burden.

[2] The idea that there is a wedge between the actual household's stochastic discount factor and the discount factor used to price risk premia has been micro-founded for example by Barillas and Nimark (2017), who priced bonds subject to speculative behavior of individual traders, and Ellison and Tischbirek (2021), who decomposed the real term premium at any maturity into covariances of realized stochastic discount factors and covariances of expectations of stochastic discount factors that differ due to informational assumptions.

and the risk-neutral present value of the bond, $\widehat{p}_{l,t}$ (i.e., discounted at the risk-free rate) is

$$\widehat{p}_{l,t} = \mathbb{E}_t \left[\frac{1 + \delta_c \widehat{p}_{l,t+1}}{1 + r_t} \right], \tag{23}$$

with the standard risk-neutral rate, $\widehat{r}_{l,t}$, given by

$$\widehat{r}_{l,t} = ln \left(\frac{\delta_c \widehat{p}_{l,t} + 1}{\widehat{p}_{l,t}} \right). \tag{24}$$

As guaranteed by the absence of arbitrage in the bond markets, we compute the term premium ($tp_{l,t}$) as the difference between the yield on the long-term bond and the yield on the equivalent risk-neutral bond:

$$tp_{l,t} = r_{l,t} - \widehat{r}_{l,t}, \tag{25}$$

reflecting the compensation that risk-averse investors require in order to be exposed to long-term risk.[3] We model risk mispricing as an AR(1) process following the empirical evidence supporting the idea that investors' perceived changes in the term premium are persistent (see Adrian et al., 2013).[4] The risk mispricing shock in Eq. (21) enters the stochastic discount factor that prices government bonds negatively, so that a positive shock lowers the marginal utility growth rate, as prospects for the future valuation of bonds worsen. Since the demand (and hence price) of long-term bonds goes down, long-term yields increase via the term premium, while the risk-neutral yield remains unaffected by the shock.

2.8 Monetary policy

Monetary policy follows a standard Taylor rule, in which the central bank sets the one-period real rate according to

$$\frac{1 + r_t}{1 + \overline{r}} = \left(\frac{1 + r_{t-1}}{1 + \overline{r}} \right)^{\rho_r} \left(\left(\frac{1 + \pi_t}{1 + \overline{\pi}} \right)^{\phi_\pi} \left(\frac{y_t}{\overline{y}} \right)^{\phi_y} \right)^{1-\rho_r} (1 + \varepsilon_{r,t}), \tag{26}$$

where ρ_r is the interest-rate smoothing coefficient, $\{\overline{r}, \overline{\pi}, \overline{y}\}$ are the real interest rate, inflation and

[3] An important assumption for a positive, time-varying term premium is that the expectations hypothesis therefore does not hold and households are allowed to be risk averse.

[4] Li et al. (2017b) empirically estimated the effect of changes in U.S. term premia, concurrent with changes in implied U.S. equity volatility and the broad dollar exchange rate index, and found that the effect of a U.S. term premium shock is persistent with a significant estimate of the autocorrelation coefficient of 0.78. Using a term structure model of interest rates, Osterrieder and Schotman (2017) found that when allowing for stronger persistence of interest rate shocks, e.g. with fractional integration I(0.89), the correlation between risk prices and the spot rate becomes negative, matching the volatility observed in the data. This assumption is also consistent with the literature on uncertainty or credit risk shocks being persistent, as in Christiano et al. (2014) and Sims and Wu (2021).

output steady states, respectively, and ϕ_π and ϕ_y are the inflation and output monetary policy parameters.[5] We allow r_t to be subject to shocks, where $\varepsilon_{r,t}$ is the i.i.d. monetary policy shock with variance σ_r^2.

The monetary policy authority is responsible for setting a target capital-asset ratio for banks to avoid an over-leveraging of the economy, similar to the Basel Tier 1 leverage ratios. ν in Eq. (11) can therefore be interpreted as a capital adequacy/leverage constraint, ensuring all loans are backed by sufficient bank capital and deposits at the beginning of the period, influencing bank profits. Moreover, the central bank also sets the steady state LTV ratio for entrepreneurs that governs their borrowing constraint and therefore the entrepreneur's equilibrium condition for the price of capital in Eq. (7).

2.9 Market clearing and aggregation

We add the budget constraints of the households, entrepreneurs, and the government—Eqs. (3), (5), and (19)—to derive the aggregate resource constraint in the economy,

$$y_t = c_t + q_{k,t}\left(k_t - (1-\delta_k)k_{t-1}\right) + G_t + \frac{\kappa_\pi}{2}\pi_t^2 y_t + \delta_b k_{b,t-1} + \frac{\theta}{2}\left(\frac{k_{b,t-1}}{b_{t-1}} - \nu\right)^2 k_{b,t-1}, \tag{27}$$

where the last three terms represent the adjustment costs for prices and bank capital-asset ratios in real terms, the clearing condition for the labor market is $\ell_{h,t} = \ell_{d,t}$, the aggregate condition for output is $y_t = y_{e,t}(j)$, aggregate consumption is defined as $c_t = c_{h,t} + c_{e,t}$, and banks' symmetry condition holds in equilibrium.[6] We end up with 37 variables and 37 equations to solve, including 7 stochastic shocks.

3 Results

We describe the solution methods and baseline calibration for our model in Section 3.1 and compare the fit of the simulated model moments to the data in Section 3.2, along with several variations of our

[5] Fuerst and Mau (2019) pointed out that the exact monetary policy rule specification is important to generate variability in the term premium in response to macroeconomic shocks. In order to achieve greater variability in the term premium, the monetary authority should respond to the level of output relative to the steady state rather than the output gap (see Rudebusch and Swanson, 2012). As an output level rule means the central bank is committing to a contractionary policy for longer, thus reducing inflation by more, the term premium is more affected than in the case of an output gap rule. Hördahl et al. (2008) showed that the degree of interest rate smoothing in the monetary policy rule is also important for matching bond and macroeconomic moments in their microfounded DSGE framework. Refer to Palomino (2012) for an analysis of the role of monetary policy regimes and central bank credibility in influencing bond risk premia in long-term bonds.

[6] Refer to the Online Appendix for all the derivations and further details of the model.

baseline specification. Section 3.3 showcases the impact of our risk mispricing shock on the economy, which we compare to a more traditional preference shock. We then study the role of different channels in propagating the risk mispricing shock in Section 3.4 and evaluate the robustness of our results to different calibration and shock specifications in Section 3.5. Finally, we describe the impact of other shocks in our model in Section 3.6.

3.1 Solution and calibration

Since the dimensions of the model are relatively high with 16 state variables, the most feasible option to solve the model is through perturbation methods.[7] We use third-order solutions and apply pruning to cut out unstable higher-order explosive terms. The advantage of using third-order solutions is that the macroeconomic responses remain mostly unchanged, and thus correspond to results in the previous literature, while the responses for the bond markets can be rendered more realistically.[8] A potential disadvantage of this methodology is that the solution method is inherently local and is only valid around the steady state, so that larger shock variances might lead to more inaccurate results. As estimation of larger-scale, non-linear models is still difficult, we follow the literature (e.g., Rudebusch and Swanson, 2012) and calibrate our model based on standard assumptions about parameter values and to fit specific moments for key macroeconomic variables and the term premium.

Table 1 reports the values of the calibrated parameters for our baseline model. For households and entrepreneurs, their discount factors are set such that β_h implies an annual interest rate of 1.4 percent to approximate the mean of the real policy rate in the data, and $\beta_h > \beta_e$ ensures that entrepreneurs are more impatient. The values of $\beta_h = 0.9965$ and $\beta_e = 0.9787$ are similar to those reported in Gerali et al. (2010), Bonciani and van Roye (2016) and Gambacorta and Signoretti (2014)–0.99, 0.9943, and 0.996, respectively–for the households, and within the 0.975-to-0.98 range reported in Iacoviello (2005) and Iacoviello and Neri (2010) for the entrepreneurs. For household preferences, ϕ is based on the Frisch elasticity being 2.5 and ψ is based on the IES being 0.95, in line with previous micro-founded studies which have found the IES to be smaller than one (e.g. Vissing-Jørgensen, 2002) so that the

[7] Caldara et al. (2012) showed that perturbation methods provide equally accurate solutions to models with recursive preferences than Chebychev polynomials and value function iterations, but are considerably faster.

[8] This feature occurs because first-order solutions imply that the expectation hypothesis holds and the term premium is zero, with second-order solutions generating a positive, yet constant term premium, both results inconsistent with empirical evidence on risk pricing dynamics (e.g. Shiller, 1979; Campbell, 1987; Longstaff, 1990; Cuthbertson, 1996; Piazzesi and Schneider, 2007).

utility function is everywhere positive and the certainty equivalence is well defined. We set the Epstein-Zin parameter $\xi = 2$ to match two term premium moments (mean and standard deviation). Using the constant relative risk aversion (CRRA) formula in Swanson (2010), this number implies an overall CRRA of 4. This is a low value relative to typical numbers found in DSGE models and more consistent with the estimates found in the empirical macro-finance literature (see Havranek et al., 2015 for a meta-study).

Table 1: Calibrated parameters for baseline specification

households, entrepreneurs, production		banks, bonds, monetary policy	
household patience (β_h)	0.9965	bank capital adj. cost (θ)	11
IES ($1/\psi$)	0.95	target capital ratio (ν)	0.09
Frisch ($1/\phi$)	2.5	loan elasticity (ϵ_b)	9
EZ risk aversion (ξ)	2	bank management cost (δ_b)	0.06
entrepreneur patience (β_e)	0.9787	consol bond decay rate (δ_c)	0.982
price adj. cost (κ_π)	40	MP rule inflation weight (ϕ_π)	1.5
investment adj. cost (κ_i)	0.55	MP rule output weight (ϕ_y)	0.25
capital depreciation rate (δ_k)	0.05	**shocks: persistence and standard deviation**	
steady states		productivity: $\rho_a = 0.9$	$\sigma_a = 0.003$
inflation ($\overline{\pi}$)	0.00855	government spending: $\rho_g = 0.9$	$\sigma_g = 0.003$
price elasticity ($\overline{\epsilon_y}$)	6	monetary policy: $\rho_r = 0.5$	$\sigma_r = 0.005$
target LTV ratio ($\overline{\Omega}$)	0.35	bond supply: $\rho_{bl} = 0.9$	$\sigma_{bl} = 0.003$
gov. spending-to-GDP ratio $\left(\frac{\overline{G}}{\overline{y}}\right)$	0.20	price markup: $\rho_{\mu y} = 0.9$	$\sigma_{\mu y} = 0.003$
bond supply-to-GDP ratio $\left(\frac{\overline{b_l}}{\overline{y}}\right)$	0.10	LTV ratio: $\rho_\Omega = 0.9$	$\sigma_\Omega = 0.003$
bank capital-asset ratio $\left(\frac{\overline{k_b}}{\overline{b}}\right)$	ν =0.09	risk mispricing: $\rho_{risk} = 0.9$	$\sigma_{risk} = 0.005$

For production, we set the adjustment cost for prices (κ_π) to 40 and for investment (κ_i) to 0.55, and later explore robustness to different values within our setup. The capital share α is assumed to be 0.3 and $\delta_k = 0.05$ implies an annual capital depreciation rate of 20 percent. The bank parameter ν is set to match the Basel target capital-asset ratio of 0.09 and the bank capital adjustment cost is $\theta = 11$ as in Gambacorta and Signoretti (2014). We set the loan elasticity coefficient to 9, such that the loan rate is 12.5 percent higher than the long-term and wholesale-loan rates combined, bank management costs δ_b to 0.06, and the decay rate for consol bonds, $\delta_c = 0.982$, is set to match the average real rate of a 10-year government bond of 2.87 percent. The monetary policy rule parameters $\phi_\pi = 1.5$ and $\phi_y = 0.25$ reflect that the central bank targets both inflation and output with a stronger weight on inflation, in line with the literature (e.g., Gertler et al., 2020; Sims and Wu, 2021).

We calibrate the steady state of inflation to match the annual rate of 3.42 percent we observe in the data, and the price elasticity of demand is assumed to be 6 in steady state, implying a 20 percent markup in the goods market steady-state price as in, for example, Bonciani and van Roye (2016). The steady-state or target LTV ratio of 0.35 follows Gerali et al. (2010), bank capital-asset ratios are equal to their target in the steady state, the ratio of government spending to output is 20 percent in the steady state as in Gertler et al. (2020), and bond supply is calibrated to be 10 percent of output in the steady state. The parameters governing our stochastic processes are also set to standard values, with the persistence of the autoregressive processes ρ being 0.9 for all but the monetary policy smoothing coefficient, which is set to 0.5. The standard deviations of the shocks σ are calibrated to 0.005 for the monetary policy and risk mispricing shocks and 0.003 for all other shocks.

3.2 Model fit

To evaluate the fit of the model, we compare key macroeconomic as well as term premium moments implied by the DSGE model to the data. We use the Hodrick-Prescott filter to compute the business cycle component of log quarterly U.S. data for chained GDP, consumption, investment, and labor. The annualized real policy rate moments are calculated from the inflation-adjusted Federal Funds rate and annual inflation is calculated using the GDP deflator. The term premium is the Adrian et al. (2013) nominal ten-year Treasury term premium from the Federal Reserve Bank of New York. Details, summary statistics, and sources can be found in the Online Appendix.

Table 2 shows the results for the moments observed in the U.S. data in the first row and the simulated moments using the baseline calibration from Table 1 in the second row. The model performs very well in matching key macroeconomic moments: the standard deviations of output, investment, and labor are under 2 percent of the data variation, and consumption deviates within less than 7 percent from its data moment. Our model inflation rate, however, has a lower standard deviation from what we observe in the data. This is a trade-off that arises when it comes to matching term premium moments. If we set the Epstein-Zin (EZ) parameter to zero, the standard deviation of inflation increases significantly, as can be observed in the "no EZ preferences" row, at the expense of not being able to match the term premium moments, particularly its variability. When EZ= 0, we can increase the flexibility of capital in the model to match the standard deviation of inflation (and investment) closer to what we observe in the data ($\kappa_i = 0.0255$ yields $\sigma(\pi_t) = 2.44$ and $\sigma(I_t) = 4.13$), yet term premium moments remain virtually unaffected. This trade-off occurs because in our model retail firms are owned by the households and therefore take their stochastic discount factor as given when maximizing the discounted value of their profits. The EZ parameter that governs the stochastic discount factor of the households with recursive preferences derived in Eq. (4), therefore influences the New Keynesian Phillips curve in Eq. (10) and regulates relative price dynamics in the model via the markup price over intermediate goods.

Table 2: Comparing data with simulated model moments

						r_t		$tp_{l,t}$	
	$\sigma\,(y_t)$	$\sigma\,(c_t)$	$\sigma\,(I_t)$	$\sigma\,(\ell_t)$	$\sigma\,(\pi_t)$	*mean*	σ	*mean*	σ
data	1.46	0.85	4.08	2.10	2.45	1.54	2.47	1.64	1.19
baseline	1.46	0.80	4.12	2.14	1.08	1.52	1.06	1.64	1.19
no TFP shock	1.85	1.10	5.73	2.49	1.12	1.58	1.22	0.50	1.29
no risk mispricing shock	1.37	0.84	3.77	2.05	1.11	1.46	1.06	0.48	1.11
no bond supply shock	1.48	0.84	4.35	2.04	1.19	1.35	1.05	0.48	1.66
no gov. spending shock	1.47	0.89	4.37	2.12	1.16	1.42	1.07	0.40	1.65
no price markup shock	1.60	1.02	4.97	2.32	1.22	1.49	1.23	0.11	1.26
no monetary policy shock	1.06	0.97	3.52	1.54	0.55	1.47	0.75	0.10	0.91
no LTV ratio shock	1.22	0.92	3.35	1.79	1.06	1.53	0.89	-0.02	1.33
only risk mispricing shock	0.23	0.23	1.24	0.30	0.06	1.43	0.17	0.18	0.65
no EZ preferences	1.51	1.13	0.52	2.14	10.5	3.65	1.38	0.58	0.01
fixed capital ($\kappa_i = 1{,}000$)	1.63	1.02	0.06	2.31	1.06	1.44	0.93	1.43	1.17

Notes: Moments for real aggregate output (y_t), consumption (c_t), investment (I_t), and labor (ℓ_t) are reported in quarterly percentage points; whereas inflation (π_t), the real policy rate (r_t), and nominal term premium ($tp_{l,t}$) are reported in annual frequency. The model moments are computed by simulating the data 224 times to be consistent with the duration of the time series. Refer to the Online Appendix for data sources and calculations. Mean and standard deviations (σ) correspond to the model with the baseline calibration from Table 1 in the row labeled "baseline," and the same model without each shock at a time in subsequent rows. The row titled "no EZ preferences" corresponds to a version of the model with CRRA preferences for the households. The final row proxies for a model with fixed capital by setting $\kappa_i = 1,000$.

When it comes to financial variables, our baseline calibration allows us to perfectly match the mean and standard deviation of the nominal term premium, and obtain a real policy rate that is on average similar to the data yet with lower variability. Again, this is a trade-off that occurs as we prioritize matching term premium moments, since under EZ$= 0$ a more flexible capital calibration increases the standard deviation of the policy rate to 2.07. The term premium is the compensation that investors require in order to hold a long-term bond instead of a series of short-term bonds during the same horizon. A high term premium therefore reflects a perceived increase in financial risk over the life of a bond. Although unobservable, and therefore difficult to measure, it has been established in the literature that this compensation for risk varies throughout time as investors update their beliefs about the future path of the economy (e.g. Campbell and Shiller, 1991), hence the model term premium variability we obtain is crucial for the purpose of our study.[9]

In order to understand the components of our model that help us match the data moments, we turn off one shock at a time to observe their contribution to modeling the term premium. With regards to matching the level of the term premium, all shocks at least partially contribute to delivering a high term premium, particularly the LTV ratio shock. In a model without LTV ratio shocks, the mean of the term premium is -2 basis points, far from the 164 basis points observed in the data. The variance of the term premium, on the other hand, does not seem to be driven by any specific shock: risk mispricing and monetary policy shocks are the most important drivers, yet without them, term premium variability is still present due to risk aversion. Under "no EZ preferences" the variance of the term premium is practically zero. This is consistent with the literature identifying that the additional parameter modeling households' risk aversion is crucial to matching the empirical features of bond moments. Fuerst and Mau (2019) explored business cycle moments for different model specifications with and without segmented markets and Epstein-Zin preferences, and found these features to also help deliver a counter-cyclical term premium. We find, consistent with their results and empirical evidence (e.g. Campbell and Cochrane, 1999; Wachter, 2006; Bauer et al., 2012; Lustig et al., 2014), that the model-implied correlation between future consumption and nominal term premia is –0.30. This counter-cyclicality suggests that we expect the term premium to rise along with the deterioration of future consumption prospects during economic downturns and fall during economic upswings, as compensation for risk increases during bad times and vice versa. Finally, we consider a model with only risk mispricing shocks and proxy for a version of the model with fixed capital by setting the investment adjustment cost parameter to a very high number, the "fixed capital ($\kappa_i = 1,000$)" row. This implies that retailers find it prohibitively costly to change investment levels relative to the previous period. We conclude that risk mispricing shocks alone cannot generate the variability we observe in the term premium data. Furthermore, while dynamic capital in our model is critical for matching the standard deviation of investment, it is not crucial for delivering a positive term premium with high variability.

[9] The term premium is typically estimated or inferred from the term structure of interest rates, forecasts, or surveys of market participants. Swanson (2007), Rudebusch et al. (2007), and Li et al. (2017a) compared different estimates of the term premium and provide excellent overviews of the challenges faced when measuring the long-term expectation of short rates. Importantly, despite the disagreement on what the level of the term premium is (e.g. Rudebusch and Swanson, 2012 report a mean of 106 b.p., whereas Adrian et al., 2013 report a mean of 169 b.p. for the 1961/2–2007/8 period), all measures broadly agree on the general movements of the term premium and are hence highly correlated.

