Evaluating Interest Rate Covariance Models within a Value-at-Risk Framework*

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Abstract

A key component of managing international interest rate portfolios is forecasts of the covariances between national interest rates and accompanying exchange rates. How should portfolio managers choose among the large number of covariance forecasting models available? We find that covariance matrix forecasts generated by models incorporating interest-rate level volatility effects perform best with respect to statistical loss functions. However, within a value-at-risk (VaR) framework, the relative performance of the covariance matrix forecasts depends greatly on the VaR distributional assumption, and forecasts based just on weighted averages of past observations perform best. In addition, portfolio variance forecasts that ignore the covariance matrix generate the lowest regulatory capital charge, a key economic decision variable for commercial banks. Our results provide empirical support for the commonly-used VaR models based on simple covariance matrix forecasts and distributional assumptions.

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The short-term interest rate is one of the most important and fundamental variables in financial markets. In particular, interest rate volatility and its forecasts are important to a wide variety of economic agents. Portfolio managers require such volatility forecasts for asset allocation decisions; option traders require them to price and hedge interest rate derivatives; and risk managers use them to compute measures of portfolio market risk, such as value-at-risk (VaR). In fact, since the late 1990s, financial regulators have been using VaR methods and their underlying volatility forecasts to monitor specific interest rate exposures of financial institutions.

Forecasts of interest rate volatility are available from two general sources. The market-based approach uses option market data to infer implied volatility, which is interpreted to be the market's best estimate of future volatility conditional on an option pricing model. Since these forecasts incorporate the expectations of market participants, they are said to be forward-looking measures. The model-based approach forecasts volatility from time series data, generating what are usually called historical forecasts. The empirical evidence in Day and Lewis (1992), Amin and Ng (1997), and others suggests that implied volatility can forecast future volatility. However, there is also some consensus that implied volatility is an upward biased forecast of future volatility and that historical volatility forecasts contain additional useful information. Moreover, the scope of available implied second moments is limited due to options contracts availability, especially with respect to covariances.[1]

Much of the work on the model-based approach to forecast second moments has focused on univariate time-varying variance models; see Bollerslev, Engle, and Nelson (1994). In practice, given the large number of models available, how should portfolio managers choose among them?[2] Most studies evaluate the performance of such volatility models using in-sample analysis. However, out-of-sample evaluation of forecasts provides a potentially more useful comparison of the alternative models, due to the possibility of overfitting. Moreover, the ability to produce useful out-of-sample forecasts is a clear test of model performance because it only uses the same information set available to agents at each point in time.[3] Previous studies across several financial markets have compared the out-of-sample forecasting ability of several GARCH type models with that of simple variance measures and found that there is no clear winner using statistical loss functions; see for example, West and Cho (1995), and Andersen and Bollerslev (1998). Figlewski (1997) shows that GARCH models fitted to daily data may be useful in forecasting stock market volatility both for short and long horizons, but they are much less useful in forecasting volatility in other markets beyond short horizons.[4] In particular, using 3-month US Treasury bill yields, sample variances generate more accurate forecasts than GARCH model. Ederington and Guan (2000) also find that sample variances dominate GARCH models in the 3-month Eurodollar market.

In contrast, relatively less work has been done on modeling and forecasting second moments using multivariate time-varying covariance models. For example, Kroner and Ng (1998) propose a general family of multivariate GARCH models, called General Dynamic Covariance (GDC) model, and Engle (2002) proposes the dynamic conditional correlation (DCC) model that is a generalization of the constant conditional correlation model of Bollerslev (1990). Focussing on interest rates, Ferreira (2001) presents a new multivariate model called the GDC-Levels model that includes both level effects and conditional heteroskedasticity effects in the volatility dynamics.[5] He finds that the interest rate variance-covariance matrix is best forecasted out-of-sample using the multivariate VECH model with a level effect, although simple models using either equally or exponentially-weighted moving averages of past observations appear to perform as well under statistical loss functions.

Given the mixed out-of-sample performance of these models with respect to statistical loss functions, some studies have compare their out-of-sample performance in terms of a specific economic application. Not surprisingly, these results may vary according to the selected economic loss function; see Lopez (2001). To date, such economic evaluations of out-of-sample volatility forecasts has been in the areas of hedging performance, asset allocation decisions, and options pricing. Regarding hedging strategies, Cecchetti, Cumby, and Figlewski (1988) find that a time-varying covariance matrix is necessary to construct an optimal hedge ratio between Treasury bonds and bond futures, while Kroner and Ng (1998) find that the choice of the multivariate specification can result in very different estimates of the optimal hedge ratio for stock portfolios. With respect to asset allocation, volatility model selection is also important as shown by Fleming, Kirby, and Ostdiek (2001), and Poomimars, Cadle, and Theobald (2002). For options pricing, Engle, Kane, and Noh (1997), Gibson and Boyer (1998), and Byström (2002) find that standard time-series models produce better correlation forecasts than simple moving average models for the purpose of stock-index option pricing and trading. Christoffersen and Jacobs (2004) also evaluate alternative GARCH covariance forecasts in terms of option pricing and find out-of-sample evidence in favor of relatively parsimonious models.

In this paper, we consider economic loss functions related to financial risk management and based on value-at-risk (VaR) measures that generally indicate the amount of portfolio value that could be lost over a given time horizon with a specified probability. Hendricks (1996) provides an extensive evaluation of alternative VaR models using a portfolio of foreign exchange rates, although he does not examine covariance matrix forecasts. Other papers - Alexander and Leigh (1997), Jackson, Maude, and Perraudin (1997), Brooks, Henry, and Persand (2002), Brooks and Persand (2003), and Wong, Cheng, and Wong (2003) - examine VaR measures for different asset portfolios using multivariate models. We extend this literature by evaluating the forecast performance of multivariate volatility models within a VaR framework using interest rate data, as per Lopez and Walter (2001).[6]

The performance of VaR for interest rate portfolios, one of the most important financial prices and of particular relevance in banks' trading portfolios, is studied using univariate models by Vlaar (2000), who examines the out-of-sample forecast accuracy of several univariate volatility models for portfolios of Dutch government bonds with different maturities. In addition, he examines the impact of alternative VaR distributional assumptions. The analysis shows that the percentage of VaR exceedances for simple models is basically equal to that for GARCH models. The distributional assumption is also important with the normal performing much better than the t-distribution. Brooks and Persand (2003) examine FTA British Government Bond Index and find that multivariate GARCH models do not contribute much under standard statistical loss functions and certain VaR evaluation measures.

Our study considers an equally-weighted portfolio of short-term fixed income positions in the US dollar, German deutschemark and Japanese yen. The portfolio returns are calculated from the perspective of a US-based investor using daily interest rate and foreign exchange data from 1979 through 2000. We examine VaR estimates for this portfolio from a wider variety of multivariate volatility models, ranging from simple sample averages to standard time-series models. We compare the out-of-sample forecasting performance of these alternative variance-covariance structures and their economic implications within the VaR framework used by Lopez and Walter (2001). The question of interest is whether more complex covariance models provide improved out-of-sample VaR forecasts.

The VaR framework consists of three sets of evaluation techniques. The first set focuses on the statistical properties of VaR estimates derived from alternative covariance matrix forecasts and VaR distributional assumptions. Specifically, the binomial test of correct unconditional and conditional coverage, which is implicitly incorporated into the aforementioned bank capital requirements, is used to examine 1%, 5%, 10%, and 25% VaR estimates. The second set of techniques focus on the magnitude of the losses experienced when VaR estimates are exceeded. To determine whether the magnitudes of observed exceptions are in keeping with the model generating the VaR estimates, we use the hypothesis test based on the truncated normal distribution proposed by Berkowitz (2001). Finally, to examine the performance of the competing covariance matrix forecasts with respect to regulatory capital requirements, we use the regulatory loss function implied by the U.S. implementation of the market risk amendment to the Basel Capital Accord. This loss function penalizes a VaR model for poor performance by using a capital charge multiplier based on the number of VaR exceptions; see Lopez (1999a) for further discussion.

Our findings are roughly in line with those of Lopez and Walter (2001), which are based on a simple portfolio of foreign exchange positions. As per Ferreira (2001), we confirm that covariance matrix forecasts generated from models that incorporate interest-rate level effects perform best under statistical loss functions. However, simple specifications, such as weighted averages of past observations, perform best with regard to the magnitude of VaR exceptions and regulatory capital requirements.

We also find that the relative performance of covariance matrix forecasts depends greatly on the VaR models' distributional assumptions. Our results provide further empirical support for the commonly-used VaR models based on simple covariance matrix forecasts and distributional assumptions, such as the normal and nonparametric distributions. Moreover, simple variance forecasts based only on the portfolio returns (i.e., ignoring the portfolio's component assets) perform as well as the best covariance matrix forecasts and, in fact, generate the minimum capital requirements in our exercise. This finding is mainly consistent with the evidence in Berkowitz and O'Brien (2002), who analyzed the daily trading revenues and VaR estimates of six US commercial banks. They find that a GARCH model of portfolio returns that ignores the component assets, when combined with a normal distributional assumption, provides VaR estimates and regulatory capital requirements that are comparable to other models and does not produce large VaR exceptions.

The paper is structured as follows. Section 1 describes the issue of forecasting variances and covariances and presents the alternative time-varying covariance specifications. We also compare the models in terms of out-of-sample forecasting power for variances and covariances. Section 2 describes the VaR models analyzed. Section 3 presents the results for the hypothesis tests that make up the VaR evaluation framework. Section 4 concludes.

1 Time-Varying Covariance Models

In this paper, we study an equally-weighted US dollar-denominated portfolio of US dollar (US$), Deutschemark (DM), and Japanese yen (JY) short-term interest rates. We take the view of a US-based investor, who is unhedged with respect to exchange rate risk. Thus, we need to estimate the (5×5) variance-covariance matrix between the three interest rates as well as DM/US$ and JY/US$ exchange rates.

1.1 Forecasting Variances and Covariances

The standard empirical model for the short-term interest rate r_{it} for i =US$,DM,JY is

$$r_{it} - r_{it-1} = \mu_i + \kappa_i r_{it-1} + \varepsilon_{it}, \tag{1}$$

and the standard model for the foreign exchange rate s_{it} for i =DM/US$,JY/US$ is

$$\log\left(\frac{s_{it}}{s_{it-1}}\right) = \varepsilon_{it}, \tag{2}$$

where $E\left[\varepsilon_{it}|\mathcal{F}_{t-1}\right] = 0$ and $\mathcal{F}_{t-1}$ is the information set up to time $t-1$. The conditional mean function for the interest rate in (1) is given by a first-order autoregressive process, and we assume a zero conditional mean for the continuously compounded foreign exchange rate return in (2). The conditional variance-covariance matrix H_t is (5×5) with elements $[h_{ijt}]$. The conditional covariances are expressed as

$$h_{ijt} = E\left[\varepsilon_{it}\varepsilon_{jt}|\mathcal{F}_{t-1}\right], \tag{3}$$

and the conditional variances are represented when $i = j$.

The first category of second moment forecasts is based on sample averages of past observations.[7] The most common estimator is the equally-weighted sample covariance over a period of T observations; i.e.,

$$\widehat{h}_{ijt} = \frac{1}{T} \sum_{s=0}^{T-1} \varepsilon_{it-s} \varepsilon_{jt-s}. \tag{4}$$

Note that this forecast assumes that all future values of h_{ijt} are constant. The sample covariance estimator weights equally each past observation included in the estimate. The first set of sample average forecasts we examine set $T = 250$ and is denoted as E250.[8] The second set of such forecasts uses all past observations as of time t (i.e., the unconditional estimate at time t) and is denoted as Eall.

A common feature of financial data is that recent observations have a greater impact on second moments than older ones. A simple way to incorporate this feature is to weight observations in inverse relation to their age. The exponentially-weighted sample covariance (EWMA) forecast assigns more weight to recent observations relative to more distant observations using a weighting parameter. Let w be the weighting factor (or decay rate) with $0 < w \leq 1$. The forecast formulated at time t weights all available observations up to that time in the following way:

$$\widehat{h}_{ijt} = \frac{\sum_{s=0}^{t-1} w^s \varepsilon_{it-s} \varepsilon_{jt-s}}{\sum_{s=0}^{t-1} w^s}. \tag{5}$$

In the limit, the forecasts generated by equation (5) include an infinite number of past observations, but their weights become infinitesimally small.[9] The weighting parameter determines the degree to which more recent data is weighted, and in this study, we assume $w = 0.94$, which is the value most commonly used in actual risk management applications.[10]

We also consider second moment forecasts based directly on portfolio returns, hence ignoring the covariance matrix completely. This approach significantly reduces computational time and estimation errors and presents comparable performance in terms of VaR estimates as reported in Lopez and Walter (2001) and Berkowitz and O'Brien (2002). In particular, we examine the equally-weighted (with 250 and all past observations) and exponentially-weighted (with a decay factor of 0.94) sample variance of portfolio returns and denote them as E250-Port, Eall-Port, and EWMA-Port, respectively. We also consider a GARCH(1,1) variance model of portfolio return, denoted GARCH-Port.

The second category of forecasts we examine explicitly is based on multivariate GARCH models. In the basic multivariate GARCH model, the components of the conditional variance-covariance matrix H_t vary through time as functions of the lagged values of H_t and of the squared ε_t innovations. We model the (5×5) covariance matrix of the relevant changes in US$, DM and JP short-term interest rates as well as DM/US$ and JY/US$ foreign exchange rates using several multivariate GARCH specifications, as described below.

1.1.1 VECH Model

The VECH model of Bollerslev, Engle, and Wooldridge (1988) is a parsimonious version of the most general multivariate GARCH model given by, for $i, j = 1, 2, ..., 5$,

$$h_{ijt} = \omega_{ij} + \beta_{ij} h_{ijt-1} + \alpha_{ij} \varepsilon_{it-1} \varepsilon_{jt-1}, \tag{6}$$

where ω_{ij}, β_{ij} and α_{ij} are parameters. In fact, the VECH model is simply an ARMA process for $\varepsilon_{it}\varepsilon_{jt}$. An important implementation problem is that the model may not yield a positive definite H_t covariance matrix. Furthermore, the VECH model does not allow for volatility spillover effects, i.e., the conditional variance of a given variable is not a function of other variables' past shocks.

