Basel II Technical Issues: A Comment

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"The Devil is in the Details."

1 Basel II Overview

This short note discusses various issues related to the new Basel II requirements in the light of current research findings. We first summarize some of the regulations that we will refer to in the course of this comment, and then proceed to present evidence from extant research that suggests more analysis is required of some of the implementation details of Basel II. This comment is agnostic about whether Basel II is likely to result in over or under-capitalization of major financial institutions. Rather, the attempt is to focus on areas that need further clarification and analysis, with a view to improve the accuracy of the amount of capital maintained.

Initially, in this section, we present briefly the benefits of the new IRB approach, structure of the capital adequacy measures and various acronyms and notation that will be used throughout this comment. The goal here is to comment on a few technical

*These comments are rough and ready discussion of various areas of Basel II rules and the U.S. notice of proposed rule-making (NPR). These comments serve the purpose of linking Basel II issues to other published work, of the author and others. Reproduced figures from other work are cited as such. The contents also contain analyses that are original and distinct from other work as well. These comments are for conference discussion only and not for publication. As always, comments on these comments are welcome.

aspects of the guidelines, as well as the NPR (notice of proposed rule making). We will discuss whether the new IRB approach improves on the previous standard approach in Basel I, and also take a look at some of technical issues surrounding an implementation of the new approach.

The change from the old system under Basel I to a new system under Basel II is well-summarized by the comments of Fed Chairman Ben Bernanke at the Federal Reserve Bank of Chicago's 42nd Annual Conference on Bank Structure and Competition, Chicago, Illinois on May 18, 2006. To quote -

"... the relatively crude method of assigning risk weights to assets, as well as an emphasis on balance-sheet risks as opposed to other risks facing financial firms, limits the overall responsiveness of capital requirements to risk under Basel I, which renders that system increasingly inadequate for supervising the largest and most complex banking organizations. For these organizations, we need to move beyond Basel I to a more risk-sensitive and more comprehensive framework for assessing capital adequacy. Basel II represents the concerted efforts of the supervisory community, in consultation with banks and other stakeholders, to develop such a framework."

There are many issues surrounding the impending implementation of the Basel II IRB (internal ratings based) approach. Proponents for this highlight many advantages such as (a) a reduction in the amount of capital being held, (b) more dynamic and realistic capital adequacy computation, (c) risk-based pricing of products, (d) a means to instill best practices, (e) the introduction of much needed analytical methods, (f) reduction in expected future charge-offs, (g) reduction in operating expenses, (h) reduction in operating losses, (i) better capital allocation amongst business units within a financial institution, (j) improved corporate governance, and (k) overall lower systemic risk in the financial system.

On the other hand, opponents of the new accord suggest many disadvantages such as (a) a high cost of implementation, (b) competitive disadvantages between banks that are not required to comply and those that are required to, (c) competitive imbalances across countries as different national supervisors impose varied levels of compliance, (d) strong opposition to operational risk charges as being a deadweight cost for imposing governance that is already legally mandated, (e) inability to obtain consistent implementation across all institutions, resulting in more noise than accurate determination of risk, (f) the propensity to increase systemic risk if the rules impose distortionary portfolio changes in one same direction across all financial institutions.

To quickly summarize, Basel II envisages three *pillars*: (1) capital adequacy, (2) regulatory review, and (3) market discipline. In this comment, we will focus mainly on capital adequacy and more specifically, on the credit risk component of capital requirements.

