STRATEGIC LOAN MODIFICATION: An Options based response to strategic default
(joint work with Ray Meadows)

sequel to:
THE PRINCIPAL PRINCIPLE: Optimal Modification of Distressed Home Loans (Why Lenders should Forgive, not Foresake Mortgages)

Papers at http://algo.scu.edu/~sanjivdas/research.htm

Sanjiv R. Das
Santa Clara University

@FDIC, October 25, 2010.

Current Statistics

- Q3 2009: 1/136 received foreclosure filing (RealtyTrac).
- 1 in 380 households received a foreclosure filing in August 2010 (RealtyTrac).
- Sep 2010: pre-sale foreclosure inventory 2.038 million homes (LPS).
- Sep 2010: 4.9 million mortgages 30-days overdue, 2.374 million 90-days past due (LPS).
- Sep 2010: 11 million borrowers (23% of households with a mortgage) have negative equity (CoreLogic).

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CitiMortgage CEO Sanjiv Das helps people keep their homes

By Stephanie Armour, USA TODAY
April 27, 2009

USA TODAY
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There is very little in Sanjiv Das' uncluttered office.

Just snapshots of his wife and his 20-year-old daughter, Natasha. No pictures of his favorite sports: golf and cricket.

Das has moved around the world, run credit card acquisitions for American Express in India, and handled the mortgage business for Citibank in Sydney. But none of it compares to what he's doing now

http://algo.scu.edu/~sanjivdas/

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Students: Obama's stimulus gives up to $2500 a year

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A tale of two Das: Citi CEO, academic and mortgages

Al Yoos
NEW YORK
Wed Feb 24, 2010 2:06pm EST

NEW YORK (Reuters) - Sanjiv Ranjan Das, a professor at California's Santa Clara University, last fall attacked the problem of "underwater" mortgages often cited as an Achilles' heel to the U.S. housing market.

HOUSING MARKET

He had a special fan: Sanjiv Das, the top executive at CitiMortgage, the nation's fourth-largest home loan lender and servicer of $723 billion in mortgages.

http://algo.scu.edu/~sanjivdas/

Preview of Some Results

- ✓ Rate reductions are value-destroying.
- ✓ Maturity extensions also destroy value.
- ✓ Principal reductions are optimal.
- ✓ Capitalization of payments into back-ended principal will also destroy loan value.
- ✓ Shared-appreciation mortgages improve ability to pay, mitigate moral hazard.
- ✓ Optimal modifications may be computed in closed-form in a reduced-form model.

Main Result …

The best way to modify an underwater loan is to reduce the principal balance, lowering the monthly payment and restoring equity. But for the most part, lenders have refused to reduce principal because it would force them to take an immediate loss on the loan. Lenders also have vehemently and successfully resisted Congressional efforts to change the law so that bankruptcy courts could reduce the mortgage balances for bankrupt borrowers.

New York Times, January 4, 2010

How Long Will Negative Equity Last?

BY: CARRIE BAY 03/29/2010

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There has been a lot of recent talk about mortgages in negative equity – underwater homes – and the impact on the housing market. In response, First American CoreLogic asked the question: When will these homes start to float?

The company estimates that the typical underwater homeowner will not begin to surface until late 2015 to early 2016. It's an even longer stretch for some of the most depressed markets, where First American CoreLogic says the typical borrower in negative equity may not experience positive equity until 2020 or later.

Even in markets with low shares of negative equity, the recovery time will still be long because the few borrowers that are upside down are deeply in negative equity and these are typically not high appreciation markets, the company has concluded.

Although house price appreciation will, over time, offset negative equity, First American CoreLogic says amortization (the paying down of loan balances) will be a more significant remedy to negative equity. The company's data shows that over

average loan balance will decrease by an annual rate of 3.3 percent; meanwhile home prices are expected to increase at a 3 percent annual rate over the next decade.

To forecast when the typical U.S. homeowner will achieve neutral and positive equity, First American CoreLogic looked at 10 key markets, plotting equity trends over the next decade, and assuming a nominal annual appreciation rate of 3 percent.

Of the markets studied, the Washington D.C. area is expected to reach positive equity by 2015.

Atlanta, Georgia; Dallas, Texas; and Riverside-San Bernardino, California are projected to rise to the surface in 2016. Boston, Massachusetts should find a balance in 2017.

Cape Coral-Fort Myers, Florida; Pittsburgh, Pennsylvania; Las Vegas; and Lancaster, Pennsylvania are forecast to reach positive territory by 2020.

Detroit, though, is not projected to recover even by 2020, because of its depressed economy.

The latest numbers from First American CoreLogic show that more than 11.3 million, or 24 percent, of all residential properties with mortgages were underwater at the end of the fourth quarter of 2009.

Among the new housing initiatives announced by the administration Friday was assistance for borrowers with negative equity. In order to deter these homeowners from strategically defaulting, the Treasury will begin requiring servicers to consider principal write-downs as part of their Home Affordable Modification Program (HAMP) evaluations for borrowers whose loan balance is more than 115 percent of the property's current value. The plan also includes a Federal Housing Administration (FHA) refinancing program for negative equity mortgages.

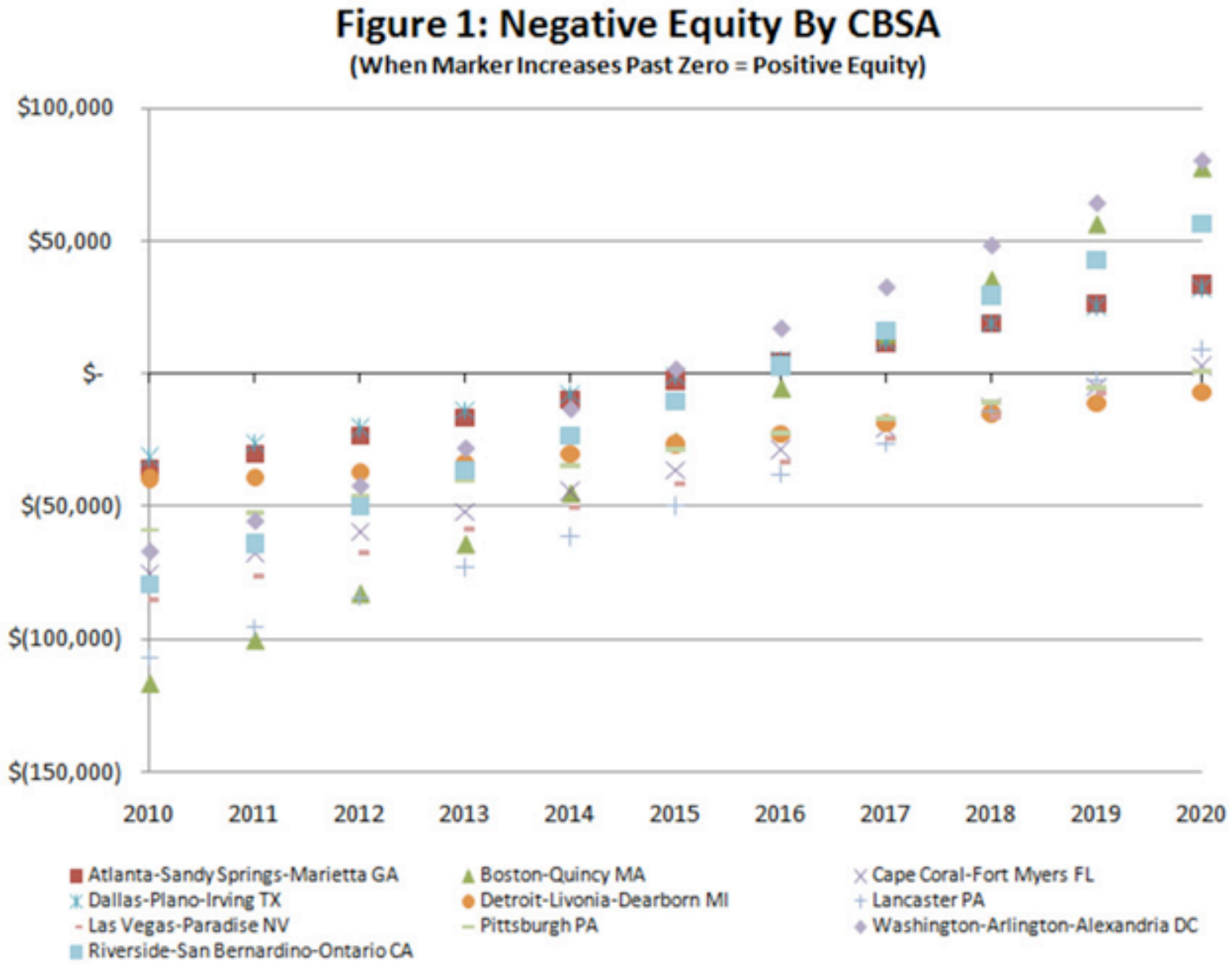


Figure 1: Negative Equity By CBSA
(When Marker Increases Past Zero = Positive Equity)
$100,000
$50,000
$-
$(50,000)
$(100,000)
$(150,000)
2010
2011
2012
2013
2014
2015
2016
2017
2018
2019
2020
Atlanta-Sandy Springs-Marietta GA
Boston-Quincy MA
Cape Coral-Fort Myers FL
Dallas-Plano-Irving TX
Detroit-Livonia-Dearborn MI
Lancaster PA
Las Vegas-Paradise NV
Pittsburgh PA
Washington-Arlington-Alexandria DC
Riverside-San Bernardino-Ontario CA

HAMP UPDATE

June 4, 2010

Announcing HAMP Principal Reduction Alternative

Yesterday, June 3, 2010, Supplemental Directive 10-05: Modification of Loans with Principal Reduction Alternative, was issued offering mortgage relief to eligible homeowners whose homes are worth significantly less than the remaining amounts owed under their first lien mortgage loans. The Principal Reduction Alternative (PRA) guidance applies to non-GSE loans eligible for the Home Affordable Modification Program (HAMP) only.