1.1.2 Constant Correlation (CCORR) Model

In the CCORR model of Bollerslev (1990), the conditional covariance is proportional to the product of the conditional standard deviations; i.e., $\rho_{ijt} = \rho_{ij}$. Consequently, the conditional correlation is constant across time. The model is described by the following equations, for $i, j = 1, 2, ..., 5$,

$$h_{iit} = \omega_{ii} + \beta_{ii} h_{iit-1} + \alpha_{ii} \varepsilon_{it-1}^2, \quad h_{ijt} = \rho_{ij} \sqrt{h_{iit-1}} \sqrt{h_{jjt-1}}, \qquad \text{for all } i \neq j. \tag{7}$$

The conditional covariance matrix of the CCORR model is positive definite if and only if the correlation matrix is positive definite. Like in the VECH model there is no volatility spillover effects across series.

1.1.3 Dynamic Conditional Correlation (DCC) Model

The DCC model of Engle (2002) preserves the simplicity of estimation of the CCORR model, but explicitly allows for time-varying correlations. The H_t covariance matrix model is decomposed as

$$H_t = D_t R_t D_t, \tag{8}$$

where D_t is a (5×5) diagonal matrix of the individual asset variances h_{iit} given by a univariate GARCH(1,1) specification, and R_t is a time-varying correlation matrix expressed as

$$\begin{aligned} R_t &= (Q_t^*)^{-1} Q_t (Q_t^*)^{-1}, \\ Q_t &= (1-a-b)\overline{Q} + a\varepsilon_{t-1}\varepsilon_{t-1}' + bQ_{t-1}, \end{aligned} \tag{9}$$

where $\overline{Q}$ is the unconditional covariance of the standardized residuals resulting from the individual D_t estimations, and Q_t^* is a diagonal matrix composed of the square root of the diagonal elements of Q_t. This model generates a positive definite covariance matrix, and there are no volatility spillover effects across series.

1.1.4 BEKK Model

The BEKK model of Engle and Kroner (1995) is given by:

$$H_t = \Omega + B' H_{t-1} B + A' \varepsilon_{t-1}\varepsilon_{t-1}' A, \tag{10}$$

where Ω, A, and B are (5×5) matrices, with Ω being symmetric. In the BEKK model, the conditional covariance matrix is determined by the outer product matrices of the vector of past innovations. As long as Ω is positive definite, the conditional covariance matrix is also positive definite because the other terms in (10) are expressed in quadratic form.

1.1.5 General Dynamic Covariance (GDC) Model

The GDC model proposed by Kroner and Ng (1998) nests several multivariate GARCH models. The conditional covariance specification is given by:

$$H_t = D_t R D_t + \Phi \circ \Theta_t, \tag{11}$$

where $\circ$ is the Hadamard product operator (element-by-element matrix multiplication), D_t is a diagonal matrix with elements $\sqrt{\theta_{iit}}$, R is the correlation matrix with components ρ_{ij}, Φ is a matrix of parameters with components $\phi_{ii} = 0$ for all i, $\Theta_t = [\theta_{ijt}]$,

$$\theta_{ijt} = \omega_{ij} + b_i' H_{t-1} b_j + a_i' \varepsilon_{t-1} \varepsilon_{t-1}' a_j, \qquad \text{for all } i, j \tag{12}$$

a_i, b_i, $i = 1, 2, ..., 5$ are (5×1) vectors of parameters, and ω_{ij} are scalars with $\Omega \equiv [\omega_{ij}]$ being a diagonal positive definite (5×5) matrix.

This model combines the constant correlation model with the BEKK model. The first term in (11) is like the constant correlation model but with variance functions given by the BEKK model. The second term in (11) has zero diagonal elements with the off-diagonal elements given by the BEKK model covariance functions scaled by the ϕ_{ij} parameters. Thus, the GDC model structure can be written as follows:

$$\begin{aligned} h_{iit} &= \theta_{iit} \\ h_{ijt} &= \rho_{ij} \sqrt{\theta_{iit}} \sqrt{\theta_{jjt}} + \phi_{ij} \theta_{ijt} \qquad \text{for all } i \neq j. \end{aligned} \tag{13}$$

The GDC model nests the multivariate GARCH models discussed above.[11] The VECH model assumes that $\rho_{ij} = 0$ for all $i \neq j$.[12] The BEKK model has the restrictions $\rho_{ij} = 0$ for all $i \neq j$ and $\phi_{ij} = 1$ for all $i \neq j$. The CCORR model considers $\phi_{ij} = 0$ for all $i \neq j$.

1.1.6 GDC-Levels (GDC-L) Model

One of the most important features of the short-term interest rate volatility is the level effect; i.e., volatility is a function of the interest rate level. Chan, Karolyi, Longstaff, and Sanders (1992) document this effect using a conditional variance specification where only the interest rate level drives the evolution of the interest rate volatility. They find that the sensitivity of interest rate variance to the interest rate level is very high and in excess of unity. Subsequent work by Brenner, Harjes, and Kroner (1996) shows that the level effect exists, but it is considerably smaller (i.e., consistent with a square root process) by incorporating into the variance function both level and conditionally heteroskedasticity effects (i.e., news effect) using a GARCH framework. Bali (2003) also finds that incorporating both GARCH and level effects into the interest rate volatility improves the pricing performance of the models.

We consider a multivariate version of the Brenner et al. (1996) model by extending the GDC model to incorporate the level effect in the variance-covariance functions for the interest rates, but not for the exchange rates. That is, we specify

$$\theta_{ijt} = \omega_{ij}^* + b_i^{'} H_{t-1}^* b_j + a_i^{'} \varepsilon_{t-1}^* \varepsilon_{t-1}^{*\prime} a_j, \qquad \text{for } i, j = 1, 2, 3 \tag{14}$$

where $\omega_{ii}^* = \omega_{ii} / r_{it-1}^{2\gamma_i}$, the diagonal elements of the matrix H_{t-1}^* are multiplied by the factor $(r_{it-1}/r_{it-2})^{2\gamma_i}$ for all i, and the elements of the vector ε_{t-1}^* are divided by $r_{it-1}^{\gamma_i}$ for all i. The GDC-Levels model nests the prior multivariate GARCH models and introduces the level effect.

For the sake of completeness, we also introduce the level effect specification into our other multivariate GARCH models. The exact form of conditional variance in the VECH model with level effect, denoted the VECH-L model for the interest rate is given by:

$$h_{iit} = \left(\omega_{ii} + \beta_{ii} h_{iit-1} r_{it-2}^{-2\gamma_i} + \alpha_{ii} \varepsilon_{it-1}^2 \right) r_{it-1}^{2\gamma_i}. \tag{15}$$

Similarly, we construct the CCORR-L, DCC-L and BEKK-L models. In summary, a total of seventeen volatility forecasting models are analyzed below.

1.2 Models Estimation

1.2.1 Data Description

The daily interest rate data examined here are US dollar (US$), Deutschemark (DM) and Japanese yen (JY) three-month money market interest rates from the London interbank market.[13] The interest rates are given in percentage and in annualized form. These interest rates are taken as proxies for the instantaneous riskless interest rate (i.e., the short rate) as per Chapman, Long, and Pearson (1999). We also require foreign exchange rates to compute U.S. dollar investor rates of return, and the JY/US$ and DM/US$ spot exchange rates used here are from the London market.[14]

As in most of the literature, we focus on nominal interest rates because generating real interest rates at high frequencies is subject to serious measurement errors. The use of daily frequency minimizes the discretization bias resulting from estimating a continuous-time process using discrete periods between observations. We use interbank interest rates in our work, instead of government securities yields, because of the lesser liquidity of these secondary markets in Germany and Japan, which is in sharp contrast with the high liquidity of the interbank money market. Concerns regarding the existence of a default premium in the interbank money market should not be material since these interest rates result from trades among highly-rated banks that are considered almost default-free at short maturities.

The period of analysis is from January 12, 1979 to December 29, 2000, providing 5731 daily observations. Table 1 presents summary statistics for the interest rate first differences and foreign exchange rate geometric return. The mean interest rate changes are all close to zero, while the standard deviations differ considerably with the US interest rate changes having an annualized standard deviation of 2.8% compared with 1.8% and 1.4% for the Japanese and German interest rate changes, respectively. The daily exchange rate changes are also close to zero, but they present a much higher degree of variation with an annualized standard deviation of about 11% for both series. Table 1 also presents summary statistics for an equally-weighted portfolio of the US$, DM and JY interest rates with returns calculated in US$; see section 2 for a detailed portfolio return definition. The mean portfolio return is almost zero with an annualized standard deviation of 19.6%. The portfolio return presents leptokurtosis, though less severe that most individual components series, with exception of the DM/US$ exchange rate return.

Figure 1 plots the daily time series of US$, DM, and JY three-month interest rates as well as the DM/US$ and JY/US$ foreign exchange rates. Both daily interest rates (foreign exchange rates) levels and first differences (geometric returns) are plotted. The daily time series of portfolio returns is also plotted. All three interest rate series present a downward trend over the sample period. However, the DM and JY interest rate present a sharp increase in the early 1990s. The US$ foreign exchange rate presents a steady depreciation after the mid 1980s, when it reached a peak against both the DM and JY.

1.2.2 Forecasting Performance

The multivariate model parameters are estimated using a rolling scheme; i.e., parameters are reestimated using a fixed window of 2861 daily observations (or approximately ten years of data) and updated once every year. For example, the first estimation period is from January 12, 1979 to December 29, 1989 to calculate the models' one-step-ahead variance-covariance forecasts over the out-of-sample period from January 1, 1990 to December 31, 1990. The overall out-of-sample period is from January 1, 1990 to December 29, 2000. [15]

Following standard practice, we estimate the models by maximizing the likelihood function under the assumption of independent and identically distributed (i.i.d.) innovations. In addition, we assume that the conditional density function of the innovations is given by the normal distribution. Conditional normality of the innovations is often difficult to justify in many empirical applications that use leptokurtic financial data, as shown for our data in Table 1. However, the maximum likelihood estimator based on the normal density has a valid quasi-maximum likelihood (QML) interpretation as shown by Bollerslev and Wooldridge (1992).[16]

Tables 2 and 3 present the various models' out-of-sample forecast performance in terms of mean prediction error (MPE) and root mean squared prediction error (RMPSE). Table 2 reports MPE and RMSPE for the variances of the five component assets and the overall portfolio, and Table 3 reports MPE and RMSPE for the ten relevant covariances. We also report Diebold and Mariano (1995) test statistics comparing the models' forecasting accuracy relative to the one with the minimum RMSPE.

These results suggest a slight bias, as measured by MPE, toward overestimating both variances and covariances. Only the JY/US$ exchange rate variance is underestimated by most models. Accordingly, the portfolio variance is overestimated by most models.

Overall, the results in Tables 2 and 3 clearly indicate that no single model dominates the others in terms of the RMSPE loss function. Across the 15 forecasted second moments, 10 different models of the 13 models examined generated the lowest RMSPE values at least once. Based on the Diebold-Mariano test results, we observe that in many cases we cannot reject the null hypothesis of equal forecast accuracy for the RMSPE-minimizing forecasts and the alternative forecasts.

Based on these results, the best performing model across the 15 moments is the VECH-L model, which minimizes RMSPE in three cases and is never rejected as less accurate than other models. Notably, the VECH model performs only slightly worse as it is the RMSPE-minimizing forecast in two cases and rejected the equal accuracy hypothesis only once. This result, in conjunction with the comparative performance of the models with and without interest rate level effects, suggests that the level effect does not contribute to out-of-sample performance here. In addition, the performance of the simple models is quite reasonable given their lack of explicit time dynamics. The sample variance of the past 250 day observations (E250 model) performs similarly to the best multivariate models, especially in terms of covariance. The equal accuracy hypothesis is only rejected at the 5% level in five cases, and it is the RMSPE minimizing forecast in three cases

Portfolio variance is best forecasted by the VECH and VECH-Levels models, confirming their performance in forecasting the individual variances and covariances. For the portfolio variance forecasts based solely on the portfolio returns, the null hypothesis of equal forecasting power relative to the best model is not rejected at the 5% level for the GARCH-Port and EWMA-Port models. This result suggests that simple portfolio variance forecasts are useful, even though full covariance matrix forecasts generally perform as well or better. Surprisingly, the E250 model is rejected in terms of forecasting portfolio variance, in contrast to its good performance for individual variances and covariances. This comes as a result of the weak performance of this models in terms of forecasting foreign exchange rate volatilities - an important component of the portfolio variance.

In summary, statistical loss functions provide a useful, although incomplete, analysis of individual assets' variance and covariance as well as portfolio variance forecasts. Our results indicate that the second moment forecasts from simple models and time-varying covariance models present similar out-of-sample forecasting performance. In addition, the most complex time-varying covariance models tend to overfit the data. Our findings are generally consistent with those of Ferreira (2001) for interest rates, and West and Cho (1995) and Lopez (2001) for foreign exchange rates. However, economic loss functions that explicitly incorporate the costs faced by volatility forecast users provide the most meaningful forecast evaluations. In the next section, we examine the performance of the covariance matrix forecasts within a value-at-risk framework.

2 Value-at-Risk Framework

2.1 Portfolio Definition

Assume that we have a US fixed-income investor managing positions in US\$, DM, and JY three-month zero-coupon bonds in terms of dollar-denominated returns.[17] From the perspective of a US-based investor, define the value of a US\$ investment tomorrow to be

$$Y_{\mathrm{US\$}t+1} = Y_{\mathrm{US\$}t} e^{R_{\mathrm{US\$}t+1}}, \tag{16}$$

where $R_{\mathrm{US\$}t+1} = -\frac{1}{4}\left(r_{\mathrm{US\$}t+1} - r_{\mathrm{US\$}t}\right)$ denotes the day $t+1$ geometric rate of return on the three-month zero-coupon bond investment and $r_{\mathrm{US\$}t}$ is the annualized US\$ three-month interest rate (continuously compounded).

The value of the DM zero-coupon bond in US\$ tomorrow is given by:

$$Y_{\mathrm{DM}t+1} = \left(Y_{\mathrm{DM}t} s_{\mathrm{DM/US\$}t}\right) e^{R_{\mathrm{DM}t+1}} s_{\mathrm{US\$/DM}t+1}, \tag{17}$$

where $s_{\mathrm{US\$/DM}t}$ is the exchange rate between US\$ and DM. Thus, the day $t+1$ geometric rate of return on the DM three-month zero-coupon bond investment given by:

$$\ln\left(Y_{\mathrm{DM}t+1}\right) - \ln\left(Y_{\mathrm{DM}t}\right) = R_{\mathrm{DM}t+1} - \Delta \ln\left(s_{\mathrm{DM/US\$}t+1}\right), \tag{18}$$

where $R_{\mathrm{DM}t+1} = -\frac{1}{4}\left(r_{\mathrm{DM}t+1} - r_{\mathrm{DM}t}\right)$ is the local-currency rate of return on the fixed income investment, $r_{\mathrm{DM}t}$ is the annualized DM three-month interest rate (continuously compounded), and $\Delta \ln\left(s_{\mathrm{DM/US\$}t+1}\right) = \left[\ln\left(s_{\mathrm{DM/US\$}t+1}\right) - \ln\left(s_{\mathrm{DM/US\$}t}\right)\right]$ is the foreign exchange rate geometric return. The analysis for the JY fixed income position is the same as for the DM fixed income.

