Basel II Technical Issues: A Comment

Sanjiv Das
Santa Clara University

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"... the relatively crude method of assigning risk weights to assets, as well as an emphasis on balance-sheet risks as opposed to other risks facing financial firms, limits the overall responsiveness of capital requirements to risk under Basel I, which renders that system increasingly inadequate for supervising the largest and most complex banking organizations. For these organizations, we need to move beyond Basel I to a more risk-sensitive and more comprehensive framework for assessing capital adequacy. Basel II represents the concerted efforts of the supervisory community, in consultation with banks and other stakeholders, to develop such a framework."

Comments of Chairman Ben Bernanke on May 18, 2006 at the Fed in Chicago, Conference on Bank Structure and Competition

Basel II has 3 pillars:

(a) Capital adequacy,

(b) Regulatory review,

(c) Market discipline.

Posited Benefits of the IRB approach

(a) a reduction in the amount of capital being held,

(b) more dynamic and realistic capital adequacy computation,

(c) risk-based pricing of products,

(d) a means to instill best practices,

(e) the introduction of much needed analytical methods,

(f) reduction in expected future charge-offs,

(g) reduction in operating expenses,

(h) reduction in operating losses,

(i) better capital allocation amongst business units within a financial institution,

(j) improved corporate governance, and

(k) overall lower systemic risk in the financial system.

Stated Drawbacks of the IRB approach

(a) high cost of implementation,

(b) competitive disadvantages between banks that are not required to comply and those that are required to,

(c) competitive imbalances across countries as different national supervisors impose varied levels of compliance,

(d) strong opposition to operational risk charges as being a deadweight cost for imposing governance that is already legally mandated,

(e) inability to obtain consistent implementation across all institutions, resulting in more noise than accurate determination of risk,

(f) the propensity to increase systemic risk if the rules impose distortionary portfolio changes in one same direction across all financial institutions.

The IRB approach allows more autonomy, which may be exploited by institutions, but these distortions may be mitigated: (a) with more oversight and, (b) the fact that the IRB approach recognizes that banks have already been using risk-based capital for almost two decades now, and (c) that this new approach is much more consistent with internal risk management.

Definitions

Expected Loss (EL)

$$EL(T-t) = E[P(T) - P(t) | P(T) - P(t) < 0]$$

Unexpected Loss (UL)

VaR at a level of α (say 1%), is defined as the tail cut off $[P_\alpha(T) - P(t)]$ for which losses in excess of this value will occur with α probability. We write this loss value as $\mathrm{VaR}(\alpha, T-t)$. Unexpected losses are then defined as:

$$UL(T-t) = \mathrm{VaR}(\alpha, T-t) - EL(T-t)$$

Extreme Losses

Losses in excess of $\mathrm{VaR}(\alpha, T-t)$ are denoted as *extreme losses*

Expected loss attracts *regulatory* capital and unexpected loss attracts *economic* capital. The latter is more sensitive to the shape of the loss distribution, and correlation assumptions across assets and counterparties.

VaR - deficiencies

(a) It is not a "coherent" risk measure, in that it fails the "sub-additivity" criterion.

(b) VaR is very hard to measure because it depends wholly on the tail of the loss distribution. At tail cut offs of 99.99%, it is hard to be confident of its value. This is popularly known as the "Star-Trek" problem, i.e. how do we estimate something in a range where we have never gone before.

(c) VaR is known to depend on the number of samples generated in Monte Carlo simulation (see the study cited by Chorafas 2004, page xxii), in that it increases as we raise the number of samples.

[Jeffery and Chen (2006) - one day VaR at 99% across major FIs is approximately $52 million.]

Credit Losses

$$\mathrm{EL} = \mathrm{PD} \times \mathrm{LGD} \times \mathrm{EAD} \times f(\mathrm{M})$$

Prob of default

Loss given default = 1 - Recovery

Exposure at default

Maturity

If each element comes from a distribution, there are issues of Jensen's inequality.

Foundation IRB (F-IRB) vs Advanced IRB (A-IRB): In the former, LGD is mandated by regulator.