Principal Reduction Alternative (PRA)
With this new guidance, servicers are required to evaluate all HAMP-eligible loans with a mark-to-market loan-to-value (MTMLTV) greater than 115% to determine if a principal reduction is beneficial. If the evaluation shows the net present value (NPV) for a HAMP modification using PRA is positive, servicers are encouraged to offer the principal reduction to the borrower. An updated NPV model reflecting principal reduction will be available to use for this evaluation. Additional details are as follows:

- **Effective Date** -- The PRA Effective Date (i.e., the date the principal reduction evaluation is required) will be either October 1, 2010, or the date of the HAMP NPV Model 4.0 release (whichever is later). However, servicers may immediately offer PRA for HAMP-eligible modifications as long as the reduction follows all PRA requirements.
- **Application** -- PRA is earned over a three-year period and is initially treated as a PRA Forbearance. Each year (for three years) that the borrower is in good standing on the anniversary of their trial period effective date, one-third of the original PRA forbearance amount will be reduced. This reduced amount will be applied to their unpaid principal balance.
- **Second Lien** -- Servicers participating in the Second Lien Program (2MP) will be required to provide a principal reduction on the borrower's second mortgage in proportion to any principal reduction offered on the borrower's first mortgage.
- **Investor Incentive** -- Investors will receive an incentive based on loan delinquency, LTV ratio, and the amount of the principal reduction. Note: Guidance on principal reduction and related investor incentives will be forthcoming for loans in active HAMP Trial Period Plans or that were permanently modified prior to June 3, 2010 (i.e., the SD 10-05 effective date).

Ability to pay

&

Willingness to pay

"Iso-Service" Surface

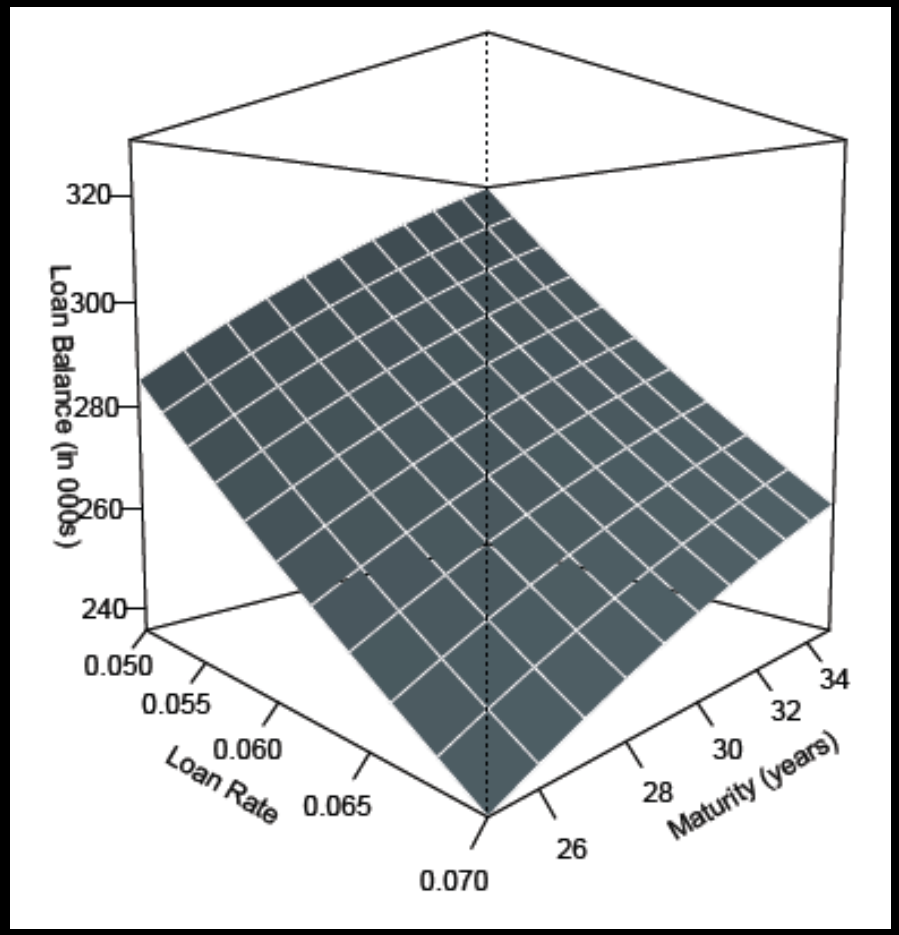
- $A \leq A_{max}$ → choose → $\{T, L_t, r_L\}$

Loan balance = $300,000
Home value = $250,000

Remaining maturity = 25 years
A = $1,933 per month

A_{max} = $20,000 per year
($1,667 per month)

$$L_0 = \frac{A_{max}}{m}\left[\frac{1-(1+r_L/m)^{-N}}{r_L/m}\right]$$



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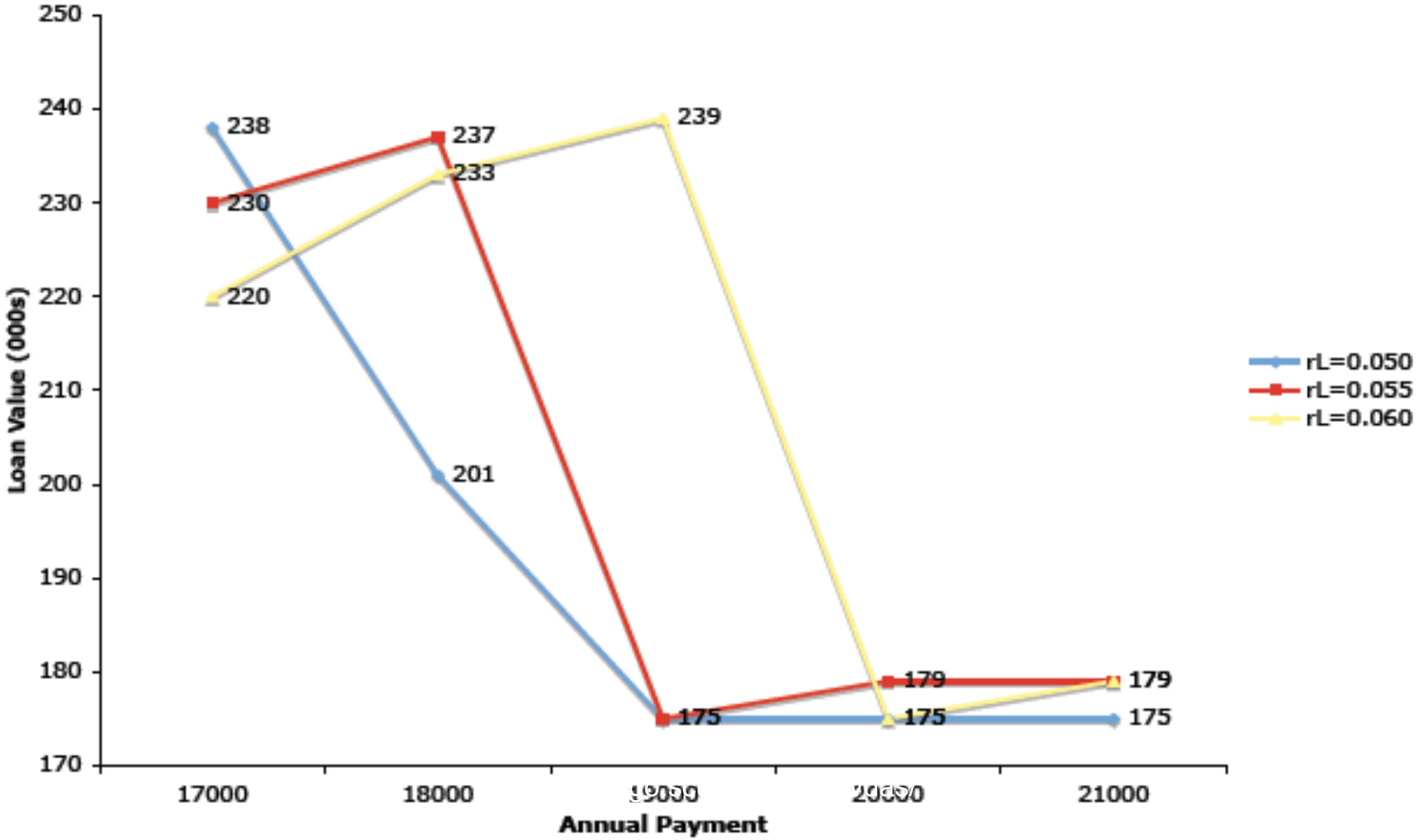
Values of Iso-Service Loans