For simplicity, we will assume that the portfolio at time t is evenly divided between the three fixed-income positions (equally-weighted portfolio). Thus, the investor's market risk exposure is the sum of the three investment returns. That is,

$$R_{pt+1} = \left[\begin{array}{ccccc} R_{\mathrm{US\$}t+1} & R_{\mathrm{DM}t+1} & \Delta \ln\left(s_{\mathrm{DM/US\$}t+1}\right) & R_{\mathrm{JY}t+1} & \Delta \ln\left(s_{\mathrm{JY/US\$}t+1}\right) \end{array}\right] w', \tag{19}$$

where $w = \begin{bmatrix} 1 & 1 & -1 & 1 & -1 \end{bmatrix}$ is the portfolio weight vector.

2.2 VaR Models and Exceptions

We examine the accuracy of several VaR models based on different covariance matrix specifications, as discussed above, and conditional densities. The VaR estimate on day t derived from model m for a k-day-ahead return, denoted $\text{VaR}_{mt}(k, \alpha)$, is the critical value that corresponds to the lower α percent tail of the forecasted return distribution f_{t+k}; that is,

$$\text{VaR}_{mt}(k, \alpha) = F^{-1}_{mt+k}(\alpha) \tag{20}$$

where F^{-1}_{mt+k} is the inverse of the cumulative distribution function to f_{mt+k}. In our analysis, a VaR model m consists of two items: a covariance matrix model and a univariate distributional assumption. Following Lopez and Walter (2001), we separately specify the portfolio variance dynamics

$$h_{mpt+1} = w' H_{mt+1} w, \tag{21}$$

and the distributional form of f_{mt+k}.[18] In terms of portfolio variance dynamics, we consider 10 multivariate covariance model forecasts, three sample variance forecasts and four portfolio variance forecasts for a total of 17 alternative h_{mpt+1} specifications. In terms of distributions, we consider four alternative assumptions: normal, t-student, generalized t-student, and nonparametric distribution. The last distributional form is the unsmoothed distribution of the standardized portfolio returns residuals, which is the distribution used in the so-called "historical simulation" approach for generating VaR estimates. This gives a total of 68 VaR models to consider.

The t-distribution assumption requires an estimate of the number of degrees of freedom, which affects the tail thickness of the distribution. This parameter is estimated using the in-sample standardized residuals generated by the 17 model specifications. The estimated number of degrees of freedom ranges from 8 and 11, which correspond to some fat tailedness, and the implied 1% quantiles range from -2.9 to -2.7 compared with -2.33 for the normal distribution. The generalized t-distribution assumption requires estimates of two parameters that control the tail thickness and width of the center of the distribution, named n and q; see Bollerslev et al. (1994). The parameters

n and q are also estimated using the in-sample standardized residuals and are found to be in the ranges of $[2.1, 2.5]$ and $[1.0, 1.1]$, respectively. These estimates imply considerably more fat tailedness than the normal distribution at the 1% quantile, but the opposite is true at higher quantiles. The nonparametric distribution quantiles are estimated using the unsmoothed distribution of the standardized portfolio return residuals arising from each of the 17 models. The estimated nonparametric critical values generally present more fat-tailedness at the 1% and 5% quantiles than the normal distribution, but again the opposite is true at higher quantiles.

The one-day-ahead VaR estimates from model m for the specified fixed-income portfolio at the α% quantile on day t is

$$\mathrm{VaR}_{mt}(\alpha) = \sqrt{h_{mpt+1}} F_m^{-1}(\alpha), \tag{22}$$

where we drop the subscript t from the portfolio distribution F_m because we assume that it is constant over time. We then compare $\mathrm{VaR}_{mt}(\alpha)$ to the actual portfolio return on day $t+1$, denoted as R_{pt+1}. If $R_{pt+1} < \mathrm{VaR}_{mt}(\alpha)$, then we have an exception. For testing purposes, we define the exception indicator variable as

$$I_{mt+1} = \begin{cases} 1 & \text{if } R_{pt+1} < \mathrm{VaR}_{mt}(\alpha) \\ 0 & \text{if } R_{pt+1} \geq \mathrm{VaR}_{mt}(\alpha) \end{cases}. \tag{23}$$

2.3 VaR Evaluation Tests

2.3.1 Unconditional and Conditional Coverage Tests

Assuming that a set of VaR estimates and their underlying model are accurate, exceptions can be modeled as independent draws from a binomial distribution with a probability of occurrence equal to α percent. Accurate VaR estimates should exhibit the property that their unconditional coverage $\widehat{\alpha} = x/T$ equals α percent, where x is the number of exceptions and T the number of observations. The likelihood ratio statistic for testing whether $\widehat{\alpha} = \alpha$ is

$$LR_{uc} = 2\left[\log\left(\widehat{\alpha}^x (1-\widehat{\alpha})^{T-x}\right) - \log\left(\alpha^x (1-\alpha)^{T-x}\right)\right], \tag{24}$$

which has an asymptotic $\chi^2(1)$ distribution.[19] Note that for this study, finite sample critical values based on the appropriate parameter values and 100,000 replications are used.

The LR_{uc} test is an unconditional test of the coverage of VaR estimates since it simply counts exceptions over the entire period without reference to the information available at each point in time. However, if the underlying portfolio returns exhibit time-dependent heteroskedasticity, the conditional accuracy of VaR estimates is probably a more important issue. In such cases, VaR models that ignore such variance dynamics will generate VaR estimates that may have correct unconditional coverage, but at any given time, will have incorrect conditional coverage.

To address this issue, Christoffersen (1998) proposed conditional tests of VaR estimates based on interval forecasts. VaR estimates are essentially interval forecasts of the lower 1% tail of f_{mt+1}, the one-step-ahead return distribution. The LR_{cc} test used here is a test of correct conditional coverage. Since accurate VaR estimates have correct conditional coverage, the I_{mt+1} variable must exhibit both correct unconditional coverage and serial independence. The LR_{cc} test is a joint test of these properties, and the relevant test statistic is $LR_{cc} = LR_{uc} + LR_{ind}$, which is asymptotically distributed $\chi^2(2)$.

The LR_{ind} statistic is the likelihood ratio statistic for the null hypothesis of serial independence against the alternative of first-order Markov dependence.[20] The likelihood function under this alternative hypothesis is

$$L_A = (1-\pi_{01})^{T_{00}}\,\pi_{01}^{T_{01}}\,(1-\pi_{11})^{T_{10}}\,\pi_{11}^{T_{11}}, \tag{25}$$

where the T_{ij} notation denotes the number of observations in state j after having been in state i in the previous period, $\pi_{01} = T_{01}/(T_{00}+T_{01})$ and $\pi_{11} = T_{11}/(T_{10}+T_{11})$. Under the null hypothesis of independence, $\pi_{01} = \pi_{11} = \pi$, and the likelihood function is

$$L_0 = (1-\pi)^{T_{00}+T_{01}}\,\pi^{T_{01}+T_{11}}, \tag{26}$$

where $\pi = (T_{01}+T_{11})/T$. The test statistic

$$LR_{ind} = 2\left[\log L_A - \log L_0\right], \tag{27}$$

has an asymptotic $\chi^2(1)$ distribution.

2.3.2 Dynamic Quantile Test

We also use the Dynamic Quantile (DQ) test proposed by Engle and Manganelli (2002), which is a Wald test of the hypothesis that all coefficients as well as the intercept are zero in a regression of the exception indicator variable on its past values (we use 5 lags) and on current VaR, i.e.,

$$I_{mt+1} = \delta_0 + \sum_{k=1}^{5} \delta_k I_{mt-k+1} + \delta_6 VaR_{mt} + \epsilon_{t+1}. \tag{28}$$

We estimate the regression model by OLS, and a good model should produce a sequence of unbiased and uncorrelated I_{mt+1} variable, so that the explanatory power of this regression should be zero. Note that here again we use finite sample critical values to test the null hypothesis.

2.3.3 Distribution Forecast Test

Since VaR models are generally characterized by their distribution forecasts of portfolio returns, several authors have suggested that evaluations should be based directly on these forecasts. Such an evaluation would use all of the information available in the forecasts. The object of interest in these evaluation methods is the observed quantile q_{mt+1}, which is the quantile under the distribution forecast f_{mt+1} in which the observed portfolio return R_{pt+1} actually falls; i.e.,

$$q_{mt+1}\left(R_{pt+1}\right) = \int_{-\infty}^{R_{pt+1}} f_{mt+1}\left(R_p\right) dR_p. \tag{29}$$

If the underlying VaR model is accurate, then its q_{mt+1} series should be independent and uniformly distributed over the unit interval.

Several hypothesis tests have been proposed for testing these two properties, but we use the likelihood ratio test proposed by Berkowitz (2001).[21] For this test, the q_{mt+1} series is transformed using the inverse of the standard normal cumulative distribution function; i.e. $z_{mt+1} = \Phi^{-1}\left(q_{mt+1}\right)$. If the VaR model is correctly specified, the z_{mt+1} series should be a series of independent and identically distributed draws from the standard normal. This hypothesis can be tested against various alternative specifications, such as

$$z_{mt+1} - \mu_m = \rho_m\left(z_{mt} - \mu_m\right) + \eta_{t+1}, \tag{30}$$

where the parameters (μ_m, ρ_m) are respectively the conditional mean and AR(1) coefficient corresponding to the z_{mt+1} series, and η_{t+1} is a normal random variable with mean zero and variance σ_m^2. Under the null hypothesis that both properties are present, $\left(\mu_m, \rho_m, \sigma_m^2\right) = (0, 0, 1)$. The appropriate LR statistic is

$$LR_{dist} = 2\left[L\left(\mu_m, \rho_m, \sigma_m^2\right) - L\left(0, 0, 1\right)\right], \tag{31}$$

where $L\left(\mu_m, \rho_m, \sigma_m^2\right)$ is the likelihood function for an AR(1) process with normally distributed errors including the likelihood for the initial observation. The LR_{dist} statistic is asymptotically distributed $\chi^2(3)$, but again finite-sample critical values are used here.

2.3.4 Exception Magnitudes

The evaluation of VaR models, both in practice and in the literature, has generally focused on the frequency of exceptions and thus has disregarded information on their magnitudes. However, the magnitudes of exceptions should be of primary interest to the various users of VaR models. To examine these magnitudes, we use the test of whether the magnitude of observed VaR exceptions are consistent with the underlying VaR model, as proposed by Berkowitz (2001). The intuition is that VaR exceptions are treated as continuous random variables, and non-exceptions are treated as censored random variables. In essence, this test provides a middle ground between the full distribution approach of the LR_{dist} test and the frequency approach of the LR_{uc} and LR_{cc} tests.

As with the LR_{dist} test, the empirical quantile series is transformed into standard normal z_{mt+1} series and then treated as censored normal random variables, where the censoring is tied to the desired coverage level of the VaR estimates. Thus, z_{mt+1} is transformed into γ_{mt+1} as follows:

$$\gamma_{mt+1} = \begin{cases} z_{mt+1} & \text{if } z_{mt+1} < \Phi^{-1}(\alpha) \\ 0 & \text{if } z_{mt+1} \geq \Phi^{-1}(\alpha) \end{cases}. \tag{32}$$

The conditional likelihood function for the right-censored observations of $\gamma_{mt+1} = 0$ (i.e., for non-exceptions) is

$$f\left(\gamma_{mt+1} | z_{mt+1} \geq \Phi^{-1}(\alpha)\right) = 1 - \Phi\left(\frac{\Phi^{-1}(\alpha) - \mu_m}{\sigma_m}\right), \tag{33}$$

where μ_m and σ_m are respectively the unconditional mean and standard deviation of the z_{mt+1} series.[22] The conditional likelihood function for $\gamma_{mt+1} = z_{mt+1}$ is that of a truncated normal distribution,

$$f\left(\gamma_{mt+1} | z_{mt+1} < \Phi^{-1}(\alpha)\right) = \frac{\phi\left(\gamma_{mt+1}\right)}{\Phi\left(\frac{\Phi^{-1}(\alpha)-\mu_m}{\sigma_m}\right)}. \tag{34}$$

If the VaR model generating the empirical quantiles is correct, the γ_{mt+1} series should be identically distributed, and (μ_m, σ_m) should equal $(0, 1)$. Thus, the relevant test statistic is

$$LR_{mag} = 2\left[L_{mag}(\mu_m, \sigma_m) - L_{mag}(0, 1)\right], \tag{35}$$

which is asymptotically distributed $\chi^2(2)$, although we use finite-sample critical values in our analysis.

2.3.5 Regulatory Loss Function

Under the 1996 Market Risk Amendment (MRA) to the Basel Capital Accord, regulatory capital for the trading positions of commercial banks is set according to the banks' own internal VaR estimates. Given its actual use by market participants, the regulatory loss function implied in the MRA is a natural way to evaluate the relative performance of VaR estimates within an economic framework; see Lopez (1999b) for further discussion.

As discussed earlier, our fixed-income portfolio return in US dollars is

$$\begin{aligned} R_{pt+1} &= [\ln(Y_{\mathrm{US\$}t+1}) - \ln(Y_{\mathrm{US\$}t})] + [\ln(Y_{\mathrm{DM}t+1}) - \ln(Y_{\mathrm{DM}t})] + [\ln(Y_{\mathrm{JY}t+1}) - \ln(Y_{\mathrm{JY}t})] \quad (36) \\ &= \ln(Y_{\mathrm{US\$}t+1} Y_{\mathrm{DM}t+1} Y_{\mathrm{JY}t+1}) - \ln(Y_{\mathrm{US\$}t} Y_{\mathrm{DM}t} Y_{\mathrm{JY}t}), \end{aligned}$$

and the portfolio value in US dollar terms is,

$$\begin{aligned} Y_{pt+1} &= Y_{pt} e^{R_{pt+1}} \quad (37) \\ &= Y_{\mathrm{US\$}t+1} Y_{\mathrm{DM}t+1} Y_{\mathrm{JY}t+1}. \end{aligned}$$

Replacing R_{pt+1} with the one-day-ahead VaR estimate, $\mathrm{VaR}_{mt}(\alpha)$, we generate what the portfolio value is expected to be at the lower α tail of the portfolio return distribution. Hence, the VaR estimate expressed in US dollar terms at time t for period $t+1$ is

$$\text{VaR\$}_{mt}(\alpha) = Y_{pt}\left[1 - e^{VaR_{mt}(\alpha)}\right]. \tag{38}$$

Note that the path dependence of the portfolio value and of the VaR$\$_{mt}$ estimates cannot be removed and must be accounted for in the calculations. In basic operational terms, we choose a starting portfolio value and simply track its value over the sample period to carry out the analysis.