Granularity and Aggregation

$n = 2^{10} = 1024$ assets (normalized assets)

$$\text{Portfolio } P : w = 1/n, \text{mean} = 0, \text{variance} = \sigma^2 = w'\Sigma w$$

$$\text{EL} = \int_{-\infty}^{0} P \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-P^2/2)\, dP$$

Granular portfolios (var): $\rho(y' \; \Omega \; y)$

Table 1: Expected loss, unexpected loss and Value-at-Risk for varying levels of granularity and aggregation. The first column shows the number of business units, and the second one the number of assets within each unit. Each asset has a standard normal distribution. "Corr" is the average pairwise correlation between portfolio values.

# portfolios	# within portfolio	Corr	EL	UL	VaR
1024	1	0.5000	0.2822	1.3635	1.6458
512	2	0.3750	0.2445	1.1814	1.4260
256	4	0.3125	0.2235	1.0796	1.3030
128	8	0.2812	0.2124	1.0261	1.2385
64	16	0.2656	0.2072	1.0011	1.2083
32	32	0.2578	0.2057	0.9938	1.1995
16	64	0.2539	0.2072	1.0011	1.2083
8	128	0.2520	0.2124	1.0261	1.2385
4	256	0.2510	0.2235	1.0796	1.3030
2	512	0.2505	0.2445	1.1814	1.4260
1	1024	0.5000	0.2822	1.3635	1.6458

As the assets get clubbed into portfolios, within unit diversification needs to be offset by higher correlations across groups; *correlation must be a function of granularity* (not fixed).

Correlation Sensitivity of Credit Portfolios

n assets each have an underlying value process as follows:

$$x_i = \sqrt{\rho}\, z + \sqrt{1-\rho}\, e_i, \quad z, e_i \sim N(0,1), \quad \forall i.$$

$$\begin{aligned} PD|z &= Prob[N(x_i) < PD|z] \\ &= Prob[x_i < N^{-1}(PD)|z] \\ &= Prob[\sqrt{\rho}\, z + \sqrt{1-\rho}\, e_i < N^{-1}(PD)|z] \\ &= Prob[e_i < \frac{N^{-1}(PD) - \sqrt{\rho}\, z}{\sqrt{1-\rho}}|z] \\ &= N\left[\frac{N^{-1}(PD) - \sqrt{\rho}\, z}{\sqrt{1-\rho}}|z\right] \\ &\equiv q_z \end{aligned}$$

The probability that there are m losses from n firms, conditional on z is denoted $p_z(m)$, given by the binomial formula

$$p_z(m) = \binom{n}{m} q_z^m (1-q_z)^{n-m}, \quad m = 0 \ldots n.$$

Noting that $z \sim N(0,1)$ we can integrate it out to get the full loss distribution, with the probability of m losses:

$$p(m) = \int_{-\infty}^{\infty} p_z(m)\, \phi(z)\, dz, \quad m = 0 \ldots n.$$

Table 2: Risk measures for varying default correlation. The PD for each firm is 5% and the number of identical firms is 100. The expected loss should be exactly 5.00 for all correlation levels, and the tiny discrepancy comes from numerical rounding error. The last column contains the adjustment term from the formula on page 405 of the draft NPR, i.e. $N\left[\frac{N^{-1}(PD)-\sqrt{\rho}\,N^{-1}(0.999)}{\sqrt{1-\rho}}\right]$. We can see how it varies with correlation.

Corr	EL	UL	CVar	Kadj
0.00	5.0000	5.2046	10.2046	0.0500
0.10	4.9991	13.1910	18.1902	0.2408
0.20	4.9984	20.7485	25.7469	0.3844
0.21	4.9984	21.5080	26.5064	0.3985
0.22	4.9983	22.2602	27.2585	0.4124
0.23	4.9982	23.0061	28.0044	0.4264
0.24	4.9982	23.7793	28.7775	0.4403
0.25	4.9981	24.5474	29.5455	0.4542
0.26	4.9981	25.3110	30.3090	0.4680
0.27	4.9980	26.0713	31.0693	0.4817
0.28	4.9980	26.8482	31.8461	0.4955
0.29	4.9979	27.6300	32.6279	0.5091
0.30	4.9979	28.4099	33.4078	0.5227
0.40	4.9975	36.4720	41.4695	0.6553
0.50	4.9973	45.1927	50.1900	0.7776
0.60	4.9972	54.8697	59.8670	0.8818