Deadweight Foreclosure Costs are critical

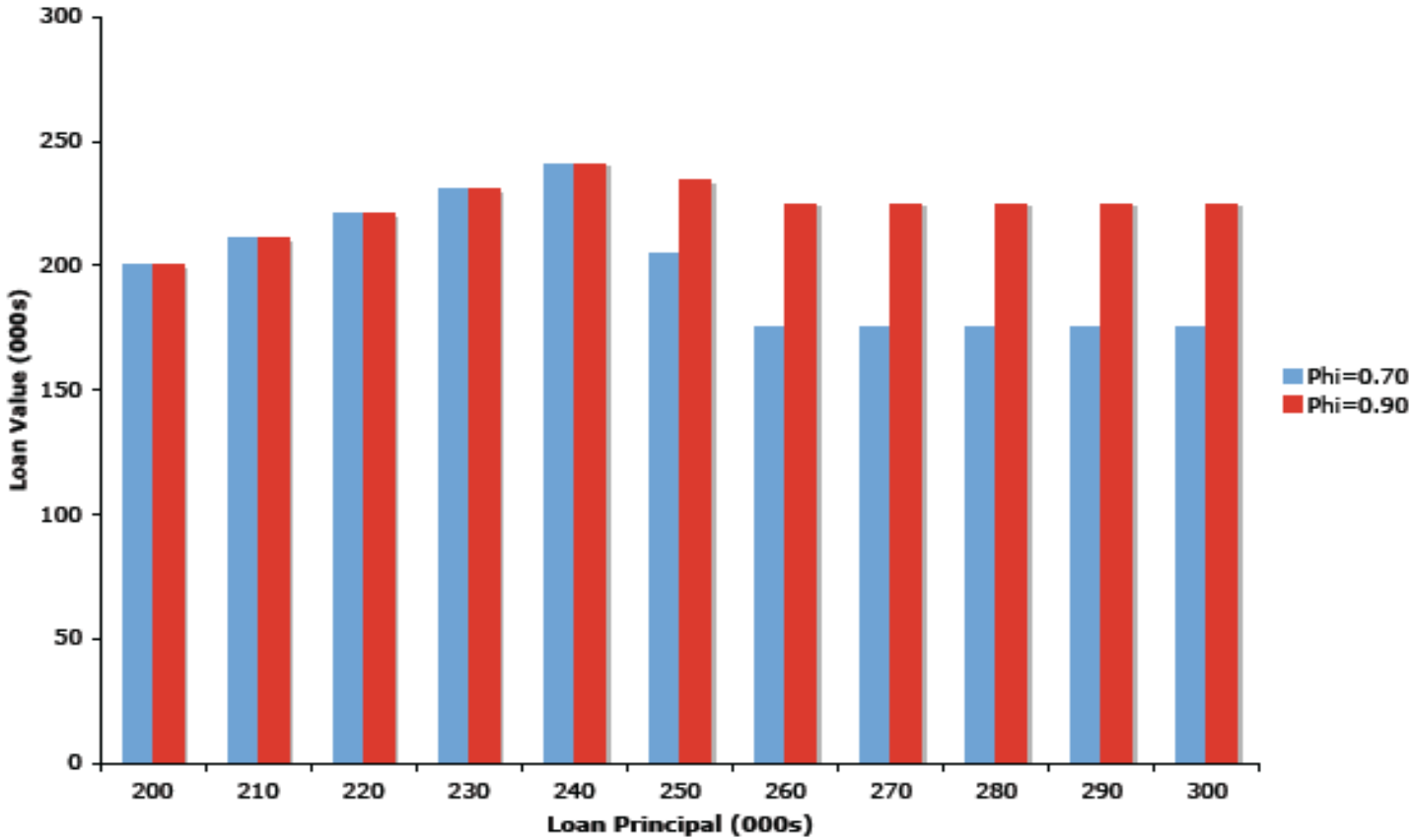
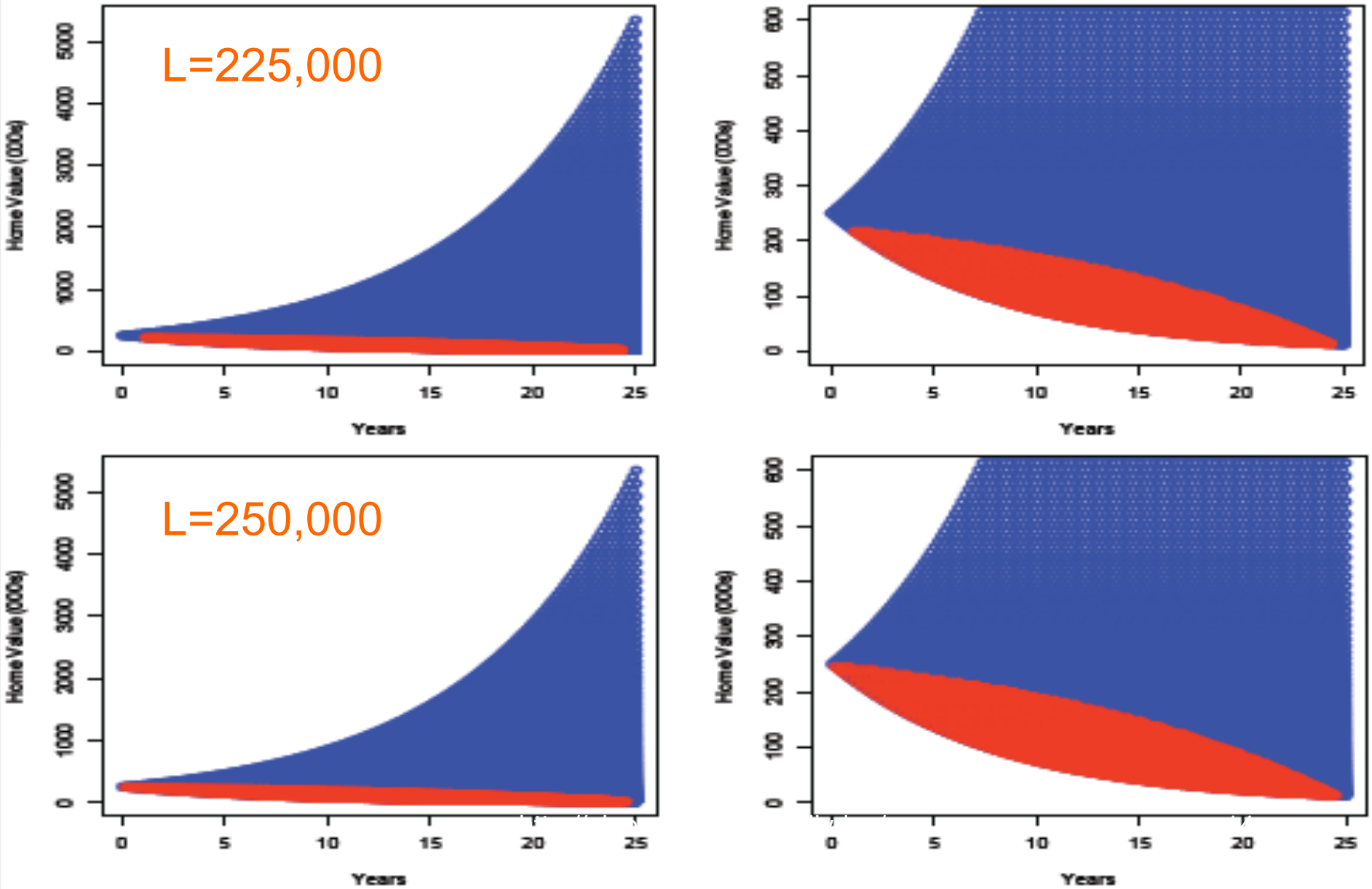
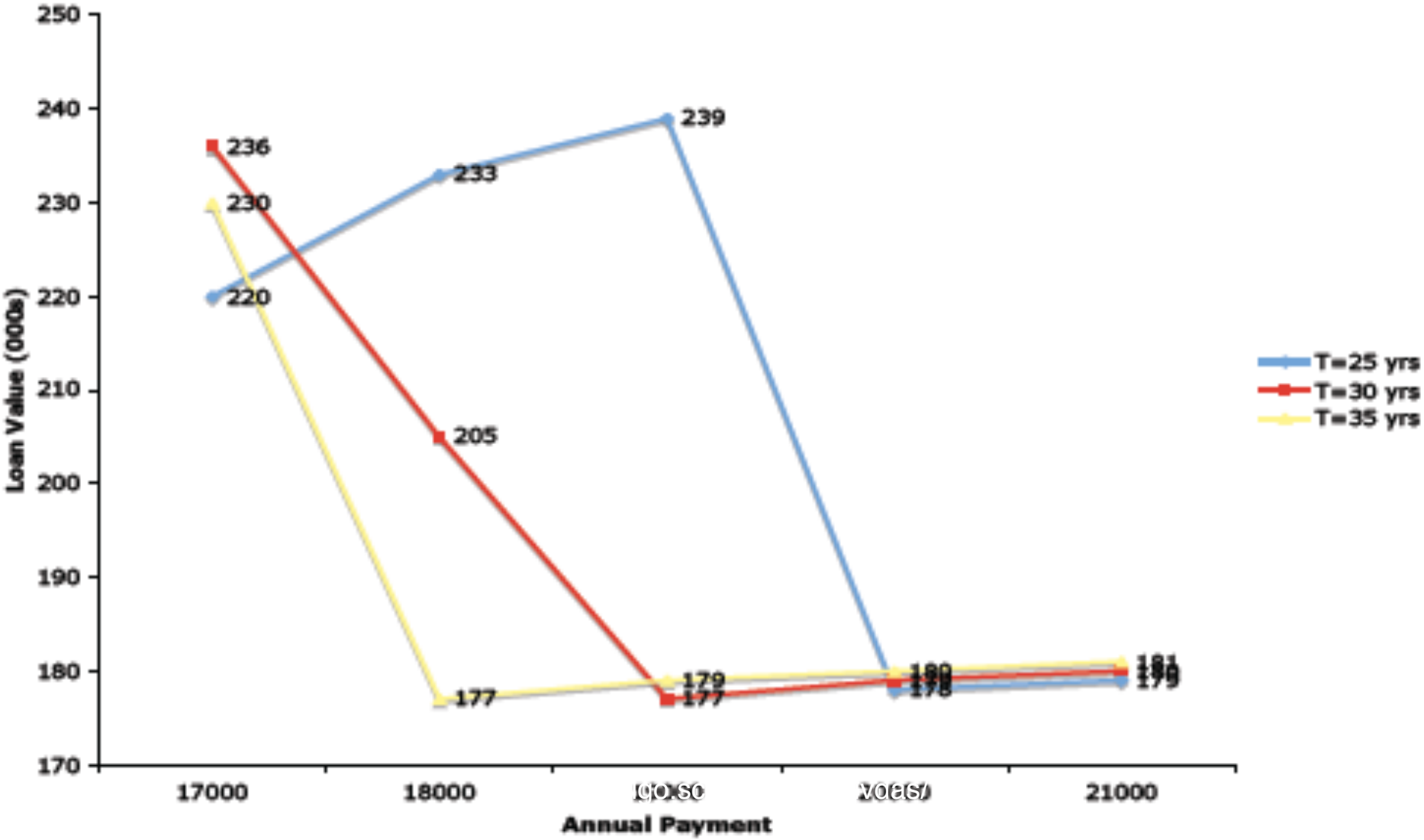


Figure 3: How loan principal and deadweight costs determine loan value. In this graph we plot the value of the loan from our model, for a range of loan principal balances, varying from \$200,000 to \$300,000, given a current home value of \$250,000. The parameters used for this graph are: home value volatility parameters $\sigma_1 = 0.02$ and $\sigma_2 = 0.03$, service flow level $\delta = 0.01$, interest rate volatility per annum $\beta = 0.0050$ (i.e., 50 bps), time step $h = 1/4$, loan rate $r_L = 0.06$, relocation costs $K_R = 0$, foreclosure recovery rate $\phi = \{0.7, 0.9\}$, loan maturity $T = 25$ years, and a flat forward rate curve at 5%.