3.3 The impact of a risk mispricing shock: When investors panic

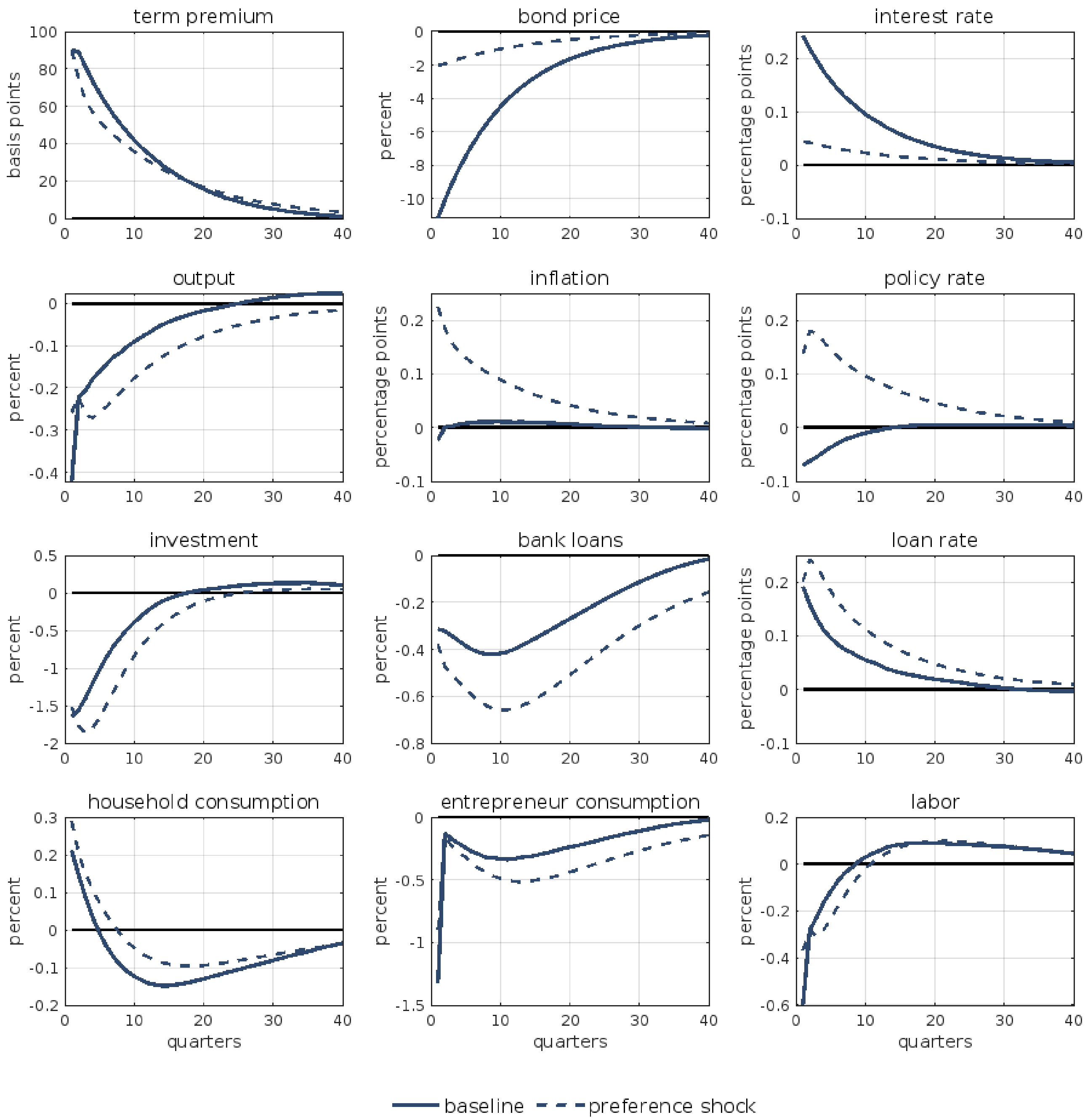
We begin by analyzing how a risk mispricing shock affects the macroeconomy as a temporary, exogenous wedge to the real price of long-term government bonds. Figure 1 reports the results for a shock that underprices long-term bonds in solid blue by generating a 90 basis point increase in the annual nominal term premium, a magnitude comparable to the increase experienced from September to October 2008, during the initial stages of the Global Financial Crisis. This shock lowers the real price of long-term government bonds by 11 percent on impact and increases the real long-term annualized interest rate by 0.24 percentage points, as investors demand higher compensation due to the perceived elevated risk.

Risk mispricing has a negative effect on the real economy with output declining by 0.42 percent on impact, returning to the steady state only after 6 years, and has a small effect on inflation, a short-lived 2 basis point decline, much as if our risk mispricing shock behaved like a demand shock. The central bank reacts to ease economic conditions by decreasing the real policy rate by 7 basis points, lowering the cost of funding for banks with an imperfect pass-through. The decline in the policy rate brings expectations of the future policy rate down, incentivizing households to increase their consumption and lower their supply of labor on impact, as well as reduce their savings. We find that, overall, the results of our theoretical model are in line with the previous empirical literature identifying sizable effects on economic activity resulting from uncertainty shocks (e.g., 1.2 percent decline in output in Baker et al., 2016; –1 percent change in output during expansions and –2 percent during contractions in Caggiano et al., 2022; a persistent effect on industrial production Carriero et al., 2018) and that our risk mispricing shock seems to correspond with a demand shock (e.g. Leduc and Liu, 2016).[10]

With the long-term interest rate on government bonds increasing, banks pass on the impact of the risk mispricing shock to the private sector in the form of higher loan rates, around 200 basis points on impact, and reduce the amount of loans to entrepreneurs, as banks protect their profitability and capital-asset ratios. The higher loan rates lead to higher bank profits and eventually higher bank net worth as banks accumulate capital out of reinvested profits in Eq. (16). Since the amount of government bonds in the bank portfolio remains the same, while loan volumes decline, total bank assets decrease.

[10] Our results also support the reduced-form evidence found in the macro-finance literature that identifies persistent effects in macroeconomic variables in response to term premia shocks (e.g. Gil-Alana and Moreno, 2012; Jardet et al., 2013). Joslin et al. (2014), for example, identified that both economic activity and inflation decline when a canonical term structure model of interest rates incorporates macroeconomic fundamentals beyond the information spanned by the yield curve. Although not a general equilibrium framework, their model allows future bond prices to be influenced by yield curve factors as well as macroeconomic risks, which in turn account for variation in the term premium.

Figure 1: Impulse responses to risk mispricing shock



Notes: The blue solid line represents the impulse responses to a risk mispricing shock, whereas the blue dashed line represents the responses to a preference shock. Both shocks are scaled to raise the nominal term premium by 90 basis points for comparison. All variables—except for the term premium—are in real terms. Responses are deviations relative to the steady state in the corresponding units indicated in the vertical axis. All variables are quarterly, except for the term premium and the interest rate, which are annualized.

Higher bank capital and lower bank assets lead to a further reduction in demand for deposits through the balance sheet structure of the bank. The decline in deposits occurs as households increase their consumption and reduce their supply of labor on impact, before the effects of the economic downturn bring household (and aggregate) consumption down.

As a response to the reduction in the amount of loans to entrepreneurs, whose consumption falls by more than 1 percent and are less willing to take out a loan at a higher rate (the *loan-price channel* in our model), investment falls by 1.65 percent. The contraction in investment is what drives the decline in output, as capital decreases by 0.1 percent on impact, reaching a 0.4 percent decline 3 years later and staying persistently below the steady state for 10 years. The fall in the stock and price of capital reduces the collateral value of the entrepreneurs, further restricting their ability to take out a loan, and amplifying the impact of the risk mispricing shock on bank lending and the real economy. This mechanism is consistent with anecdotal evidence on bank behavior: The Euro area bank lending survey providing qualitative information on bank loan demand and supply across Euro area enterprises and households, for example, identified 'risk perceptions' as one of the most important factors in periods of net tightening of credit standards on housing loans and loans to enterprises (Köhler Ulbrich et al., 2016).

To further understand our shock, Figure 1 also displays the results of a preference shock in blue dashed lines. In order to transform our shock into a preference shock, we incorporate risk mispricing into the household value function Eq. (1), such that it multiplies the consumption term in the intra-period utility function Eq. (2).[11] This leads to a different equation for the stochastic discount factor of the households featuring a preference shock:

$$\frac{1}{1+r_t} = \mathbb{E}_t \underbrace{\beta_h \underbrace{\frac{\epsilon_{risk,t+1}}{\epsilon_{risk,t}}}_{\text{preference shock}} \frac{c_{h,t}^{\psi}}{c_{h,t+1}^{\psi}} \left(\frac{\left(\mathbb{E}_t V_{t+1}^{1-\xi}\right)^{\frac{1}{1-\xi}}}{V_{t+1}} \right)^{\xi}}_{m^h_{t,t+1}\text{: stochastic discount factor}}, \tag{28}$$

where we make two additional modifications. First, we change the exogenous process defining risk mispricing to have a mean of 1 and make the shock enter Eq. (21) positively, such that a positive shock reduces the stochastic discount factor on impact, lowers the real price of bonds, and elevates the nominal term premium. Second, we set IES$=0.5$ to avoid running into the asymptote that can arise

[11] Our version of a preference shock is similar in principle to the uncertainty shock of Basu and Bundick (2017) in that it impacts the EZ preferences of the households, lowering their stochastic discount factor. Their uncertainty shock, however, impacts the *volatility* or second moment of the exogenous process, as opposed to the first moment.

when the IES is close to unity under this modified specification, as explained in De Groot et al. (2018).

Finally, we scale the preference shock to induce a 90 basis point increase in the nominal term premium to be in line with our risk mispricing shock, although an apples-to-apples comparison is difficult given the different nature of the shocks. The first important difference is that a preference shock lowers the stochastic discount factor of the households on impact, a feature that is not present in our baseline specification. Via Eq. (28), we can observe that the preference shock therefore influences $m^h_{t,t+1}$ as much as r_t, breaking the segmentation previously featured in our bond markets. Without the wedge between $\frac{1}{1+r_t}$ pricing the risk-neutral bond in Eq. (23) and $m^h_{t,t+1}$ pricing the risky bond in Eq. (20), the real term premium as defined by the difference between the risky and risk-neutral yields in Eq. (25) is zero. The nominal term premium therefore elevates in this context driven solely by the increase in inflation, since the decline in the bond price of 2 percent is offset by the decrease in the risk-neutral bond price of the same magnitude. The initial contraction in output is smaller, 0.25 percent below the steady state, and the monetary policy increases the rate this time in response to higher inflation. The reason behind the more pronounced impact on inflation is that retail firms, owned by the households such that retailers take their stochastic discount factor as given in Eq. (9), are now exposed to the shock, influencing the New Keynesian Phillips curve in Eq. (10) that determines relative prices in the economy. Banks reduce lending and increase the loan rate even more, since the tighter monetary policy increases banks' funding costs. Although a preference shock delivers similar macroeconomic responses—output and investment decline— all the effect is transmitted via inflation as opposed to the bond markets, a very different mechanism from the risk mispricing shock we want to study.

3.4 The role of different channels in propagating the risk mispricing shock

There is recent evidence to support the importance of financial frictions in restricting credit availability during the early 2020 outbreak of the COVID-19 pandemic. Between March 9th and March 19th, the 10-year U.S. Treasury yield rose by 60 basis points, reflecting a pronounced increase in the Kim and Wright (2005) daily 10-year term premium of almost 40 basis points, as estimated by the Board of Governors.[12] During the following weeks, consumer lending by all commercial banks declined, a higher percentage of banks tightened their standards for issuing all types of loans (e.g. commercial, consumer, credit card, mortgage) as well as increased the spread of loan rates over the cost of funds, further

[12]See https://www.federalreserve.gov/pubs/feds/2005/200533/200533abs.html.

restricting the availability of credit between 2020-Q1 and 2020-Q2.[13] While the COVID-19 shock was not a risk mispricing shock—and was followed by lock-down policies and other interventions that we do not model in our framework—it is a clear recent example of a disruption to the economy whose origins were unrelated to fundamentals, was reflected in a higher term premium elevating long-term interest rates, and at least partially impacted the availability of credit via the types of channels we identify in our model.

We hence asses the relative importance of different channels in allowing risk mispricing shocks to feed back to the real economy by considering specifications of our model in which we remove financial frictions one at a time. We begin by illustrating the main transmission mechanism in our baseline specification by changing the retail bank problem in Eq. (13), such that banks no longer incorporate long-term government bonds into their portfolios, effectively removing the *loan-price channel*. In this case, the loan rate in Eq. (14) is defined as a constant markup over the wholesale-loan rate: $r_{b,t} = r_{w,t}\left(\frac{\epsilon_b}{\epsilon_b - 1}\right)$, changing bank profits in Eq. (15) to $J_{b,t} = r_{b,t}b_{e,t} - r_t d_t - \frac{\theta}{2}\left(\frac{k_{b,t}}{b_t} - \nu\right)^2 k_{b,t}$, and rendering the risk mispricing shock inconsequential for the banking system. Figure 2 illustrates that the presence of this channel is of first order importance for the transmission of the shock to the real economy (gold dashes). With the risk mispricing shock no longer impacting the loan rate, loans to entrepreneurs, investment, and therefore output, there is no longer a need for the monetary policy authority to intervene. Therefore, the bank's cost of funding, capital, and profits remain unaffected by risk mispricing. These results illustrate that our risk mispricing shock is only transmitted via bond prices held by the banks, as opposed to the case of a preference shock, for example, which can still impact the bank's balance sheet via inflation. In fact, the responses to a preference shock remain virtually unaltered, even after removing the loan-price channel entirely.

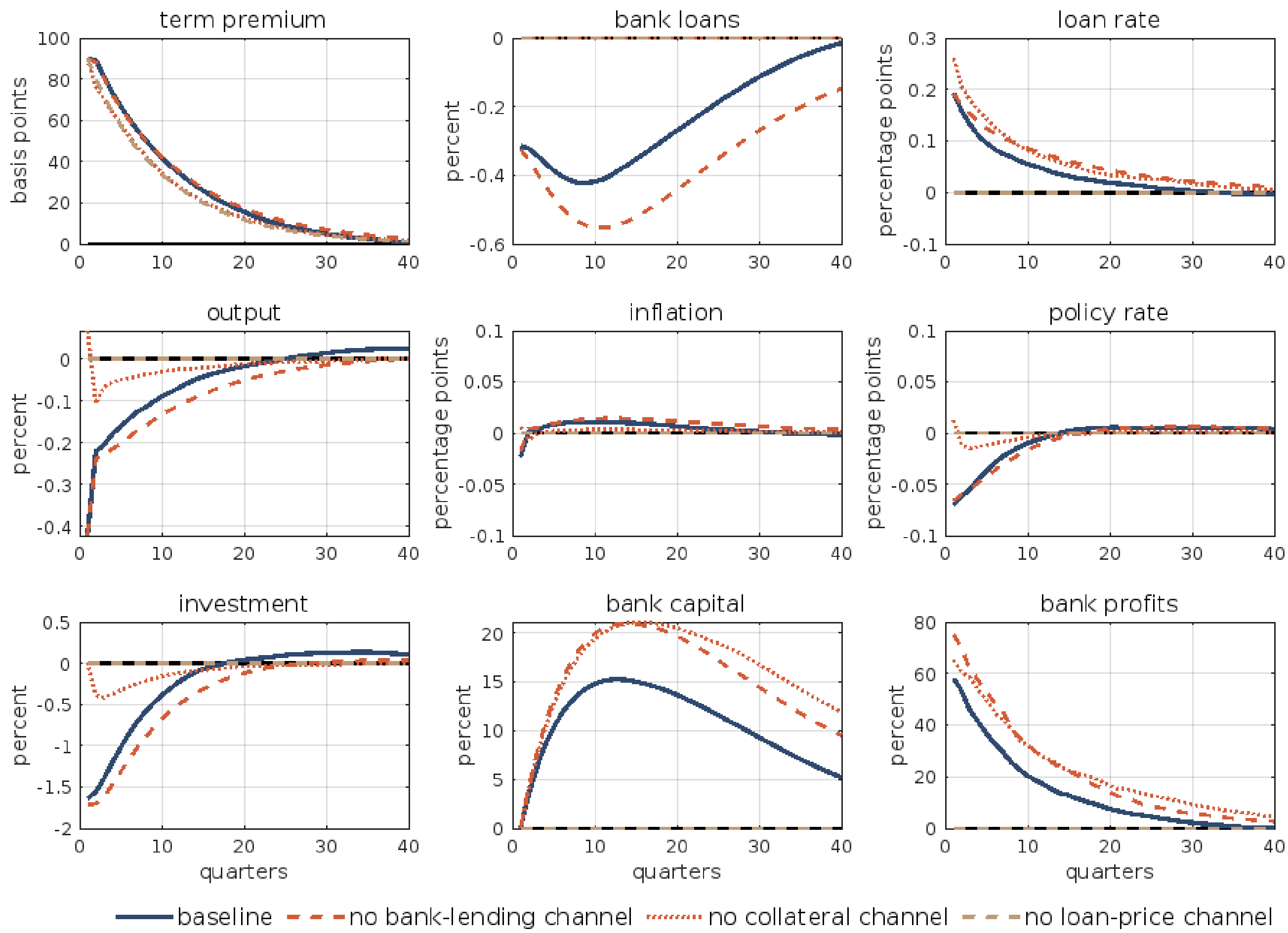
We next investigate the role of the *bank-lending channel* in propagating our risk mispricing shock. To this end we set the parameter that governs the importance of the capital-asset ratio deviations from its target to zero, by letting $\theta = 0$ in Eq. (9), such that the wholesale-loan rate in Eq. (12) now moves one-to-one relative to the policy rate; yet the imperfect pass-through of monetary policy to the loan rate in Eq. (14) remains. This modification changes the equation for bank profits and eliminates the adjustment term for aggregate output in Eq. (27). Figure 2 (orange dashes) shows that without a *bank-*

[13]Board of Governors of the Federal Reserve System's April 2020 Senior Loan Officer Opinion Survey on Bank Lending Practices release, May 4, 2020, available online at http://www.federalreserve.gov/boarddocs/SnLoanSurvey/.

lending channel, banks are no longer concerned about deviations from their target capital-asset ratios, hence when the risk mispricing shock hits and the loan rate increases, banks protect their profitability by contracting lending even more and charging persistently higher loan rates than under the baseline case. The banks' more pronounced reduction in their balance sheet leads to higher bank profits on impact and a more sustained and persistent accumulation of bank capital over time as those higher profits get reinvested. With a stronger reduction of credit in the economy, investment's return to the steady state after the initial decline is slower relative to the baseline case, making the effect of a risk mispricing shock on output more persistent. The presence of this channel therefore suggests that actively managing their capital-asset ratios to achieve a specific target can be beneficial in mitigating the effects of risk mispricing on the real economy, at the expense of reduced profitability for banks.

Lastly, we evaluate the impact of the *collateral channel* on amplifying the effects of the risk mispricing shock (orange dots). Similar to the "no asset price channel" scenario of Iacoviello (2005), we consider the case in which the entrepreneur's borrowing limit is present but no longer tied to the value of the assets, such that the borrowing constraint in Eq. (5) is set to $b_{e,t}(j) \leq z$ for $z > 0$. This removes the last term in the investment Euler equation of the entrepreneur and eliminates the presence of the LTV ratio in the model. When the risk mispricing shock increases the loan rate via the long-term interest rate, banks raise rates by an additional 5 basis points relative to the baseline scenario to increase profits, leading eventually to higher bank net worth. Since banks do not reduce their supply of credit in response to the shock, the changes in capital, investment, and output are much more muted than in the baseline case. We find that in the absence of the *collateral channel*, changes in the stock and price of capital no longer constrain the entrepreneurs' ability to borrow, hence the deterioration in their consumption and investment opportunities is less pronounced: only the price of credit changes, not the quantity. This result is independent from the presence/absence of the LTV shock; it is solely driven by bank lending no longer being tied to the value of entrepreneurs' capital. Given the lack of change in the quantity of credit availability, the overall effect in the macroeconomy is more modest, with the policy rate reacting to the milder changes in output and inflation. It may seem that banks requiring loans to be backed by collateral amplifies the transmission of risk mispricing as it acts as a financial accelerator; however, is important to recall that our model features no loan default and hence no upside to collateralized debt. Incorporating credit risk in this type of model by allowing lenders to default on their payments is outside the scope of this project but it would allow one to better assess these relative trade-offs.

Figure 2: Impulse responses to risk mispricing shock: NO channels



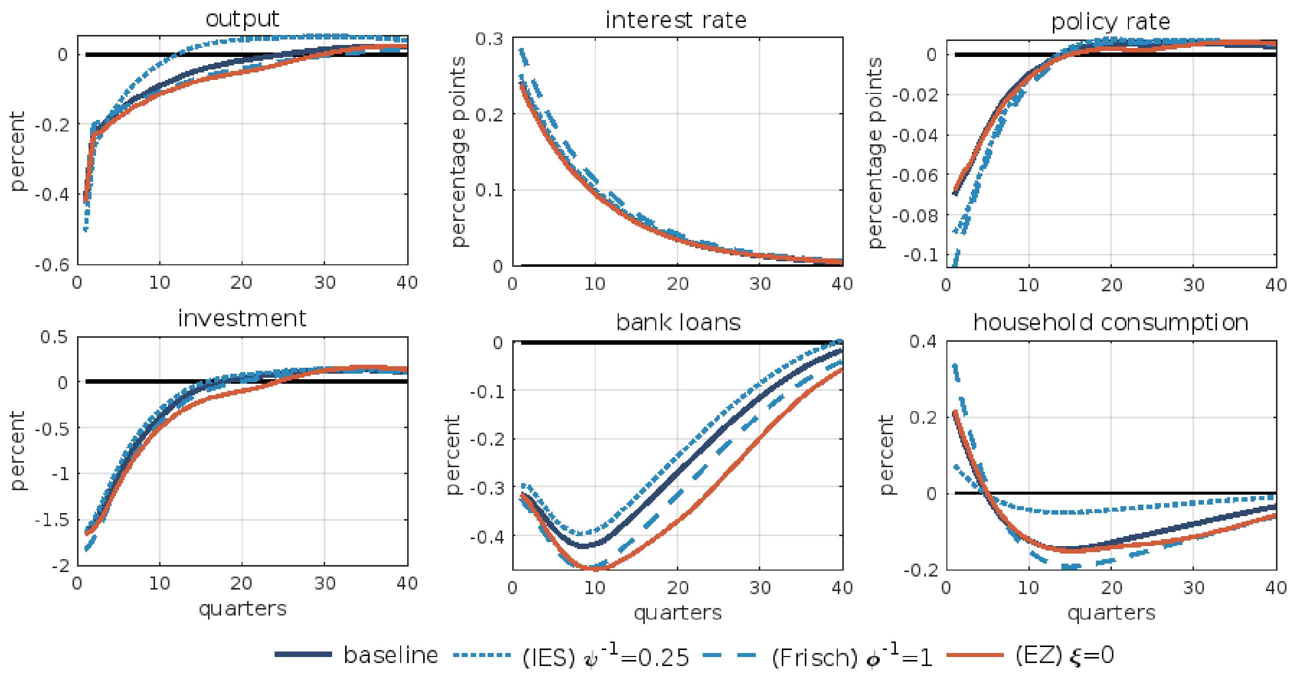
Notes: The blue solid line represents the impulse responses to a risk mispricing shock (scaled to raise the nominal term premium by 90 basis points) under the calibration in Table 1, which we label our "baseline" specification. The orange dashes represent a "no bank-lending channel" scenario by setting $\theta = 0$. The orange dots represent a "no collateral channel" scenario by setting the entrepreneur's borrowing constraint to $b_{e,t}(j) \leq z$, for $z > 0$. The gold dashes represent a "no loan-price channel" scenario, which delivers a loan rate no longer tied to the long-term interest rate, deactivating the transmission of the risk mispricing shock. All variables are quarterly and in real terms, except for the term premium, which is annualized and in nominal terms. Responses are deviations relative to the steady state in the corresponding units indicated in the vertical axis.