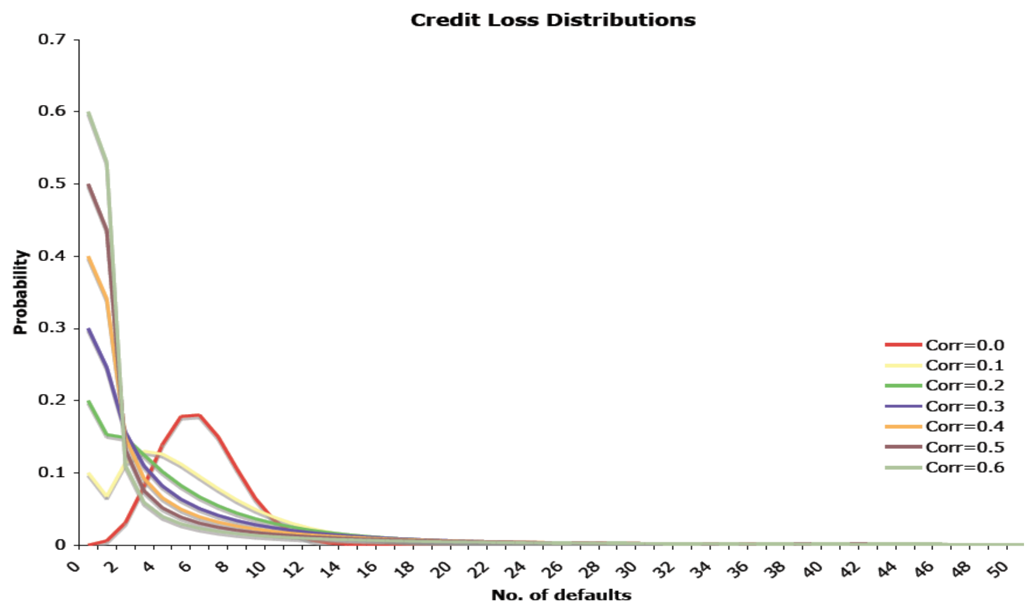


Figure 2: Credit Loss distributions under varied default correlation levels. We only present the loss levels out to 50 on the x-axis, even though the maximum number of defaults is 100, as the probabilities become very low thereafter. Note the distribution is mostly symmetric under the zero correlation assumption, and then becomes sharply skewed rapidly as we increase the level of correlation.

PD Correlations: is the NPR realistic?

Low PD portfolios are more correlated than high PD ones (pg 67 draft NPR).

Table 4: Correlations of Defaults Intensities.

The table reports the median correlation ρ_{ij} computed using default intensities from Moody's PDs. The correlation is defined as the correlation between the shocks in each of the regressions,

$$\lambda_i(t) - \lambda_i(t-1) = \epsilon_i(t), \qquad \text{MODEL 1}$$

$$\lambda_i(t) = \alpha_i + \beta_i \lambda_i(t-1) + \tilde{\epsilon}_i(t), \qquad \text{MODEL 2}$$

where $\lambda_i(t)$ is the default intensity of firm i in month t. Correlations are estimated pairwise for each pair of firms in the sample, and the median is reported. Panel A reports the results by credit class. The first line reports the Pearson correlation coefficient. The second line reports the rank correlation coefficient. Panel B reports results by SIC code; to conserve space, only the Pearson correlation coefficient is reported.

Panel A

Group	Period I		Period II		Period III		Period IV	
	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2
High Grade	0.36	0.37	0.10	0.10	0.02	0.01	0.37	0.38
	0.33	0.30	0.12	0.11	0.04	0.04	0.34	0.31
Medium Grade	0.22	0.23	0.10	0.10	0.03	0.02	0.24	0.25
	0.25	0.23	0.14	0.12	0.06	0.06	0.27	0.24
Low Grade	0.16	0.16	0.06	0.07	0.02	0.02	0.17	0.17
	0.19	0.17	0.13	0.13	0.05	0.05	0.20	0.18
Not Rated	0.16	0.16	0.05	0.06	0.02	0.02	0.16	0.17
	0.15	0.13	0.07	0.07	0.02	0.02	0.16	0.13
	(late 1980s)						(early 2000s)	

Figure 3: Reproduction of Table 4 from Das, Freed, Geng and Kapadia (2001) showing that low-PD (high quality) firms have higher correlations than high-PD (low quality) firms.