Default Put Exercise Region



Modifying Maturity



Logit: Explaining Re-default

2007	Monthly Payment Amount after Modification									
	< 450		450-620		620-820		820-1120		> 1120	
	Est.	χ^2	Est.	χ^2	Est.	χ^2	Est.	χ^2	Est.	χ^2
Intercept	-2.91090	9.935	-3.23960	14.896	-1.75030	5.092	-2.71130	9.844	-1.47930	5.899
ΔRate	-0.11310	0.796	-0.28570	4.212	-0.31070	3.931	-0.33240	5.760	-0.36810	8.224
ΔTerm	-0.00640	5.427	-0.00133	0.220	0.00114	0.079	0.00016	0.002	-0.00038	0.008
ΔPrincipal	-0.00017	5.323	-0.00007	2.303	-0.00007	5.633	-0.00004	3.567	-0.00004	7.157
LTV	0.02270	6.143	0.02530	8.212	0.00163	0.040	0.02200	6.358	0.00141	0.043
Debt Ratio	-0.02110	2.021	-0.01060	0.802	-0.00291	0.056	-0.01790	3.182	-0.01150	1.357
PPC	-0.01000	0.421	-0.01340	1.888	-0.00324	0.033	-0.00463	0.106	0.00077	0.004
Re-Default	43		59		66		89		93	
No Default	230		330		382		459		545	
Wald Stat	18.9061	0.0043	18.8779	0.0044	15.1021	0.0195	23.4206	0.0007	20.8749	0.0019

2008	Monthly Payment Amount after Modification									
	< 450		450-620		620-820		820-1120		> 1120	
	Est.	χ^2	Est.	χ^2	Est.	χ^2	Est.	χ^2	Est.	χ^2
Intercept	-0.66310	3.595	-1.77870	18.532	-2.47180	39.303	-1.77290	22.438	-1.89300	39.145
ΔRate	0.13490	7.877	-0.16530	4.650	-0.01360	0.037	-0.30900	14.547	-0.14070	4.826
ΔTerm	0.00077	0.408	0.00028	0.042	0.00024	0.038	-0.00138	1.399	0.00035	0.143
ΔPrincipal	-0.00005	2.328	-0.00008	18.808	-0.00006	21.597	-0.00007	47.587	-0.00004	41.344
LTV	-0.00267	0.675	0.00806	4.207	0.01370	13.809	0.00653	3.627	0.00859	12.609
Debt Ratio	-0.01710	5.000	-0.00693	1.027	-0.00356	0.344	-0.00025	0.002	-0.00113	0.068
PPC	-0.02540	12.026	-0.00572	0.453	-0.01130	1.826	0.00540	0.417	-0.00754	1.286
Re-Default	177		205		286		375		522	
No Default	747		819		1000		1071		1564	
Wald Stat	32.3832	< 0.0001	42.1224	< 0.0001	46.1375	< 0.0001	83.5345	< 0.0001	81.2047	< 0.0001

Shared-Appreciation Mortgages

Exercise value = $B^b(t) - [V(t) - \theta \cdot C(V(t), K, t)] - K_R$

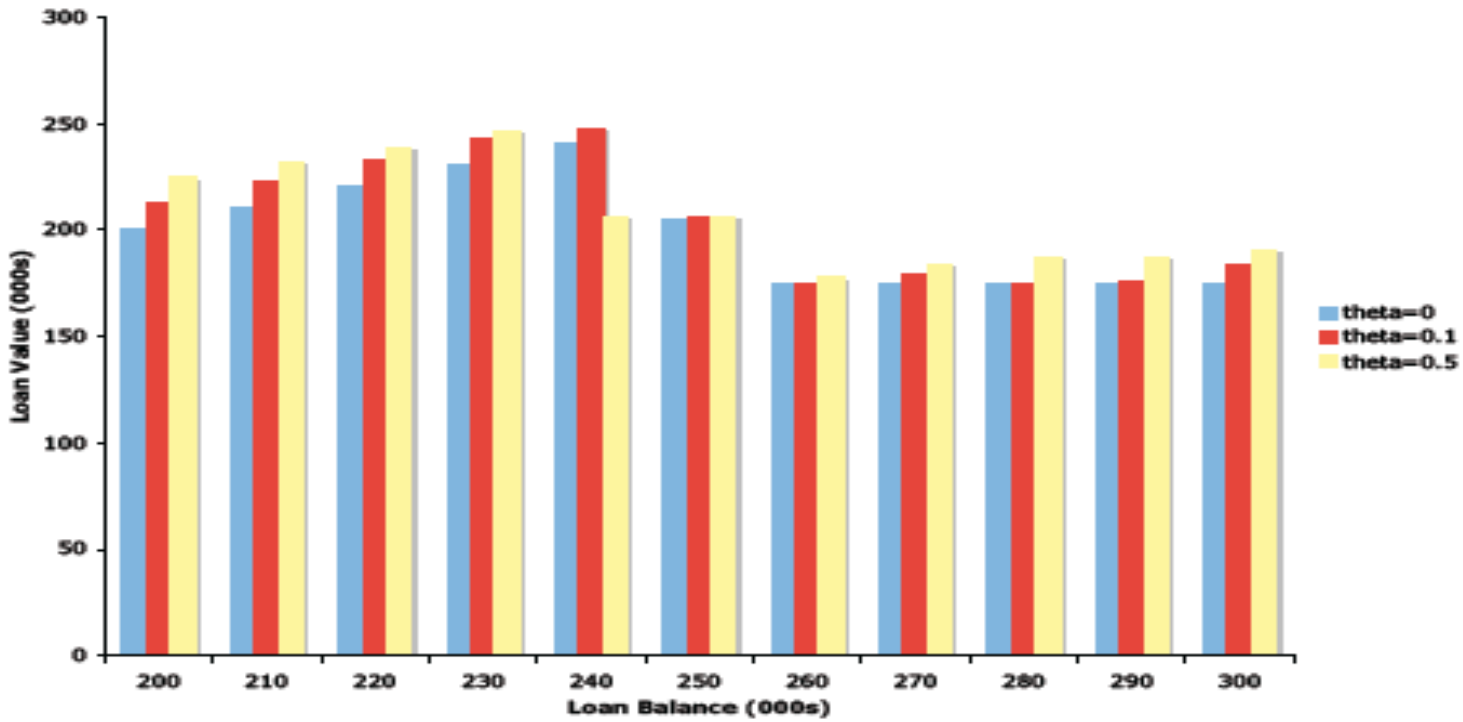


Figure 8: Loan values with appreciation sharing. The annual payment is $A = 19,000$. The parameters used for this graph are: home value volatility parameters $\sigma_1 = 0.02$ and $\sigma_2 = 0.03$, service flow level $\delta = 0.01$, interest rate volatility per annum $\beta = 0.0050$ (i.e., 50 bps), time step $h = 1/4$, loan rate $r_L = 0.06$, relocation costs $K_R = 0$, foreclosure recovery rate $\phi = 0.7$, loan maturity $T = 25$ years, and a flat forward rate curve at 5%. Appreciation share θ takes values in the set $\{0, 0.1, 0.5\}$, and the strike of the appreciation sharing agreement is \$250,000.

Reduced-Form Analysis of SAMs

$$dH_t = \mu H_t \, dt + \sigma H_t \, dZ_t$$

Home values

Normalize initial home value to 1. The option to default is ITM when ($H > L$).

There is a home value D at which the borrower will default. D is a "default level" or default exercise barrier.

D is a function of the lender share θ, we write it as $D(L, \theta)$.

D increases in L and in θ.

Foreclosure recovery as a fraction of H is ϕ .

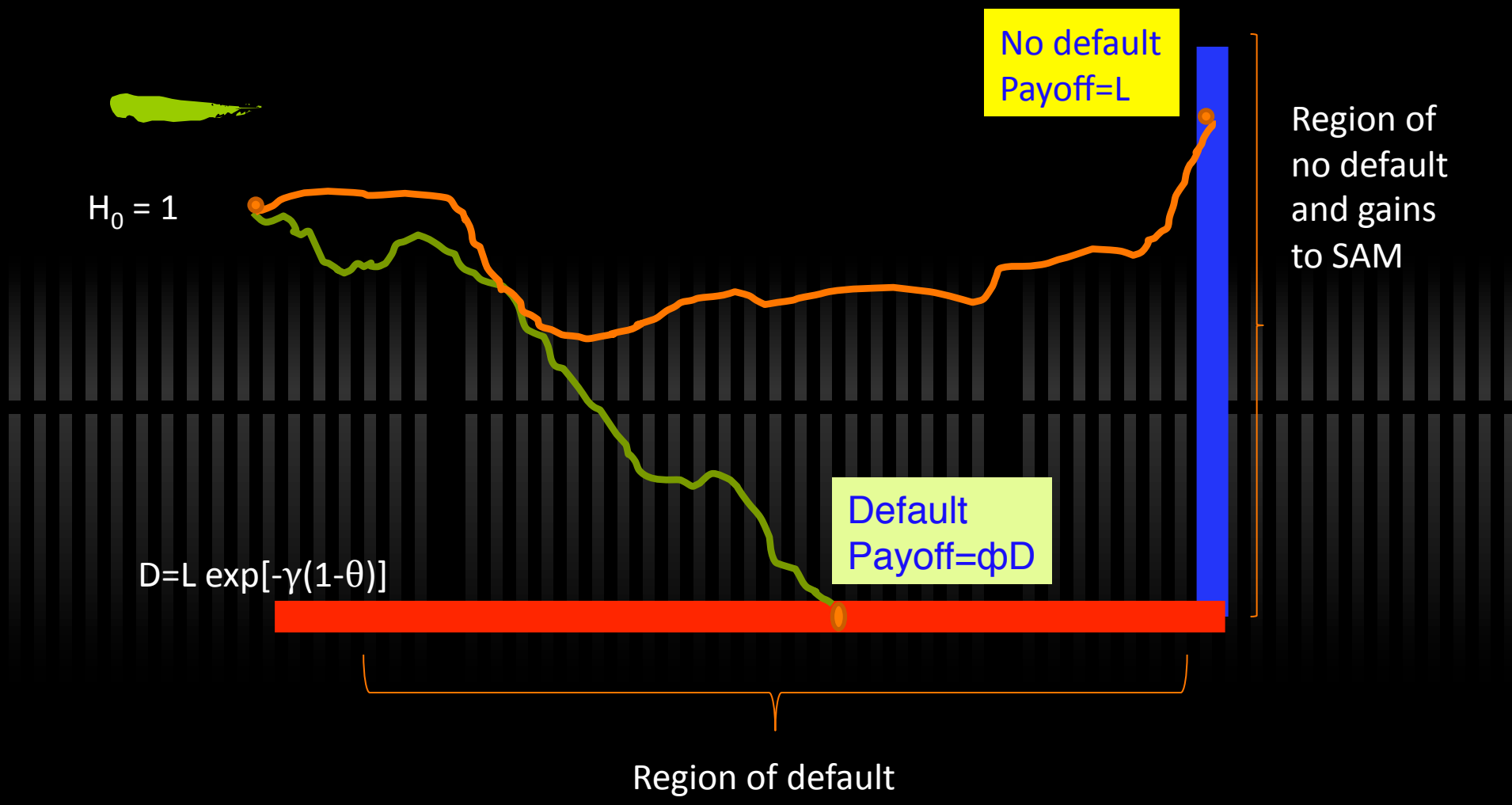
Default Barrier and Lender Share

$$D = L \exp[-\gamma(1-\theta)]$$

(1) The greater the willingness to pay (γ), the lower is the default level of home value D.
(2) When $\gamma = \infty$, the willingness to pay is infinite, the default level $D = 0$. The borrower never defaults unless the home value goes to zero.
(3) When $\gamma = 0$, there is no willingness to pay and the default level is $D = L$, i.e., the borrower defaults the moment the home value drops infintesimally below LTV at the time zero.