3.5 Robustness to parametrization and shock specification

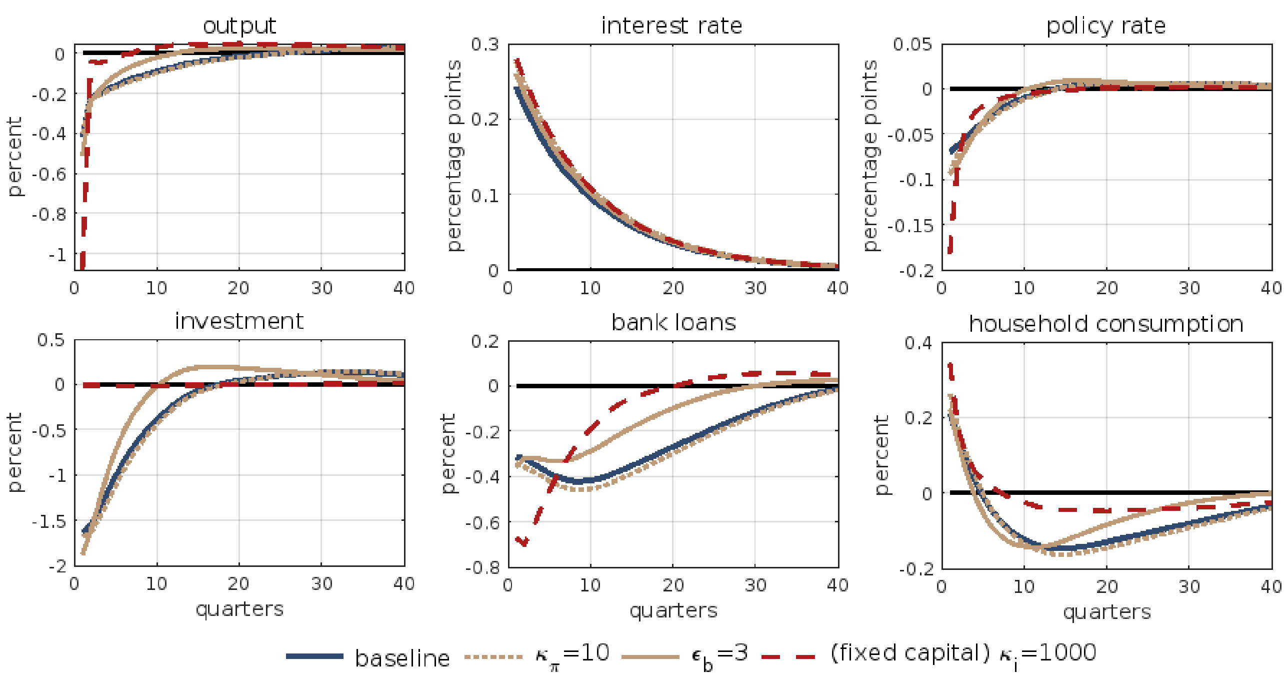
We next evaluate the robustness of the responses to a risk mispricing shock when changing key preference, production, and bank parameters. For reference, Figure 3 displays the impulse response functions for the baseline model calibration in solid blue. In panel (a), we lower the household IES to 0.25 (light blue dots), increasing the curvature of households' utility with respect to consumption from 1.05 to 4.

Figure 3: Impulse responses to risk mispricing shock: Robustness

(a) preferences



(b) production and banking



Notes: The blue solid line represents the impulse responses to a risk mispricing shock (scaled to raise the nominal term premium by 90 basis points) under the calibration in Table 1, which we label our "baseline" specification. We change different parameters regulating household preferences in panel (a) and production and banking in panel (b). All variables are quarterly and in real terms, except for the interest rate, which is annualized. Responses are deviations relative to the steady state in the corresponding units indicated in the vertical axis.

Responses to a risk mispricing shock are similar to the baseline scenario, with both inflation (-0.3 percentage points) and output (-0.5 percent) decreasing more on impact and hence inducing a stronger policy rate response (2 basis points lower than the baseline). Since a higher curvature makes consumption growth less sensitive to changes in the policy rate, household consumption raises by less, while entrepreneur consumption goes down by just as much as in the baseline scenario. Aggregate consumption hence contracts more on impact, magnifying the response of output, with more muted effects in the long run. The impact on bank loans is more subdued, as the change in the IES coefficient affects the quantity and not the price of capital, with all other responses relatively unchanged. Second, we lower the elasticity of substitution of labor supply (light blue dashes), increasing households' utility curvature with respect to labor from 0.4 to 1. While with a lower Frisch elasticity most responses remain relatively the same, the risk mispricing shock lowers wages by more on impact (-0.2 percent), and generates a stronger policy rate response stemming from a more pronounced decline in inflation (5 additional basis points). Impulse response functions are also robust to a model without EZ preferences (solid orange line). While crucial for matching the term premium variability observed in the data, macroeconomic responses to the risk mispricing shock are quantitatively the same, albeit slightly more persistent in the long run, with the effect on credit being more pronounced over time. Analogous to the baseline specification, the risk mispricing shock is transmitted to the economy via the *loan-price channel* and amplified through the *collateral channel* under standard preferences.

In regards to production in panel (b), we make price adjustments more flexible by lowering κ_π from 40 to 10 (gold dots), making inflation more sensitive to the risk mispricing shock (4 additional basis points), yet all other responses to the shock remain the same as in the baseline scenario, since our shock works through the real pricing of risky bonds. We also evaluate the importance of investment adjustment costs by considering a fully flexible capital model where $\kappa_i = 0$ and a quasi-fixed capital model where $\kappa_i = 1000$. When we set the investment adjustment cost parameter to zero (not shown), the capital accumulation equation collapses to a standard version without adjustment costs and the price of capital in Eq. (8) is fixed. Investment is therefore more sensitive (2 additional percentage points) and hence the response of output to the risk mispricing shock is stronger, with bank capital and loans exhibiting a more pronounced decline via the change in the *quantity* of capital. On the other hand, we create a quasi-fixed capital model by setting the investment adjustment cost parameter to a very high value, such that capital-good producers find it prohibitively costly to change investment relative to the previous period, making capital a "fixed" rather than a dynamic variable that only changes due to depreciation (red dashes). Since investment cannot adjust, the sharper contraction in output is driven by the decline in entrepreneurs' consumption. In this case the *quantity* of capital cannot change in response to the shock, yet the decline in the *price* of capital diminishes the ability of entrepreneurs to borrow (the value of the collateral required to take out a loan plummets), exacerbating the decline in credit, which more than doubles on impact. These effects, although more severe on impact, are shorter-lived relative to the baseline specification we present. Lastly, we lower the loan elasticity coefficient to 3, such that the loan rate features a 50 percent markup over the long-term and wholesale-loan rates combined (gold solid line). With a higher loan rate as a response to the risk mispricing shock, the banks contract lending by the same amount on impact, but their balance sheet, lending, and deposits revert back to their steady states faster.

The effects of the risk mispricing shock are not driven by different calibration choices nor its additive specification. To this final point, risk mispricing enters the stochastic discount factor while pricing long-term bonds in an additive fashion, which entails generating an up or downward parallel shift in the term premium. We therefore allow this shock to act as a wedge between the stochastic discount factor of the households and the price of the risky bond. We test whether this additive specification drives our results by changing Eq. (20) such that risk mispricing multiplies the stochastic discount factor instead.[14] Unlike Kim (2000) additive vs. multiplicative TFP shock to firms' production function, for example, we find our results to be robust to either shock specification (not shown), since our shock only operates via asset prices.

3.6 Other shocks

We analyze the impulse responses of traditional macroeconomic shocks, including their effects on the term premium, to cross-check that their responses are consistent with economic theory and our model reproduces standard results that are well understood in the literature. The responses to a positive, one standard deviation productivity (a), government spending (b), and monetary policy (c) shocks are

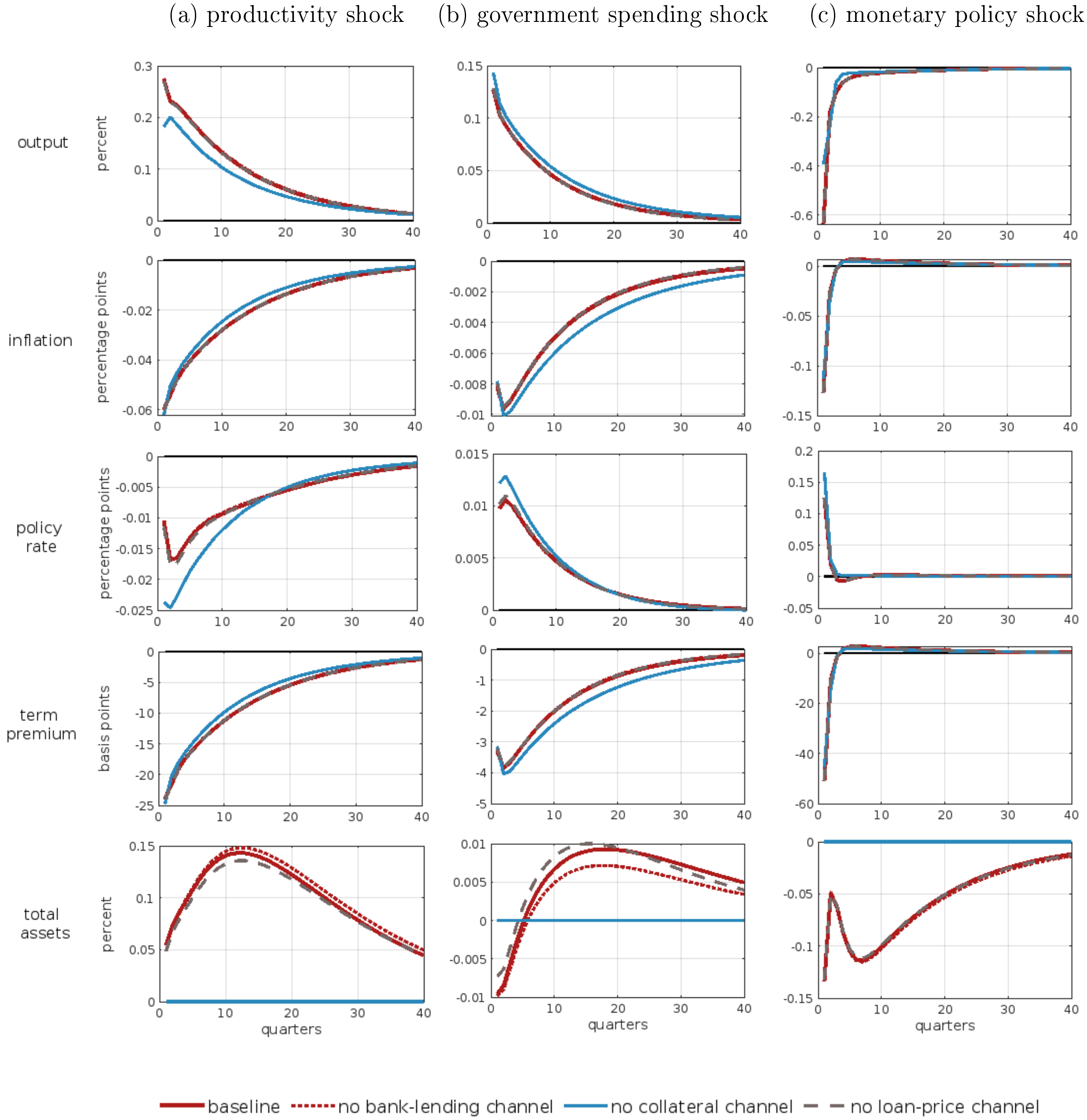
[14] This modification generates two changes in the interpretation of our risk mispricing shock. First, the shock still affects the level of the stochastic discount factor, but it also alters its slope, steepening or flattening the rate of growth instead of adding a wedge. Second, because a multiplicative shock is proportional to the stochastic discount factor, when the pricing kernel is very high (the good times), then the effect of the risk mispricing shock is larger than during the bad times, when the pricing kernel is smaller. Therefore, the impact of a risk mispricing shock on the term premium is no longer independent from the level of the stochastic discount factor.

reported in Figure 4 with red solid lines.

As is standard in the literature, a productivity shock persistently increases output (0.3 percent) and lowers inflation (6 basis points on impact), inducing the monetary policy authority to marginally lower the policy rate, given its stronger emphasis on managing inflation. Driven by a boost in investment, wages go up and consumption increases, and banks meet the higher demand for credit by issuing more loans, expanding their balance sheet. Since the loan rate falls on impact due to easier monetary policy, bank capital and profits go down, and investment is thus supported both by the productivity boost and easier access to credit. Furthermore, higher capital induces a higher valuation of entrepreneurs' collateral and thus higher demand for credit, amplifying the impact on investment. In fact, when we turn the *collateral channel* off (light blue solid lines), investment expands by half the value under the baseline model, leading to a more muted increase in output, since credit is no longer constrained. The productivity shock also boosts bond prices, putting downward pressure on interest rates and compressing the term premium, which declines by 24 basis points, consistent with the qualitative findings outlined in Rudebusch and Swanson (2012) revealing a negative, albeit weaker, relationship between output and the term premium. This correlation is robust to removing the financial frictions in our model, since most macroeconomic and term premium responses remain unchanged, despite bank capital and profit losses being somewhat amplified in the absence of the *loan-price* and *bank-lending channels*, similar to the amplification effects observed in the case of a risk mispricing shock.

A one standard deviation positive shock to real government spending in Eq. (17) boosts real output by 0.12 percent, with almost no effect on inflation, leading the monetary policy authority to marginally raise the policy rate, tightening economic conditions. The 1 percent increase in government spending is financed through higher taxes to the households, for which they lower their consumption (0.1 percent on impact), reduce their deposits, and work more despite lower wages. The small decline in term premium occurs as the average expected future policy rate from Eq. (24)—due to the monetary policy response to higher output—is higher than the increase in the long-term rate due to reduced bond prices from Eq. (22). We therefore find a small 4 basis point impact on the term premium from such a shock. Bank lending initially declines and then increases and remains persistently above the steady state, while banks increase their net worth and profits on the back of higher loan rates. Since investment goes up (0.06 percent), entrepreneurs' consumption increases 0.6 percent on impact and capital accumulates. Results are again, robust to removing our financial frictions.

Figure 4: Impulse responses to other shocks



Notes: The red solid lines represent the impulse responses to a one standard deviation shock to (a) productivity, (b) government spending, and (c) monetary policy, which we label our "baseline" specification. The red dots represent a "no bank-lending channel" scenario by setting $\theta = 0$. The light blue solid lines represent a "no collateral channel" scenario by setting the entrepreneur's borrowing constraint to $b_{e,t}(j) \leq z$, for $z > 0$. The gray dashes represent a "no loan-price channel" scenario, which delivers a loan rate no longer tied to the long-term interest rate, deactivating the transmission of the risk mispricing shock. All variables are quarterly and in real terms, except for the term premium, which is annualized and in nominal terms. Responses are deviations relative to the steady state in the corresponding units indicated in the vertical axis.

We consider a one-standard deviation monetary policy shock when $\sigma_r = 0.003$, such that the real policy rate in Eq. (26) increases by 130 basis points. As expected, this shock has a contractionary effect in the economy, with both output and inflation declining on impact, yet returning back to the steady state after only 3 quarters. In this scenario, the term premium goes down 50 basis points driven by the increase in the expected future policy rate surpassing the increase in the long-term interest rate, and returns back to the steady state as monetary policy quickly unwinds. The increase in the policy rate gets directly passed onto the loan rates, such that bank loans decline on impact and remain persistently below the steady state, while bank profits and bank capital benefit from the higher loan rates. We also find that financial frictions play no role in the transmission of the monetary policy shock to the macroeconomy, as expected.

Three other shocks are featured in our model, not shown in Figure 4. While a bond supply shock in Eq. (18) has almost no impact on output, inflation, nor the term premium, a positive final-good markup price shock increases inflation while lowering output, driving the term premium up by 10 basis points. This is consistent with the effects of inflationary supply-side shocks identified in Rudebusch and Swanson (2012) that cause the term premium to rise at the same time that they generate declines in consumption, investment and output. Since the policy rate remains relatively unchanged, so does the loan rate, and entrepreneurs react to the higher relative prices by lowering their demand for credit, their consumption, and investment. Finally, we consider a shock to the LTV ratio that relaxes the entrepreneurs' budget constraint, increasing the demand for credit by 0.43 percent and their consumption by more than 2.5 percent on impact. This is in essence a short-lived demand-side shock, for which we see higher investment drive an expansion in both production and inflation simultaneously. The monetary policy authority reacts by increasing the policy rate by 150 basis points, which increases the loan rate charged by the banks, increasing bank profits and net worth, and putting upward pressure on the term premium, which goes up by 26 basis points on impact.

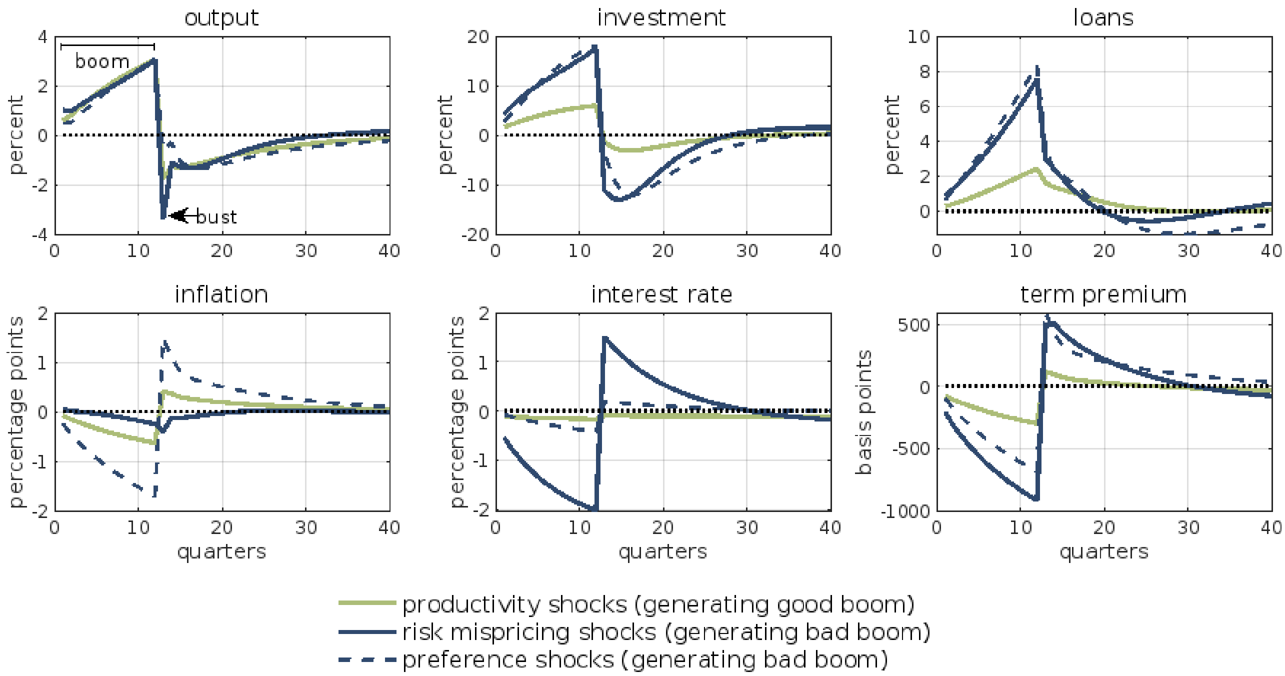
4 Application: When investors underprice risk

With the unfolding of the Financial Crisis, there has been a surge in both empirical and theoretical models trying to explain the underlying causes of financial market fluctuations. While recessions are a normal feature of business cycle dynamics, a consensus emerged that recessions following a credit fueled boom are particularly damaging to the economy (Minsky, 1986; Borio and Lowe, 2002; Kindleberger and Aliber, 2011; Claessens et al., 2012; Jordà et al., 2013). Dell'Ariccia et al. (2016) found that one third of all credit booms are followed by a financial crisis and 60 percent are followed by lower economic performance. As such, a distinction that is often made is between a 'good' credit boom and a 'bad' credit boom. While both booms increase output and the availability of credit, a good boom is based on fundamentals in the economy improving, such as higher productivity or technological innovation (e.g., Kydland and Prescott, 1982). In contrast, a bad boom can be defined as a credit boom that is solely based on sentiment or 'animal spirits' and is thus likely to mean revert, once agents realize their mistake (e.g., Azariadis, 1981; De Grauwe, 2011; De Grauwe and Macchiarelli, 2015; Bianchi and Melosi, 2016). An event that triggers the reversal in expectation, i.e., a Minsky moment, could likely set off a chain reaction ultimately inducing a financial crisis, a recession, or both. From a financial stability perspective, a bad boom could therefore have devastating consequences on financial markets and the real economy. Beaudry and Willems (2022), for example, found cross-country empirical evidence that over-optimism about the economic prospects of a country that later on fail to materialize, lead to excessive borrowing and is therefore associated with future economic recessions.