The previous approach to capital adequacy relied on taking the size of the portfolio and ascribing to it a risk-factor, based on which a capital requirement was imposed from a table. Clearly, this suffers from a basic fallacy that ignores portfolio specific risk, that portfolios tend to be quite different in their individual characteristics, even when they are of the same asset class, leverage and maturity issues. By suggesting that we move on to a Value-at-Risk (VaR) like system, where the loss distribution is explicitly modeled is clearly going to determine capital adequacy better, provided that the calculations involved and the modeling assumptions are practical and reasonably accurate. However, moving to the IRB approach allows banks greater flexibility in making a wide range of assumptions to "cook" the numbers to achieve internal target capital levels. Yet, one may be optimistic that this is unlikely to occur (a) with more oversight and, (b) the fact that the IRB approach recognizes that banks have already been using risk-based capital for almost two decades now, and (c) that this new approach is much more consistent with internal risk management. By all counts, this will reduce the costs of internal and regulatory risk management in the long run, though in the short run, the need to produce both Basel I and II reports is no doubt an onerous imposition.

To briefly summarize, Basel envisions two types of losses: (a) *expected loss* (EL), and (b) *unexpected loss* (UL). If the horizon for the analysis is denoted T, and the current value of a portfolio today (at time t) is $P(t)$, then expected loss (EL) is:

$$EL(T-t) = E[P(T) - P(t) | P(T) - P(t) < 0]$$

The new accord envisages the horizon $(T-t)$ to be one year (the old accord looked more at horizons of 10 trading days, i.e. 2 weeks). For possible horizons greater than a year, a maturity adjustment is envisaged through a factor, denoted M.

VaR at a level of α (say 1%), is defined as the tail cut off $[P_\alpha(T) - P(t)]$ for which losses in excess of this value will occur with α probability. We write this loss value as $\text{VaR}(\alpha, T-t)$. Unexpected losses are then defined as:

$$UL(T-t) = \text{VaR}(\alpha, T-t) - EL(T-t)$$

Losses in excess of $\text{VaR}(\alpha, T-t)$ are denoted as *extreme losses* and may also be reserved for. However, the guidelines appear to only look at EL and UL.

Inextricably tied up with the concepts of expected and unexpected loss are the notions of *regulatory* capital and *economic* capital. Regulatory capital (Tier 1 and Tier 2) is meant to buttress expected losses, and economic capital is for unexpected losses. One would also expect that economic capital plays a bigger role in maintaining the credit rating of a financial institution. Further, regulatory capital is applied to losses that are expected to occur but are of smaller consequence. Economic capital is applied towards low frequency losses, but that have significant magnitude. Again, for obvious reasons, EL is not sensitive to the shape of the loss distribution as much, whereas UL clearly is. UL is also susceptible to all the ills that risk measures like VaR suffer from [such as failure to be a "coherent" risk measure, per Artzner, Delbaen, Eber and Heath (1999)].

This may be a good point at which to recap that whereas VaR has widespread use, it has some well-recognized flaws: (a) It is not a "coherent" risk measure, in that it fails the "sub-additivity" criterion, which simply put, says that a risk measure should always be lower when a portfolio is diversified. In the case of VaR, this is not guaranteed; indeed, taking a weighted average of two portfolios may result in an increase in the risk measure. Intuitively, this comes from the fact that VaR is a percentile measure, and not a moment of the loss distribution. (b) VaR is very hard to measure because it depends wholly on the tail of the loss distribution. At tail cut offs of 99.99%, it is hard to be confident of its value. This is popularly known as the "Star-Trek" problem, i.e. how do we estimate something where we have never even gone before. There is really no data to validate the efficacy of such cut-offs. (c) VaR is known to depend on the number of samples generated in Monte Carlo simulation (see the study cited by Chorafas (2004), page xxii), in that it increases as we raise the number of samples (my attempts at replicating this were not successful, and in fact an examination of mean loss in the tail resulted in reducing mean loss as sample size increased). Intuitively, greater sampling increases the number of outlier observations seen, and hence stretches out the tail. It almost necessitates that risk managers specify the sample size to make VaR meaningful. Conversely, if you want to reduce UL, simply simulate less. The NPR might comment on this.

To get a sense of the magnitudes of risk in the financial sector, it is interesting to examine the data on VaR reported by Jeffery and Chen (2006). They show that VaR in the world's leading financial institutions for 2005 was $51.9 million (one-day VaR, at a 99% level). We present the table from their paper in Figure 1 showing the breakdown of VaR by major bank. Of this, the biggest component is interest rate VaR, then equity and commodity VaR respectively.