(1) The greater the lender's share (θ), the higher is the default level of home value D. The likelihood of default is therefore greater.
(2) When the lender share $\theta = 0$, the default level is $Le^{-\gamma}$.
(3) When $\theta = 1$, the default level is $D = L$. The borrower defaults the moment there is negative equity.

Barrier Model Intuition



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A Barrier Option Decomposition

Non-default component

$$Le^{-rT}\int_{D(L,\theta)}^{\infty} p(H_T|H_t > D, \forall t < T)\, dH_T$$

where $p(H_T|H_t > D, \forall t < T)$ is the density of the terminal home value conditional on no interim default.

Default component

$$\phi D \int_0^T e^{-rt} f(t; D)\, dt$$

where $f(t; D)$ is the first-passage time density for $H_t = D$.

Shared Appreciation component

$$e^{-rT}\int_K^{\infty} (H_T - K)\, p(H_T|H_t > D, \forall t < T)\, dH_T$$

PDE

$$\frac{\partial F}{\partial H}[\mu - \lambda\sigma]H + \frac{1}{2}\frac{\partial^2 F}{\partial H^2}\sigma^2 H^2 + \frac{\partial F}{\partial t} = rF$$

The Closed-Form Solution

$$\text{LOANVAL} \equiv V(H, L, K, r, T, \phi, \theta, \mu, \lambda, \sigma, \gamma)$$

$$\begin{aligned}
&= Le^{-rT}\left[N(d_2') - (D/H)^{2(R/\sigma^2)-1} \cdot N(d_{2b}')\right] \\
&\quad + \phi D\left[(D/H)^{b_1} \cdot N(a_1) + (D/H)^{b_2} \cdot N(a_2)\right] \\
&\quad + \theta\left[C_{SAM}(H, K) - D^{2(R/\sigma^2)-1} \cdot C_{SAM}(D^2/H, K)\right]
\end{aligned}$$

$$\begin{aligned}
D &= L\exp[-\gamma(1-\theta)] \\
d_2' &= \frac{\ln(H/D) + (R - 0.5\sigma^2)T}{\sigma\sqrt{T}} \\
d_{2b}' &= \frac{\ln(D/H) + (R - 0.5\sigma^2)T}{\sigma\sqrt{T}} \\
a_1 &= \frac{\ln(D/H) + \sqrt{2r\sigma^2 + (R - 0.5\sigma^2)^2} \cdot T}{\sigma\sqrt{T}} \\
a_2 &= \frac{\ln(D/H) - \sqrt{2r\sigma^2 + (R - 0.5\sigma^2)^2} \cdot T}{\sigma\sqrt{T}} \\
b_1 &= \frac{(R - 0.5\sigma^2) + \sqrt{2r\sigma^2 + (R - 0.5\sigma^2)^2}}{\sigma^2} \\
b_2 &= \frac{(R - 0.5\sigma^2) - \sqrt{2r\sigma^2 + (R - 0.5\sigma^2)^2}}{\sigma^2} \\
C_{SAM}(x, y) &= xe^{-(r-R)T}N(d_1') - ye^{-rT}N(d_1' - \sigma\sqrt{T}) \\
d_1' &= \frac{\ln(x/y) + (R + 0.5\sigma^2)T}{\sigma\sqrt{T}}
\end{aligned}$$

SAM or not?

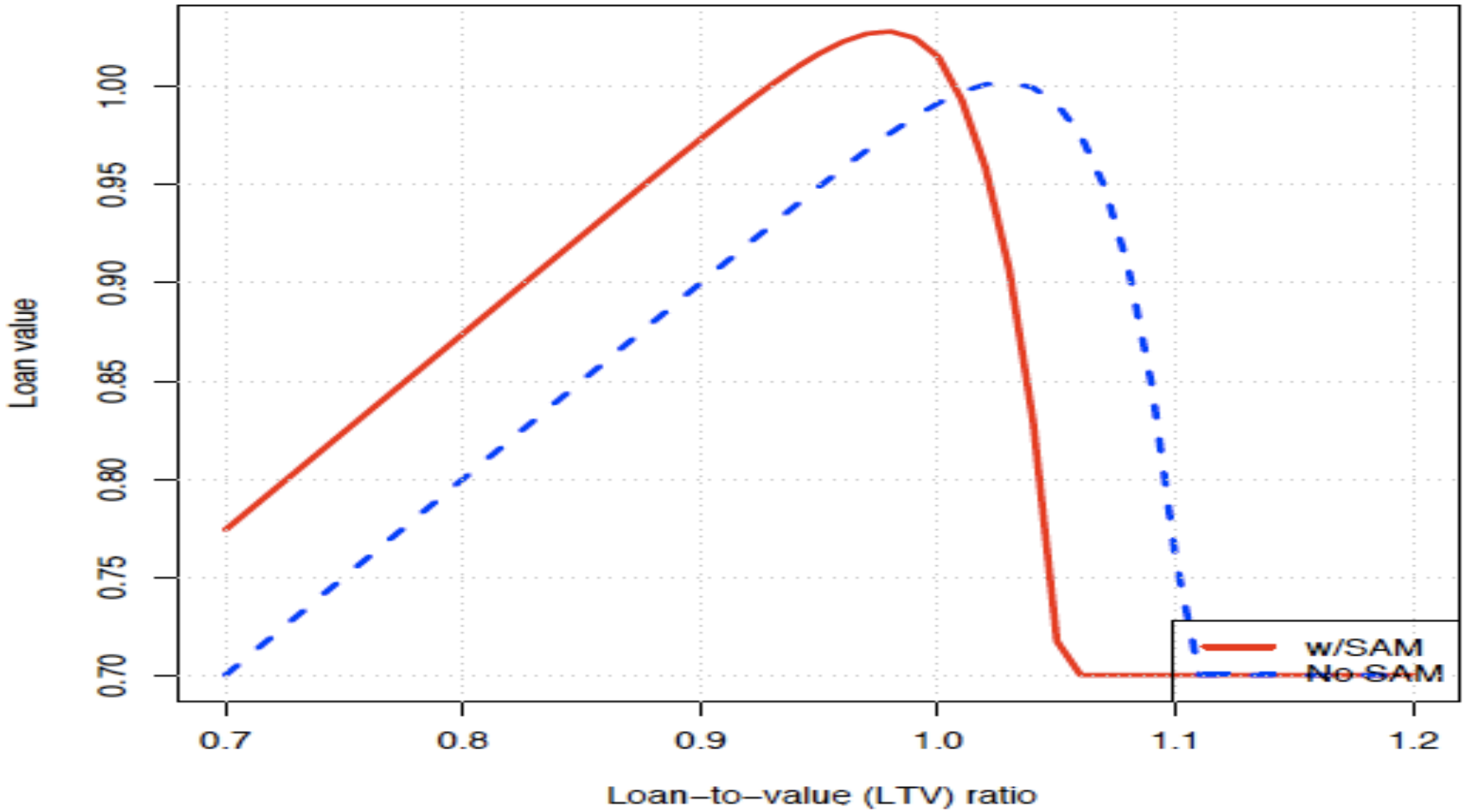

Fig. 2. Loan value as LTV is varied for loans with and without appreciation sharing. The parameters for the plot are as follows: willingness to pay coefficient $\gamma = 0.1$, home price volatility $\sigma = 0.04$, foreclosure fraction $\phi = 0.7$, risk-free rate $r = 0.02$, the house value growth rate $\mu = 0.04$, price of risk $\lambda = 0.25$, and the horizon of the model $T = 5$ years. The appreciation share fraction is $\theta = 0.50$ for the case when a SAM is applied, and $\theta = 0$ when there is no share appreciation.