4.1 Good and bad boom-bust scenarios

We simulate different boom-bust scenarios with our baseline specification, labeling productivity shocks as 'good' booms and risk mispricing or preference shocks as 'bad' booms. In all three cases, we generate shocks every period scaled such that output increases by 1 percent every year for three consecutive years resulting in an economic boom that induces banks to lend more. Figure 5 shows that all three shocks are scaled to lead output to be 3.03 percent above the steady state by quarter 12. During this economic boom, investment and credit expand, while inflation, interest rates, and the term premium go down. The magnitudes of the responses, yet, depend on the type of shocks driving the boom. The booms generated by risk mispricing (solid blue lines) or preference shocks (dashed blue lines) lead to credit and investment growth around three times higher than a boom generated by technological improvements (solid green lines). These booms, however, are labeled 'bad' booms since they are not driven by stronger economic fundamentals, but misperceptions about the current and future state of the economy. In the case of a risk-mispricing boom, agents underprice the actual risk in the economy because they perceive risks, for some exogenous reason, to be lower; significantly compressing the term premium and pushing long-term interest rates 2 percentage points below their steady state. The effect of the preference-shock boom is driven by changes in inflation, generating a similarly-sized term premium compression via the inflation channel instead. Therefore, expansions fueled by credit growth that are accompanied by pronounced changes in interest rates or inflation can indicate that the nature of the credit boom is driven by unjustified perceptions of the economy or financial market returns.

Figure 5: Impulse responses to different boom-bust scenarios



Notes: Economic expansion (boom) is generated by risk mispricing shocks in solid blue lines, preference shocks in dashed blue lines, and productivity shocks in solid green lines, scaled such that output increases by 1 percent every year for 3 consecutive years using continuous compounding. Each boom is then counteracted in period 13, labeled a bust. All variables—except for the term premium—are in real terms. Responses are deviations relative to the steady state in the corresponding units indicated in the vertical axis. All variables are quarterly, except for the term premium and the interest rate, which are annualized.

In order to evaluate the potential consequences of credit fueled expansion via different shocks, we generate a severe bust that counteracts the entire three-year boom on period 13. For the good boom scenario, we model the bust as a negative productivity shock, and for the bad boom scenarios, we assume that an exogenous event occurs making agents realize that they have not priced risk correctly or their consumption preferences reverse back to the baseline. All busts lead to a contraction in output and investment and an increase in interest rates and the term premium.[15] The decline in output is,

[15]It should also be noted that the only non-linearity we assume in the model is the one arising due to the higher order

however, more pronounced for the scenario in which the economic booms are driven by risk mispricing, justifying the notion of this being a particularly 'bad' boom. Although the depth of the recession is about halved by the following quarter, output does not revert back to the steady state for about four additional years. This exercise suggests that the transmission from the financial sector to the real economy can be amplified if bank lending is driven by unfounded optimism or mispricing instead of economic fundamentals.

4.2 Macroprudential policies

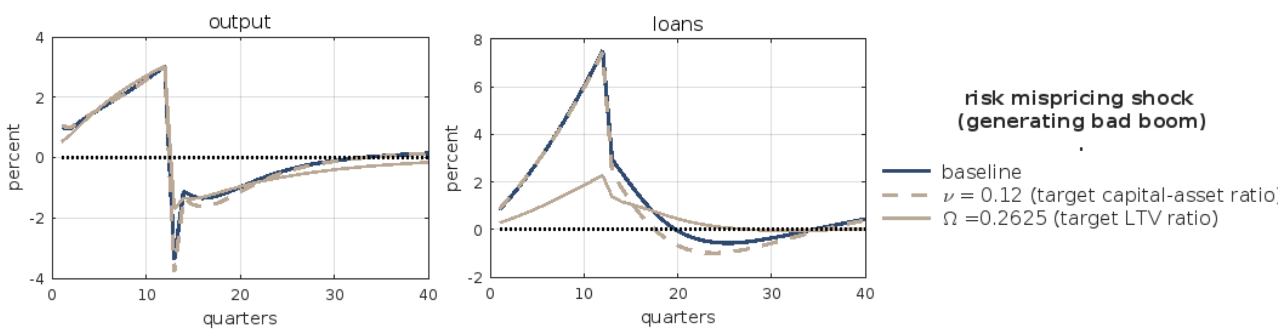
Financial stability policy makers are tasked with identifying bad credit booms in advance, and building up resilience against a potential burst that could be harmful for the economy, such as experienced in the 2008-2009 Great Recession. Policy makers are hence given powers over macroprudential policies to ensure the resilience of the financial system. We test two specific types of macroprudential policies that the financial stability authority can implement in our model: (a) a macroprudential policy that increases banks' target capital-asset ratio, ν in Eq. (11), and (b) a macroprudential policy that decreases the steady state of the LTV ratio of entrepreneurs, $\overline{\Omega}$ in Eq. (5). While bank capital-asset ratios are targeted in particular at strengthening lenders' solvency, the LTV ratio aims to improve borrowers' solvency and should help to avoid unsustainable debt levels. As an illustrative example, we assume a 33 percent increase in the target bank capital-asset ratio from 0.09 to 0.12, so that banks are encouraged to have a larger capital buffer with respect to their assets. In a similar vein, we analyze a 25 percent reduction of the LTV target ratio for entrepreneurs from 0.35 to 0.2625 implying that entrepreneurs need to back up the same quantity of loans with more collateral than before, reasonable values within those discussed in Gerali et al. (2010). Both measures are intended to make the financial system more resilient.

Figure 6 displays the boom-bust scenario driven by risk mispricing shocks under the baseline calibration (solid blue lines), compared to a scenario with higher target capital-asset ratios (dashed gray lines) and lower target LTV ratios (solid gray lines). During the economic boom, higher bank capital-asset ratios reduce the ability of banks to issue credit, leading to loan volume increases similar to those that would occur under a good boom scenario driven by productivity shocks. During the bust, this policy

perturbation method with which we solve the model. The effects of a bust are likely to be larger when accounting for other non-linearities like occasionally binding constraints (see e.g., Bluwstein, 2017).

also helps reduce the severity of the decline in output by almost half, attenuating the contractionary effect on the economy and the volatility of credit growth. A similar result is found by Caballero and Simsek (2020), who showed that macroprudential policies can be welfare improving by reducing the risk-taking behavior of overly optimistic agents. In our simulation, the macroprudential policy targeting lender solvency is deemed more effective in reducing the depth of the recession during a bad-boom reversal driven by risk mispricing of financial markets. Which policy is preferred, of course, will depend on the exact specification of the calibration and the policy maker's preference function which is beyond the scope of this paper.

Figure 6: Impulse responses to bad boom-bust scenario under different macroprudential policies



Notes: Economic expansion (boom) is generated by risk mispricing shocks with the baseline calibration in solid blue lines, under a higher target capital-asset ratio in dashed gray lines, and under a lower target LTV ratio in solid gray lines, scaled such that output increases by 1 percent every year for 3 consecutive years using continuous compounding. The boom is then counteracted in period 13, labeled a bust. All variables are quarterly, in real terms, and in percent deviations relative to the steady state.

5 Conclusion

We construct a unifying model of the macroeconomy with a financial sector to show that incorporating a feedback loop via bank lending helps quantify the general equilibrium effects of risk mispricing shocks. Our model generates a time-varying term premium, consistent with empirical evidence, which allows us to match both macro and finance moments with standard calibrated parameters. We find that shocks that might occur during a panic and lead investors to overprice risk, reduce the supply of credit via the loan price and collateral channels, and significantly reduce the bank's balance sheet, investment, and output. Our simulation also shows that a bad credit boom driven by agents underpricing risks is very different from a good credit boom driven by economic fundamentals. Furthermore, we demonstrate how our model can be used for macroprudential policy analysis to foster financial stability over the business cycle.

There are many avenues in which the model can be extended. In terms of the banking sector, one useful addition would be to allow for private loans to default endogenously and thus generate another source of risk in the model beyond duration risk. By introducing the possibility of corporate default, one could endogenize the risk premium that is charged on top of private loans based on the relative riskiness of private debt over government debt. An example could be modeling the empirical relationship between credit spreads and economic activity found in Faust et al. (2013), in the spirit of Görtz and Tsoukalas (2017). Note also that in our setup, the reason for risk mispricing, and thus the under- or over-valuation of long-term bond prices via the term premium, is exogenous. In reality, these perceptions might have an endogenous cyclical nature. Another interesting avenue to pursue would be to estimate the model formally. Especially, the household parameters, which are crucial to pin down both macroeconomic as well as asset price behavior, would benefit from estimation. As methods that allow to estimate a model to the third order (see e.g. Andreasen et al., 2018) are still difficult to implement for high-dimensional models, we shall leave this possible extension for future investigation.

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Back to the Real Economy: The Effects of Risk Mispricing on the Term Premium and Bank Lending

- Appendix -

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1 A DSGE model with bank lending and risk mispricing

1.1 Households

Equations

Recursive utility function for household i, each of unit mass:

$$V_t(i) = U(c_{h,t}(i), \ell_{h,t}(i)) + \beta_h \left(\mathbb{E}_t V_{t+1}^{1-\xi}(i)\right)^{\frac{1}{1-\xi}}. \tag{1}$$

Intra-period utility function:

$$U(c_{h,t}(i), \ell_{h,t}(i)) = \frac{c_{h,t}^{1-\psi}(i)}{1-\psi} - \frac{\ell_{h,t}^{1+\phi}(i)}{1+\phi}. \tag{2}$$

Budget constraint:

$$c_{h,t}(i) + d_t(i) \leq w_t \ell_{h,t}(i) + (1+r_{t-1})d_{t-1}(i) - T_t(i) + J_{R,t}(i). \tag{3}$$

Solution

Households choose consumption, labor, and deposits to maximize V_0 subject to the utility function (1) and the budget constraint (3), with $\lambda_{1,t}^h(i)$ and $\lambda_{2,t}^h(i)$ being the respective multipliers of the Lagrangean function ($\mathcal{L}$) and substituting Eq. (2) for the intra-period utility function:

$$\begin{aligned}
\mathcal{L} = {} & V_0(i) - \mathbb{E}_0 \sum_{t=0}^{\infty} \lambda_{1,t}^h(i) \left[V_t(i) - U(c_{h,t}(i), \ell_{h,t}(i)) - \beta_h \left(\mathbb{E}_t V_{t+1}^{1-\xi}(i)\right)^{\frac{1}{1-\xi}}\right] \\
& - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_h^t \lambda_{2,t}^h(i) \left[c_{h,t}(i) + d_t(i) - w_t \ell_{h,t}(i) - (1+r_{t-1})d_{t-1}(i) - J_{R,t}(i) + T_t(i)\right] \\
\mathcal{L} = {} & V_0(i) - \mathbb{E}_0 \sum_{t=0}^{\infty} \lambda_{1,t}^h(i) \left[V_t(i) - \frac{c_{h,t}^{1-\psi}(i)}{1-\psi} + \frac{\ell_{h,t}^{1+\phi}(i)}{1+\phi} - \beta_h \left(\mathbb{E}_t V_{t+1}^{1-\xi}(i)\right)^{\frac{1}{1-\xi}}\right] \\
& - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_h^t \lambda_{2,t}^h(i) \left[c_{h,t}(i) + d_t(i) - w_t \ell_{h,t}(i) - (1+r_{t-1})d_{t-1}(i) - J_{R,t}(i) + T_t(i)\right].
\end{aligned}$$

First order necessary conditions are:

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial c_t(i)} &= \lambda_{1,t}^h(i)\frac{1-\psi}{1-\psi}c_{h,t}^{-\psi}(i) - \beta_h^t \lambda_{2,t}^h(i) = 0 \Longrightarrow \lambda_{2,t}^h(i) = \frac{\lambda_{1,t}^h(i)}{\beta_h^t c_{h,t}^{\psi}(i)} && (4)\\
\frac{\partial \mathcal{L}}{\partial \ell_t(i)} &= -\lambda_{1,t}^h(i)\frac{1+\phi}{1+\phi}\ell_{h,t}^{\phi}(i) + \beta_h^t \lambda_{2,t}^h(i) w_t = 0 \Longrightarrow \lambda_{2,t}^h(i) = \frac{\lambda_{1,t}^h(i)\ell_{h,t}^{\phi}(i)}{\beta_h^t w_t} && (5)\\
\frac{\partial \mathcal{L}}{\partial d_t(i)} &= -\beta_h^t \lambda_{2,t}^h(i) + \mathbb{E}_t \beta_h^{t+1}\lambda_{2,t+1}^h(i)(1+r_t) = 0 \Longrightarrow \lambda_{2,t}^h(i) = \beta_h \mathbb{E}_t \lambda_{2,t+1}^h(i)(1+r_t) && (6)\\
\frac{\partial \mathcal{L}}{\partial V_t(i)} &= -\lambda_{1,t}^h(i) + \lambda_{1,t-1}^h(i)\left[\frac{1}{1-\xi}\beta_h\left(\mathbb{E}_{t-1}V_t^{1-\xi}(i)\right)^{\frac{\xi}{1-\xi}}(1-\xi)V_t^{-\xi}(i)\right] = 0 &&\\
&\Longrightarrow \frac{\lambda_{1,t}^h(i)}{\lambda_{1,t-1}^h(i)} = \beta_h\left(\frac{\left(\mathbb{E}_{t-1}V_t^{1-\xi}(i)\right)^{\frac{1}{1-\xi}}}{V_t(i)}\right)^{\xi} && (7)
\end{aligned}
$$

Obtain the labor supply decision from Eq. (4) = (5):

$$
\frac{\lambda_{1,t}^h(i)\ell_{h,t}^{\phi}(i)}{\beta_h^t w_t} = \frac{\lambda_{1,t}^h(i)}{\beta_h^t c_{h,t}^{\psi}(i)} \Longrightarrow \ell_{h,t}^{\phi}(i) = \frac{w_t}{c_{h,t}^{\psi}(i)}
$$

Set Eq. (4) = (6), iterate Eqs. (4) and (7) forward, and substitute to obtain the consumption Euler equation:

$$
\begin{aligned}
\frac{\lambda_{1,t}^h(i)}{\beta_h^t c_{h,t}^{\psi}(i)} &= \beta_h \mathbb{E}_t \lambda_{2,t+1}^h(i)(1+r_t)\\
\frac{1}{c_{h,t}^{\psi}(i)} &= \mathbb{E}_t \frac{\beta_h^{t+1}\lambda_{2,t+1}^h(i)(1+r_t)}{\lambda_{1,t}^h(i)}\\
\frac{1}{c_{h,t}^{\psi}(i)} &= \mathbb{E}_t \frac{\beta_h^{t+1}\lambda_{1,t+1}^h(i)(1+r_t)}{\lambda_{1,t}^h(i)\beta_h^{t+1}c_{h,t+1}^{\psi}(i)}\\
\frac{1}{c_{h,t}^{\psi}(i)} &= \mathbb{E}_t\left[\beta_h \frac{(1+r_t)}{c_{h,t+1}^{\psi}(i)}\left(\frac{\left(\mathbb{E}_t V_{t+1}^{1-\xi}(i)\right)^{\frac{1}{1-\xi}}}{V_{t+1}(i)}\right)^{\xi}\right]\\
1 &= \mathbb{E}_t\left[m_{t,t+1}^h(i)(1+r_t)\right],\\
\text{where } m_{t,t+1}^h(i) &= \beta_h \frac{c_{h,t}^{\psi}(i)}{c_{h,t+1}^{\psi}(i)}\left(\frac{\left(\mathbb{E}_t V_{t+1}^{1-\xi}(i)\right)^{\frac{1}{1-\xi}}}{V_{t+1}(i)}\right)^{\xi}.
\end{aligned}
$$

1.2 Entrepreneurs

Problem

Entrepreneur j of unit mass chooses consumption, labor demand, loan amount, and the stock of physical capital in order to maximize utility subject to budget and borrowing constraints, where $y_{e,t}(j) = A_t k_{t-1}^{\alpha}(j)\,\ell_{d,t}^{1-\alpha}(j)$

$$\begin{aligned}
\max_{\{c_{e,t}(j),\ell_{d,t}(j),b_{e,t}(j),k_t(j)\}} \quad & \mathbb{E}_0\sum_{t=0}^{\infty}\beta_e^t \log\left(c_{e,t}(j)\right) \\
\text{s.t.} \quad & c_{e,t}(j) + (1+r_{b,t-1})b_{e,t-1}(j) + w_t\ell_{d,t}(j) + q_{k,t}k_t(j) \\
& \leq \frac{y_{e,t}(j)}{x_t} + b_{e,t}(j) + q_{k,t}(1-\delta_k)k_{t-1}(j) \qquad (8)\\
& b_{e,t}(j) \leq \frac{\Omega_t\mathbb{E}_t\left[q_{k,t+1}k_t(j)(1-\delta_k)\right]}{1+r_{b,t}}
\end{aligned}$$

Solution

$$\begin{aligned}
\mathcal{L} = {} & \mathbb{E}_0\sum_{t=0}^{\infty}\beta_e^t \log\left(c_{e,t}(j)\right) - \lambda_{1,t}^e(j)\Big[c_{e,t}(j) + (1+r_{b,t-1})b_{e,t-1}(j) + w_t\ell_{d,t}(j) \\
& + q_{k,t}k_t(j) - \frac{A_t k_{t-1}^{\alpha}(j)\,\ell_{d,t}^{1-\alpha}(j)}{x_t} - b_{e,t}(j) - q_{k,t}(1-\delta_k)k_{t-1}(j)\Big] \\
& - \lambda_{2,t}^e(j)\left[b_{e,t}(j) - \frac{\Omega_t\mathbb{E}_t\left[q_{k,t+1}k_t(j)(1-\delta_k)\right]}{1+r_{b,t}}\right]
\end{aligned}$$

$$\begin{aligned}
\frac{\partial\mathcal{L}}{\partial c_{e,t}(j)} &= \beta_e^t\frac{1}{c_{e,t}(j)} - \lambda_{1,t}^e(j) = 0 \Longrightarrow \lambda_{1,t}^e(j) = \beta_e^t\frac{1}{c_{e,t}(j)} \qquad (9)\\
\frac{\partial\mathcal{L}}{\partial \ell_{d,t}(j)} &= -\lambda_{1,t}^e(j)\,w_t + \lambda_{1,t}^e(j)(1-\alpha)\left[\frac{A_t k_{t-1}^{\alpha}(j)\,\ell_{d,t}^{-\alpha}(j)}{x_t}\right] = 0 \\
& \Longrightarrow w_t = (1-\alpha)\left[\frac{A_t k_{t-1}^{\alpha}(j)\,\ell_{d,t}^{-\alpha}(j)}{x_t}\right] \qquad (10)\\
\frac{\partial\mathcal{L}}{\partial b_{e,t}(j)} &= \lambda_{1,t}^e(j) - \mathbb{E}_t\lambda_{1,t+1}^e(j)(1+r_{b,t}) - \lambda_{2,t}^e(j) = 0 \\
& \Longrightarrow \lambda_{2,t}^e(j) = \lambda_{1,t}^e(j) - \mathbb{E}_t\lambda_{1,t+1}^e(j)(1+r_{b,t}) \qquad (11)
\end{aligned}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial k_t(j)} &= -\lambda^e_{1,t}(j)\, q_{k,t} + \mathbb{E}_t \lambda^e_{1,t+1}(j)\left[\alpha \frac{A_{t+1} k_t^{\alpha-1}(j)\, \ell_{d,t+1}^{1-\alpha}(j)}{x_{t+1}} + q_{k,t+1}(1-\delta_k)\right] \\ &\quad + \lambda^e_{2,t}(j)\left[\frac{\Omega_t \mathbb{E}_t\left[q_{k,t+1}(1-\delta_k)\right]}{1+r_{b,t}}\right] = 0 \end{aligned} \tag{12}$$

From Eq. (10), the labor demand schedule is

$$w_t = (1-\alpha)\left[\frac{A_t k_{t-1}^{\alpha}(j)\, \ell_{d,t}^{-\alpha}(i)}{x_t}\right] \Longrightarrow w_t = \frac{(1-\alpha) y_{e,t}(j)}{\ell_{d,t}(j)\, x_t}$$

From substituting Eq. (9) into Eq. (11), and defining $\lambda_{e,t}(j) \equiv \frac{\lambda^e_{2,t}(j)}{\beta_e^t}$, the consumption Euler equation is

$$\begin{aligned} \lambda^e_{2,t}(j) &= \lambda^e_{1,t}(j) - \mathbb{E}_t \lambda^e_{1,t+1}(j)(1+r_{b,t}) \\ \lambda^e_{2,t}(j) &= \beta_e^t \frac{1}{c_{e,t}(j)} - \mathbb{E}_t \beta_e^{t+1} \frac{1}{c_{e,t+1}(j)}(1+r_{b,t}) \\ \lambda^e_{2,t}(j) &= \beta_e^t \left(\frac{1}{c_{e,t}(j)} - \mathbb{E}_t \beta_e \frac{1}{c_{e,t+1}(j)}(1+r_{b,t})\right) \\ 1 &= \mathbb{E}_t\left[m^e_{t,t+1}(j)(1+r_{b,t})\right] \\ \text{where } m^e_{t,t+1}(j) &= \beta_e \left(\frac{1}{c_{e,t}(j)} - \lambda_{e,t}(j)\right)^{-1} \mathbb{E}_t \frac{1}{c_{e,t+1}(j)} \end{aligned}$$