When speaking specifically about *credit losses*, the accord envisages four *risk components*, i.e., (a) probability of default (PD), (b) loss given default (LGD), (c) exposure at default (EAD), and (d) a maturity adjustment (M). For credit losses,

$$\mathrm{EL} = \mathrm{PD} \times \mathrm{LGD} \times \mathrm{EAD} \times f(\mathrm{M})$$

Whereas the document presents the formula as above (more or less), what is hidden is that all these four risk components above may be stochastic and drawn from dis-

A. Average VAR, one-day, 99%

Financial institution	2005 ($m)[1]	2004 ($m)[2]	Change in VAR (%)	Change in reporting currency VAR (%)[3]	VAR rank 2005	VAR rank 2004	VAR rank 2003
UBS[4]	122.8	95.2	29.0	32.5	1	2	1
Citigroup	109.0	101.0	7.9		2	1	3
Goldman Sachs	99.1	94.9	4.5		3	3	2
JP Morgan[5]	86.0	85.0	1.2		4	5	4
Morgan Stanley	85.0	73.0	16.4		5	6	7
Deutsche Bank	83.5	87.4	-4.5	-8.2	6	4	6
ABN Amro	63.4	32.2	97.1		7	17	19
Bank of America	62.2	48.0	29.6		8	11	13
Barclays	62.1	71.7	-13.5	7.0	9	7	9
Commerzbank	60.4	64.8	-6.8	3.1	10	8	5
Credit Suisse	54.2	55.6	-2.6	0.0	11	9	8
Merrill Lynch	53.8	49.6	8.6		12	10	12
Lehman Brothers	44.5	38.0	17.2		13	14	15
HSBC	37.3	35.6	4.8		14	15	14
ING	35.5	30.1	18.0	13.4	15	18	16
RBS	33.5	28.3	18.4	20.4	16	20	20
BNP Paribas	29.7	38.8	-23.5	-20.4	17	13	11
Bear Stearns	29.0	22.4	29.7		18	23	21
Santander	26.5	n/a	n/a		19	n/a	n/a
BBVA	24.2	n/a	n/a		20		
SG	24.1	29.3	-17.6	-20.8	21	19	17
Wachovia	24.1	23.5	2.7		22	22	24
WestLB	21.7	32.3	-33.0	-30.3	23	16	18
Dresdner	19.7	39.6	-50.3	-48.4	24	12	10
Lloyds TSB	5.4	3.1	72.2	75.0	25	26	27
Average	51.9	51.3	1.2				

Note 1 \$1=Sfr1.228, \$1=€0.791, \$1=£0.552
Note 2 \$1=Sfr1.119, \$1=€0.758, \$1=£0.541
Note 3 Not relevant for institutions that report in US dollars, including HSBC
Note 4 UBS 2005 figures reported at one-day holding period, 2004 figures converted to one-day holding period
Note 5 Trading VAR

Figure 1: Reproduction of table A from Jeffery and Chen (2006), *Risk*.

tributions as well (with correlation amongst them). Hence, the formula above really suggests that the expected values of these risk components be used to determine EL (with the concomitant result of running afoul of Jensen's inequality, though it is unclear in which direction.). In order to ascertain UL, a distribution of losses needs to be generated under many scenarios accounting for the fact that these inputs vary, and that the occurrence of default is also subject to the specific realization of the value of PD. The actual LGD also may be variable. The devil lies very much in the details here. As we will soon see, credit loss distributions are far more tail dependent than that for market risk, making correlation assumptions difficult to stipulate, validate and implement.

The Basel II framework suggests two levels of IRB implementation: (i) *foundation*, or F-IRB and (ii) *advanced*, or A-IRB. In the former, banks use their own PD, but take LGD and EAD as provided by regulators. In the latter, banks use their internal estimates of all input parameters.