The market-risk capital loss function expressed in the MRA is specified as

$$\text{MRC}_{mt} = \max\left[\text{VaR\$}_{mt}(10,1), \frac{S_{mt}}{60}\sum_{k=0}^{59}\text{VaR\$}_{mt-k}(10,1)\right] + SR_{mt}, \tag{39}$$

where VaR$\$_{mt}(10,1)$ is the 1% VaR estimate generated on day t for a ten-day holding period and expressed in dollar ters, S_{mt} is the MRA's multiplication factor (i.e., from 3 to 4 depending on the number of exceptions over the past 250 days) and SR_{mt} is a specific risk charge that is not associated with the VaR modeling (and not examined here). That is, MRC$_{mt}$ is the amount of regulatory capital a bank must hold with respect to its market risk exposure. The MRA capital loss function has several elements that reflect the bank regulators' concerns. First, their focus on the lower 1% quantile of the banks' market risk returns suggests a high degree of concern with extreme portfolio losses.

Second, the use of ten-day VaR estimates reflects the desire of the regulators to impose a ten-day (or two-week) period during which a bank's market exposure can be diminished. Instead of using actual ten-day VaR estimates, it is common for banks and regulators to rely on the simplifying assumption that one-day VaR estimates can adequately be scaled up using the square root of the time horizon or, in this case, $\sqrt{10}$; see Christoffersen, Diebold, and Shuermann (1998) for further discussion. Hence, the loss function used in our analysis is

$$\text{MRC}_{mt} = \sqrt{10}\max\left[\text{VaR\$}_{mt}(1), \frac{S_{mt}}{60}\sum_{k=0}^{59}\text{VaR\$}_{mt-k}(1)\right]. \tag{40}$$

Third, the max function is used to capture instances when the most recent VaR estimate (in dollar terms) exceeds its trailing sixty-day average. The objective here is to make sure that the regulatory capital requirement adjusts quickly to large changes in a bank's market risk exposure.

Finally, the number of exceptions is explicitly included in the capital requirement via the S_{mt} multiplier in order to assure that banks do not generate inappropriately low VaR estimates.

Under the current framework, $S_{mt} \geq 3$, and it is a step function that depends on the number of exceptions observed over the previous 250 trading days.[23] This initial value can be viewed as a regulatory preference parameter with respect to the degree of market risk exposure within the banking system. The possible number of exceptions is divided into three zones. Within the green zone of four or fewer exceptions, a VaR model is deemed "acceptably accurate" to the regulators, and S_{mt} remains at its minimum value of three. Within the yellow zone of five to nine exceptions, S_{mt} increases incrementally with the number of exceptions.[24] Within the red zone of ten or more exceptions, the VaR model is deemed to be "inaccurate" for regulatory purposes, and S_{mt} increases to its maximum value of four. The institution must also explicitly take steps to improve its risk management system. Thus, banks look to minimize exceptions (in order to minimize the multiplier) without reporting VaR estimates that are too large and raise the average term in the loss function.

3 VaR Evaluation Results

This section presents the forecast evaluation results of our 68 VaR models. Tables 4 and 5 report, respectively, mean VaR estimates and the percentage of exceptions observed for each of the models for the 1%, 5%, 10% and 25% quantiles over the entire out-of-sample period from January 1, 1990 to December 29, 2000. These summary statistics are key components of the VaR evaluation results reported in Tables 6-13. Both the tables and the discussion below are framed with respect to the distributional assumption first and then with respect to the relative performance of the 17 models' second-moment specifications. Note that 13 of the models specify the multivariate dynamics of the conditional variance-covariance matrix and that four models only specify the dynamics of the portfolio's conditional variance.

Table 4 show the mean VaR estimates for our alternative models. The most striking feature is that the mean VaR estimates depends mainly on the VaR distributional assumption, rather than on the covariance matrix forecast. The average 1% VaR estimates for the t- and generalized t-distributions lies outside the first percentile of the portfolio return distribution; see Table 1. In contrast, the average 1% VaR estimates under the normal distribution is greater than the first percentile of the portfolio return. The nonparametric mean VaR estimate at the 1% quantile is quite close to the first percentile of the portfolio return for most covariance forecasts.

The observed frequency of exceptions reported in Table 5 confirms that the VaR normal estimates at the 1% quantile are too low, with the observed frequency of exceptions above 1% for all covariance matrix forecasts. However, at higher quantiles, the normal distribution tend to overestimate the VaR estimates. While the t-distribution VaR estimates are too conservative at all quantiles, the generalized t-distribution VaR estimates do not provide a clear pattern. The nonparametric distribution assumption VaR estimates perform reasonable well at all quantiles, but they show a tendency to generate VaR estimates that are too conservative.

3.1 Unconditional and Conditional Coverage Tests

Tables 6 and 7 report results for the unconditional (LR_{uc}) and conditional coverage (LR_{cc}) tests, respectively. Note that the LR_{uc} and LR_{cc} test results are qualitatively similar. VaR models based on the standard normal distributional assumption perform relatively well at the 1% quantile, in that only a few sets of VaR estimates fail the LR_{uc} and LR_{cc} tests at the 5% significance level. However, almost all fail at the higher quantiles (5%, 10%, and 25%). It is interesting to find that the standard normal distribution generates VaR estimates that perform well at such a low quantile, especially since portfolio returns are commonly found to be distributed non-normally, in particular with fatter tails. This result appears to be in line with those of Lucas (2000), who find that the upward bias in the estimated dispersion measure under such models (see MPE in Panel A of Tables 2 and 3) appears to partially offset the neglected leptokurtosis.

VaR models based on the t-distribution assumption perform only modestly well for the 1% quantile, i.e., only about a third of the covariance matrix specifications do not reject the null hypotheses of the LR_{uc} and LR_{cc} tests. All specifications reject the null hypotheses at the higher quantiles. Similarly, the results for the VaR models based on the generalized t-distribution, indicate that most of the covariance matrix specifications fail at the lowest quantile, although they have some very limited success at the higher quantiles. This result suggests that the tail thickness of these distributional assumptions probably limits their use to the lowest VaR coverage levels.

The nonparametric distributional results coverage test results provide evidence that these VaR estimates do relatively well across all four quantiles. For the 1% quantile, all the specifications (except DCC models) do not reject the null hypotheses of correct unconditional and conditional coverage. While the performance for the 5% quantiles is relatively poor and similar to alternative distributional assumptions, the performance for the 10% and 25% quantiles is quite reasonable with non-rejections of the null hypotheses in about 50% and 60% of the cases, respectively.

Although we can conclude that the standard normal and the nonparametric distributional assumptions produce better VaR estimates, inference regarding the relative forecast accuracy of the 13 covariance matrix specifications and four portfolio variance specifications is limited. As shown in Tables 6 and 7, no strong conclusions can be drawn across the forecasts using these two distributional assumptions, especially at the 1% quantile. Among the multivariate models, the BEKK, GDC and VECH specifications perform better than the CCORR and DCC specifications. In addition, there is some evidence that the level effect is important for VaR model performance, especially for the BEKK and GDC specifications. However, the simple covariance forecasts seem to present similar performance to the best multivariate models, in particular the EWMA and E250 specifications. Furthermore, the performance of the univariate portfolio variance specifications suggests that one could reasonably simplify the generation of VaR estimates by ignoring the underlying covariance matrix without sacrificing the accuracy of the estimates' unconditional and conditional coverage.

In summary, these results indicate that the dominant factor in determining the relative accuracy of VaR estimates with respect to coverage is the distributional assumption, which is consistent with the Lopez and Walter (2001) findings. The specification of the portfolio variance, whether based on covariance matrix forecasts or univariate forecasts, appears to be of second-order importance. These results provide support for the common industry practice of using the simple portfolio variance specifications in generating VaR estimates. However, it does not completely explain why practitioners have generally settled on the standard normal distributional assumption. Although it did perform well for the lower coverage levels that are usually of interest, the nonparametric distribution did relatively better across the four quantiles examined.

3.2 Dynamic Quantile Test

Table 8 presents the results of the VaR estimates evaluation using the Dynamic Quantile test proposed by Engle and Manganelli (2002). The results are mainly consistent with the ones using the unconditional and conditional coverage tests. In fact, VaR models based on the standard normal and nonparametric distributional assumption perform relatively well at the 1% quantile. Under the normal distributional assumption, all but one of the specifications do not reject the null hypotheses of correct 1% VaR estimates, and the number of rejections under the nonparametric assumption is just two. Furthermore, the nonparametric distribution presents the best performance at higher quantiles. The VaR models based on the t-distribution and generalized t-distribution perform slightly better for the 1% VaR estimates than in the unconditional and conditional coverage tests, even though the performance is still clearly worse than for the alternative distributional assumptions. Once again, inference across model specifications is limited due to similar performance over the various quantiles and distributional assumptions.

3.3 Distribution Forecast Test

Table 9 contains the LR_{dist} test results for the 68 VaR models. These results again show that the standard normal and the nonparametric distributional assumptions generate VaR estimates that perform relatively well. The null hypothesis of correct conditional distributional form is not rejected in about half of these cases. For the entire distribution, the generalized-t distributional assumption performs equally well, suggesting that these models' forecasted quantiles closer to the median perform better than their tail quantiles. The t-distributional assumption again performs poorly.

These results provide further evidence that the distributional assumption appears to drive the VaR forecast evaluation results and that the portfolio variance specification is of secondary importance. Once again, inference on the relative performance of the model specifications is limited. The clearest result is that the CCORR and DCC specifications reject the null hypothesis in all cases, while the BEKK and VECH specifications perform best among the multivariate models. The roughly equivalent performance of the multivariate covariance matrix specifications and the simple univariate specifications confirms the prior result that one may simplify the VaR calculations without being penalized in terms of performance.

3.4 Exception Magnitudes

The previous tests do not take into account the magnitude of the exceptions, but only their frequency. Analysis of this forecast property should be of clear interest to users of VaR estimates that must determine actual capital requirements, whether economic or regulatory. In this section, we analyze model performance by considering the size of the loss when an exception occurs. Table 10 reports the mean exception; i.e., the average loss in excess of the VaR estimate conditional on the portfolio return being less than the VaR estimate. The mean exception beyond the VaR estimates is approximately one-half of the portfolio returns' standard deviation. As expected, the models based on the t-student distribution generate the lowest mean exceptions, but only at the 1% quantile. The models based on the normal distribution seem to have the largest mean exceptions at the 1% quantile, but at the 5% and higher quantiles, they have similar mean exceptions to the other distributional assumptions. The distributional assumption does not play as large a role with respect to exception magnitudes as it did for exception frequencies.

The results of the LR_{mag} tests for the four sets of VaR estimates using alternative distributional assumptions are reported in Table 11. Since this is a joint null hypothesis regarding the exception frequencies and magnitudes, we should expect it to be rejected for the 191 cases in which the VaR estimates rejected the binomial null hypothesis alone. This result occurs in 72% (138 of 191) of the cases with the bulk of the unexpected non-rejections of the joint null hypothesis (33 of 53) occurring for VaR estimates based on the generalized t-distribution. Overall, the two sets of test results are in agreement for 66% of the cases (180 of 272), and the joint null hypothesis is rejected after the binomial null hypothesis is not rejected in another 8% of cases (39 of 272). Thus, the LR_{mag} results are consistent with the LR_{uc} results in 74% of the cases, which seems to be a reasonable rate of cross-test correspondence.

The majority of the VaR estimates examined here reject the null hypothesis that the magnitude of their VaR exceptions are consistent with the model, suggesting that the models are misspecified. In 65% of the cases (177 of 272), the null hypothesis is rejected. The 95 cases in which the null hypothesis is not rejected are roughly uniformly distributed across the standard normal, generalized-t and nonparametric distributional assumptions. As before, the t-distributional assumption performs poorly, with the null hypothesis rejected in all of its cases. If we only consider the 45 (of 95) cases that pass both the LR_{uc} and the LR_{mag} tests, the nonparametric distribution is represented 62% (28 of 45) of the time.

Focusing on the different specifications of the portfolio variance forecasts, slightly more inference across specifications using the LR_{mag} test is possible than under prior tests. Across the three aforementioned distributional assumptions, four multivariate specification (BEKK-Levels, GDC-Levels, E250 and EWMA) and two univariate specifications (E250-Port and EWMA-Port) perform well across all four quantiles. Again, focusing on the 45 specifications that do not reject the null hypothesis of the LR_{uc} and the LR_{mag} tests, the EWMA-Port specification performs best under the normal and nonparametric distributional assumptions. However, the multivariate BEKK-Levels and GDC-Levels specifications as well as the univariate E250-Port specification also perform well, particularly under the nonparametric assumption.

In summary, distributional assumptions also play an important role in the results of the LR_{mag} test, as in the other hypothesis tests. However, these results in combination with the the LR_{uc} tests allow more inference across the portfolio variance specifications, and the results indicate that simple VaR estimates perform well. Specifically, the VaR estimates based on the EWMA-Port specification, which ignores the portfolio's components, performs well across the distributional assumptions and especially for the nonparametric distribution. This outcome presents further support and validation for the current use of these simple VaR estimates by financial institutions, if only because of the lower cost of generating these simple VaR estimates.

3.5 Regulatory Loss Function

As currently specified, the regulatory loss function is based on ten-day VaR estimates at the 1% quantile. However, since we are examining one-step-ahead portfolio variance forecasts, we evaluate one-day VaR estimates using this loss function as it is common practice in backtesting procedures. As this loss function does not correspond to a well-defined statistical test, the EWMA-Normal model is selected as the benchmark model against which we compare the others. Given the common use of this model in practice, we choose its capital charges to be the benchmark against which the other models' capital charges are compared.

Table 12 presents the percentage of the 2870 out-of-sample trading days for which the MRC_{mt+1} capital charges for the EWMA-Normal model are less than those of the other 67 VaR models. This model clearly has lower capital requirements than all of the portfolio variance specifications under the t-distribution and generalized-t distribution with the EWMA-Normal performing better than these alternatives more than 70% of the time. The only VaR estimates that generates smaller capital charges than those of the EWMA-Normal model for more than 40% of the trading days are those of the EWMA-Port model under the normal distribution (EWMA-Port-Normal). Recall that the EWMA-Port specification completely ignores the covariance matrix dynamics of the portfolios' component securities.