Substitute Eq. (9) into Eq. (12) to obtain the investment Euler equation:

$$\begin{aligned} \lambda^e_{1,t}(j)\, q_{k,t} &= \mathbb{E}_t \lambda^e_{1,t+1}(j)\left[\alpha \frac{A_{t+1} k_t^{\alpha-1}(j)\, \ell_{d,t+1}^{1-\alpha}(j)}{x_{t+1}} + q_{k,t+1}(1-\delta_k)\right] + \lambda^e_{2,t}(j)\left[\frac{\Omega_t \mathbb{E}_t\left[q_{k,t+1}(1-\delta_k)\right]}{1+r_{b,t}}\right] \\ \beta_e^t \frac{1}{c_{e,t}(j)} q_{k,t} &= \mathbb{E}_t \beta_e^{t+1} \frac{1}{c_{e,t+1}(j)}\left[\alpha \frac{A_{t+1} k_t^{\alpha-1}(j)\, \ell_{d,t+1}^{1-\alpha}(j)}{x_{t+1}} + q_{k,t+1}(1-\delta_k)\right] + \lambda^e_{2,t}(j)\left[\frac{\Omega_t \mathbb{E}_t\left[q_{k,t+1}(1-\delta_k)\right]}{1+r_{b,t}}\right] \\ \frac{q_{k,t}}{c_{e,t}(j)} &= \mathbb{E}_t \beta_e \frac{1}{c_{e,t+1}(j)}\left[\underbrace{\alpha \frac{A_{t+1} k_t^{\alpha-1}(j)\, \ell_{d,t+1}^{1-\alpha}(j)}{x_{t+1}}}_{r_{k,t+1}(j)} + q_{k,t+1}(1-\delta_k)\right] + \lambda_{e,t}(j)\left[\frac{\Omega_t \mathbb{E}_t\left[q_{k,t+1}(1-\delta_k)\right]}{1+r_{b,t}}\right] \end{aligned}$$

1.3 Capital good producers

Problem

Capital producers maximize their expected discounted profits:

$$
\begin{aligned}
\max_{\{I_t\}} \quad & \mathbb{E}_{t-1} \sum_{t=1}^{\infty} m^e_{t-1,t} \left(q_{k,t} \left(k_t - (1-\delta_k) k_{t-1} \right) - I_t \right) \\
\text{s.t.} \quad & k_t = (1-\delta_k) k_{t-1} + \left[1 - \frac{\kappa_i}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right] I_t \\
& m^e_{t-1,t} = \left(\frac{1}{c_{e,t-1}} - \lambda_{e,t-1} \right)^{-1} \beta_e \frac{1}{c_{e,t}}
\end{aligned}
$$

Solution

Let the change in the marginal value of one unit increase in the budget be equal to the change in the marginal value of one unit of additional borrowing.

$$
\mathcal{L} = \mathbb{E}_{t-1} \sum_{t=1}^{\infty} m^e_{t-1,t} \left(q_{k,t} \left[1 - \frac{\kappa_i}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right] I_t - I_t \right)
$$

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial I_t} &= m^e_{t-1,t} q_{k,t} - m^e_{t-1,t} - m^e_{t-1,t} q_{k,t} \frac{\kappa_i}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - m^e_{t-1,t} q_{k,t} I_t \kappa_i \left(\frac{I_t}{I_{t-1}} - 1 \right) \frac{1}{I_{t-1}} \\
&\quad + m^e_{t,t+1} q_{k,t+1} \kappa_i \left(\frac{I_{t+1}}{I_t} - 1 \right) \frac{I_{t+1}}{I_t^2} I_{t+1} = 0 \\
0 &= m^e_{t-1,t} \Bigg(q_{k,t} - 1 - q_{k,t} \frac{\kappa_i}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - q_{k,t} I_t \kappa_i \left(\frac{I_t}{I_{t-1}} - 1 \right) \frac{1}{I_{t-1}} \\
&\quad + \mathbb{E}_t \frac{m^e_{t,t+1}}{m^e_{t-1,t}} q_{k,t+1} \kappa_i \left(\frac{I_{t+1}}{I_t} - 1 \right) \frac{I_{t+1}}{I_t^2} I_{t+1} \Bigg) \\
1 &= q_{k,t} \left[1 - \frac{\kappa_i}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \kappa_i \left(\frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} \right] + \mathbb{E}_t \beta_e \frac{c_{e,t}}{c_{e,t+1}} q_{k,t+1} \kappa_i \left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2
\end{aligned}
$$

1.4 Retailers

Let $y_t(v)$ be the quantity of output sold by retailer v and $P_t(v)$ the associated price. Given the final-goods production technology: $y_t = \left(\int_0^1 y_t(v)^{\frac{\epsilon_{y,t}-1}{\epsilon_{y,t}}} dv \right)^{\frac{\epsilon_{y,t}}{\epsilon_{y,t}-1}}$, where $\epsilon_{y,t}$ represents the elasticity of substitution between differentiated final goods, we derive the retailers demand function

$$
\begin{aligned}
&\max_{\{y_t(v)\}} P_t y_t - \int_{i=0}^{1} P_t(v)\, y_t(v)\, dv \\
&\text{s.t.} \quad y_t = \left(\int_0^1 y_t(v)^{\frac{\epsilon_{y,t}-1}{\epsilon_{y,t}}}\, dv\right)^{\frac{\epsilon_{y,t}}{\epsilon_{y,t}-1}}
\end{aligned}
$$

$$
\mathcal{L} = P_t \left(\int_0^1 y_t(v)^{\frac{\epsilon_{y,t}-1}{\epsilon_{y,t}}}\, dv\right)^{\frac{\epsilon_{y,t}}{\epsilon_{y,t}-1}} - \int_{i=0}^{1} P_t(v)\, y_t(v)\, dv
$$

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial y_t(v)} &= P_t \frac{\epsilon_{y,t}}{\epsilon_{y,t}-1} \left(\int_0^1 y_t(v)^{\frac{\epsilon_{y,t}-1}{\epsilon_{y,t}}}\, dv\right)^{\frac{\epsilon_{y,t}}{\epsilon_{y,t}-1}-1} \frac{\epsilon_{y,t}-1}{\epsilon_{y,t}} y_t(v)^{\frac{\epsilon_{y,t}-1}{\epsilon_{y,t}}-1} - P_t(v) = 0 \\
P_t(v) &= P_t \left(\int_0^1 y_t(v)^{\frac{\epsilon_{y,t}-1}{\epsilon_{y,t}}}\, dv\right)^{\frac{1}{\epsilon_{y,t}-1}} y_t(v)^{\frac{-1}{\epsilon_{y,t}}} \\
P_t(v) &= P_t \left(\left(\int_0^1 y_t(v)^{\frac{\epsilon_{y,t}-1}{\epsilon_{y,t}}}\, dv\right)^{\frac{\epsilon_{y,t}}{\epsilon_{y,t}-1}}\right)^{\frac{1}{\epsilon_{y,t}}} y_t(v)^{\frac{-1}{\epsilon_{y,t}}} \\
P_t(v) &= P_t y_t^{\frac{1}{\epsilon_{y,t}}} y_t(v)^{\frac{-1}{\epsilon_{y,t}}} \\
y_t(v)^{\frac{-1}{\epsilon_{y,t}}} &= \frac{P_t(v)}{P_t} y_t^{-\frac{1}{\epsilon_{y,t}}} \\
y_t(v) &= \left(\frac{P_t(v)}{P_t}\right)^{-\epsilon_{y,t}} y_t.
\end{aligned}
$$

We then substitute the demand function into the final-goods production technology to obtain the aggregate price level P_t

$$
\begin{aligned}
y_t &= \left(\int_0^1 \left(\left(\frac{P_t(v)}{P_t}\right)^{-\epsilon_{y,t}} y_t\right)^{\frac{\epsilon_{y,t}-1}{\epsilon_{y,t}}} dv\right)^{\frac{\epsilon_{y,t}}{\epsilon_{y,t}-1}} \\
1 &= \left(\int_0^1 \left(\frac{P_t(v)}{P_t}\right)^{1-\epsilon_{y,t}} dv\right)^{\frac{\epsilon_{y,t}}{\epsilon_{y,t}-1}} \\
P_t^{\epsilon_{y,t}} &= \left(\int_0^1 P_t(v)^{1-\epsilon_{y,t}}\, dv\right)^{\frac{\epsilon_{y,t}}{1-\epsilon_{y,t}}} \\
P_t &= \left(\int_0^1 P_t(v)^{1-\epsilon_{y,t}}\, dv\right)^{\frac{1}{1-\epsilon_{y,t}}}.
\end{aligned}
$$

Problem

Each retailer v chooses its price to maximize the expected discounted value of profits subject to the demand for consumption goods, which we express in real dollars:

$$\begin{array}{ll} \max\limits_{\{P_t(v)\}} & \mathbb{E}_{t-1}\sum_{t=1}^{\infty} m_{t-1,t}^h \left[P_t\left(v\right)y_t\left(v\right) - P_{W,t}y_t\left(v\right) - \frac{\kappa_\pi}{2}\left(\frac{P_t\left(v\right)}{P_{t-1}\left(v\right)} - 1\right)^2 P_t y_t\right] \\ \text{s.t.} & y_t\left(v\right) = \left(\frac{P_t\left(v\right)}{P_t}\right)^{-\epsilon_{y,t}} y_t \end{array} \tag{13}$$

Solution

$$\mathcal{L} = \mathbb{E}_{t-1}\sum_{t=1}^{\infty} m_{t-1,t}^h \left[P_t\left(v\right)\left(\frac{P_t\left(v\right)}{P_t}\right)^{-\epsilon_{y,t}} \frac{y_t}{P_t} - P_{W,t}\left(\frac{P_t\left(v\right)}{P_t}\right)^{-\epsilon_{y,t}} \frac{y_t}{P_t} - \frac{\kappa_\pi}{2}\left(\frac{P_t\left(v\right)}{P_{t-1}\left(v\right)} - 1\right)^2 P_t \frac{y_t}{P_t}\right]$$

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial P_t\left(v\right)} &= m_{t-1,t}^h \left[\left(\frac{P_t\left(v\right)}{P_t}\right)^{-\epsilon_{y,t}} \frac{y_t}{P_t} - \epsilon_{y,t}P_t\left(v\right)\frac{y_t}{P_t}P_t^{\epsilon_{y,t}}P_t\left(v\right)^{-\epsilon_{y,t}-1} + \epsilon_{y,t}P_{W,t}P_t^{\epsilon_{y,t}}\frac{y_t}{P_t}P_t\left(v\right)^{-\epsilon_{y,t}-1}\right. \\
&\quad \left. -\kappa_\pi y_t\left(\frac{P_t\left(v\right)}{P_{t-1}\left(v\right)} - 1\right)\frac{1}{P_{t-1}\left(v\right)}\right] + \mathbb{E}_t m_{t,t+1}^h\left[\kappa_\pi y_{t+1}\left(\frac{P_{t+1}\left(v\right)}{P_t\left(v\right)} - 1\right)\frac{P_{t+1}\left(v\right)}{P_t^2\left(v\right)}\right] = 0 \\
0 &= \left(\frac{P_t\left(v\right)}{P_t}\right)^{-\epsilon_{y,t}}\left(1-\epsilon_{y,t}\right) + \epsilon_{y,t}P_{W,t}P_t^{\epsilon_{y,t}}P_t\left(v\right)^{-\epsilon_{y,t}-1} - \kappa_\pi P_t\left(\frac{P_t\left(v\right)}{P_{t-1}\left(v\right)} - 1\right)\frac{1}{P_{t-1}\left(v\right)} \\
&\quad +\mathbb{E}_t\left[\frac{m_{t,t+1}^h}{m_{t-1,t}^h}\kappa_\pi\left(\frac{P_{t+1}\left(v\right)}{P_t\left(v\right)} - 1\right)\left(\frac{P_{t+1}\left(v\right)}{P_t^2\left(v\right)}\right)\frac{y_{t+1}}{y_t}P_t\right] \\
0 &= \left(\frac{P_t\left(v\right)}{P_t}\right)^{-\epsilon_{y,t}}\left(1-\epsilon_{y,t}\right) + \epsilon_{y,t}P_{W,t}P_t^{\epsilon_{y,t}}P_t\left(v\right)^{-\epsilon_{y,t}-1} - \kappa_\pi\left(\frac{P_t\left(v\right)}{P_{t-1}\left(v\right)} - 1\right)\frac{P_t}{P_{t-1}\left(v\right)} \\
&\quad +\mathbb{E}_t\left[\frac{m_{t,t+1}^h}{m_{t-1,t}^h}\kappa_\pi\left(\frac{P_{t+1}\left(v\right)}{P_t\left(v\right)} - 1\right)\frac{P_{t+1}\left(v\right)}{P_t\left(v\right)}\frac{y_{t+1}}{y_t}\right].
\end{aligned}$$

In equilibrium all retailers behave identically, charging the same price, such that the optimality condition for prices can be written in terms of inflation rates, where $\frac{P_t}{P_{t-1}} = \pi_t + 1$ is the gross inflation rate, $\frac{1}{x_t} = \frac{P_{W,t}}{P_t}$ and $\epsilon_{y,t} = \frac{\mu_{y,t}}{\mu_{y,t}-1}$:

$$0 = \left(1 - \frac{\mu_{y,t}}{\mu_{y,t}-1}\right) + \frac{\mu_{y,t}}{\mu_{y,t}-1}\frac{1}{x_t} - \kappa_\pi\left(\pi_t + 1\right)\pi_t + \mathbb{E}_t\left[\frac{m_{t,t+1}^h}{m_{t-1,t}^h}\kappa_\pi\left(\pi_{t+1}+1\right)\pi_{t+1}\frac{y_{t+1}}{y_t}\right].$$

1.5 Banks

1.5.1 Wholesale branch

The wholesale branch collects deposits d_t from the households that pay r_t interest rate and issue wholesale loans b_t that receive net interest rate $r_{w,t}$. The branch manages bank capital $k_{b,t}$, accumulated out of reinvested profits, subject to a quadratic cost function that penalizes capital-asset ratio deviations from their target ν. The optimization problem is to choose the amount of wholesale loans that maximize the discounted sum of cashflows subject to the balance sheet constraint:

$$\begin{aligned} \max_{\{b_t\}} \quad & \mathbb{E}_{t-1} \sum_{t=1}^{\infty} m_{t-1,t}^{h} \left[(1+r_{w,t})\, b_t - b_{t+1} + d_{t+1} + \Delta k_{b,t+1} - (1+r_t)\, d_t - \frac{\theta}{2} \left(\frac{k_{b,t}}{b_t} - \nu \right)^2 k_{b,t} \right] \\ \text{s.t.} \quad & b_t = d_t + k_{b,t}. \end{aligned} \tag{14}$$

Profits are discounted with the stochastic discount factor of the households that own the banks, but our setup collapses to a per-period static problem, leading to the equilibrium condition for the wholesale (real) rate in net terms:

$$\mathcal{L} = r_{w,t} b_t - r_t \left(b_t - k_{b,t} \right) - \frac{\theta}{2} \left(\frac{k_{b,t}}{b_t} - \nu \right)^2 k_{b,t}$$

$$\frac{\partial \mathcal{L}}{\partial b_t} = r_{w,t} - r_t + \theta \left(\frac{k_{b,t}}{b_t} - \nu \right) \frac{k_{b,t}}{b_t^2} k_{b,t} = 0$$

$$r_{w,t} = r_t - \theta \left(\frac{k_{b,t}}{b_t} - \nu \right) \left(\frac{k_{b,t}}{b_t} \right)^2 .$$

1.5.2 Retail branch

To model market power, we assume a Dixit-Stiglitz framework for the entrepreneur loans market, such that units of loan contracts bought by entrepreneurs are a composite CES basket of slightly differentiated products supplied by a branch of a bank n with constant elasticities of substitution equal to $\epsilon_b > 1$. The demand function for entrepreneur j seeking an amount of real loans equal to $\overline{b}_{e,t}(j)$ can be derived from minimizing the due total repayment:

$$\min_{\{b_{e,t}(j,n)\}} \int_{n=0}^{1} r_{b,t}(n)\, b_{e,t}(j,n)\, dn \quad \text{s.t.} \quad \left[\int_{n=0}^{1} b_{e,t}(j,n)^{\frac{\epsilon_b-1}{\epsilon_b}}\, dn\right]^{\frac{\epsilon_b}{\epsilon_b-1}} \geq \bar{b}_{e,t}(j)$$

$$\mathcal{L} = \int_{n=0}^{1} r_{b,t}(n)\, b_{e,t}(j,n)\, dn + \lambda_{b,t}\left[\bar{b}_{e,t}(j) - \left[\int_{n=0}^{1} b_{e,t}(j,n)^{\frac{\epsilon_b-1}{\epsilon_b}}\, dn\right]^{\frac{\epsilon_b}{\epsilon_b-1}}\right]$$

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial b_{e,t}(j,n)} &= r_{b,t}(n) - \lambda_{b,t}\frac{\epsilon_b}{\epsilon_b-1}\left[\int_{n=0}^{1} b_{e,t}(j,n)^{\frac{\epsilon_b-1}{\epsilon_b}}\, dn\right]^{\frac{\epsilon_b}{\epsilon_b-1}-1} \frac{\epsilon_b-1}{\epsilon_b} b_{e,t}(j,n)^{\frac{\epsilon_b-1}{\epsilon_b}-1} = 0 \\
r_{b,t}(n) &= \lambda_{b,t}\left[\int_{n=0}^{1} b_{e,t}(j,n)^{\frac{\epsilon_b-1}{\epsilon_b}}\, dn\right]^{\frac{1}{\epsilon_b-1}} b_{e,t}(j,n)^{\frac{-1}{\epsilon_b}} \\
b_{e,t}(j,n)^{\frac{-1}{\epsilon_b}} &= \frac{r_{b,t}(n)}{\lambda_{b,t}}\left[b_{e,t}(j)^{\frac{\epsilon_b-1}{\epsilon_b}}\right]^{-\frac{1}{\epsilon_b-1}} \\
b_{e,t}(j,n) &= \left(\frac{r_{b,t}(n)}{\lambda_{b,t}}\right)^{-\epsilon_b} b_{e,t}(j).
\end{aligned}$$

This equation can also be rewritten as: $r_{b,t}(n) = \left(\frac{b_{e,t}(j,n)}{b_{e,t}(j)}\right)^{\frac{1}{-\epsilon_b}} \lambda_{b,t}$. Let $r_{b,t} = \left[\int_{n=0}^{1} r_{b,t}(n)^{1-\epsilon_b}\, dn\right]^{\frac{1}{1-\epsilon_b}}$ to find that

$$\begin{aligned}
r_{b,t} &= \left[\int_{n=0}^{1} r_{b,t}(n)^{1-\epsilon_b}\, dn\right]^{\frac{1}{1-\epsilon_b}} \\
r_{b,t} &= \left[\int_{n=0}^{1} \left(\left(\frac{b_{e,t}(j,n)}{b_{e,t}(j)}\right)^{\frac{1}{-\epsilon_b}} \lambda_{b,t}\right)^{1-\epsilon_b} dn\right]^{\frac{1}{1-\epsilon_b}} \\
r_{b,t} &= \lambda_{b,t},
\end{aligned}$$

such that the final demand condition for loans to entrepreneurs $b_{e,t}(n)$ at bank n depends on overall volumes and on the interest rates charged by bank n relative to the average rates in the economy:

$$b_{e,t}(n) = \left(\frac{r_{b,t}(n)}{r_{b,t}}\right)^{-\epsilon_b} b_{e,t}.$$

The retail branch problem is then defined as choosing the loan interest rate subject to the demand for loan rates in a monopolistically competitive environment:

$$\max_{\{r_{b,t}(n)\}} \mathbb{E}_{t-1} \sum_{t=1}^{\infty} m_{t-1,t}^{h} \left[r_{b,t}(n) \, b_{e,t}(n) + r_{l,t} b_{l,t}(n) - r_{w,t} b_t(n) \right]$$

$$\text{s.t. } b_t(n) = b_{e,t}(n) + b_{l,t}(n) \tag{15}$$

$$b_{e,t}(n) = \left(\frac{r_{b,t}(n)}{r_{b,t}} \right)^{-\epsilon_b} b_{e,t}$$

$$\mathcal{L} = r_{b,t}(n) \, b_{e,t}(n) + r_{l,t} \left(b_t(n) - b_{e,t}(n) \right) - r_{w,t} \left(b_{e,t}(n) + b_{l,t}(n) \right)$$

$$\mathcal{L} = r_{b,t}(n) \left(\frac{r_{b,t}(n)}{r_{b,t}} \right)^{-\epsilon_b} b_{e,t} + r_{l,t} \left(b_t(n) - \left(\frac{r_{b,t}(n)}{r_{b,t}} \right)^{-\epsilon_b} b_{e,t} \right) - r_{w,t} \left(\left(\frac{r_{b,t}(n)}{r_{b,t}} \right)^{-\epsilon_b} b_{e,t} + b_{l,t}(n) \right)$$

$$\mathcal{L} = r_{b,t}^{1-\epsilon_b}(n) \left(\frac{1}{r_{b,t}} \right)^{-\epsilon_b} b_{e,t} + r_{l,t} \left(b_t(n) - \left(\frac{r_{b,t}(n)}{r_{b,t}} \right)^{-\epsilon_b} b_{e,t} \right) - r_{w,t} \left(\left(\frac{r_{b,t}(n)}{r_{b,t}} \right)^{-\epsilon_b} b_{e,t} + b_{l,t}(n) \right)$$

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial r_{b,t}(n)} &= (1-\epsilon_b) \left(\frac{r_{b,t}(n)}{r_{b,t}} \right)^{-\epsilon_b} b_{e,t} + \epsilon_b r_{l,t} r_{b,t}^{-\epsilon_b - 1}(n) \left(\frac{1}{r_{b,t}} \right)^{-\epsilon_b} b_{e,t} + \epsilon_b r_{w,t} b_{e,t} r_{b,t}^{-\epsilon_b - 1}(n) \left(\frac{1}{r_{b,t}} \right)^{-\epsilon_b} \\
0 &= (1-\epsilon_b) + \epsilon_b r_{b,t}^{-1}(n) \left(r_{l,t} + r_{w,t} \right) \\
r_{b,t} &= \left(r_{l,t} + r_{w,t} \right) \frac{\epsilon_b}{\epsilon_b - 1}.
\end{aligned}$$