2 Granularity and Aggregation

Fixing asset value correlations between business segments based on empirical correlation studies may result in perverse results for the overall capital to be maintained across a franchise. Correlation assumptions need to depend on the choice of granularity of the analysis, which can make a substantive difference to the computed risk measure.

In a single risk factor framework, each transaction has systematic and idiosyncratic risk. At one extreme, we may have each transaction as a separate portfolio or business unit (perfect granularity). As transactions are grouped into sub-portfolios (units), diversification reduces the risk within each sub-portfolio. Overall risk for the franchise should remain unchanged, since sub-portfolios now become more correlated as the ratio of systematic risk to idiosyncratic risk across sub-portfolios will increase. This implies that the correlation between asset classes (sub-portfolios) depends on the extent of granularity chosen. In a situation where the correlations are *fixed* based on empirical estimates independent of granularity, aggregate franchise risk is in fact distorted. A simple numerical experiment here shows that there is an optimal level of granularity that a franchise may choose to minimize its required capital. Clearly, this concern has been recognized within the NPR, since it suggests as high a level of granularity as possible.

To illustrate the critical nature of the granularity and aggregation decision in determining capital adequacy, we conduct the following experiment. Assume we have

$n = 2^{10} = 1024$ assets in our portfolio and each asset has mean value 0 and variance 1, i.e. we may describe them as standard normal variables (this is without loss of generality). We also assume that the correlation between these variables is the same for each pair, and is denoted ρ. Hence the covariance matrix of asset values (Σ) is of dimension $n \times n$ with the value 1 on the diagonal and ρ off-diagonal. Assume an equally weighted portfolio (P) of these assets, i.e. w is a vector of weights, each of value $1/n$. The mean value of this portfolio is 0 and its variance is $\sigma^2 = w' \, \Sigma \, w$. We will compute the EL, UL, and VaR of P.

$$\mathrm{EL} = \int_{-\infty}^{0} P \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-P^2/2) \, dP$$

where the formula above is the expected value of P conditional on it being less than 0, and assuming it is distributed normally.

The 1% VaR of this portfolio is obtained by inverting the cumulative normal distribution for the left tail area of 0.01. Finally, the UL is determined using EL and VaR. If we assume that $\rho = 0.5$, we obtain the following values:

$$\mathrm{EL} = 0.2822, \quad \mathrm{UL} = 1.3635, \quad \mathrm{VaR} = 1.6458$$

where of course, the values are taken with positive sign since we are interested in the loss distribution, even though the integral above results in a negative value. We might imagine that what we have here are 1024 separate business portfolios and that we aggregate them all equally weighted into one enterprise portfolio and compute the risk measures above. This set up assumes a very high level of granularity, i.e. each asset is a distinct business unit.

Next, suppose we construct each business unit as comprising $m = 2$ assets, so that we have $n = 512$ business units or portfolios. Our basis for computing EL, UL and VaR now requires the covariance matrix of these 512 portfolios. Note that each portfolio has mean 0 as before, but the variance is $y' \, \Omega \, y$ where y is a vector of length m of values $1/m$. Ω is a matrix of dimension $m \times m$, with 1s on the diagonal and ρ off-diagonal.

We construct the covariance matrix Σ of the entire franchise (this is of dimension $n \times n$ or 512×512) [the correlation parameter ρ here between portfolios may actually vary, depending on the actual factor model used. But our goal here is more to demonstrate the perverseness of risk measurement for varying aggregation levels]. The diagonal elements are $y' \, \Omega \, y$, and the off-diagonal elements are $\rho(y' \, \Omega \, y)$. This is the crux of the problem at hand, the parameter ρ is not adjusted for the change in granularity. After running the computations as before, we obtain the following values:

$$\text{EL} = 0.2445, \quad \text{UL} = 1.1814, \quad \text{VaR} = 1.4260$$

The capital required here is lower, as all three risk measures are smaller than before. This is not surprising as the correlation across business units has been dampened as they are made up of portfolios. We undertook the same computation for a changing number of portfolios of the 1024 assets, by dividing the number of business units progressively by by 2, and multiplying the number of assets within each portfolio by 2. The results for EL, UL, and VaR are shown in Figure 1.