However, contagion effects exist, especially in high PD assets

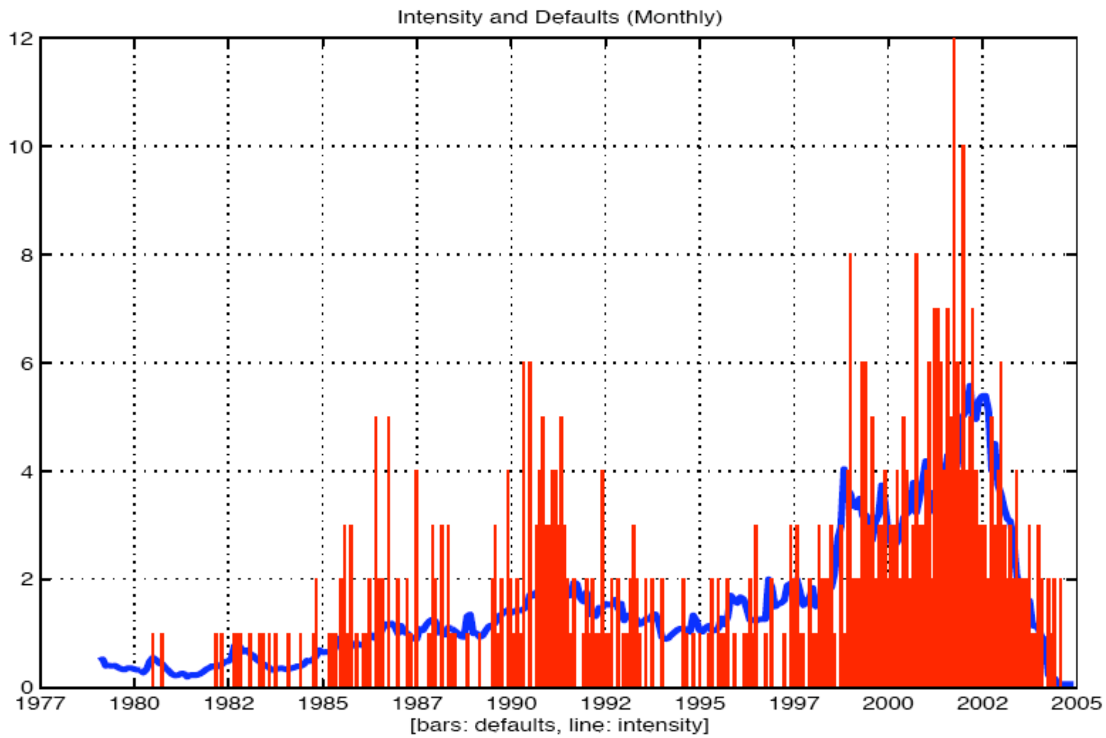


Figure 2. Intensities and Defaults. Aggregate (across firms) of monthly default intensities and number of defaults by month, from 1979-2004. The vertical bars represent the number of defaults, and the line depicts the intensities.

Figure 4: Reproduction of Figure 2 from Das, Duffie, Kapadia and Saita (2004). The figure shows the number of defaults as well as the aggregate of default intensity (probability) over time. The two series track each other very well, and hence, the aggregate intensity, based on the model of Duffie, Saita, and Wang (2005) may be used to detect pro-cyclicality.

(a) Portfolio invariance assumption?
(b) Detecting pro-cyclicality [use Duffie, Saita & Wang (2004)]
(c) Economic capital may be less for high PD portfolios?