To more carefully examine these regulatory loss function results, we examine the differences between the capital charges for the EWMA-Normal model and the other models using the Diebold-Mariano test statistic. The null hypothesis that we investigate is whether the mean difference between the two sets of capital charges is equal to zero. If we do not reject the null hypothesis, then the alternative model performs equally well as the benchmark EWMA-Normal model. If we reject the null hypothesis and the mean difference is negative, then the EWMA-Normal model and its VaR estimates perform better because they generate lower capital charges on average. If we reject the null hypothesis and the mean difference is positive, then the alternative model and its VaR estimates perform better on average.

Table 12 presents the asymptotic p-values for the Diebold-Mariano statistics. In about 75% of the cases (50 of 67), we reject the null hypothesis. We reject all the alternative VaR models based on the t-distribution and generalized-t distributional assumption. For the nonparametric distribution, we do not reject the null hypothesis for only the BEKK-Levels specification. Under the normal distribution, for all but one portfolio variance specifications, we do not reject the null hypothesis, indicating that they perform as well as the EWMA-Normal model under this regulatory loss function.

The most noteworthy case rejecting the null hypothesis of equal performance is the EWMA-Port-Normal distribution with lower capital charges than the benchmark model for about 64% of the trading days. In fact, when the EWMA-Port-Normal model is treated as the benchmark as shown in Table 13, we reject the null hypothesis in all but 9 cases; all but one case under the normal distribution. This result implies that these VaR models perform as well as the EWMA-Port-Normal model under the regulatory loss function, but they generate higher capital charges in more than 50% of the trading days. However, the EWMA-Port-Normal model is much easier to estimate than these other models. Hence, we can clearly state that this simple practitioner model that ignores the covariance matrix dynamics of the portfolio's component securities and the fat-tailed nature of the portfolio returns is the best performing model to use when the objective is to minimize the MRA regulatory capital requirements.

4 Conclusion

We examine VaR estimates for an international interest rate portfolio using a variety of multivariate volatility models, ranging from naive averages to standard time-series models. The question of interest is whether the most complex covariance models provide improved out-of-sample forecasts for use in VaR analysis.

We find that covariance matrix forecasts generated from models that incorporate interest-rate level effects perform best under statistical loss functions, such as mean-squared error. Within a VaR framework, the relative performance of covariance matrix forecasts depends greatly on the VaR models' distributional assumptions. Of the forecasts examined, simple specifications, such as weighted averages of past observations, perform best with regard to the magnitude of VaR exceptions and regulatory capital requirements. Our results provide empirical support for the commonly-used VaR models based on simple covariance matrix forecasts and distributional assumptions, such as the standard normal. Moreover, we find that VaR estimates based on portfolio variance forecasts that ignore the covariance matrix dynamics of the portfolio's component securities and on distributional assumptions that ignore the fat-tailed nature of the portfolio returns perform best when the objective is to minimize certain regulatory capital requirements.

Overall, our results are consistent with several studies, such as Lopez and Walter (2001), Berkowitz and O'Brien (2002) and Brooks and Persand (2003), for foreign-exchange rate portfolios, US commercial bank trading portfolios, and a variety of financial market indexes, respectively. Also, note that our results closely parallel those of Beltratti and Morana (1999) and Lehar, Scheicher, and Schittenkopf (2002), who find that volatility forecasts from more complex models perform as well as those from computationally simpler GARCH models. Similarly, Christoffersen and Jacobs (2004) find that an objective function based on pricing stock options favors relatively parsimonious models. Hence, our results are in line with recent other findings that for economic purposes, as expressed via economic loss functions associated with option pricing and risk management, volatility forecasts from simpler models are found to outperform the forecasts from more complex models.

The reasons for these findings range from the issue of parameter uncertainty, as per Skintzi, Skiadopoulos, and Refenes (2004), to overfitting the data in-sample. Another potential reason for our findings is the sensitivity of the relevant economic loss functions (i.e., their first derivatives) with respect to the volatility forecast inputs. As suggested by Fleming et al. (2001), volatility forecasts that are smoother than optimal under standard statistical goodness-of-fit criteria are too variable to perform well under investment loss functions. Similarly, the greater smoothness of the VaR estimates generated by our EWMA-normal and EWMA-Port-normal models is of value within our VaR framework.

An interesting paper that addresses this question more directly with respect to VaR estimates is Lucas (2000), who finds that VaR models based on simple measures of portfolio variance and the normal distribution generate smaller discrepancies between actual and postulated VaR estimates than more sophisticated VaR models. He argues that this outcome is based on offsetting biases in the variance and VaR estimates of simple models that cannot be captured by more sophisticated models that attempt to capture the actual (but unknown) degree of leptokurtosis in the portfolio returns. The reasons for our parallel results are worth further study.

Figure 1. Time Series of Interest and Foreign Exchange Rates.

The figure plots the daily time series of interest rate and foreign exchange rate levels (black line) and interest rate first differences, foreign exchange rate geometric returns, and portfolio returns (grey line) for the sample period between January 12, 1979 and December 29, 2000.

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Notes

[1]Campa and Chang (1998) and Walter and Lopez (2000) find that implied correlations from foreign exchange options tend to be more accurate forecasts of realized correlation than the ones based on historical prices. However, implied correlations do not fully incorporate the information in historical data because bivariate GARCH models of correlation are able to improve forecast accuracy.

[2]In fact, Engle (2004) states that the best method for modeling covariance matrices ”has not yet been discovered.”

[3]Lopez (2001) finds in the foreign exchange markets that out-of-sample analysis of volatility forecasts is useful because the best set of forecasts varies not only among foreign exchange rates but also from in-sample to out-of-sample evaluation. Ederington and Guan (2000) also find evidence supporting that in-sample volatility results may not be confirmed out-of-sample.

[4]This sensitivity of results to the forecast horizon can also be found in West and Cho (1995) with the GARCH model having a slight advantage for short forecast horizons in foreign exchange markets, but this advantage disappears for longer horizons.

[5]The sensitivity of interest rate volatility to the interest rate level was first documented by Chan et al. (1992).

[6]Berkowitz and O’Brien (2002) report that the poor performance of banks trading portfolios during the 1998-2000 period is mainly due to interest rate positions.

[7]The sample averages of past observations are calculated using first differences for interest rates and continuously compounded returns for exchange rates. For simplicity, interest rate first differences are denoted by ε_{it} in equations (4) and (5).

[8]This window corresponds to the minimum amount of data allowed to formulate VaR forecasts for regulatory capital purposes.

[9]Note that the exponentially-weighted sample variance forecast is equivalent to a GARCH(1,1) model with $\beta_0 = 0$, $\beta_1 = w$ and $\beta_2 = 1 - w$; i.e., corresponds to an IGARCH model with zero

constant.

[10]The EWMA model with $w = 0.94$ was popularized for risk management applications by the Riskmetrics software. Foster and Nelson (1996) as well as Fleming et al. (2001) find similar values using nonparametric estimation techniques.

[11]See proposition 1 in Kroner and Ng (1998).

[12]The GDC model does not nest exactly the VECH model but a version of it with restrictions $\beta_{ij} = \beta_{ii}\beta_{jj}$ and $\alpha_{ij} = \alpha_{ii}\alpha_{jj}$.

[13]These are middle rates as reported in Datastream under the codes ECUSD3M, ECWGM3M and ECJAP3M, respectively. Note that the the only exception is the JY interest rate in the period from January 1995 to December 2000 in which we use the three-month offered rate, reported under code JPIBK3M.

[14]The data again is drawn from Datastream. For earlier years of the sample (1979-1983), the direct per US$ quotes are not available, but we construct the series from British pound (BP) exchange rates quoted by WM/Reuters with the following codes: USDOLLR (US$ per BP rate); DMARKER (DM per BP rate); and JAPAYEN (JY per BP rate). After 1983 we use the direct quotes per US$ with codes: BBDEMSP (DM per US$ rate); and BBJPYSP (JY per US$ rate)

[15]We also consider a fixed estimation scheme for the multivariate model parameters; i.e., we define an in-sample period and do not re-estimate the parameters as we move forward into the out-of-sample period. These results, available upon request, are similar to the ones using the rolling scheme, although indicating slight deterioration in some indicators of forecast accuracy.

[16]Estimated model parameters are not presented to conserve space, but are available upon request.

[17]We use bond returns implied by interbank money market rates (LIBOR) instead of bond returns. This is common to other studies such as Duffie and Singleton (1997) and presents several advantages. First, the interbank money market is more liquid than the Treasury bill market in most countries. Second, most interest rate derivatives are priced using interbank interest rates.

Third, credit risk minimally affects these rates as they are subject to special contractual netting features and correspond to trades among high-grade banks.

[18]The intuition for specifying a VaR model as a separate portfolio variance (whether based on a modeled covariance matrix or portfolio variance) and a distributional assumption arises from the two-step procedure proposed by Engle and Gonzalez-Rivera (1991). Note that an alternative approach is to estimate the parameters of a multivariate volatility model using a distributional form other than the multivariate normal. However, such distributional forms are difficult to specify and use in estimation.

[19]The finite sample distribution of the LR_{uc} statistic as well as the others in this study are of interest in actual practice; see Lopez (1999a) and Lopez (1999b). For example, with respect to size, the finite sample distribution of LR_{uc} for specified (α, T) values may be sufficiently different from a $\chi^2(1)$ distribution that the asymptotic critical values would be inappropriate. As for the power of this test, Kupiec (1995) shows how this test has a limited ability to distinguish among alternative hypotheses and thus has low power in the typical samples of size 250 used for risk management purposes. However, 2,870 out-of-sample observations are used in this exercise. The finite-sample critical values used for this analysis are available upon request.

[20]As discussed in Christoffersen (1998), several other forms of dependence, such as second-order Markov dependence, can be specified. For the purposes of this paper, only first-order Markov dependence is used.

[21]Crnkovic and Drachman (1996) suggest that these two properties be examined separately and thus propose two separate hypothesis tests. Diebold, Gunther, and Tay (1998) propose the use of CUSUM statistic to test for these properties simultaneously. Christoffersen and Pelletier (2004) propose duration-based tests.

[22]Note that this test does not examine the autocorrelation coefficient ρ_m discussed before, since the transformation z_{mt+1} into the censored random variable γ_{mt+1} disrupts the time sequence of the series. Thus, we only examine the two unconditional moments of the z_{mt+1} series.

[23]Note that the portfolio returns reported to the regulators, commonly referred to as the profit &

loss numbers, will usually not directly correspond to R_{pt+1}. The profit & loss numbers are usually polluted by the presence of customer fees and intraday trade results, which are not captured in standard VaR models.

[24]In the yellow zone, the multiplier values for five through nine exceptions are 3.4, 3.5, 3.75 and 3.85, respectively.

Table 1: **Summary Statistics of Interest and Foreign Exchange Rates**

	US$ Interest Rate	DM Interest Rate	JY Interest Rate	DM/US$ FX Rate	JY/US$ FX Rate	Portfolio Return
Panel A: $r_{it} - r_{it-1}$ **(interest rates) and log**(s_{it}/s_{it-1}) **(foreign exchange rates)**						
Mean	-0.001	0.000	0.000	0.002	-0.010	0.008
Standard Deviation	0.173	0.085	0.114	0.670	0.695	1.213
Skewness	-0.207	0.320	0.352	-0.261	-0.664	0.418
Kurtosis	28.85	18.53	48.91	2.770	5.055	3.109
Minimum	-2.375	-1.000	-1.813	-5.785	-5.835	-6.046
1-th Percentile	-0.031	-0.016	-0.004	-0.349	-0.334	-0.661
Median	0.000	0.000	0.000	0.000	0.000	-0.004
99-th Percentile	0.034	0.031	0.004	0.364	0.360	0.639
Maximum	2.500	1.125	1.625	3.037	3.385	10.725
$Q(10)$	489.6	31.73	65.16	13.17	18.63	13.48
	[0.000]	[0.000]	[0.000]	[0.214]	[0.045]	[0.198]
$Q(30)$	1146.4	88.58	217.6	50.37	48.62	39.09
	[0.000]	[0.000]	[0.000]	[0.011]	[0.017]	[0.124]
$Q(50)$	1789.1	125.1	332.0	74.81	63.97	56.91
	[0.000]	[0.000]	[0.000]	[0.013]	[0.089]	[0.233]
Panel B: $(r_{it} - r_{it-1})^2$ **(interest rates) and** $[\log(s_{it}/s_{it-1})]^2$ **(foreign exchange rates)**						
Mean	0.030	0.007	0.013	0.449	0.483	1.472
Standard Deviation	0.166	0.032	0.093	0.980	1.285	3.330
Skewness	19.79	20.24	19.23	9.893	10.79	10.70
Kurtosis	605.0	619.6	494.4	241.6	195.3	262.1
Minimum	0.000	0.000	0.000	0.000	0.000	0.000
1-th Percentile	0.000	0.000	0.000	0.024	0.020	0.078
Median	0.001	0.001	0.000	0.128	0.122	0.426
99-th Percentile	0.004	0.004	0.004	0.469	0.449	1.506
Maximum	6.250	1.266	3.285	33.47	34.05	14.16
$Q(10)$	1851.2	2417.5	2773.2	266.33	415.89	153.1
	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
$Q(30)$	4700.5	2889.9	4958.3	519.5	663.0	283.4
	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
$Q(50)$	6458.3	3221.2	6684.1	590.8	715.8	313.9
	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]

This table reports summary statistics on daily interest rate first differences $(r_{it} - r_{it-1})$ and foreign exchange rate geometric returns $[\varepsilon_t = \log(s_{it}/s_{it-1})]$. $Q(k)$ is the Ljung-Box test statistics of order k with p-values in brackets.