Table 1: Expected loss, unexpected loss and Value-at-Risk for varying levels of granularity and aggregation. The first column shows the number of business units, and the second one the number of assets within each unit. Each asset has a standard normal distribution. "Corr" is the average pairwise correlation between portfolio values.

# portfolios	# within portfolio	Corr	EL	UL	VaR
1024	1	0.5000	0.2822	1.3635	1.6458
512	2	0.3750	0.2445	1.1814	1.4260
256	4	0.3125	0.2235	1.0796	1.3030
128	8	0.2812	0.2124	1.0261	1.2385
64	16	0.2656	0.2072	1.0011	1.2083
32	32	0.2578	0.2057	0.9938	1.1995
16	64	0.2539	0.2072	1.0011	1.2083
8	128	0.2520	0.2124	1.0261	1.2385
4	256	0.2510	0.2235	1.0796	1.3030
2	512	0.2505	0.2445	1.1814	1.4260
1	1024	0.5000	0.2822	1.3635	1.6458

The results are interesting. As we reduce the level of granularity the risk measures fall. This is because there are two types of diversification involved here: (a) diversification *within* portfolio or business unit, and (b) diversification *across* units. When the portfolio is the same as the asset, there is no within unit diversification, only across units. As we increase the number of assets in each portfolio, we get diversification within unit, and also across unit. Think of the original covariance matrix being halved in dimension and block diagonalized so as to lose some of the correlation between individual securities *across* units. Hence, this leads to *Problem #1*, i.e. that the level of granularity affects measures of risk, even though the total risk has not been changed.

As granularity is reduced further, again there is a trade-off between diversification within and across portfolios, until at some point, we begin to lose diversification across units, as the number of portfolios becomes too small, and the risk measures begin to rise once again. We can see that there is a material difference between the risk measures at varied granularity levels. This leads to *Problem #2*, i.e. when the regulators (internal or external) provide AVC levels, what granularity level do they have in mind?

The simple correction required is to increase ρ across business units as the level of granularity declines. But by how much? In our example, where all assets have the same distribution, it is easy to compute the correction. But when the assets within each unit are heterogeneous, there is no simple way to do this. Again, the NPR clearly envisages this problem in requiring that units be defined for highly homogeneous assets. In short, granularity of risk measures complicates risk aggregation on account of correlation assumptions. Imposing infinite granularity makes computation difficult since the number of assets in the VaR simulation becomes very large. Clubbing assets into sub-classes helps computationally, but careful corrections need to be applied to correlation assumptions.

3 Correlation Sensitivity of Credit Portfolios

For credit portfolios, risk measures, based on loss distributions are highly sensitive to the correlation parameter. Credit portfolios are essentially based on binary outcomes, and hence the joint distribution is quite different than with portfolios where the outcomes reside on a wide range of values. Intuitively we will see that a portfolio of binary outcomes has a distribution that changes very quickly when correlations change than say, a portfolio where the assets are distributed multivariate normal.

In the previous section, we saw that the risk measures EL, UL and VaR are sensitive to granularity. The analysis there was simple and assumed distributions over a continuous range of values. However, when dealing with credit losses, the value tends to be either zero or a loss value within some smaller range. Intuitively, each asset follows more or less a Bernoulli distribution, with one outcome being zero. When we construct portfolios of such assets, the distributions become even more sensitive to correlation assumptions, implying that the risk measures will also be much more variable when correlations are changed.

To illustrate, we work in the standard single risk factor framework that is now very popular in an analyzing correlated default risk. Assume there are n assets in a portfolio. Each asset is identical with a Bernoulli outcome over values $\{0, LGD\}$ with probability $\{1 - PD, PD\}$ respectively. For normalization assume that $EAD = 1$, and that $LGD = 1$. In this example, we assume that only PD is stochastic and that all other input variables are constant.