LGD assumption: Das and Hanouna (2006) model

The model identifies PD and LGD jointly

Reduced-form intensities from CDS spreads

$$\lambda_k = \frac{-1}{h} \ln \left[\frac{S(T_{k-1})D(T_k)(1-\phi_j) + \sum_{j=1}^{k-1} G_j - C_k \, h \, \sum_{j=1}^{k} H_j}{S(T_{k-1})D(T_k)(1-\phi_j)} \right]$$

$$G_j = S(T_{j-1}) \left(1 - e^{-\lambda_j h}\right) D(T_j)(1-\phi_j)$$

$$H_j = S(T_{j-1})D(T_j)$$

Identification

$$\phi(T) = \phi[\lambda(T)] = g[\lambda(T);\theta]$$

$$\begin{aligned} \phi(T) &= E\left\{\frac{V(T)}{F} \mid V(T) < F\right\} \\ &= \frac{1}{F}\frac{1}{Prob[V(T)<F]}\int_0^F V(T)\, f[V(T)]\, dV(T) \end{aligned}$$

$$\phi(T) \equiv g[\lambda(T);\theta] = \frac{1}{\lambda(T)} \times \frac{[\text{Forward value of asset-or-nothing put}]}{F}$$

$$\phi(T) = e^{rT}\,\frac{V}{F}\,\frac{N[-d_1]}{\lambda(T)}$$ (Merton model)

Application uses an iterative, fixed-point algorithm.