Table 2: **Out-of-Sample MPE and RMSPE for Variances Forecasts**

	Interest Rate Variance			FX Rate Variance		Portfolio
	US$	DM	JY	DM/US$	JY/US$	Variance ($\times 10^2$)
			Panel A: MPE			
BEKK	-0.00085	-0.00105	-0.00094	-0.01722	0.03208	-0.00010
BEKK-L	-0.00033	-0.00190	-0.00062	-0.00278	0.02607	0.00014
CCORR	-0.00090	-0.00088	-0.00085	-0.01650	0.04720	-0.00133
CCORR-L	-0.00053	-0.00115	-0.00033	-0.01515	0.03416	-0.00146
DCC	-0.00456	-0.00116	-0.00096	-0.00990	0.04387	0.00031
DCC-L	-0.00050	-0.00122	-0.00037	-0.00313	0.04164	0.00019
GDC	-0.00093	-0.00084	-0.00110	-0.01839	0.00546	-0.00055
GDC-L	-0.00035	-0.00090	-0.00061	0.00233	0.03094	0.00036
VECH	-0.00090	-0.00086	-0.00085	-0.02054	0.01000	-0.00079
VECH-L	-0.00058	-0.00108	-0.00035	-0.02017	0.00765	-0.00082
E250	-0.00019	-0.00024	-0.00019	0.00381	-0.00521	-0.00029
Eall	-0.03710	-0.00542	-0.01609	-0.02864	0.07717	-0.00071
EWMA	-0.00002	-0.00004	-0.00004	0.00209	0.00080	0.00006
Eall-Port						-0.00035
E250-Port						-0.00150
GARCH-Port						-0.00068
EWMA-Port						0.00001
			Panel B: RMSPE			
BEKK	0.01459**	0.01374	0.00994	0.85339	1.34896	0.02876
BEKK-L	**0.01453**	0.01418***	0.00991	**0.85291**	1.34596	**0.02871**
CCORR	0.01464	0.01372	0.00999	0.85337	1.34934	0.02878
CCORR-L	**0.01453**	**0.01371**	0.00988	0.85329	1.34351	0.02876
DCC	0.02845***	0.01386**	0.01006	0.85462	<u>**1.34321**</u>	0.02872
DCC-L	**0.01453**	0.01373	**0.00990**	0.85293	1.34339	0.02875
GDC	0.01461	0.01386	0.01014*	<u>**0.85284**</u>	1.34573	0.02872
GDC-L	<u>**0.01452**</u>	0.01378	0.01003	0.85411	1.34596*	0.02874
VECH	0.01463	**0.01371**	0.00998	0.85371	**1.34328**	<u>**0.02870**</u>
VECH-L	**0.01453**	<u>**0.01369**</u>	0.00987	0.85352	1.34373	<u>**0.02870**</u>
E250	0.01476*	0.01423	<u>**0.00980**</u>	0.86828***	1.37766***	0.02880***
Eall	0.04049***	0.01519***	0.01893***	0.87222***	1.38265**	0.02930**
EWMA	0.01461*	0.01395	0.00987	0.85694**	1.34830	0.02921
Eall-Port	-	-	-	-	-	0.02930***
E250-Port	-	-	-	-	-	0.02921***
GARCH-Port	-	-	-	-	-	**0.02870**
EWMA-Port	-	-	-	-	-	0.02881*

This table presents mean prediction error (MPE) and root mean squared prediction error (RMSPE) for the daily interest rate changes variance and foreign exchange rate geometric returns variance using alternative model specifications. Panel A reports the MPE. Panel B reports the RMSPE with the minimum value in each column is in bold and underlined, and the second smallest value is just in bold. *** denotes significance at the 1% level, ** at the 5% level and * at the 10% level of the Diebold-Mariano test statistic, which test the null hypothesis of equal loss functions values between the forecast and the minimizing forescat.

Table 3: **Out-of-Sample MPE and RMSPE for Covariances Forecasts**

	Interest Rate Covariances			Covariances between Interest Rates and FX Rates						FX Rates Covariances
	US$ DM	US$ JY	DM JY	US$ DM/US$	US$ JY/US$	DM DM/US$	DM JY/US$	JY DM/US$	JY JY/US$	DM/US$ JY/US$
Panel A: MPE										
BEKK	-0.00034	-0.00030	-0.00022	0.00062	0.00060	-0.00291	-0.00226	-0.00087	-0.00034	-0.01607
BEKK-L	-0.00022	-0.00027	-0.00036	0.00179	0.00094	-0.00316	-0.00250	-0.00096	-0.00104	-0.00850
CCORR	-0.00048	-0.00025	-0.00027	-0.00178	-0.00136	-0.00473	-0.00291	-0.00185	-0.00178	-0.08817
CCORR-L	-0.00046	-0.00020	-0.00025	-0.00141	-0.00111	-0.00513	-0.00320	-0.00156	-0.00143	-0.08875
DCC	-0.00093	-0.00008	-0.00005	-0.00571	-0.00121	-0.00111	0.00067	-0.00103	-0.00054	-0.00597
DCC-L	-0.00044	-0.00007	-0.00012	-0.00284	-0.00140	-0.00203	-0.00121	-0.00066	-0.00032	-0.01467
GDC	-0.00041	-0.00031	-0.00031	-0.00007	-0.00097	-0.00445	-0.00302	-0.00253	-0.00215	-0.02713
GDC-L	-0.00033	-0.00025	-0.00029	0.00139	0.00157	-0.00411	-0.00305	-0.00175	-0.00110	-0.00288
VECH	-0.00032	-0.00018	-0.00020	-0.00085	-0.00103	-0.00568	-0.00223	-0.00147	-0.00113	-0.03996
VECH-L	-0.00030	-0.00012	-0.00020	-0.00088	-0.00101	0.00024	-0.00215	-0.00156	-0.00117	-0.03875
E250	-0.00005	-0.00001	-0.00003	0.00006	0.00009	0.00007	-0.00025	-0.00061	-0.00051	-0.01629
Eall	-0.00389	-0.00185	-0.00143	-0.02836	-0.01853	-0.00891	-0.00712	-0.00408	-0.00702	-0.07848
EWMA	-0.00001	0.00000	0.00000	0.00000	0.00009	0.00010	0.00010	-0.00018	-0.00007	-0.00090
Panel B: RMSPE										
BEKK	0.00752*	0.00351***	0.00385***	0.05385***	0.04110***	0.05741	0.04444	0.03470***	0.03768***	**0.64989**
BEKK-L	**0.00749**	0.00347***	0.00390**	0.05329	0.04093***	0.05658	0.04456*	0.03477***	0.03771***	**<u>0.64968</u>**
CCORR	0.00751	0.00342**	0.00378**	**<u>0.05303</u>**	**<u>0.04055</u>**	0.05708	0.04435	0.03419	0.03738	0.65826***
CCORR-L	0.00750	0.00342**	0.00378**	**0.05307**	**0.04058**	0.05718	0.04438	0.03420	0.03739	0.65837***
DCC	0.00863***	**0.00340****	0.00378	0.05552***	**0.04058**	0.05665*	0.04515***	0.03431*	**<u>0.03735</u>**	0.65201
DCC-L	0.00757***	**<u>0.00339</u>**	**<u>0.00376</u>**	0.05355**	0.04068	0.05645	**0.04434**	0.03421	**0.03736**	0.65403***
GDC	**0.00749**	0.00344***	0.00381***	0.05310	0.04080*	0.05679	**<u>0.04430</u>**	0.03424	0.03741	0.65024
GDC-L	0.00750	0.00343***	0.00383***	0.05310	0.04096**	0.05694	0.04436	0.03424	0.03737	0.65087
VECH	**<u>0.00748</u>**	0.00340*	**0.00377**	0.05312	0.04068	0.05617	0.04440	**<u>0.03417</u>**	0.03738	0.65049
VECH-L	**0.00749**	**<u>0.00339</u>**	**<u>0.00376</u>**	0.05312	0.04061	0.05613	0.04439	**0.03418**	0.03739	0.65031
E250	0.00750	**<u>0.00339</u>**	**<u>0.00376</u>**	0.05356***	0.04080**	**0.05579**	**0.04434**	0.03428	0.03746*	0.65753***
Eall	0.00846***	0.00388***	0.00402***	0.06050***	0.04476***	0.05666	0.04488***	0.03439***	0.03802***	0.66157***
EWMA	0.00754*	0.00343**	0.00382***	0.05355*	0.04111**	**<u>0.05507</u>**	0.04500*	0.03474***	0.03788**	0.65214

This table presents mean prediction error (MPE) and root mean squared prediction error (RMSPE) for the daily interest rate changes and foreign exchange rate geometric returns covariances using alternative model specifications. Panel A reports the MPE. Panel B reports the RMSPE with the minimum value in each column is in bold and underlined, and the second smallest value is just in bold. *** denotes significance at the 1% level, ** at the 5% level and * at the 10% level of the Diebold-Mariano test statistic, which test the null hypothesis of equal loss functions values between the forecast and the minimizing forescat.

Table 4: **Mean Value-at-Risk**

	Normal distribution				t-distribution				Generalized t-distribution				Nonparametric distribution			
	1%	5%	10%	25%	1%	5%	10%	25%	1%	5%	10%	25%	1%	5%	10%	25%
BEKK	-2.708	-1.915	-1.492	-0.785	-3.164	-2.091	-1.587	-0.812	-3.264	-1.567	-1.086	-0.520	-2.957	-1.888	-1.388	-0.676
BEKK-L	-2.679	-1.894	-1.476	-0.777	-3.130	-2.068	-1.570	-0.803	-2.970	-1.431	-1.000	-0.490	-2.759	-1.848	-1.365	-0.667
CCORR	-1.992	-1.992	-1.552	-0.817	-3.347	-2.195	-1.662	-0.848	-4.309	-2.166	-1.500	-0.714	-3.176	-2.044	-1.491	-0.734
CCORR-L	-2.826	-1.998	-1.557	-0.819	-3.358	-2.202	-1.667	-0.850	-4.322	-2.172	-1.505	-0.716	-3.168	-2.049	-1.491	-0.735
DCC	-2.652	-1.875	-1.461	-0.769	-3.836	-2.297	-1.682	-0.829	-5.040	-2.729	-1.909	-0.997	-3.647	-2.339	-1.702	-0.841
DCC-L	-2.665	-1.884	-1.468	-0.773	-3.855	-2.308	-1.691	-0.833	-5.065	-2.743	-1.918	-1.002	-3.656	-2.348	-1.704	-0.842
GDC	-2.736	-1.935	-1.507	-0.793	-3.251	-2.132	-1.614	-0.823	-3.297	-1.583	-1.097	-0.526	-3.063	-1.956	-1.435	-0.708
GDC-L	-2.645	-1.870	-1.457	-0.767	-3.142	-2.061	-1.560	-0.796	-3.188	-1.530	-1.060	-0.508	-2.960	-1.914	-1.406	-0.687
VECH	-2.747	-1.942	-1.513	-0.796	-3.209	-2.120	-1.610	-0.823	-3.045	-1.468	-1.025	-0.502	-3.026	-1.943	-1.393	-0.685
VECH-L	-2.749	-1.944	-1.514	-0.797	-3.212	-2.122	-1.611	-0.824	-3.047	-1.469	-1.026	-0.503	-3.028	-1.944	-1.397	-0.686
E250	-2.718	-1.922	-1.497	-0.788	-3.296	-2.142	-1.616	-0.821	-4.157	-2.089	-1.447	-0.689	-2.905	-1.967	-1.430	-0.683
Eall	-2.793	-1.975	-1.538	-0.810	-3.477	-2.232	-1.677	-0.848	-4.663	-2.431	-1.681	-0.795	-3.141	-2.040	-1.488	-0.681
EWMA	-2.618	-1.851	-1.442	-0.759	-3.259	-2.092	-1.572	-0.795	-4.371	-2.279	-1.576	-0.746	-3.155	-1.950	-1.400	-0.626
E250-Port	-2.724	-1.926	-1.500	-0.790	-3.182	-2.103	-1.596	-0.816	-3.013	-1.452	-1.015	-0.497	-2.868	-1.911	-1.378	-0.653
Eall-Port	-2.869	-2.028	-1.580	-0.832	-3.408	-2.235	-1.692	-0.863	-3.457	-1.660	-1.150	-0.551	-3.131	-2.027	-1.493	-0.719
EWMA-Port	-2.622	-1.854	-1.444	-0.760	-3.115	-2.043	-1.546	-0.789	-3.159	-1.517	-1.051	-0.504	-3.030	-1.899	-1.388	-0.665
GARCH-Port	-2.757	-1.949	-1.519	-0.799	-3.344	-2.173	-1.639	-0.833	-3.323	-1.595	-1.105	-0.530	-2.956	-1.956	-1.418	-0.697

This table shows mean VaR estimates in percentage for the alternative VaR models under the 1%, 5%, 10% and 25% significance levels.

Table 5: **Observed Frequency of Exceptions**

	Normal distribution				t-distribution				Generalized t-distribution				Nonparametric distribution			
	1%	5%	10%	25%	1%	5%	10%	25%	1%	5%	10%	25%	1%	5%	10%	25%
BEKK	1.15	4.22	7.91	21.43	0.63	3.10	6.62	20.73	0.56	6.93	14.15	29.90	0.87	4.39	9.30	24.29
BEKK-L	1.32	4.32	8.19	21.78	0.77	3.21	6.86	21.15	0.80	8.50	16.10	31.05	1.12	4.70	9.48	25.02
CCORR	1.08	3.69	7.25	20.73	0.49	2.68	5.89	20.00	0.07	2.86	7.91	23.59	0.77	3.31	7.94	22.89
CCORR-L	1.05	3.69	7.21	20.70	0.49	2.65	5.85	19.90	0.07	2.82	7.84	23.48	0.77	3.34	7.87	22.79
DCC	1.29	4.46	8.26	22.02	0.28	2.44	5.68	20.59	0.03	1.15	4.15	16.03	0.31	2.26	5.51	20.38
DCC-L	1.39	4.43	8.15	22.02	0.24	2.44	5.61	20.52	0.03	1.18	4.11	15.85	0.31	2.26	5.47	20.35
GDC	1.29	4.22	7.80	21.43	0.63	2.89	6.72	20.59	0.52	6.93	14.15	29.86	0.80	3.97	8.61	23.83
GDC-L	1.32	4.70	8.36	22.09	0.66	3.38	7.28	21.32	0.59	7.56	14.98	30.59	0.91	4.29	9.13	24.32
VECH	1.11	4.18	7.70	21.25	0.63	3.17	6.72	20.73	0.73	8.19	15.64	30.73	0.73	4.18	9.09	24.53
VECH-L	1.11	4.18	7.67	21.25	0.63	3.17	6.69	20.63	0.73	8.22	15.68	30.70	0.73	4.18	9.02	24.53
E250	1.57	4.36	8.36	21.01	0.66	3.28	7.21	20.24	0.14	3.48	9.02	24.29	1.32	4.22	9.16	24.56
Eall	1.36	4.01	7.70	20.59	0.52	2.72	5.99	19.79	0.07	2.09	5.99	20.98	0.73	3.55	8.22	23.83
EWMA	1.71	5.02	8.85	22.86	0.63	3.52	7.49	21.32	0.10	2.75	7.49	23.55	0.73	4.32	9.34	26.69
E250-Port	1.53	4.36	8.26	20.98	0.84	3.38	7.39	20.35	1.11	8.99	16.10	30.45	1.36	4.36	9.79	25.54
Eall-Port	1.25	3.69	7.04	20.03	0.56	2.72	5.75	19.48	0.52	6.17	12.93	28.19	0.77	3.69	8.08	22.61
EWMA-Port	1.57	5.09	8.95	22.89	0.80	3.87	7.91	21.53	0.77	8.19	15.12	31.15	0.84	4.60	9.51	25.64
GARCH-Port	1.08	4.11	7.53	21.18	0.66	3.14	6.69	20.59	0.80	8.01	15.44	30.07	0.94	4.04	8.64	23.87

This table shows the observed frequency in percentage of exceptions for the alternative VaR models under 1%, 5%, 10% and 25% signficance levels. An exception is defined by the indicator variable I_{mt+1} for whether an exception occurrence is constructed as 1 if $R_{pt+1} < VaR_{mt}(\alpha)$, where R_{pt+1} is the portfolio return on day $t+1$ and $VaR_{mt}(\alpha)$ is the Value-at-Risk calculated on the previous business day with significance level α and using covariance specification m.