1.5.3 Profits

Bank profits $J_{b,t}$ from both branches,

$$\begin{aligned}
J_{b,t} &= r_{w,t} b_t - r_t \left(b_t - k_{b,t} \right) - \frac{\theta}{2} \left(\frac{k_{b,t}}{b_t} - \nu \right)^2 k_{b,t} \\
&\quad + r_{b,t} b_{e,t} + r_{l,t} b_{l,t} - r_{w,t} b_t \\
J_{b,t} &= r_{b,t} b_{e,t} + r_{l,t} b_{l,t} - r_t d_t - \frac{\theta}{2} \left(\frac{k_{b,t}}{b_t} - \nu \right)^2 k_{b,t}, \qquad (16)
\end{aligned}$$

get reinvested as bank capital, which depreciates at rate δ_b in the following accumulation equation:

$$k_{b,t} = (1-\delta_b) \, k_{b,t-1} + J_{b,t-1}. \tag{17}$$

1.6 Government sector

Government budget constraint:

$$G_t + (1 + r_{l,t-1})\, b_{l,t-1} = T_t + b_{l,t} \tag{18}$$

Government spending (exogenous):

$$G_t = (1 - \rho_g)\, \overline{G} + \rho_g G_{t-1} + \varepsilon_{g,t}$$

Supply of government bonds (exogenous):

$$b_{l,t} = (1 - \rho_{bl})\, \overline{b_l} + \rho_{bl} b_{l,t-1} + \varepsilon_{bl,t}$$

1.7 Asset pricing equations and risk mispricing

Long-term bond price formula (including risk mispricing shocks):

$$p_{l,t} = \mathbb{E}_t \left[\left(m^h_{t,t+1} + \epsilon_{risk,t} \right) (1 + \delta_c p_{l,t+1}) \right].$$

In equilibrium, this collapses to the standard asset pricing equation for long-term bonds:

$$\overline{p_l} = \frac{\beta_h}{1 - \beta_h \delta_c}.$$

The rate of return on the long-term bond ($r_{l,t}$) equates the price of the long-term bond today to the present discounted value of cashflows on perpetuity paying a coupon normalized to 1 that decays at rate δ_c. Using the formula for the infinite geometric series and $ln\,(1 + r_{l,t}) \approx r_{l,t}$:

$$\begin{aligned}
p_{l,t} &= \frac{1}{1 + r_{l,t}} + \frac{\delta_c}{(1 + r_{l,t})^2} + \frac{\delta_c^2}{(1 + r_{l,t})^3} + ... \\
p_{l,t} &= \frac{1}{1 + r_{l,t}} \left(\frac{1 + r_{l,t}}{1 + r_{l,t} - \delta_c} \right) \\
r_{l,t} &= ln \left(\frac{\delta_c p_{l,t} + 1}{p_{l,t}} \right).
\end{aligned}$$

Similarly for the risk-neutral price of the long-term bond (discounted by the risk-free rate) and its rate of return:

$$\begin{aligned} \widehat{p}_{l,t} &= \mathbb{E}_t\left[\frac{1+\delta_c \widehat{p}_{l,t+1}}{1+r_t}\right], \\ \widehat{r}_{l,t} &= ln\left(\frac{\delta_c \widehat{p}_{l,t}+1}{\widehat{p}_{l,t}}\right). \end{aligned}$$

The real term premium is then defined as:

$$tp_{l,t} = r_{l,t} - \widehat{r}_{l,t}.$$

Let risk mispricing be an exogenous process:

$$\epsilon_{risk,t} = \rho_{risk}\epsilon_{risk,t-1} - \varepsilon_{risk,t}.$$

Finally, to calibrate the decay rate for the long-term government bond, we set the duration of a 10-year bond (40 quarters) equal to the discounted present value of cashflows of a perpetual bond, and solve for δ_c by setting $\overline{r}_l = 0.007175$, such that we match the average of a 10-year real rate on a government bond of 2.87 percent:

$$\delta_c = 1 + \overline{r}_l - \frac{1+\overline{r}_l}{40} \simeq 0.982.$$

1.8 Monetary policy

$$\frac{1+r_t}{1+\overline{r}} = \left(\frac{1+r_{t-1}}{1+\overline{r}}\right)^{\rho}\left(\left(\frac{1+\pi_t}{1+\overline{\pi}}\right)^{\phi_\pi}\left(\frac{y_t}{\overline{y}}\right)^{\phi_y}\right)^{1-\rho}(1+\varepsilon_{r,t})$$

1.9 Aggregation

For the aggregate resource constraint, we combine the budget constraint of the households Eq. (3), the budget constraint of the entrepreneurs Eq. (8), and the government budget constraint Eq. (18). We substitute retailers' profits $J_{R,t} = y_t - \frac{y_t}{x_t} - \frac{\kappa_\pi}{2}\left(\frac{P_t}{P_{t-1}} - 1\right)^2 y_t$ from Eq. (13), the bank balance sheet constraint $b_t - k_{b,t} = d_t$ from Eq. (14), and the bank assets definition $b_t = b_{e,t} + b_{l,t}$ from Eq. (15). The clearing condition for the labor market is $\ell_{h,t} = \ell_{d,t}$ and the aggregate condition for output is

$y_t = y_{e,t}(j)$. We use Eq. (17) and the iterated version of Eq. (16) to obtain the aggregate condition:

$$
\begin{aligned}
c_{h,t} + d_t + c_{e,t} + (1+r_{b,t-1})b_{e,t-1} + w_t\ell_{d,t} + q_{k,t}k_t + G_t + (1+r_{l,t-1})\,b_{l,t-1} &= w_t\ell_{h,t} + (1+r_{t-1})d_{t-1} \\
-T_t + J_{R,t} + \frac{y_{e,t}}{x_t} + b_{e,t} + q_{k,t}(1-\delta_k)k_{t-1} + T_t + b_{l,t}
\end{aligned}
$$

$$
\begin{aligned}
c_{h,t} + c_{e,t} + G_t + \frac{\kappa_\pi}{2}\pi_t^2 y_t &= y_t + q_{k,t}(1-\delta_k)k_{t-1} - q_{k,t}k_t + k_{b,t} + (1+r_{t-1})d_{t-1} \\
&\quad -(1+r_{b,t-1})b_{e,t-1} - (1+r_{l,t-1})\,b_{l,t-1} \\
c_{h,t} + c_{e,t} + G_t + \frac{\kappa_\pi}{2}\pi_t^2 y_t &= y_t - q_{k,t}k_t + q_{k,t}(1-\delta_k)k_{t-1} + (1-\delta_b)\,k_{b,t-1} + r_{b,t-1}b_{e,t-1} \\
&\quad + r_{l,t-1}b_{l,t-1} - r_{t-1}d_{t-1} - \frac{\theta}{2}\left(\frac{k_{b,t-1}}{b_{t-1}} - \nu\right)^2 k_{b,t-1} \\
&\quad +(1+r_{t-1})d_{t-1} - (1+r_{b,t-1})b_{e,t-1} - (1+r_{l,t-1})\,b_{l,t-1} \\
c_{h,t} + c_{e,t} + G_t + \frac{\kappa_\pi}{2}\pi_t^2 y_t &= y_t - q_{k,t}k_t + q_{k,t}(1-\delta_k)k_{t-1} - \delta_b k_{b,t-1} - \frac{\theta}{2}\left(\frac{k_{b,t-1}}{b_{t-1}} - \nu\right)^2 k_{b,t-1} \\
&\quad + k_{b,t-1} + d_{t-1} - b_{e,t-1} - b_{l,t-1}
\end{aligned}
$$

$$
y_t = \underbrace{c_{h,t} + c_{e,t}}_{c_t} + q_{k,t}\left(k_t - (1-\delta_k)k_{t-1}\right) + G_t + \frac{\kappa_\pi}{2}\pi_t^2 y_t + \delta_b k_{b,t-1} + \frac{\theta}{2}\left(\frac{k_{b,t-1}}{b_{t-1}} - \nu\right)^2 k_{b,t-1}
$$

1.10 Equilibrium conditions

$$
\begin{aligned}
&(HH) \qquad V_t = \frac{c_{h,t}^{1-\psi}}{1-\psi} - \frac{\ell_{h,t}^{1+\phi}}{1+\phi} + \beta_h\left(\mathbb{E}_t V_{t+1}^{1-\xi}\right)^{\frac{1}{1-\xi}} \\
&(HH) \qquad \frac{1}{1+r_t} = \mathbb{E}_t\left[\beta_h \frac{c_{h,t}^{\psi}}{c_{h,t+1}^{\psi}}\left(\frac{\left(\mathbb{E}_t V_{t+1}^{1-\xi}\right)^{\frac{1}{1-\xi}}}{V_{t+1}}\right)^{\xi}\right] \\
&(HH) \qquad c_{h,t}^{\psi} = \frac{w_t}{\ell_{h,t}^{\phi}} \\
&(HH) \qquad J_{R,t} = c_{h,t} + d_t - w_t\ell_{h,t} - (1+r_{t-1})d_{t-1} + T_t \\
&(Ent) \qquad 1 = \mathbb{E}_t\left[\beta_e\left(\frac{1}{c_{e,t}} - \lambda_{e,t}\right)^{-1}\frac{1}{c_{e,t+1}}(1+r_{b,t})\right] \\
&(Ent) \qquad y_{e,t} = A_t k_{t-1}^{\alpha}\ell_{d,t}^{1-\alpha} \\
&(Ent) \qquad r_{k,t} = \alpha\frac{A_t k_{t-1}^{\alpha-1}\ell_{d,t}^{1-\alpha}}{x_t} \\
&(Ent) \qquad w_t = \frac{(1-\alpha)y_{e,t}}{\ell_{d,t}x_t}
\end{aligned}
$$

$$(Ent) \qquad \frac{q_{k,t}}{c_{e,t}} = \mathbb{E}_t\left[\beta_e \frac{1}{c_{e,t+1}}\left(r_{k,t+1} + q_{k,t+1}(1-\delta_k)\right)\right] + \lambda_{e,t}\left(\frac{\Omega_t \mathbb{E}_t\left[q_{k,t+1}(1-\delta_k)\right]}{1+r_{b,t}}\right)$$

$$(Ent) \qquad b_{e,t} = \frac{\Omega_t \mathbb{E}_t\left[q_{k,t+1}k_t(1-\delta_k)\right]}{1+r_{b,t}}$$

$$(Ent) \qquad c_{e,t} = \frac{y_t}{x_t} + b_{e,t} - (1+r_{b,t-1})b_{e,t-1} - w_t\ell_{d,t} - q_{k,t}\left(k_t - (1-\delta_k)k_{t-1}\right)$$

$$(Cap) \qquad k_t = (1-\delta_k)k_{t-1} + \left(1 - \frac{\kappa_i}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right) I_t$$

$$(Cap) \qquad 1 = q_{k,t}\left(1 - \frac{\kappa_i}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 - \kappa_i\left(\frac{I_t}{I_{t-1}} - 1\right)\frac{I_t}{I_{t-1}}\right) + \mathbb{E}_t\left[\beta_e \frac{c_{e,t}}{c_{e,t+1}} q_{k,t+1}\kappa_i\left(\frac{I_{t+1}}{I_t} - 1\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right]$$

$$(Ret) \qquad 0 = 1 - \frac{\mu_{y,t}}{\mu_{y,t}-1} + \frac{\mu_{y,t}}{\mu_{y,t}-1}\frac{1}{x_t} - \kappa_\pi\left(\pi_t+1\right)\pi_t + \mathbb{E}_t\left[\frac{m^h_{t,t+1}}{m^h_{t-1,t}}\kappa_\pi\left(\pi_{t+1}+1\right)\pi_{t+1}\frac{y_{t+1}}{y_t}\right]$$

$$(Bank) \qquad b_t = b_{e,t} + b_{l,t}$$

$$(Bank) \qquad d_t = b_t - k_{b,t}$$

$$(Bank) \qquad k_{b,t} = (1-\delta_b)\,k_{b,t-1} + J_{b,t-1}$$

$$(Bank) \qquad r_{w,t} = r_t - \theta\left(\frac{k_{b,t}}{b_t} - \nu\right)\left(\frac{k_{b,t}}{b_t}\right)^2$$

$$(Bank) \qquad r_{b,t} = \left(r_{l,t} + r_{w,t}\right)\frac{\epsilon_b}{\epsilon_b - 1}$$

$$(Bank) \qquad J_{b,t} = r_{b,t}b_{e,t} + r_{l,t}b_{l,t} - r_t d_t - \frac{\theta}{2}\left(\frac{k_{b,t}}{b_t} - \nu\right)^2 k_{b,t}$$

$$(MP) \qquad \frac{1+r_t}{1+\overline{r}} = \left(\frac{1+r_{t-1}}{1+\overline{r}}\right)^\rho \left(\left(\frac{1+\pi_t}{1+\overline{\pi}}\right)^{\phi_\pi}\left(\frac{y_t}{\overline{y}}\right)^{\phi_y}\right)^{1-\rho}(1+\varepsilon_{r,t})$$

$$(Bond) \qquad p_{l,t} = \mathbb{E}_t\left[\left(m^h_{t,t+1} + \epsilon_{risk,t}\right)\left(1 + \delta_c p_{l,t+1}\right)\right]$$

$$(Bond) \qquad \widehat{p}_{l,t} = \mathbb{E}_t\left[\frac{1+\delta_c\widehat{p}_{l,t+1}}{1+r_t}\right]$$

$$(Bond) \qquad r_{l,t} = ln\left(\frac{\delta_c p_{l,t} + 1}{p_{l,t}}\right)$$

$$(Bond) \qquad \widehat{r}_{l,t} = ln\left(\frac{\delta_c \widehat{p}_{l,t} + 1}{\widehat{p}_{l,t}}\right)$$

$$(Bond) \qquad tp_{l,t} = r_{l,t} - \widehat{r}_{l,t}$$

$$(Gov) \qquad T_t = G_t - b_{l,t} + \left(1 + r_{l,t-1}\right)b_{l,t-1}$$

$$(Agg) \qquad c_t = c_{h,t} + c_{e,t}$$

$$(Agg) \qquad y_{e,t} = y_t$$

$$(Agg) \qquad \ell_{h,t} = \ell_{d,t}$$

$$(Agg) \qquad y_t = c_t + q_{k,t}\left(k_t - (1-\delta_k)k_{t-1}\right) + G_t + \frac{\kappa_\pi}{2}\pi_t^2 y_t + \delta_b k_{b,t-1} + \frac{\theta}{2}\left(\frac{k_{b,t-1}}{b_{t-1}} - \nu\right)^2 k_{b,t-1}$$

Exogenous Processes

$$\begin{aligned}
&(TFP) && A_t = (1-\rho_a)\,\overline{A} + \rho_a A_{t-1} + \varepsilon_{a,t} \\
&(LTV) && \Omega_t = (1-\rho_\Omega)\,\overline{\Omega} + \rho_\Omega \Omega_{t-1} + \varepsilon_{\Omega,t} \\
&(MARKUP) && \mu_{y,t} = (1-\rho_{\mu y})\,\overline{\mu_y} + \rho_{\mu y}\mu_{y,t-1} + \varepsilon_{\mu y,t} \\
&(RISK) && \epsilon_{risk,t} = \rho_{risk}\epsilon_{risk,t-1} - \varepsilon_{risk,t} \\
&(BOND) && b_{l,t} = (1-\rho_{bl})\,\overline{b_l} + \rho_{bl} b_{l,t-1} + \varepsilon_{bl,t} \\
&(GOV) && G_{l,t} = (1-\rho_g)\,\overline{G} + \rho_g G_{t-1} + \varepsilon_{g,t}
\end{aligned}$$

2 Data

Quarterly from 1961-Q1 to 2016-Q4, expressed in annual terms.

1. Consumption*. Real personal consumption is computed as the period-to-period log growth rates of real expenditures of non-durable goods and services (SAAR, Bil.$), averaged using their shares in nominal expenditures. The weighted average growth rate is applied to the sum of nominal expenditures in both categories in 1961-Q1 to produce chained real consumption with a base of 1961-Q1.

2. Investment*. SSAR, Chn.2009$ log growth of the private domestic investment of chained real GDP.

3. Labor*. Computed as the amount of aggregate weekly hours of total private production and non-supervisory employees (SA, Thous.), multiplied by number of weeks in the quarter to produce quarterly hours of labor. Since the data start in 1964-Q1, business sector compensation per hour (SA) from the Bureau of Labor Statistics is used to extend the series backwards to the start of the dataset.

4. Inflation. Inflation is annualized log growth rate of the chain price index of GDP.

5. Output*. Seasonally adjusted annual log growth rate of chained real GDP.

6. Policy Rate. The short-term nominal interest rate is computed as the average discount rate from 1961-Q1 to 1961-Q4; the end-of-period discount rate at Federal Reserve Bank of New York from 1962-Q1 to 1982-Q2; and the Federal Funds target rate from 1982-Q3 to 2016-Q4; converted to real terms using the inflation rate described above.

7. Term Premium. The nominal ten-year Treasury average term premium from the Federal Reserve Bank of New York, developed by Tobias Adrian, Richard Crump, and Emanuel Moench, which can be downloaded at https://www.newyorkfed.org/research/data_indicators/term_premia.html. For details on the methodology refer to Adrian et al. (2013). Data from January 1961 to May 1961 are extended back using the growth rate of the ten-year Treasury note yield at constant maturity from the Federal Reserve Board.

* HP filtered to extract the cyclical component.

Table: Data sources and summary statistics (1961—2016)

Variable Name	Obs.	Mean	St. Dev.	Min	Max	Source
Consumption (c_t)	224	0.00	0.85	-2.00	2.92	Bureau of Economic Analysis
Investment (I_t)	224	0.00	4.08	-12.62	9.50	FRB St. Louis
Labor (ℓ_t)	224	0.00	2.10	-6.71	4.71	Bureau of Labor Statistics
Inflation (π_t)	224	3.42	2.45	-0.62	12.77	Bureau of Economic Analysis
Output (y_t)	224	0.00	1.46	-4.78	3.75	FRB St. Louis
Policy Rate (r_t)	224	1.37	2.73	-5.06	7.66	FRB, FRB Atlanta
Term Premium ($tp_{l,t}$)	224	1.64	1.19	-0.59	4.94	Federal Reserve Board

3 Robustness

3.1 Different model specifications

- No *loan price channel*: the following equilibrium conditions change:

$$(Bank) \qquad b_t = b_{e,t}$$

$$(Bank) \qquad r_{b,t} = r_{w,t}\frac{\epsilon_b}{\epsilon_b - 1}$$

$$(Bank) \qquad J_{b,t} = r_{b,t}b_{e,t} - r_t d_t - \frac{\theta}{2}\left(\frac{k_{b,t}}{b_t} - \nu\right)^2 k_{b,t}$$

- No *bank-lending channel*: the following equilibrium conditions change when $\theta = 0$:

$$\begin{aligned}
&(Bank) \qquad r_{w,t} = r_t \\
&(Bank) \qquad r_{b,t} = (r_{l,t} + r_t)\,\frac{\epsilon_b}{\epsilon_b - 1} \\
&(Bank) \qquad J_{b,t} = r_{b,t} b_{e,t} + r_{l,t} b_{l,t} - r_t d_t \\
&(Agg) \qquad y_t = c_t + q_{k,t}\,(k_t - (1-\delta_k)k_{t-1}) + G_t + \frac{\kappa_\pi}{2}\pi_t^2 y_t + \delta_b k_{b,t-1}
\end{aligned}$$

- No *collateral channel*: We now have 36 variables and 36 equations to solve for, including 6 stochastic shocks, after removing the exogenous process for the LTV ratio, we set $z = 0.64$. The following equilibrium conditions change:

$$\begin{aligned}
&(Ent) \qquad \frac{q_{k,t}}{c_{e,t}} = \mathbb{E}_t\left[\beta_e \frac{1}{c_{e,t+1}}\,(r_{k,t+1} + q_{k,t+1}(1-\delta_k))\right] \\
&(Ent) \qquad b_{e,t} = z
\end{aligned}$$

- Preference shock:

Equations

Recursive utility function for household i, each of unit mass:

$$V_t(i) = \epsilon_{risk,t} U(c_{h,t}(i), \ell_{h,t}(i)) + \beta_h \left(\mathbb{E}_t V_{t+1}^{1-\xi}(i)\right)^{\frac{1}{1-\xi}}. \tag{19}$$

Intra-period utility function:

$$U(c_{h,t}(i), \ell_{h,t}(i)) = \frac{c_{h,t}^{1-\psi}(i)}{1-\psi} - \frac{\ell_{h,t}^{1+\phi}(i)}{1+\phi}. \tag{20}$$

Budget constraint:

$$c_{h,t}(i) + d_t(i) \leq w_t \ell_{h,t}(i) + (1 + r_{t-1}) d_{t-1}(i) - T_t(i) + J_{R,t}(i). \tag{21}$$

Solution

Households choose consumption, labor, and deposits to maximize V_0 subject to the utility function

(19) and the budget constraint (21), with $\lambda^h_{1,t}(i)$ and $\lambda^h_{2,t}(i)$ being the respective multipliers of the Lagrangean function ($\mathcal{L}$) and substituting Eq. (20) for the intra-period utility function:

$$\mathcal{L} = V_0(i) - \mathbb{E}_0 \sum_{t=0}^{\infty} \lambda^h_{1,t}(i) \left[V_t(i) - \epsilon_{risk,t} U(c_{h,t}(i), \ell_{h,t}(i)) - \beta_h \left(\mathbb{E}_t V_{t+1}^{1-\xi}(i) \right)^{\frac{1}{1-\xi}} \right]$$
$$- \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_h^t \lambda^h_{2,t}(i) \left[c_{h,t}(i) + d_t(i) - w_t \ell_{h,t}(i) - (1+r_{t-1}) d_{t-1}(i) - J_{R,t}(i) + T_t(i) \right]$$
$$\mathcal{L} = V_0(i) - \mathbb{E}_0 \sum_{t=0}^{\infty} \lambda^h_{1,t}(i) \left[V_t(i) - \epsilon_{risk,t} \left(\frac{c_{h,t}^{1-\psi}(i)}{1-\psi} + \frac{\ell_{h,t}^{1+\phi}(i)}{1+\phi} \right) - \beta_h \left(\mathbb{E}_t V_{t+1}^{1-\xi}(i) \right)^{\frac{1}{1-\xi}} \right]$$
$$- \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_h^t \lambda^h_{2,t}(i) \left[c_{h,t}(i) + d_t(i) - w_t \ell_{h,t}(i) - (1+r_{t-1}) d_{t-1}(i) - J_{R,t}(i) + T_t(i) \right].$$