In order to inject correlation amongst defaults, we examine the following set up. Assume that the n assets each have an underlying value process as follows:

$$x_i = \sqrt{\rho}\, z + \sqrt{1-\rho}\, e_i, \quad z, e_i \sim N(0,1), \quad \forall i.$$

Hence, $E(x_i) = 0$, and $Var(x_i) = 1$, for all assets, assuming that z is independent of all e_i, and that the e_is are independent. As we can see correlation amongst the assets is generated from the common random variable z. Note that $Cov(e_i, e_j) = \rho$ for all pairs (i, j). Since means are zero and variances are 1, the covariance is also the correlation. This is a standard set up and for more examples of credit loss computations, see Kupiec (2005).

For asset i, default occurs if $N(x_i) < PD$, where $N(.)$ stands for the cumulative normal distribution. We will find it easier to build up the loss distribution if we condition on various values of z. Suppose we fix a value of z. Then the probability of default, conditional on z is denoted $PD|z$, and is

$$\begin{aligned}
PD|z &= Prob[N(x_i) < PD|z] \\
&= Prob[x_i < N^{-1}(PD)|z] \\
&= Prob[\sqrt{\rho}\, z + \sqrt{1-\rho}\, e_i < N^{-1}(PD)|z] \\
&= Prob[e_i < \frac{N^{-1}(PD) - \sqrt{\rho}\, z}{\sqrt{1-\rho}}|z] \\
&= N\left[\frac{N^{-1}(PD) - \sqrt{\rho}\, z}{\sqrt{1-\rho}}|z\right] \\
&\equiv q_z
\end{aligned}$$

The probability that there are m losses from n firms, conditional on z is denoted $p_z(m)$, given by the binomial formula

$$p_z(m) = \binom{n}{m} q_z^m (1-q_z)^{n-m}, \quad m = 0...n.$$

Noting that $z \sim N(0,1)$ we can integrate it out to get the full loss distribution, with the probability of m losses:

$$p(m) = \int_{-\infty}^{\infty} p_z(m)\, \phi(z)\, dz, \quad m = 0...n.$$

Table 2: Risk measures for varying default correlation. The PD for each firm is 5% and the number of identical firms is 100. The expected loss should be exactly 5.00 for all correlation levels, and the tiny discrepancy comes from numerical rounding error. The last column contains the adjustment term from the formula on page 405 of the draft NPR, i.e. $N\left[\frac{N^{-1}(PD)-\sqrt{\rho}\,N^{-1}(0.999)}{\sqrt{1-\rho}}\right]$. We can see how it varies with correlation.

Corr	EL	UL	CVar	Kadj
0.00	5.0000	5.2046	10.2046	0.0500
0.10	4.9991	13.1910	18.1902	0.2408
0.20	4.9984	20.7485	25.7469	0.3844
0.21	4.9984	21.5080	26.5064	0.3985
0.22	4.9983	22.2602	27.2585	0.4124
0.23	4.9982	23.0061	28.0044	0.4264
0.24	4.9982	23.7793	28.7775	0.4403
0.25	4.9981	24.5474	29.5455	0.4542
0.26	4.9981	25.3110	30.3090	0.4680
0.27	4.9980	26.0713	31.0693	0.4817
0.28	4.9980	26.8482	31.8461	0.4955
0.29	4.9979	27.6300	32.6279	0.5091
0.30	4.9979	28.4099	33.4078	0.5227
0.40	4.9975	36.4720	41.4695	0.6553
0.50	4.9973	45.1927	50.1900	0.7776
0.60	4.9972	54.8697	59.8670	0.8818

where $\phi(t)$ is the normal pdf. This is easily computed using a fast quadrature routine or discrete integral. Once we have the loss distribution we can compute EL, UL and CVaR (credit VaR). Note that this approach is fairly standard and correctly produces credit loss distributions with the desired correlation. The risk measures are shown in Table 2. It is evident that the UL risk measure (and hence economic capital) is very sensitive to correlation assumptions. To get a visual feel for how quickly the loss distributions change, see Figure 2. Also note the last column in Table 2. It contains the term that varies as correlation changes in the capital formula from page 405 of the draft NPR. It complements this analysis in that the correlation adjustment tracks the vastly changing risk measures quite well.