Table 6: **Unconditional Coverage Test Results**

	Normal distribution				t-distribution				Generalized t-distribution				Nonparametric distribution			
	1%	5%	10%	25%	1%	5%	10%	25%	1%	5%	10%	25%	1%	5%	10%	25%
BEKK	0.431	**0.048**	**0.000**	**0.000**	**0.031**	**0.000**	**0.000**	**0.000**	**0.009**	**0.000**	**0.000**	**0.000**	0.478	0.126	0.209	0.375
BEKK-L	0.096	0.088	**0.001**	**0.000**	0.190	**0.000**	**0.000**	**0.000**	0.268	**0.000**	**0.000**	**0.000**	0.543	0.462	0.347	0.983
CCORR	0.670	**0.001**	**0.000**	**0.000**	**0.002**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	0.079	0.190	**0.000**	**0.000**	**0.008**
CCORR-L	0.809	**0.001**	**0.000**	**0.000**	**0.002**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	0.059	0.190	**0.000**	**0.000**	**0.006**
DCC	0.136	0.177	**0.001**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**
DCC-L	**0.045**	0.150	**0.001**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**
GDC	0.136	**0.048**	**0.000**	**0.000**	**0.031**	**0.000**	**0.000**	**0.000**	**0.005**	**0.000**	**0.000**	**0.000**	0.268	**0.009**	**0.011**	0.147
GDC-L	0.096	0.462	**0.003**	**0.000**	0.053	**0.000**	**0.000**	**0.000**	**0.018**	**0.000**	**0.000**	**0.000**	0.607	0.072	0.115	0.399
VECH	0.543	**0.039**	**0.000**	**0.000**	**0.031**	**0.000**	**0.000**	**0.000**	0.129	**0.000**	**0.000**	**0.000**	0.129	**0.039**	0.101	0.560
VECH-L	0.543	**0.039**	**0.000**	**0.000**	**0.031**	**0.000**	**0.000**	**0.000**	0.129	**0.000**	**0.000**	**0.000**	0.129	**0.039**	0.077	0.560
E250	**0.005**	0.106	**0.003**	**0.000**	0.053	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	0.077	0.375	0.097	**0.048**	0.130	0.589
Eall	0.067	**0.012**	**0.000**	**0.000**	**0.005**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	0.129	**0.000**	**0.001**	0.147
EWMA	0.001	**0.966**	0.037	0.007	**0.031**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	0.072	0.129	0.088	0.232	**0.038**
E250-Port	**0.008**	0.106	**0.001**	**0.000**	0.364	**0.000**	**0.000**	**0.000**	0.543	**0.000**	**0.000**	**0.000**	0.067	0.106	0.708	0.505
Eall-Port	0.188	**0.001**	**0.000**	**0.000**	**0.009**	**0.000**	**0.000**	**0.000**	**0.005**	**0.006**	**0.000**	**0.000**	0.190	**0.001**	**0.000**	**0.003**
EWMA-Port	**0.005**	0.831	0.058	**0.008**	0.268	**0.004**	**0.000**	**0.000**	0.190	**0.000**	**0.000**	**0.000**	0.364	0.318	0.380	0.427
GARCH-Port	0.670	**0.024**	**0.000**	**0.000**	0.053	**0.000**	**0.000**	**0.000**	0.268	**0.000**	**0.000**	**0.000**	0.747	**0.015**	**0.013**	0.159

This table shows asymptotic p-values for the LR_{uc} statistics for the alternative VaR models under 1%, 5%, 10% and 25% significance levels. The LR_{uc} statistics are asymptotically distributed $\mathcal{X}^2(1)$. The cells in bold font indicate rejection of the null hypothesis of correct VaR estimates at the 5% significance level using bootstrap critical values.

Table 7: **Conditional Coverage Test Results**

	Normal distribution				t-distribution				Generalized t-distribution				Nonparametric distribution			
	1%	5%	10%	25%	1%	5%	10%	25%	1%	5%	10%	25%	1%	5%	10%	25%
BEKK	0.499	**0.018**	**0.000**	**0.000**	0.088	**0.000**	**0.000**	**0.000**	**0.031**	**0.000**	**0.000**	**0.000**	0.625	0.062	0.191	0.636
BEKK-L	0.150	0.125	**0.000**	**0.000**	0.358	**0.000**	**0.000**	**0.000**	0.451	**0.000**	**0.000**	**0.000**	0.579	0.615	0.147	0.912
CCORR	0.650	**0.001**	**0.000**	**0.000**	**0.009**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	0.212	0.358	**0.000**	**0.000**	**0.030**
CCORR-L	0.707	**0.001**	**0.000**	**0.000**	**0.009**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	0.171	0.358	**0.000**	**0.000**	**0.021**
DCC	0.202	0.093	**0.000**	**0.001**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**
DCC-L	**0.076**	0.076	**0.000**	**0.001**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**
GDC	0.202	**0.037**	**0.000**	**0.000**	0.088	**0.000**	**0.000**	**0.000**	**0.017**	**0.000**	**0.000**	**0.000**	0.451	**0.018**	**0.017**	0.347
GDC-L	0.206	0.739	**0.002**	**0.001**	0.135	**0.000**	**0.000**	**0.000**	**0.054**	**0.000**	**0.000**	**0.000**	0.691	0.151	0.121	0.616
VECH	0.579	0.081	**0.000**	**0.000**	0.088	**0.000**	**0.000**	**0.000**	0.272	**0.000**	**0.000**	**0.000**	0.272	0.081	0.053	0.805
VECH-L	0.579	0.081	**0.000**	**0.000**	0.088	**0.000**	**0.000**	**0.000**	0.272	**0.000**	**0.000**	**0.000**	0.272	0.081	**0.037**	0.805
E250	**0.008**	**0.010**	**0.000**	**0.000**	0.135	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.015**	0.578	0.206	**0.008**	**0.022**	0.735
Eall	0.109	**0.000**	**0.000**	**0.000**	**0.017**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	0.272	**0.000**	**0.001**	0.338
EWMA	**0.001**	0.384	0.053	**0.028**	0.088	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	0.201	0.272	0.125	0.279	0.110
E250-Port	**0.027**	**0.010**	**0.000**	**0.000**	0.542	**0.000**	**0.000**	**0.000**	0.556	**0.000**	**0.000**	**0.000**	0.157	**0.010**	0.065	0.784
Eall-Port	0.265	**0.000**	**0.000**	**0.000**	**0.031**	**0.000**	**0.000**	**0.000**	**0.017**	**0.004**	**0.000**	**0.000**	0.358	**0.000**	**0.000**	**0.012**
EWMA-Port	**0.017**	0.421	0.090	**0.032**	0.451	**0.007**	**0.000**	**0.000**	0.358	**0.000**	**0.000**	**0.000**	0.542	0.186	0.389	0.706
GARCH-Port	0.650	0.051	**0.000**	**0.000**	0.135	**0.000**	**0.000**	**0.000**	0.451	**0.000**	**0.000**	**0.000**	0.735	**0.030**	**0.007**	0.339

This table shows asymptotic p-values for the LR_{cc} statistics for the alternative VaR models under 1%, 5%, 10% and 25% significance levels. The LR_{cc} statistics are asymptotically distributed $\mathcal{X}^2(2)$. The cells in bold font indicate rejection of the null hypothesis of correct VaR estimates at the 5% significance level using bootstrap critical values.

Table 8: **Dynamic Quantile Test Results**

	Normal distribution				t-distribution				Generalized t-distribution				Nonparametric distribution			
	1%	5%	10%	25%	1%	5%	10%	25%	1%	5%	10%	25%	1%	5%	10%	25%
BEKK	0.807	**0.058**	**0.000**	**0.000**	0.177	**0.000**	**0.000**	**0.000**	0.032	**0.000**	**0.000**	**0.000**	0.910	0.108	0.077	0.466
BEKK-L	0.402	0.185	**0.000**	**0.003**	0.692	**0.000**	**0.000**	**0.000**	0.802	**0.000**	**0.000**	**0.000**	0.863	0.593	0.132	0.510
CCOR	0.967	**0.003**	**0.000**	**0.000**	**0.007**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.001**	0.235	0.822	**0.000**	**0.000**	0.063
CCOR-L	0.978	**0.003**	**0.000**	**0.000**	**0.007**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	0.208	0.822	**0.000**	**0.000**	**0.042**
DCC	0.752	0.146	**0.001**	**0.007**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**
DCC-L	0.629	0.103	**0.000**	**0.009**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**
GDC	0.771	0.099	**0.000**	**0.000**	0.278	**0.000**	**0.000**	**0.000**	0.027	**0.001**	**0.000**	**0.000**	0.892	**0.056**	**0.031**	0.354
GDC-L	0.725	0.940	**0.000**	**0.012**	0.436	**0.000**	**0.000**	**0.001**	0.154	**0.000**	**0.000**	**0.000**	0.958	0.474	0.115	0.517
VECH	0.960	0.104	**0.000**	**0.000**	0.283	**0.000**	**0.000**	**0.000**	0.718	**0.000**	**0.000**	**0.000**	0.718	0.104	0.193	0.409
VECH-L	0.960	0.175	**0.000**	**0.000**	0.283	**0.000**	**0.000**	**0.000**	0.718	**0.000**	**0.000**	**0.000**	0.718	**0.175**	**0.135**	0.410
E250	**0.017**	**0.000**	**0.000**	**0.000**	0.145	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.004**	0.438	**0.000**	**0.000**	**0.014**
Eall	0.213	**0.000**	**0.000**	**0.000**	0.042	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	0.383	**0.000**	**0.000**	0.333
EWMA	0.043	0.104	**0.018**	**0.021**	0.173	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	0.132	0.516	**0.013**	**0.035**	**0.024**
E250-Port	0.035	**0.000**	**0.000**	**0.000**	0.832	**0.000**	**0.000**	**0.000**	0.356	**0.000**	**0.000**	**0.000**	0.367	**0.000**	**0.001**	**0.042**
Eall-Port	0.140	**0.000**	**0.000**	**0.000**	0.066	**0.000**	**0.000**	**0.000**	0.027	**0.000**	**0.000**	**0.001**	0.486	**0.000**	**0.000**	0.059
EWMA-Port	0.053	0.120	**0.023**	**0.023**	0.597	**0.000**	**0.000**	**0.000**	0.528	**0.000**	**0.000**	**0.000**	0.772	**0.019**	**0.044**	0.135
GARCH-Port	0.972	**0.055**	**0.000**	**0.000**	0.386	**0.000**	**0.000**	**0.000**	0.858	**0.000**	**0.000**	**0.000**	0.975	**0.054**	**0.026**	0.368

This table shows asymptotic p-values for the *DQ* statistic for the alternative VaR models under 1%, 5%, 10% and 25% signficance levels. The *DQ* statistics are asymptotically distributed $\mathcal{X}^2(7)$. The cells in bold font indicate rejection of the null hypothesis of correct VaR estimates at the 5% significance level using bootstrap critical values.

Table 9: **Distribution Forecast Test Results**

	Normal distribution	t-distribution	Generalized t-distribution	Nonparametric distribution
BEKK	0.051	**0.000**	0.606	0.168
BEKK-L	0.194	**0.000**	0.673	0.761
CCORR	**0.000**	**0.000**	**0.000**	**0.000**
CCORR-L	**0.000**	**0.000**	**0.000**	**0.000**
DCC	**0.000**	**0.000**	**0.000**	**0.000**
DCC-L	**0.000**	**0.000**	**0.000**	**0.000**
GDC	**0.012**	**0.000**	**0.000**	**0.001**
GDC-L	0.604	**0.000**	0.812	0.203
VECH	**0.007**	**0.000**	0.830	0.083
VECH-L	**0.006**	**0.000**	0.830	0.137
E250	**0.001**	**0.000**	**0.000**	**0.002**
Eall	0.678	**0.000**	0.142	0.812
EWMA	0.297	**0.000**	0.231	0.805
E250-Port	0.622	**0.000**	0.570	0.847
Eall-Port	**0.000**	**0.000**	**0.002**	**0.000**
EWMA-Port	0.317	**0.000**	0.188	0.810
GARCH-Port	**0.001**	**0.000**	0.183	**0.040**

This table shows asymptotic p-value for the LR_{dist} statistics for the alternative VaR models. The LR_{dist} statistic is asymptotically distributed $\mathcal{X}^2(3)$. The cells in bold font indicate rejection of the null hypothesis of correct VaR estimates at the 5% significance level using bootstrap critical values.