Foreclosure Recovery Rate

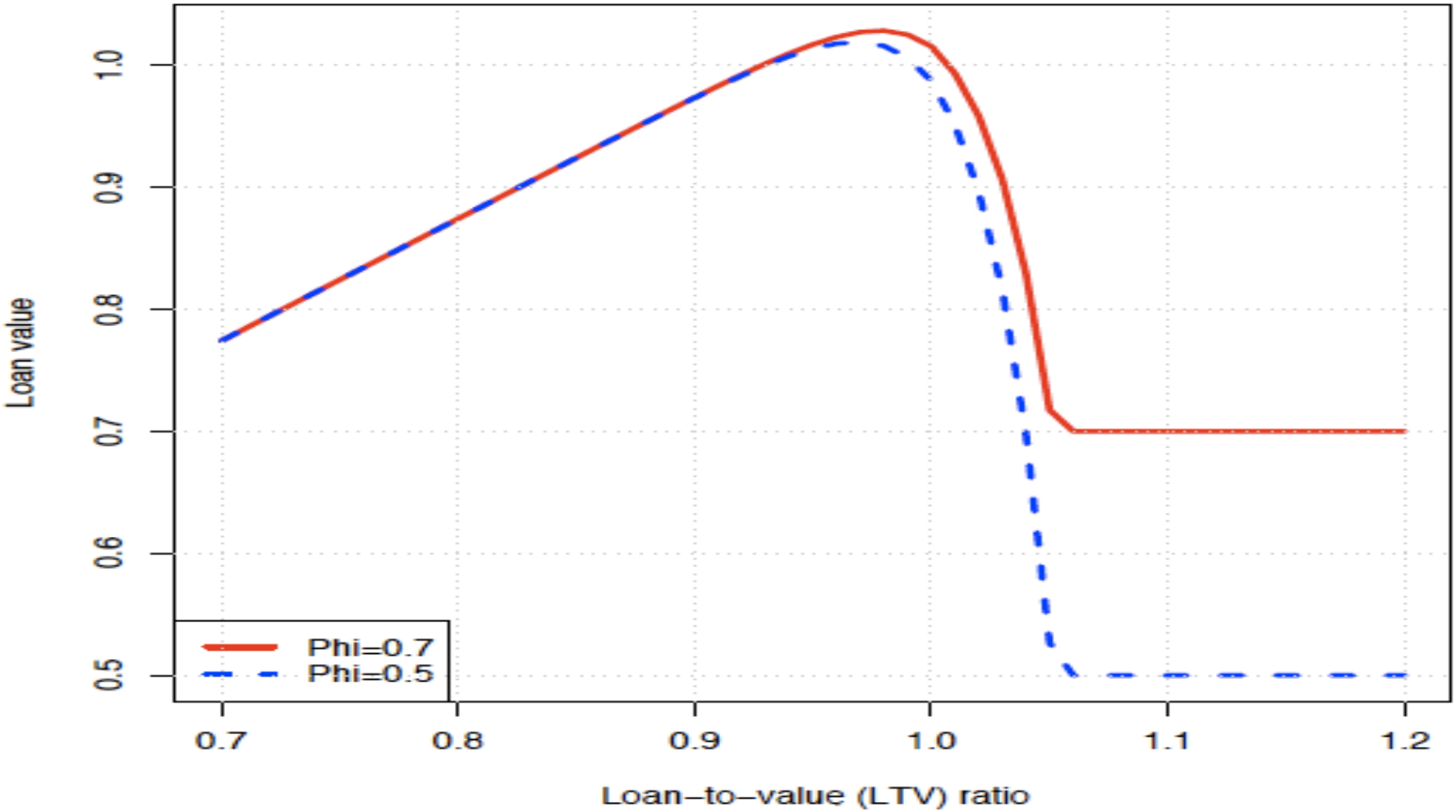

Fig. 3. Loan value as LTV is varied for loans with SAMs and the foreclosure recovery rate is varied across $\{\phi = 0.5, \phi = 0.7\}$. Both cases are with appreciation sharing. The parameters for the plot are as follows: willingness to pay coefficient $\gamma = 0.1$, home price volatility $\sigma = 0.04$, risk-free rate $r = 0.02$, the house value growth rate $\mu = 0.04$, price of risk $\lambda = 0.25$, and the horizon of the model $T = 5$ years. The appreciation share fraction is $\theta = 0.50$.

Home Price Volatility

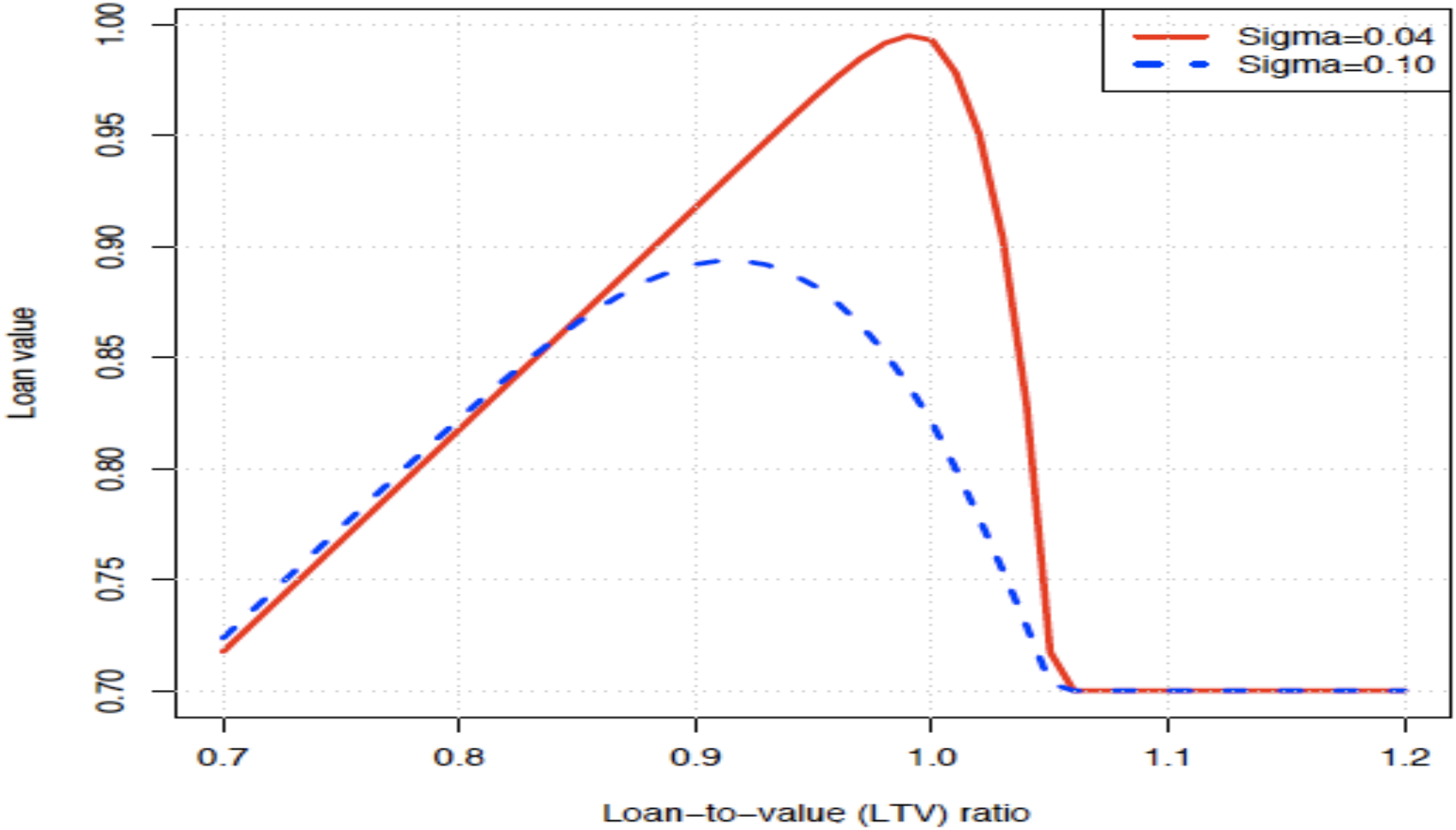

Fig. 4. Loan value as LTV is varied for loans with SAMs and housing price volatility is varied across $\{\sigma = 0.04, \sigma = 0.10\}$. Both cases are with appreciation sharing. The parameters for the plot are as follows: willingness to pay coefficient $\gamma = 0.1$, foreclosure percentage $\phi = 0.7$, risk-free rate $r = 0.02$, the house value growth rate $\mu = 0.04$, price of risk $\lambda = 0.25$, and the horizon of the model $T = 1$ year. The appreciation share fraction is $\theta = 0.50$.