LGD algorithm example

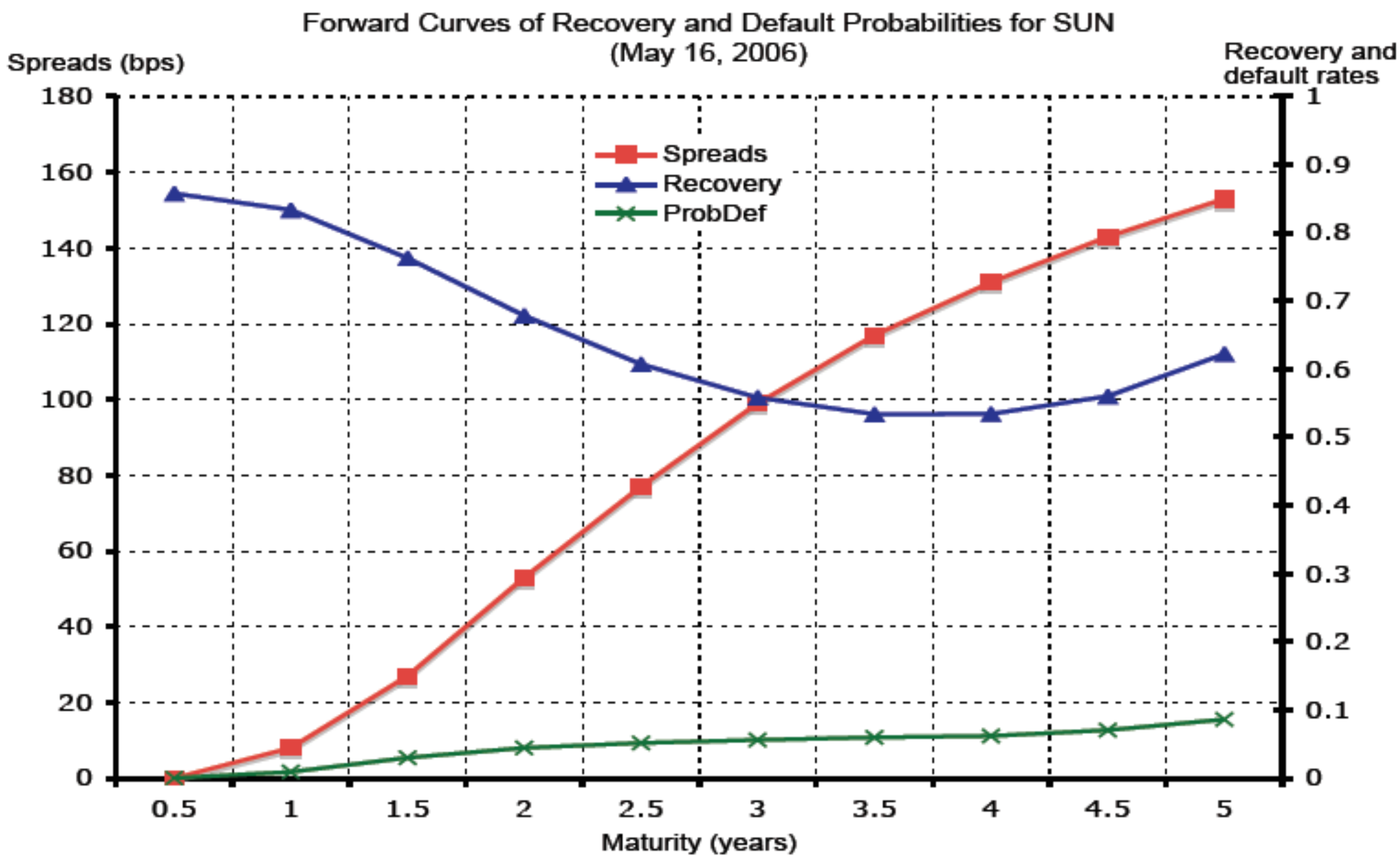
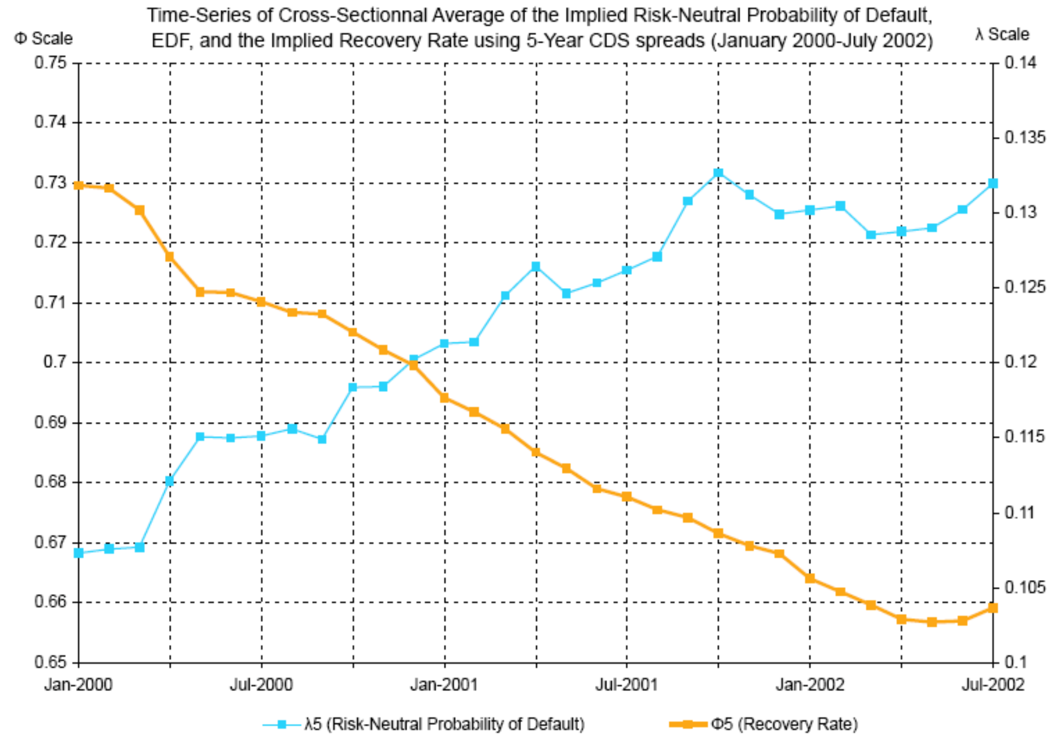
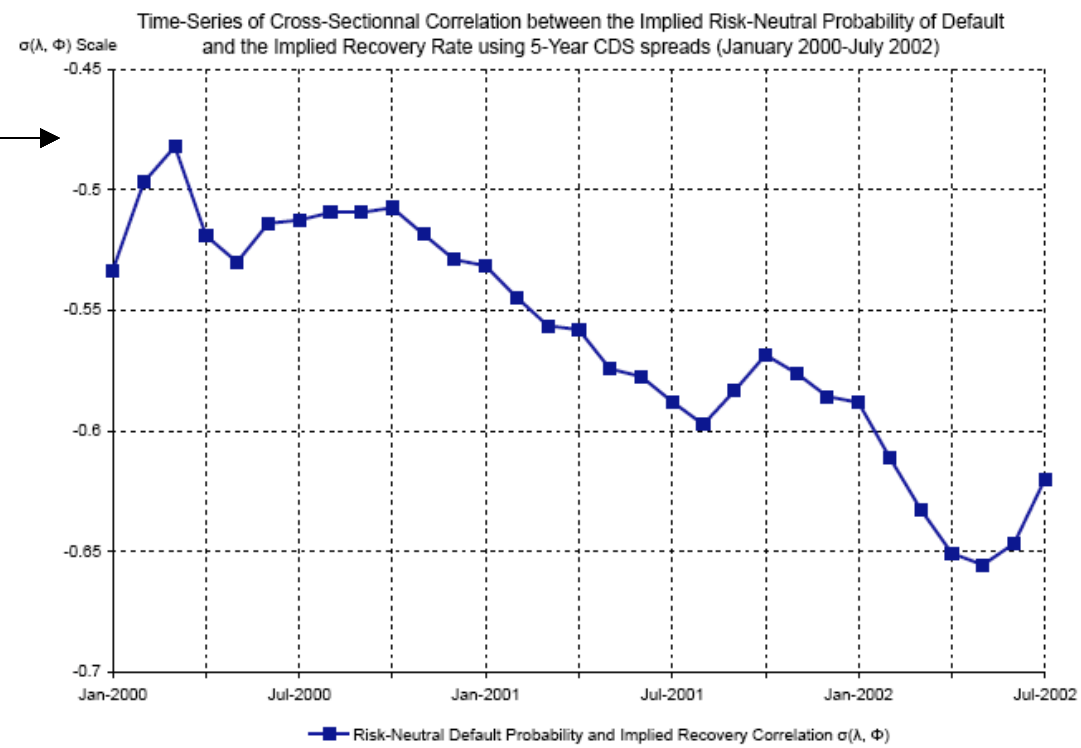