First order necessary conditions are:

$$\frac{\partial \mathcal{L}}{\partial c_t(i)} = \lambda^h_{1,t}(i) \frac{1-\psi}{1-\psi} \epsilon_{risk,t} c_{h,t}^{-\psi}(i) - \beta_h^t \lambda^h_{2,t}(i) = 0 \Longrightarrow \lambda^h_{2,t}(i) = \frac{\lambda^h_{1,t}(i)\, \epsilon_{risk,t}}{\beta_h^t c_{h,t}^{\psi}(i)} \tag{22}$$

$$\frac{\partial \mathcal{L}}{\partial \ell_t(i)} = -\lambda^h_{1,t}(i) \frac{1+\phi}{1+\phi} \epsilon_{risk,t} \ell_{h,t}^{\phi}(i) + \beta_h^t \lambda^h_{2,t}(i)\, w_t = 0$$

$$\Longrightarrow \lambda^h_{2,t}(i) = \frac{\lambda^h_{1,t}(i)\, \ell_{h,t}^{\phi}(i)\, \epsilon_{risk,t}}{\beta_h^t w_t} \tag{23}$$

$$\frac{\partial \mathcal{L}}{\partial d_t(i)} = -\beta_h^t \lambda^h_{2,t}(i) + \mathbb{E}_t \beta_h^{t+1} \lambda^h_{2,t+1}(i)(1+r_t) = 0$$

$$\Longrightarrow \lambda^h_{2,t}(i) = \beta_h \mathbb{E}_t \lambda^h_{2,t+1}(i)(1+r_t) \tag{24}$$

$$\frac{\partial \mathcal{L}}{\partial V_t(i)} = -\lambda^h_{1,t}(i) + \lambda^h_{1,t-1}(i) \left[\frac{1}{1-\xi} \beta_h \left(\mathbb{E}_{t-1} V_t^{1-\xi}(i) \right)^{\frac{\xi}{1-\xi}} (1-\xi) V_t^{-\xi}(i) \right] = 0$$

$$\Longrightarrow \frac{\lambda^h_{1,t}(i)}{\lambda^h_{1,t-1}(i)} = \beta_h \left(\frac{\left(\mathbb{E}_{t-1} V_t^{1-\xi}(i) \right)^{\frac{1}{1-\xi}}}{V_t(i)} \right)^{\xi} \tag{25}$$

Obtain the labor supply decision from Eq. (22) = (23):

$$\frac{\lambda^h_{1,t}(i)\, \ell_{h,t}^{\phi}(i)\, \epsilon_{risk,t}}{\beta_h^t w_t} = \frac{\lambda^h_{1,t}(i)\, \epsilon_{risk,t}}{\beta_h^t c_{h,t}^{\psi}(i)} \Longrightarrow \ell_{h,t}^{\phi}(i) = \frac{w_t}{c_{h,t}^{\psi}(i)}$$

Set Eq. (22) = (24), iterate Eqs. (22) and (25) forward, and substitute to obtain the consumption

Euler equation:

$$
\begin{aligned}
\frac{\lambda_{1,t}^{h}(i)\,\epsilon_{risk,t}}{\beta_h^t c_{h,t}^{\psi}(i)} &= \beta_h \mathbb{E}_t \lambda_{2,t+1}^{h}(i)(1+r_t) \\
\frac{1}{c_{h,t}^{\psi}(i)} &= \mathbb{E}_t \frac{\beta_h^{t+1}\lambda_{2,t+1}^{h}(i)(1+r_t)}{\lambda_{1,t}^{h}(i)\,\epsilon_{risk,t}} \\
\frac{1}{c_{h,t}^{\psi}(i)} &= \mathbb{E}_t \frac{\beta_h^{t+1}\lambda_{1,t+1}^{h}(i)\,\epsilon_{risk,t+1}(1+r_t)}{\lambda_{1,t}^{h}(i)\,\epsilon_{risk,t}\beta_h^{t+1}c_{h,t+1}^{\psi}(i)} \\
\frac{1}{c_{h,t}^{\psi}(i)} &= \mathbb{E}_t \left[\beta_h \frac{\epsilon_{risk,t+1}(1+r_t)}{\epsilon_{risk,t}c_{h,t+1}^{\psi}(i)} \left(\frac{\left(\mathbb{E}_t V_{t+1}^{1-\xi}(i)\right)^{\frac{1}{1-\xi}}}{V_{t+1}(i)}\right)^{\xi}\right] \\
1 &= \mathbb{E}_t \left[m_{t,t+1}^{h}(i)(1+r_t)\right], \\
\text{where } m_{t,t+1}^{h}(i) &= \beta_h \frac{\epsilon_{risk,t+1}c_{h,t}^{\psi}(i)}{\epsilon_{risk,t}c_{h,t+1}^{\psi}(i)} \left(\frac{\left(\mathbb{E}_t V_{t+1}^{1-\xi}(i)\right)^{\frac{1}{1-\xi}}}{V_{t+1}(i)}\right)^{\xi}.
\end{aligned}
$$

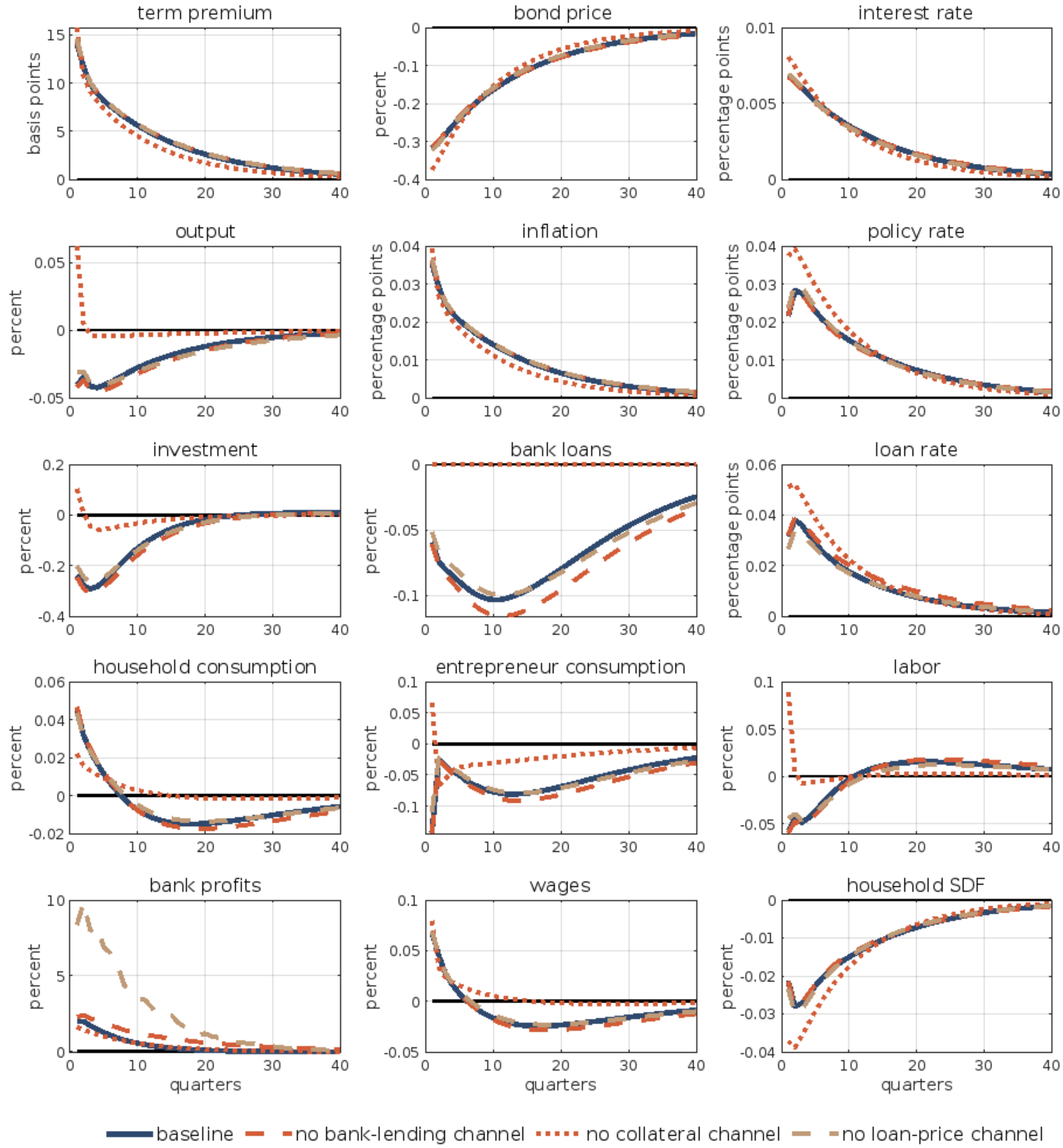
Final conditions that change

- (IES) $\psi = 0.50$
- $\bar{\epsilon}_{risk} = 1$
- $\varepsilon_{risk,t}$ enters the exogenous process as a plus such that a positive shock reduces the SDF:

$$
\begin{aligned}
&(HH) \qquad V_t = \epsilon_{risk,t}\frac{c_{h,t}^{1-\psi}}{1-\psi} - \frac{\ell_{h,t}^{1+\phi}}{1+\phi} + \beta_h \left(\mathbb{E}_t V_{t+1}^{1-\xi}\right)^{\frac{1}{1-\xi}} \\
&(HH) \qquad \frac{1}{1+r_t} = \mathbb{E}_t \left[\beta_h \frac{\epsilon_{risk,t+1}}{\epsilon_{risk,t}} \frac{c_{h,t}^{\psi}}{c_{h,t+1}^{\psi}} \left(\frac{\left(\mathbb{E}_t V_{t+1}^{1-\xi}\right)^{\frac{1}{1-\xi}}}{V_{t+1}}\right)^{\xi}\right] \\
&(RISK) \qquad \epsilon_{risk,t} = (1-\rho_{risk})\,\bar{\epsilon}_{risk} + \rho_{risk}\epsilon_{risk,t-1} + \varepsilon_{risk,t} \\
&(Bond) \qquad p_{l,t} = \mathbb{E}_t \left[m_{t,t+1}^{h}(1+\delta_c p_{l,t+1})\right]
\end{aligned}
$$

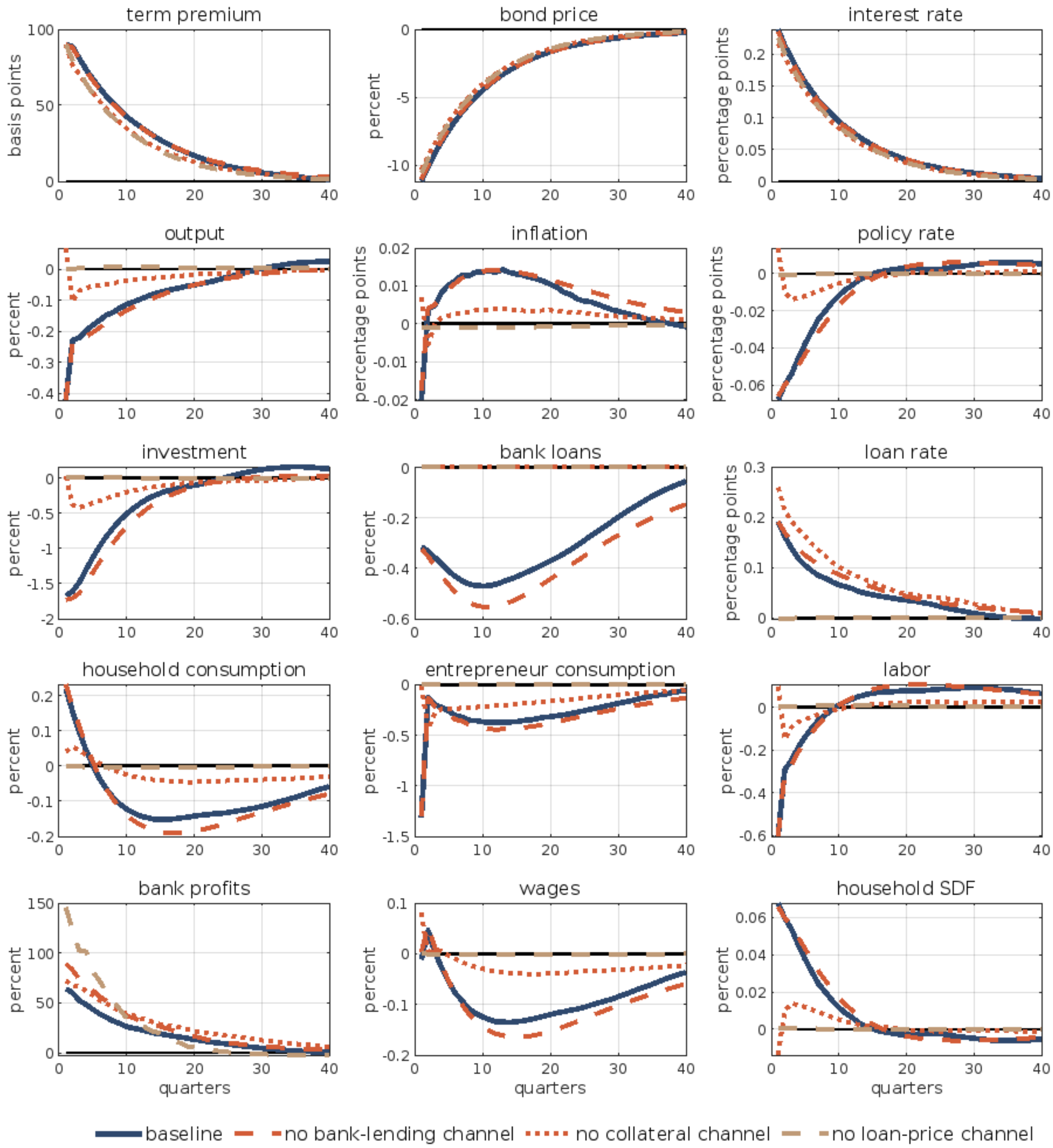
3.2 Additional impulse response functions

Figure 1: Impulse responses to preference shock



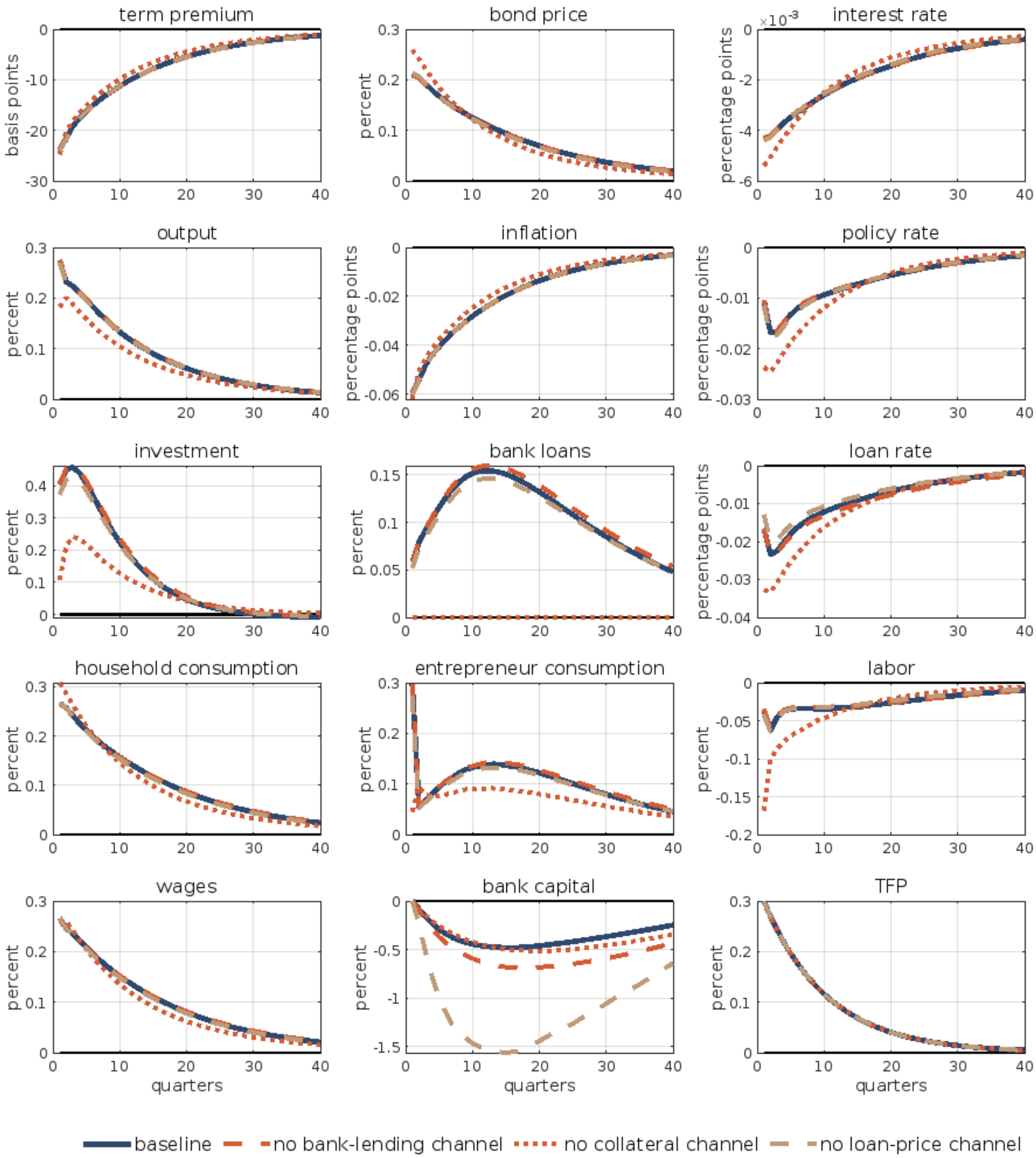
Notes: The blue solid line represents the impulse responses to a positive one standard deviation preference shock under the calibration in Table 1 of the main draft, which we label our "baseline" specification. The orange dashes represent a "no bank-lending channel" scenario by setting $\theta = 0$. The orange dots represent a "no collateral channel" scenario by setting the entrepreneur's borrowing constraint to $b_{e,t}(j) \leq z$, for $z > 0$. The gold dashes represent a "no loan-price channel" scenario, which delivers a loan rate no longer tied to the long-term interest rate, deactivating the transmission of risk mispricing. All variables are quarterly and in real terms, except for the term premium, which is annualized and in nominal terms, and the interest rate, which is annualized. Responses are deviations relative to the steady state in the corresponding units indicated in the vertical axis.

Figure 2: Impulse responses to risk mispricing shock with standard preferences



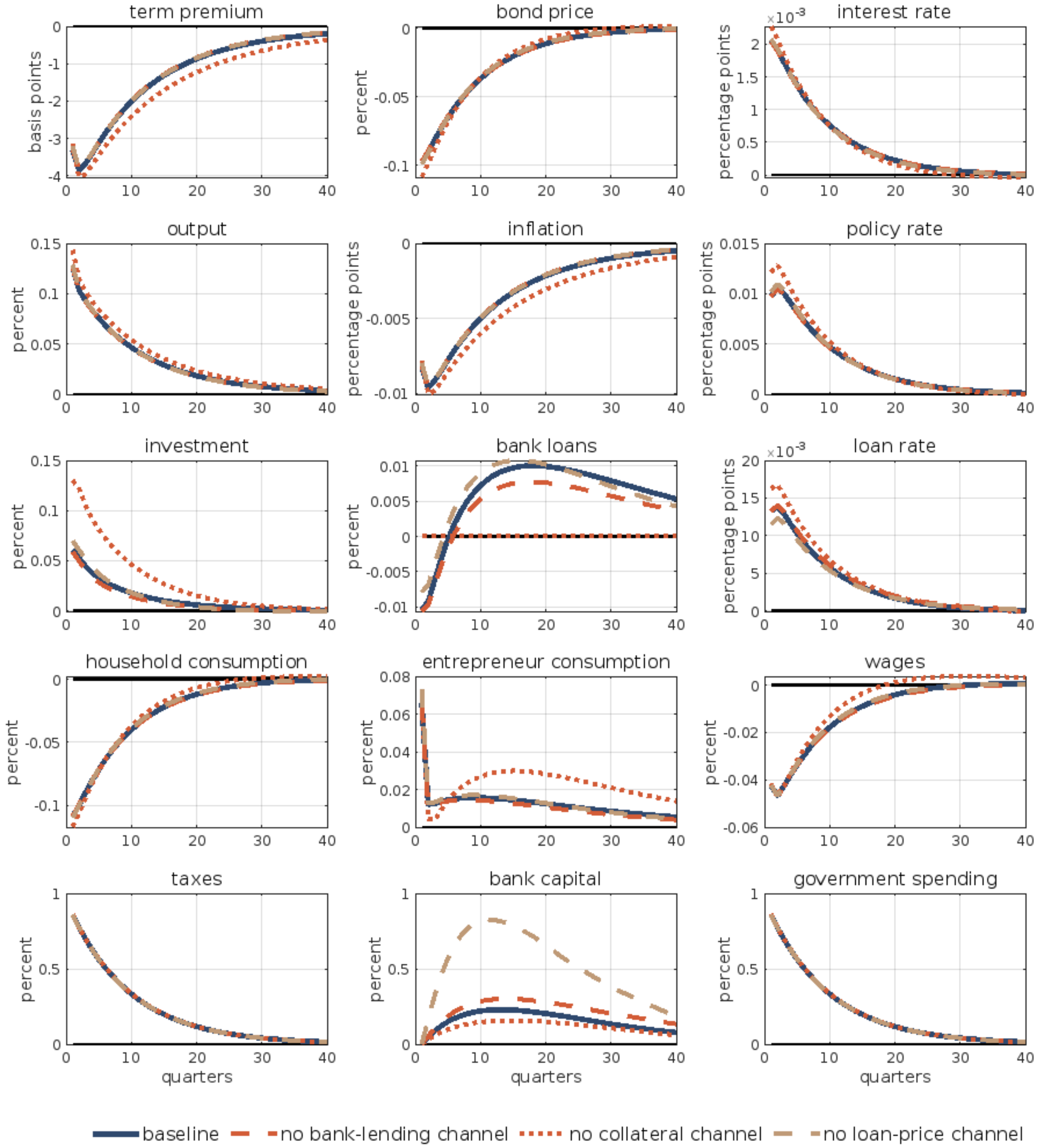
Notes: The blue solid line represents the impulse responses to a risk mispricing shock when (EZ preferences) $\xi = 0$ (scaled to raise the nominal term premium by 90 basis points) under the calibration in Table 1 of the main draft, which we label our "baseline" specification. The orange dashes represent a "no bank-lending channel" scenario by setting $\theta = 0$. The orange dots represent a "no collateral channel" scenario by setting the entrepreneur's borrowing constraint to $b_{e,t}(j) \leq z$, for $z > 0$. The gold dashes represent a "no loan-price channel" scenario, which delivers a loan rate no longer tied to the long-term interest rate, deactivating the transmission of risk mispricing. All variables are quarterly and in real terms, except for the term premium, which is annualized and in nominal terms, and the interest rate, which is annualized. Responses are deviations relative to the steady state in the corresponding units indicated in the vertical axis.