Home Value Growth Rates

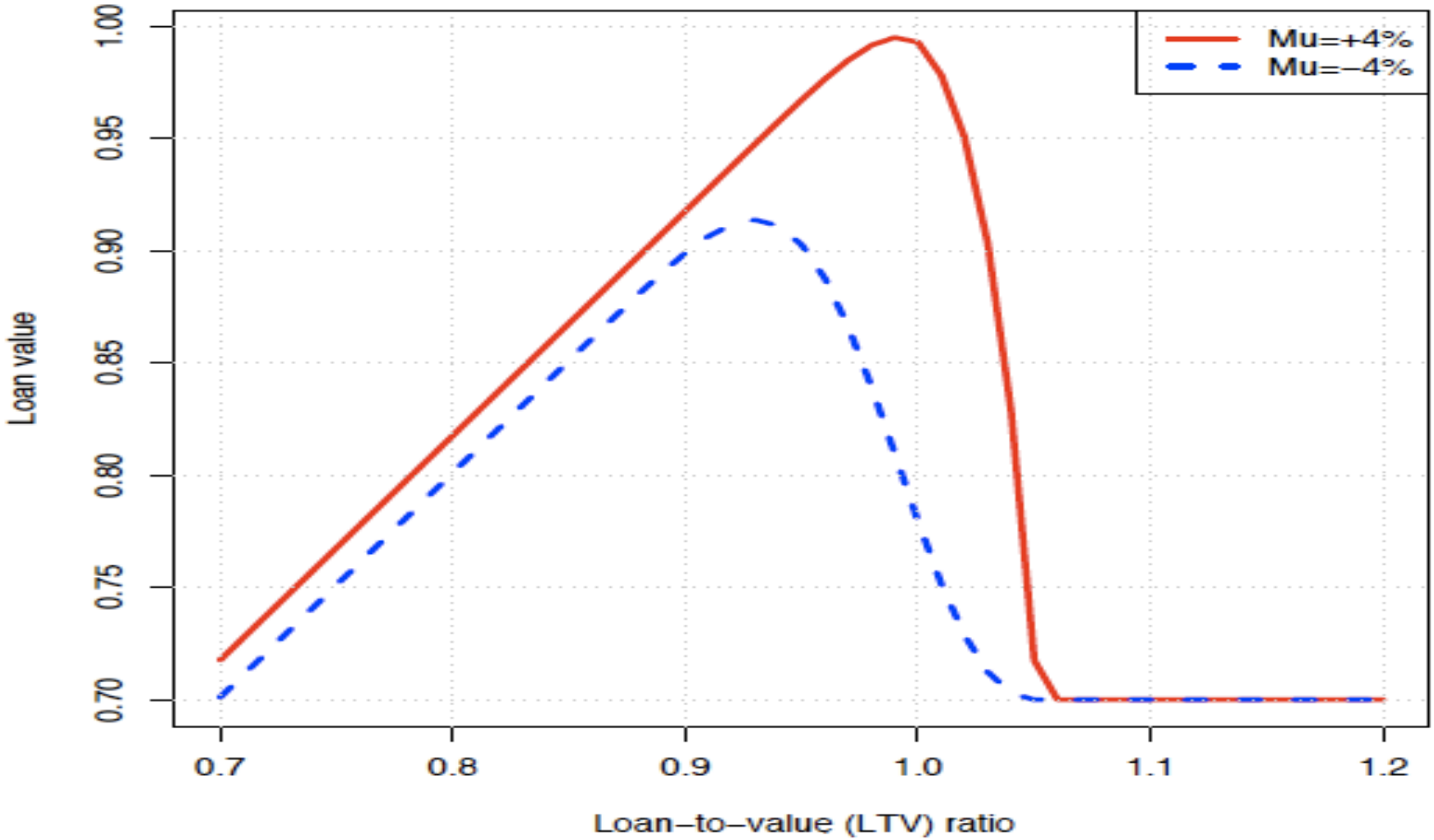

Fig. 5. Loan value as LTV is varied for loans with SAMs and housing value growth rate is varied across $\{\mu = -0.04, \mu = +0.04\}$. Both cases are with appreciation sharing. The parameters for the plot are as follows: willingness to pay coefficient $\gamma = 0.1$, foreclosure percentage $\phi = 0.7$, risk-free rate $r = 0.02$, housing price volatility $\sigma = 0.04$, price of risk $\lambda = 0.25$, and the horizon of the model $T = 1$ year. The appreciation share fraction is $\theta = 0.50$.

Willingness to Pay

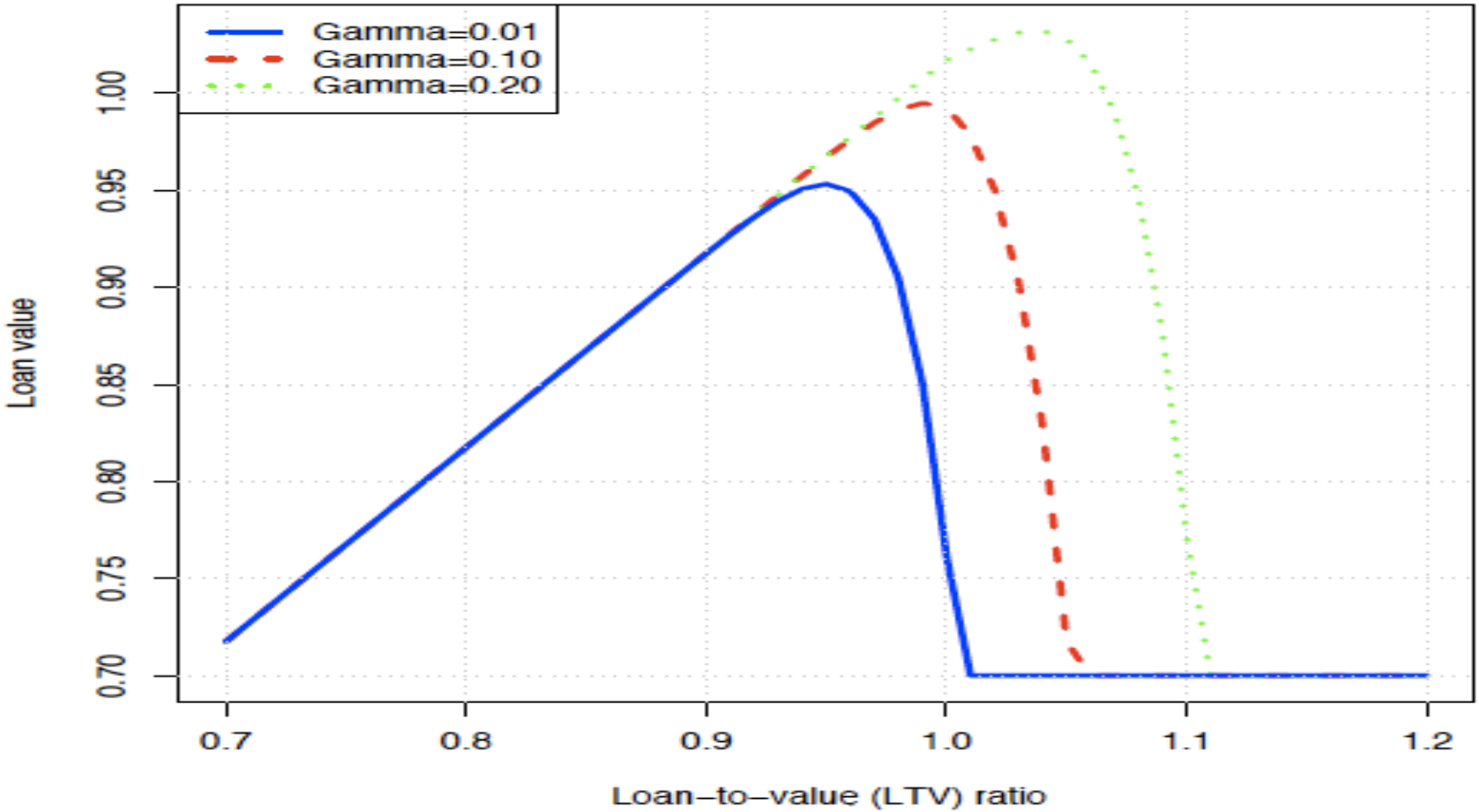

Fig. 6. Loan value as LTV is varied for loans with SAMs and willingness to pay is varied across $\{\gamma = 0.01, \gamma = 0.10, \gamma = 0.20\}$. All cases are with appreciation sharing. The parameters for the plot are as follows: the house value growth rate $\mu = 0.04$, price of risk $\lambda = 0.25$, foreclosure percentage $\phi = 0.7$, risk-free rate $r = 0.02$, housing price volatility $\sigma = 0.04$, and the horizon of the model $T = 1$ year. The appreciation share fraction is $\theta = 0.50$.

SAM Share

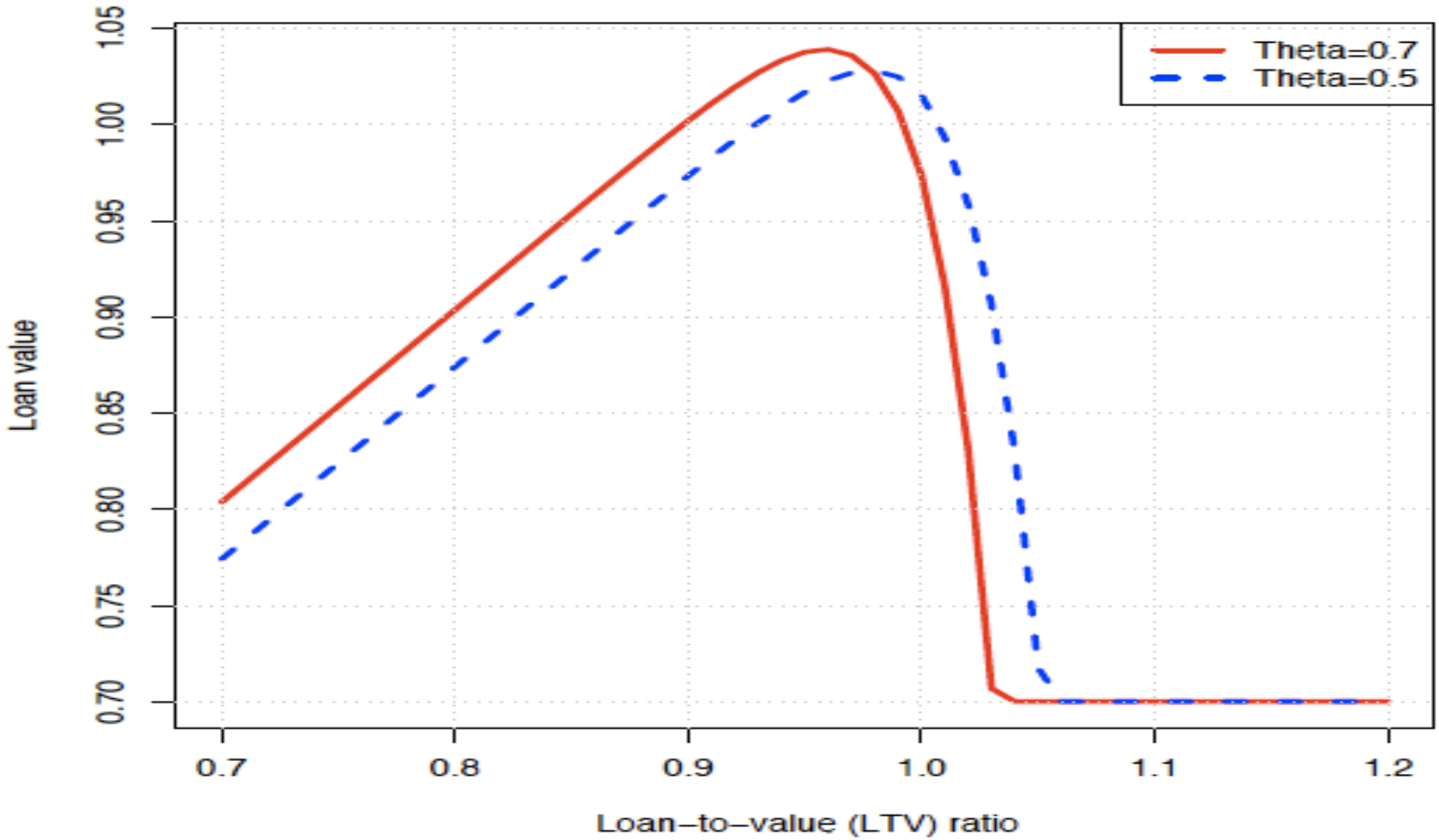

Fig. 7. Loan value as LTV is varied for loans with SAMs and appreciation is varied across $\{\theta = 0.70, \theta = 0.50\}$. The parameters for the plot are as follows: willingness to pay coefficient $\gamma = 0.1$, foreclosure percentage $\phi = 0.7$, risk-free rate $r = 0.02$, housing price volatility $\sigma = 0.04$, and the horizon of the model $T = 5$ years. The growth rate in home values is $\mu = 0.04$, and the price of risk $\lambda = 0.25$.