Figure 1: Implied forward recovery rates and default probabilities for Sun Microsystems on May 16, 2006. These are computed from the application of the algorithm in section 5.3 . The default swap spread curve from `CreditGrades` is also shown. The stock price for SUNW was \$4.75, the forward curve was assumed flat at 5.43%, and the asset volatility was taken to be 0.46. Debt per share is 1.11.

Aggregate PD and Recovery for over 3000 firms.



Evidence of contagion in LGD



Overall 4 correlations: (a) between PDs, (b) PD and LGD, (c) conditional default correlations, (d) PD and EAD (credit & market risk)

Non-Gaussian Assumptions

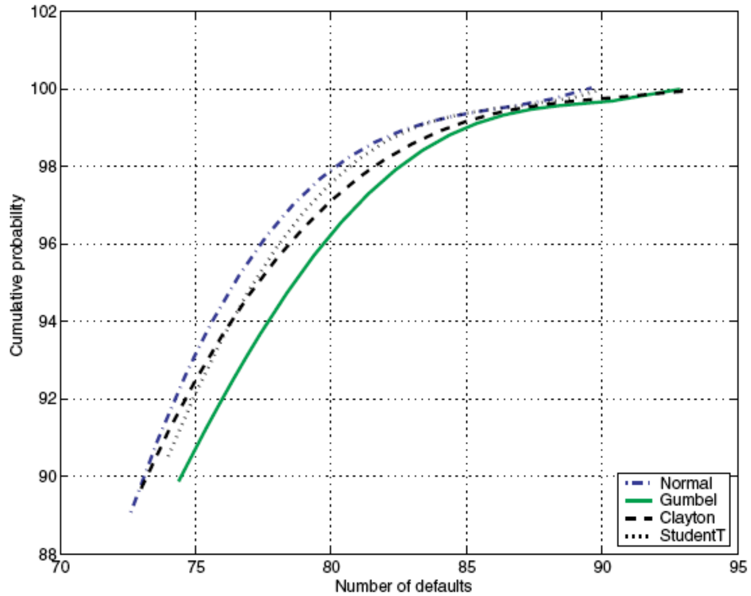


Figure 5 Comparing copula tail loss distributions: this figure presents plots of the tail loss distributions for four copulas, when the marginal distribution is normal. The *x*-axis shows the number of losses out of more than 600 issuers, and the *y*-axis depicts the percentiles of the loss distribution. The simulation runs over a horizon of 5 years and accounts for regime shifts as well. The copulas used are: normal, Gumbel, Clayton, Student's *t*.