Figure 3: Impulse responses to productivity shock



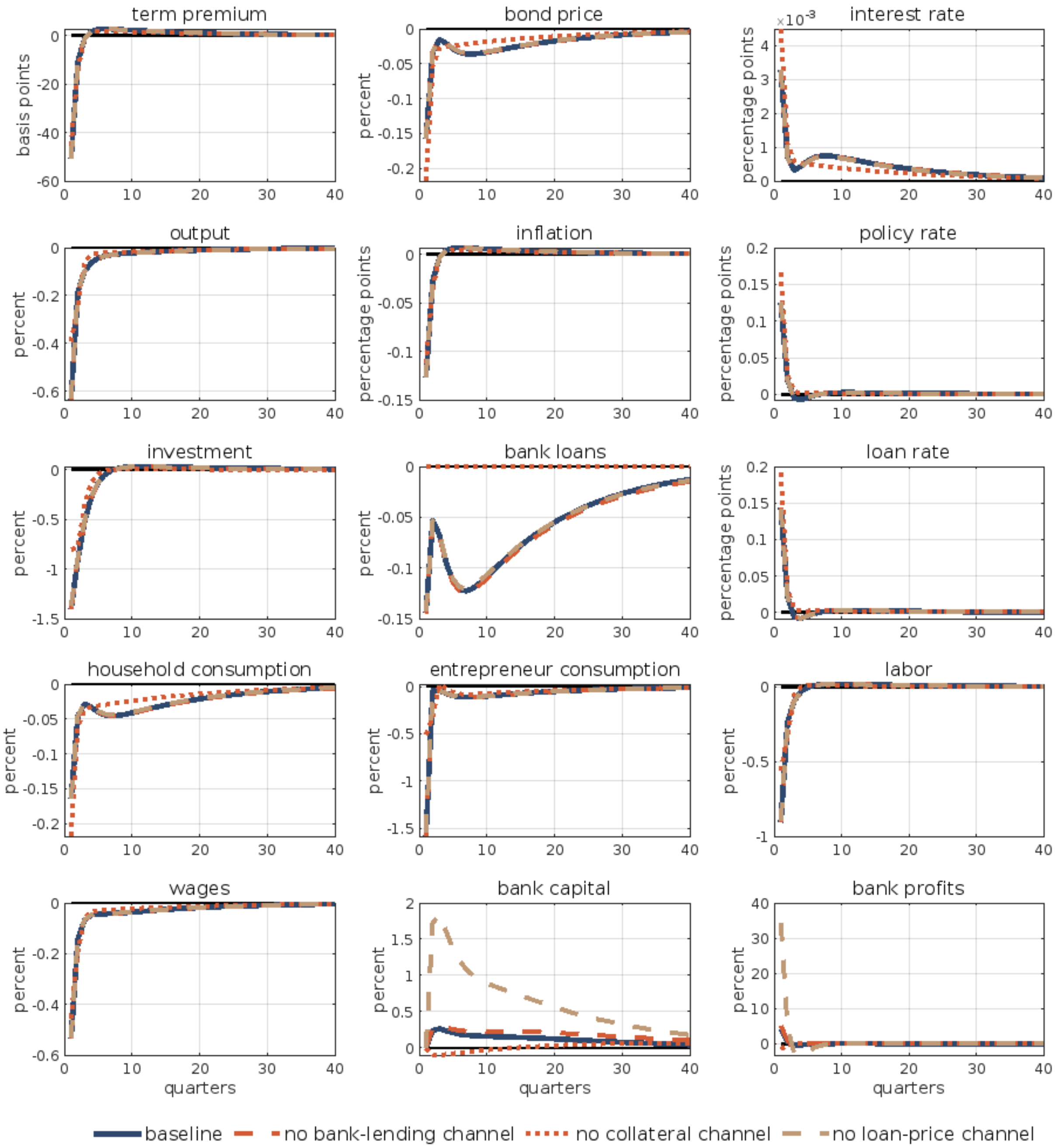
Notes: The blue solid line represents the impulse responses to a positive one standard deviation productivity shock under the calibration in Table 1 of the main draft, which we label our "baseline" specification. The orange dashes represent a "no bank-lending channel" scenario by setting $\theta = 0$. The orange dots represent a "no collateral channel" scenario by setting the entrepreneur's borrowing constraint to $b_{e,t}(j) \leq z$, for $z > 0$. The gold dashes represent a "no loan-price channel" scenario, which delivers a loan rate no longer tied to the long-term interest rate, deactivating the transmission of risk mispricing. All variables are quarterly and in real terms, except for the term premium, which is annualized and in nominal terms, and the interest rate, which is annualized. Responses are deviations relative to the steady state in the corresponding units indicated in the vertical axis.

Figure 4: Impulse responses to government spending shock



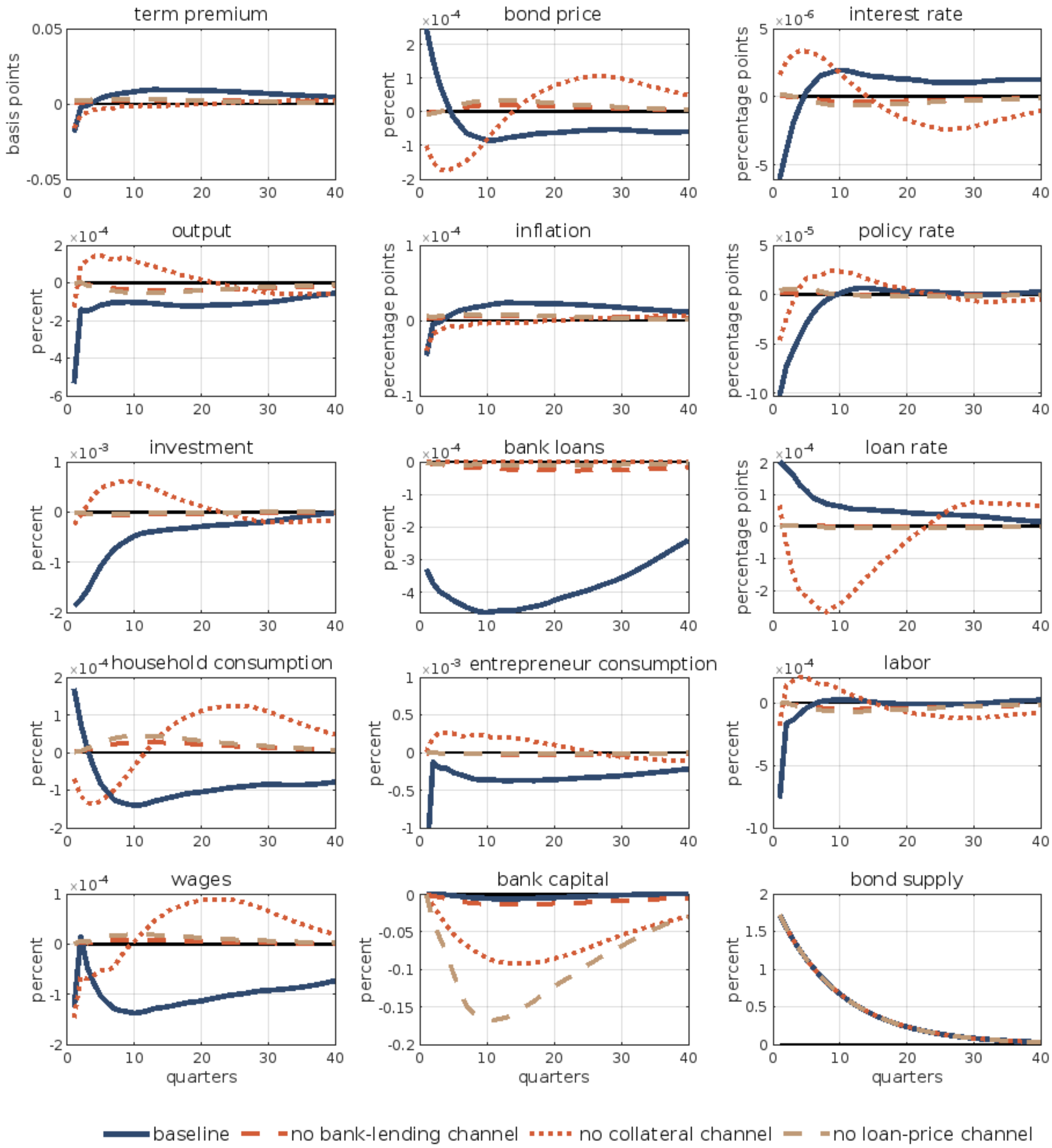
Notes: The blue solid line represents the impulse responses to a positive one standard deviation government spending shock under the calibration in Table 1 of the main draft, which we label our "baseline" specification. The orange dashes represent a "no bank-lending channel" scenario by setting $\theta = 0$. The orange dots represent a "no collateral channel" scenario by setting the entrepreneur's borrowing constraint to $b_{e,t}(j) \leq z$, for $z > 0$. The gold dashes represent a "no loan-price channel" scenario, which delivers a loan rate no longer tied to the long-term interest rate, deactivating the transmission of risk mispricing. All variables are quarterly and in real terms, except for the term premium, which is annualized and in nominal terms, and the interest rate, which is annualized. Responses are deviations relative to the steady state in the corresponding units indicated in the vertical axis.

Figure 5: Impulse responses to monetary policy shock



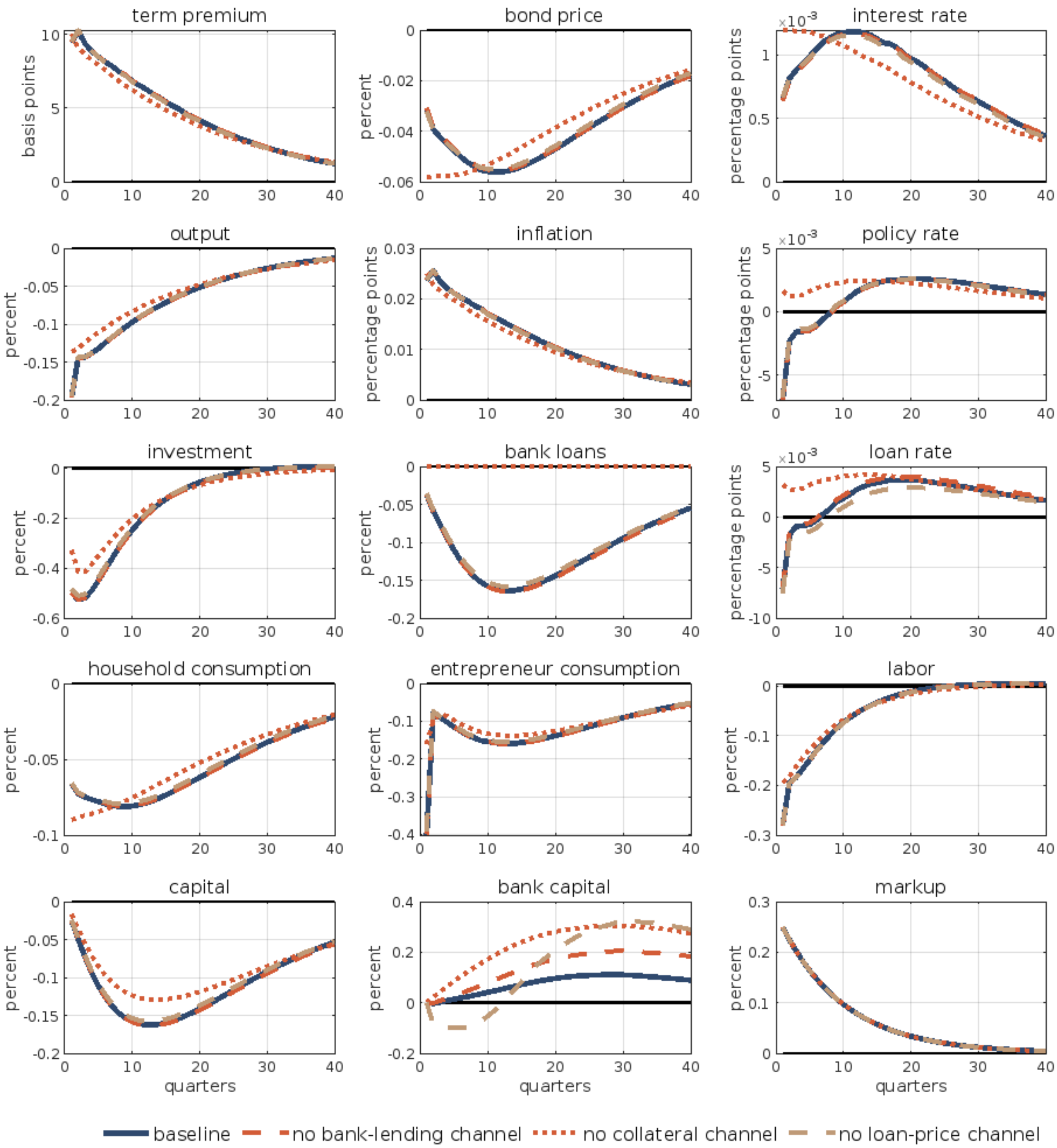
Notes: The blue solid line represents the impulse responses to a positive one standard deviation monetary policy shock under the calibration in Table 1 of the main draft, which we label our "baseline" specification. The orange dashes represent a "no bank-lending channel" scenario by setting $\theta = 0$. The orange dots represent a "no collateral channel" scenario by setting the entrepreneur's borrowing constraint to $b_{e,t}(j) \leq z$, for $z > 0$. The gold dashes represent a "no loan-price channel" scenario, which delivers a loan rate no longer tied to the long-term interest rate, deactivating the transmission of risk mispricing. All variables are quarterly and in real terms, except for the term premium, which is annualized and in nominal terms, and the interest rate, which is annualized. Responses are deviations relative to the steady state in the corresponding units indicated in the vertical axis.

Figure 6: Impulse responses to bond supply shock



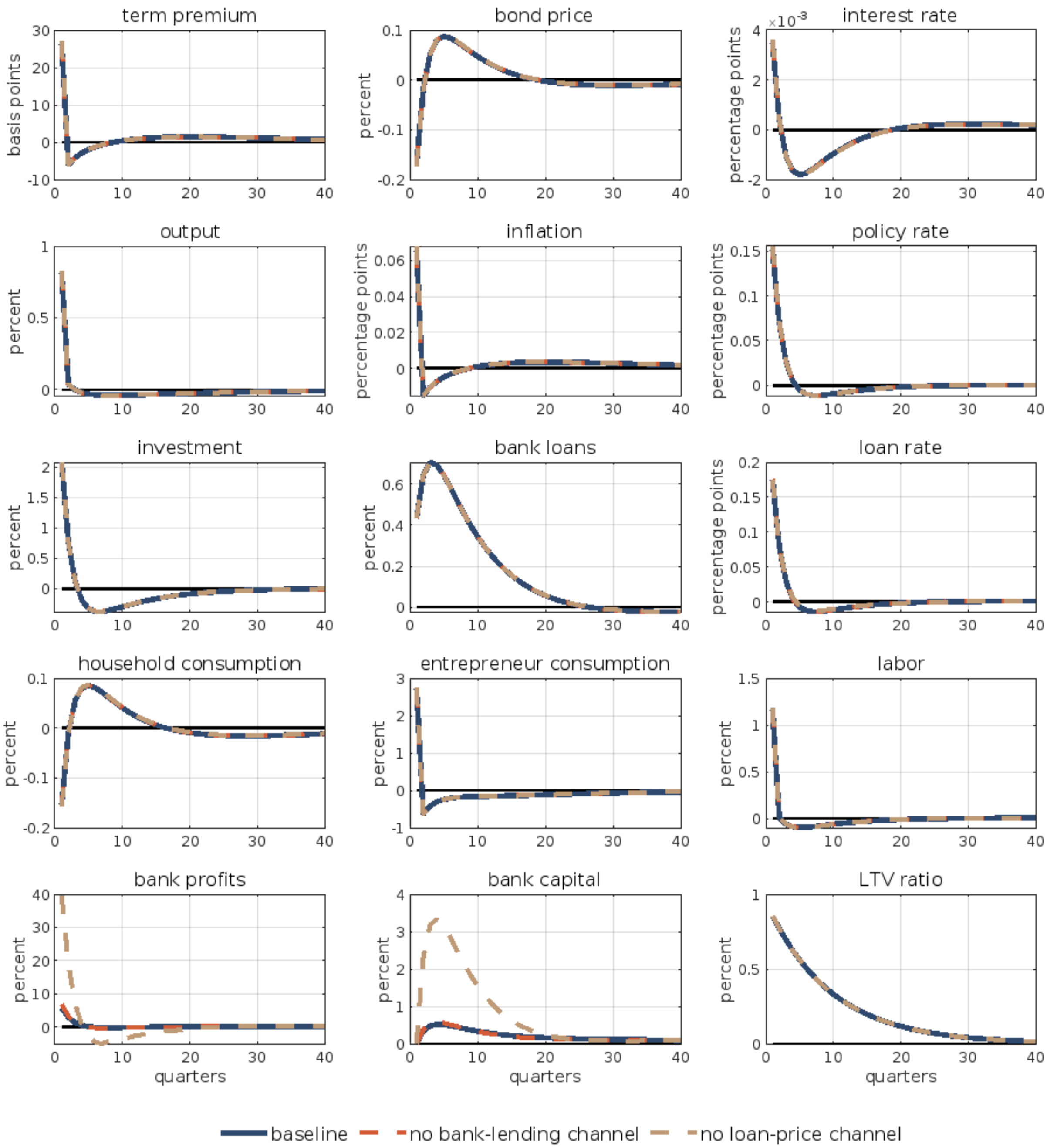
Notes: The blue solid line represents the impulse responses to a positive one standard deviation government bond supply shock under the calibration in Table 1 of the main draft, which we label our "baseline" specification. The orange dashes represent a "no bank-lending channel" scenario by setting $\theta = 0$. The orange dots represent a "no collateral channel" scenario by setting the entrepreneur's borrowing constraint to $b_{e,t}(j) \leq z$, for $z > 0$. The gold dashes represent a "no loan-price channel" scenario, which delivers a loan rate no longer tied to the long-term interest rate, deactivating the transmission of risk mispricing. All variables are quarterly and in real terms, except for the term premium, which is annualized and in nominal terms, and the interest rate, which is annualized. Responses are deviations relative to the steady state in the corresponding units indicated in the vertical axis.

Figure 7: Impulse responses to price markup shock



Notes: The blue solid line represents the impulse responses to a positive one standard deviation price markup shock under the calibration in Table 1 of the main draft, which we label our "baseline" specification. The orange dashes represent a "no bank-lending channel" scenario by setting $\theta = 0$. The orange dots represent a "no collateral channel" scenario by setting the entrepreneur's borrowing constraint to $b_{e,t}(j) \leq z$, for $z > 0$. The gold dashes represent a "no loan-price channel" scenario, which delivers a loan rate no longer tied to the long-term interest rate, deactivating the transmission of risk mispricing. All variables are quarterly and in real terms, except for the term premium, which is annualized and in nominal terms, and the interest rate, which is annualized. Responses are deviations relative to the steady state in the corresponding units indicated in the vertical axis.

Figure 8: Impulse responses to LTV ratio shock



Notes: The blue solid line represents the impulse responses to a positive one standard deviation LTV ratio shock under the calibration in Table 1 of the main draft, which we label our "baseline" specification. The orange dashes represent a "no bank-lending channel" scenario by setting $\theta = 0$. The orange dots represent a "no collateral channel" scenario by setting the entrepreneur's borrowing constraint to $b_{e,t}(j) \leq z$, for $z > 0$. The gold dashes represent a "no loan-price channel" scenario, which delivers a loan rate no longer tied to the long-term interest rate, deactivating the transmission of risk mispricing. All variables are quarterly and in real terms, except for the term premium, which is annualized and in nominal terms, and the interest rate, which is annualized. Responses are deviations relative to the steady state in the corresponding units indicated in the vertical axis.