Table 10: **Mean Exception**

	Normal distribution				t-distribution				Generalized t-distribution				Nonparametric distribution			
	1%	5%	10%	25%	1%	5%	10%	25%	1%	5%	10%	25%	1%	5%	10%	25%
BEKK	-0.639	-0.640	-0.652	-0.697	-0.519	-0.660	-0.673	-0.693	-0.474	-0.663	-0.677	-0.725	-0.555	-0.642	-0.651	-0.719
BEKK-L	-0.580	-0.642	-0.647	-0.694	-0.447	-0.657	-0.668	-0.688	-0.584	-0.667	-0.675	-0.728	-0.599	-0.634	-0.662	-0.706
CCORR	-0.632	-0.663	-0.652	-0.688	-0.527	-0.670	-0.681	-0.682	-0.662	-0.657	-0.648	-0.701	-0.461	-0.684	-0.654	-0.703
CCORR-L	-0.652	-0.657	-0.651	-0.687	-0.527	-0.670	-0.680	-0.683	-0.660	-0.657	-0.650	-0.702	-0.477	-0.671	-0.660	-0.704
DCC	-0.661	-0.650	-0.656	-0.695	-0.411	-0.617	-0.680	-0.681	-0.074	-0.661	-0.664	-0.679	-0.530	-0.621	-0.681	-0.675
DCC-L	-0.612	-0.658	-0.663	-0.692	-0.475	-0.622	-0.690	-0.680	-0.072	-0.638	-0.673	-0.684	-0.542	-0.629	-0.694	-0.677
GDC	-0.591	-0.623	-0.650	-0.688	-0.473	-0.666	-0.641	-0.685	-0.519	-0.652	-0.665	-0.721	-0.534	-0.639	-0.658	-0.699
GDC-L	-0.632	-0.622	-0.659	-0.696	-0.527	-0.639	-0.647	-0.692	-0.541	-0.653	-0.666	-0.722	-0.539	-0.636	-0.652	-0.709
VECH	-0.613	-0.618	-0.650	-0.690	-0.494	-0.605	-0.643	-0.680	-0.573	-0.655	-0.667	-0.723	-0.592	-0.617	-0.660	-0.702
VECH-L	-0.609	-0.616	-0.652	-0.689	-0.493	-0.603	-0.645	-0.683	-0.571	-0.651	-0.665	-0.723	-0.590	-0.616	-0.661	-0.701
E250	-0.650	-0.700	-0.675	-0.714	-0.569	-0.718	-0.713	-0.703	-0.761	-0.707	-0.708	-0.715	-0.684	-0.719	-0.681	-0.736
Eall	-0.614	-0.698	-0.668	-0.724	-0.531	-0.681	-0.651	-0.719	-0.593	-0.689	-0.666	-0.717	-0.533	-0.676	-0.672	-0.714
EWMA	-0.510	-0.601	-0.638	-0.677	-0.498	-0.587	-0.618	-0.689	-0.366	-0.552	-0.614	-0.670	-0.514	-0.598	-0.643	-0.701
E250-Port	-0.622	-0.694	-0.674	-0.724	-0.513	-0.697	-0.656	-0.719	-0.522	-0.663	-0.690	-0.742	-0.556	-0.708	-0.678	-0.716
Eall-Port	-0.621	-0.703	-0.695	-0.711	-0.593	-0.713	-0.727	-0.700	-0.582	-0.709	-0.702	-0.743	-0.659	-0.704	-0.687	-0.737
EWMA-Port	-0.556	-0.590	-0.630	-0.676	-0.507	-0.578	-0.610	-0.689	-0.488	-0.617	-0.667	-0.713	-0.558	-0.608	-0.645	-0.691
GARCH-Port	-0.662	-0.629	-0.665	-0.692	-0.512	-0.636	-0.655	-0.687	-0.533	-0.669	-0.676	-0.738	-0.549	-0.633	-0.673	-0.711

This table shows mean exception in excess of the VaR conditional on the portfolio return being lower than the VaR estimate in each day. Mean exception is shown in percentage for the alternative VaR models under the 1%, 5%, 10% and 25% significance levels.

Table 11: **Normal Transform Test Results**

	Normal distribution				t-distribution				Generalized t-distribution				Nonparametric distribution			
	1%	5%	10%	25%	1%	5%	10%	25%	1%	5%	10%	25%	1%	5%	10%	25%
BEKK	**0.021**	**0.026**	**0.028**	**0.031**	**0.000**	**0.000**	**0.000**	**0.000**	0.356	0.386	0.399	0.414	0.066	0.079	0.084	0.091
BEKK-L	0.113	0.131	0.139	0.149	**0.000**	**0.000**	**0.000**	**0.000**	0.597	0.552	0.534	0.512	0.585	0.625	0.642	0.663
CCORR	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**
CCORR-L	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**
DCC	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**
DCC-L	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**
GDC	**0.004**	**0.006**	**0.006**	**0.007**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**
GDC-L	0.672	0.707	0.721	0.739	**0.000**	**0.000**	**0.000**	**0.000**	0.765	0.737	0.726	0.711	0.095	0.111	0.118	0.128
VECH	**0.003**	**0.004**	**0.005**	**0.005**	**0.000**	**0.000**	**0.000**	**0.000**	0.815	0.811	0.808	0.804	**0.034**	**0.041**	**0.044**	**0.049**
VECH-L	**0.003**	**0.004**	**0.004**	**0.005**	**0.000**	**0.000**	**0.000**	**0.000**	0.808	0.806	0.804	0.801	0.064	0.076	0.081	0.088
E250	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.001**	**0.001**
Eall	0.492	0.530	0.547	0.567	**0.000**	**0.000**	**0.000**	**0.000**	**0.046**	0.055	0.059	0.064	0.730	0.691	0.676	0.659
EWMA	0.563	0.515	0.496	0.473	**0.000**	**0.000**	**0.000**	**0.000**	0.116	0.132	0.138	0.147	1.000	1.000	1.000	1.000
E250-Port	0.421	0.458	0.473	0.493	**0.000**	**0.000**	**0.000**	**0.000**	0.462	0.420	0.403	0.383	0.770	0.734	0.720	0.703
Eall-Port	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.001**	**0.000**	**0.000**	**0.000**	**0.000**
EWMA-Port	0.591	0.541	0.522	0.499	**0.000**	**0.000**	**0.000**	**0.000**	0.170	0.148	0.139	0.130	1.000	1.000	1.000	1.000
GARCH-Port	**0.000**	**0.001**	**0.001**	**0.001**	**0.000**	**0.000**	**0.000**	**0.000**	0.075	0.087	0.092	0.099	**0.014**	**0.018**	**0.019**	**0.021**

This table shows the p-value for the LR_{mag} test statistic for the alternative VaR models under 1%, 5%, 10% and 25% significance levels. The LR_{mag} statistic is asymptotically distributed $\mathcal{X}^2(2)$. The cells in bold font indicate rejection of the null hypothesis of correct VaR estimates at the 5% significance level using bootstrap critical values.

Table 12: **Capital Charge Relative to Benchmark Model - EWMA-Normal**

	Normal dist.		t-distribution		Generalized t-distrib		Nonparametric distrib.	
	% trading	p-value	% trading	p-value	% trading	p-value	% trading	p-value
BEKK	56.16	0.629	**82.755**	**0.000**	**85.921**	**0.000**	**73.789**	**0.001**
BEKK-L	54.56	0.696	**81.267**	**0.000**	**73.560**	**0.001**	55.284	0.485
CCORR	62.76	0.173	**90.271**	**0.000**	**100.000**	**0.000**	**83.823**	**0.000**
CCORR-L	63.11	0.138	**91.339**	**0.000**	**100.000**	**0.000**	**84.815**	**0.000**
DCC	45.29	0.867	**98.207**	**0.000**	**100.000**	**0.000**	**95.574**	**0.000**
DCC-L	52.38	0.786	**98.207**	**0.000**	**100.000**	**0.000**	**95.460**	**0.000**
GDC	56.92	0.474	**86.532**	**0.000**	**88.630**	**0.000**	**81.992**	**0.000**
GDC-L	46.62	0.861	**83.022**	**0.000**	**85.158**	**0.000**	**75.239**	**0.000**
VECH	60.97	0.462	**89.470**	**0.000**	**78.634**	**0.000**	**76.612**	**0.000**
VECH-L	62.04	0.436	**89.470**	**0.000**	**79.512**	**0.000**	**77.146**	**0.000**
E250	65.01	0.211	**84.739**	**0.000**	**97.596**	**0.000**	**71.957**	**0.010**
Eall	58.64	0.256	**79.550**	**0.000**	**97.482**	**0.000**	**67.646**	**0.028**
EWMA	-	-	**99.351**	**0.000**	**100.000**	**0.000**	**92.560**	**0.000**
E250-Port	57.76	0.240	**77.451**	**0.000**	**70.889**	**0.007**	65.815	0.050
Eall-Port	68.45	0.123	**83.937**	**0.000**	**84.586**	**0.000**	**71.919**	**0.012**
EWMA-Port	**36.28**	**0.996**	**91.835**	**0.000**	**93.018**	**0.000**	**82.068**	**0.000**
GARCH-Port	59.98	0.415	**87.486**	**0.000**	**78.596**	**0.000**	**71.995**	**0.001**

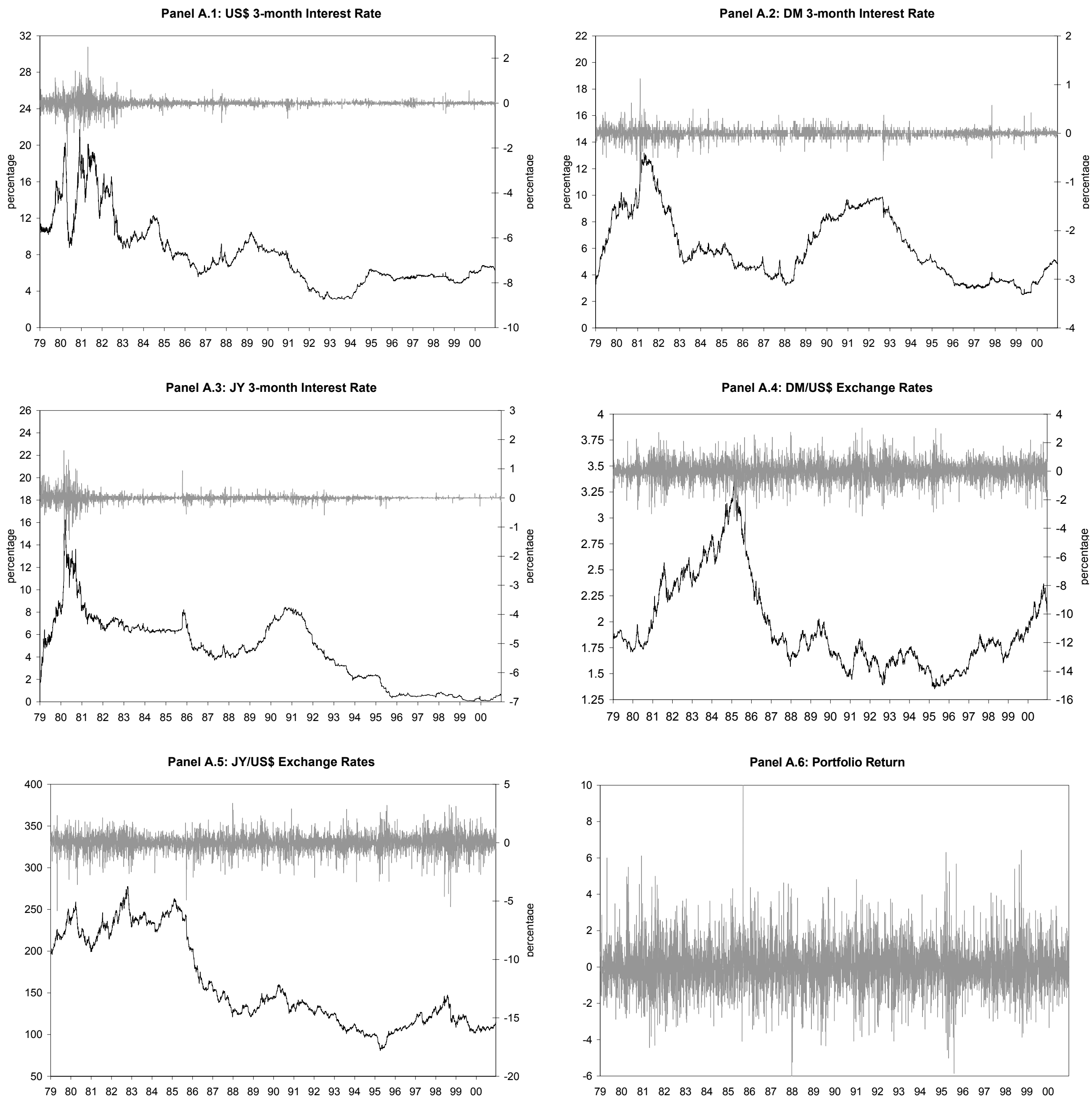
This table shows the percentage of trading days for which the benchmark VaR model (EWMA-Normal) has lower capital charge than the one given by the each model. The cells in bold font indicate rejection of the null hypothesis of equal capital charges at the 5% significance level using the Diebold-Mariano test statistic.

Table 13: **Capital Charge Relative to Benchmark Model - EWMA Portfolio-Normal**

	Normal dist.		t-distribution		Generalized t-distrib		Nonparametric distrib.	
	% trading	p-value	% trading	p-value	% trading	p-value	% trading	p-value
BEKK	61.50	0.168	87.98	**0.000**	90.767	**0.000**	80.16	**0.000**
BEKK-L	61.73	0.214	88.29	**0.000**	82.793	**0.000**	62.11	0.060
CCORR	68.75	**0.004**	93.67	**0.000**	100.000	**0.000**	92.75	**0.000**
CCORR-L	69.17	**0.002**	94.85	**0.000**	100.000	**0.000**	93.21	**0.000**
DCC	52.84	0.393	100.00	**0.000**	100.000	**0.000**	98.70	**0.000**
DCC-L	58.99	0.260	100.00	**0.000**	100.000	**0.000**	98.66	**0.000**
GDC	63.68	**0.022**	93.21	**0.000**	93.934	**0.000**	87.22	**0.000**
GDC-L	51.28	0.375	88.33	**0.000**	90.233	**0.000**	83.33	**0.000**
VECH	70.09	**0.006**	96.57	**0.000**	88.859	**0.000**	87.18	**0.000**
VECH-L	70.43	**0.004**	96.57	**0.000**	89.584	**0.000**	87.37	**0.000**
E250	68.49	0.054	88.36	**0.000**	99.733	**0.000**	75.43	**0.001**
Eall	63.41	0.074	82.49	**0.000**	98.550	**0.000**	71.65	**0.003**
EWMA	63.72	**0.004**	100.00	**0.000**	100.000	**0.000**	99.85	**0.000**
E250-Port	62.99	0.066	80.69	**0.000**	74.208	**0.001**	70.16	**0.007**
Eall-Port	71.69	**0.025**	86.88	**0.000**	88.020	**0.000**	75.39	**0.001**
EWMA-Port	-	-	99.85	**0.000**	99.847	**0.000**	93.82	**0.000**
GARCH-Port	64.78	**0.036**	91.64	**0.000**	84.739	**0.000**	80.77	**0.000**

This table shows the percentage of trading days for which the benchmark VaR model (EWMA–PortfolioNormal) has lower capital charge than the one given by the each model. The cells in bold font indicate rejection of the null hypothesis of equal capital charges at the 5% significance level using the Diebold-Mariano test statistic.

Figure 1: **Time Series of Interest and Foreign Exchange Rates**



The figure plots the daily time series of interest rate and foreign exchange rate levels (black line) and interest rate first differences, foreign exchange rate geometric returns, and portfolio returns (grey line) for the sample period between January 12, 1979 and December 29, 2000.