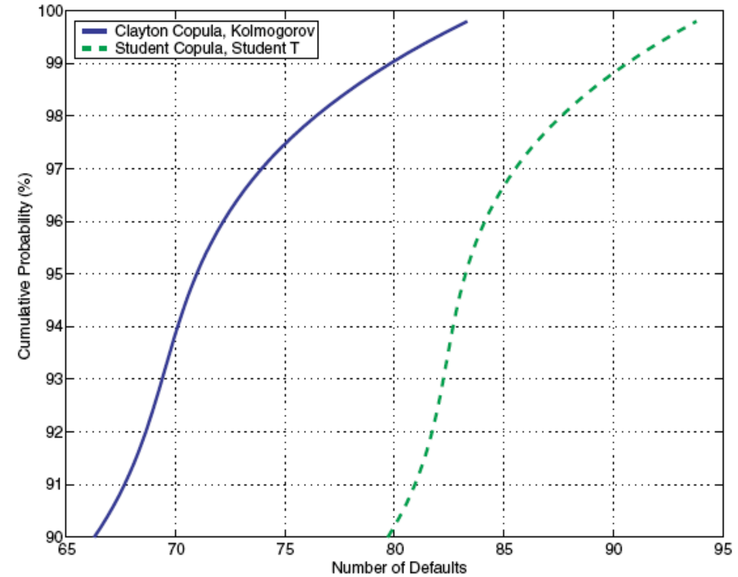


Figure 6 Comparing copula tail loss distributions: in this figure we plot the tail loss distributions for two models, the best fitting one and the worst. The best fit model combines the Clayton copula, and marginal distributions based on the Kolmogorov criterion. The worst fit copula combines the Student's *t* copula with Student's *t* marginals. The simulation runs over a horizon of 5 years and accounts for regime shifts as well.

(from Das and Geng 2004)

Moody's data for 14 years shows that the joint density of PDs is best matched with a Clayton copula over double-exponential marginal distributions.

Regimes and VaR horizon

There is clear evidence of regimes in credit risk.

A one-year VaR horizon in a regime-switching model results in higher switching probability.

a) If in the low risk state, more capital required if the horizon is greater than that needed to trade away from risk.

b) If in the high risk state, less capital maintained.

Hence, we either keep too little or too much capital.

See Gore (2006) for a discussion of similar issues in the UK for retail portfolios.

Top-Down: Apply Merton's (1977) Model

Merton (1977) showed that risk-based capital per dollar of liabilities for a financial or depository institution was the same as a put option on the bank's assets A with a strike price of the liabilities L plus interest thereon, i.e. Le^{rT}, where T is the maturity of the liabilities. This liability insurance is equal to risk-based capital C.

$$C = N(d_2) - \frac{A}{L} N(d_1)$$

where

$$d_1 = \frac{\ln(L/A) - 0.5\sigma^2 T}{\sigma\sqrt{T}}, \quad d_2 = d_1 + \sigma\sqrt{T}$$

Might it be possible to simply compute and report the Merton model capital required using this simple formula directly at the firm level? Implementation would be undertake exactly in the same way as is done with the Merton (1974) model in various market implementations. We might think of this as a "top-down" approach to capital requirements. This easily reported and also provides another point of comparison with the more detailed "bottom-up" approach.

Merton, Robert C. (1977). An Analytic Derivation of the Cost of Deposit Insurance and Loan Guarantees: An Application of Modern Option Pricing Theory, *Journal of Banking and Finance* 1, 3-11.

Merton, Robert C., and Andre F. Perold (1993). Theory of Risk Capital in Financial Firms, *Journal of Applied Corporate Finance* 6(3), Fall, 16-32.

Minimum Floor Requirements

(a) Discourages moving to IRB if the forecasted reduction in capital is small.

(b) Banks that would avail of large capital reductions will be disincentivized and disappointed.

(c) Penalizes banks that take active measures towards risk reduction, not those that do not.

(d) However, there are benefits in implementing the new risk based capital requirements in a controlled environment.

Leverage requirements

How is this handled for off balance sheet items?

May be accommodated by stating the on balance sheet equivalent replicating portfolio, but this is non-trivial to do.

Distance to Default (DTD) as a basis for capital required:

1. An alternate way to implement capital floors.
2. It accounts for leverage since it is a volatility adjusted measure of leverage.
3. Since all large FIs are publicly traded, this is easy to implement.