Extension: A Two-Horizon Model

We accommodate varying growth rates over short and long horizons.

This captures mean-reversion in house price levels.

The model is embedded on a Cox-Ross-Rubinstein tree that accommodates American option features and risk premia for non-tradeability of housing assets.

$u = \exp(\sigma\sqrt{h})$ Up move factor

$d = \exp(-\sigma\sqrt{h})$ Down move factor

$\mathcal{R} = \exp(rh)$ Risk-neutral drift

$\delta = r - \mu(t) + \lambda\sigma$ Adjustment for the growth rate and price of risk

$q = \dfrac{\mathcal{R}e^{-\delta h} - d}{u - d}$ Probability of an up move

Mean-reversion in home value

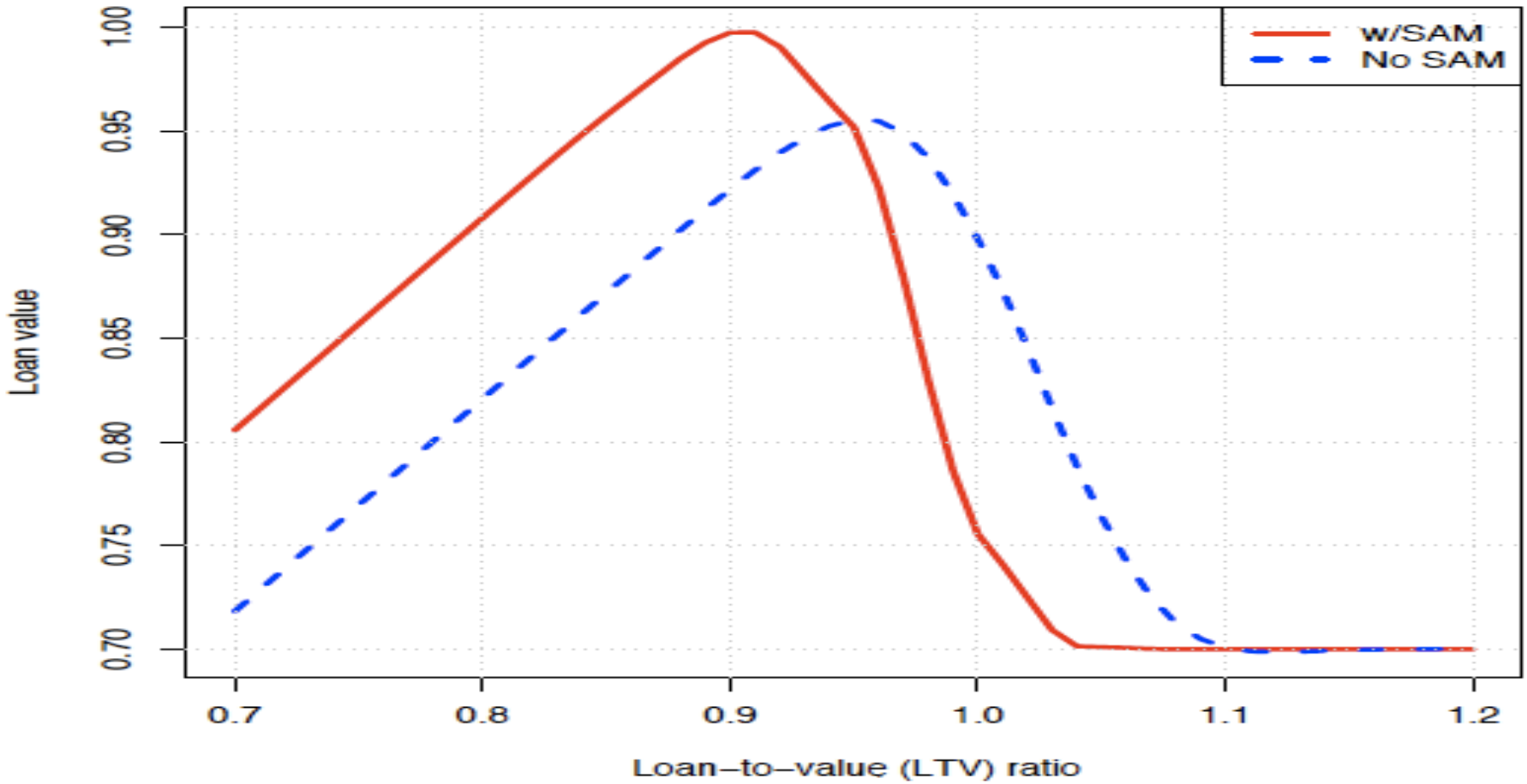

Fig. 8. Two-horizon model: Loan value as LTV is varied for loans with and without appreciation sharing. The parameters for the plot are as follows: willingness to pay coefficient $\gamma = 0.1$, home price volatility is $\sigma = 0.04$, foreclosure recovery fraction $\phi = 0.7$, risk-free rate $r = 0.03$, the house value growth rate in each period is $\mu_1 = -0.05$ and $\mu_2 = +0.04$, price of risk $\lambda = 0.25$, and the two horizons of the model are $\tau = 1$ and $T = 10$ years. The appreciation share fraction is $\theta = 0.50$ for the case when a SAM is applied, and $\theta = 0$ when there is no share appreciation. The coupon rate on the loan is 3.3%. The coupon rate on the loan has been set such that at the optimal LTV, the loan with a SAM prices up close to par.

Macroeconomic Sequencing

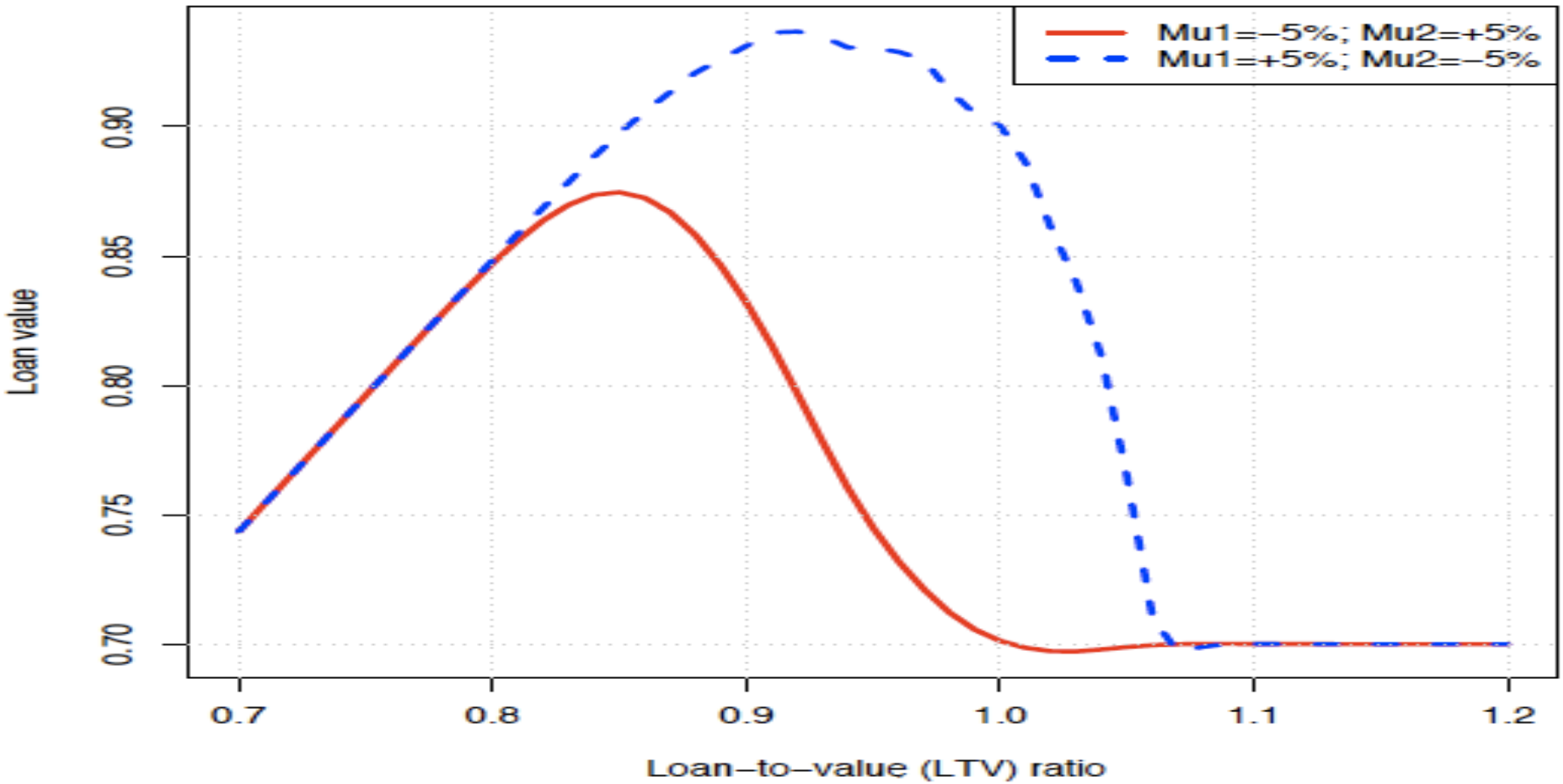

Fig. 9. Two-horizon model: Loan value as LTV is varied for loans when there are periods of positive and negative growth in home values. The parameters for the plot are as follows: willingness to pay coefficient $\gamma = 0.1$, home price volatility is $\sigma = 0.04$, foreclosure recovery fraction $\phi = 0.7$, risk-free rate $r = 0.03$, the house value growth rate in each period is $\mu_1 = -0.05$ and $\mu_2 = +0.05$ (reversed in the second case), price of risk $\lambda = 0.25$, and the two horizons of the model are $\tau = 2$ and $T = 4$ years. The appreciation share fraction is $\theta = 0.50$. The coupon rate on the loan is 4.4%. The coupon rate on the loan has been set such that at the optimal LTV, the loan for the first up then down scenario prices up close to par.

Summary

- ✓ Using a dynamic game-theoretic optimization problem, we see that principal write-downs are indeed optimal and much preferred to interest-rate reductions, or maturity extensions of loans.
- ✓ We derive a mathematical decomposition of the loan value into options that admit closed-form solutions, that may be used to optimize loan modification in practice.