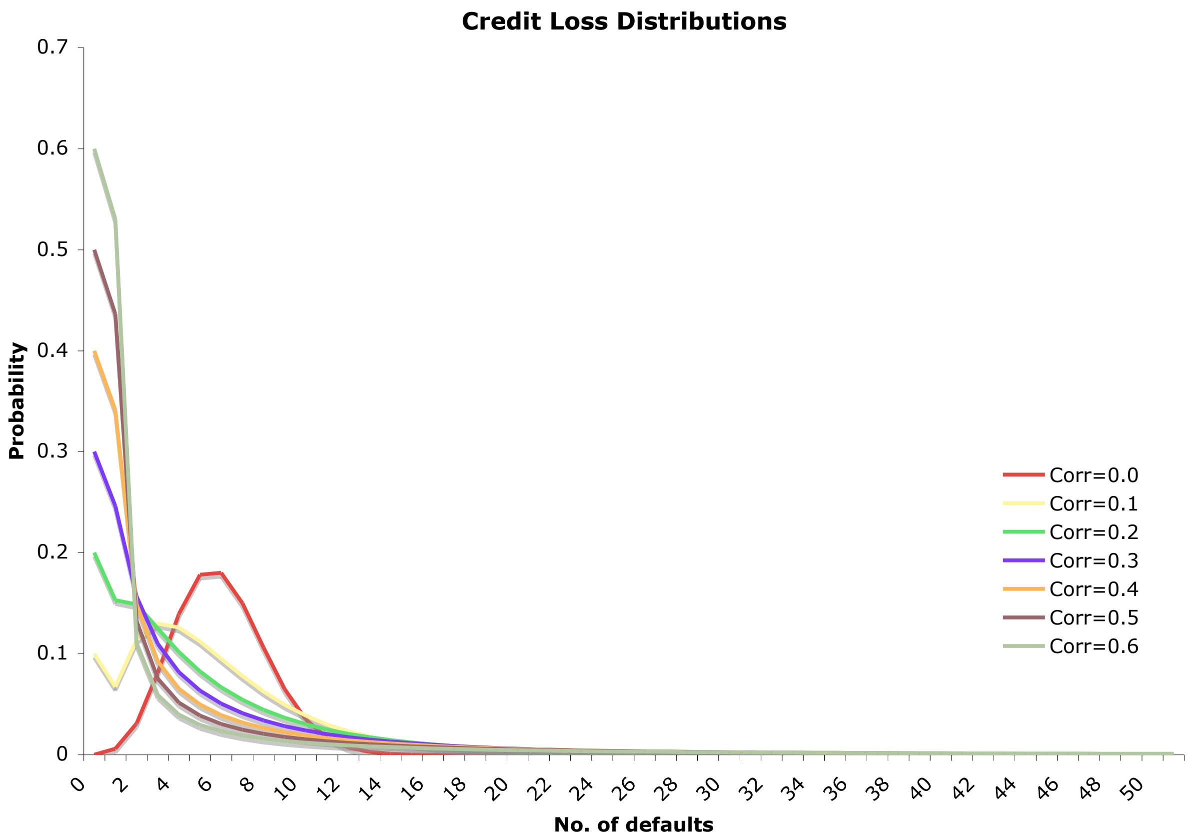

Figure 2: Credit Loss distributions under varied default correlation levels. We only present the loss levels out to 50 on the x-axis, even though the maximum number of defaults is 100, as the probabilities become very low thereafter. Note the distribution is mostly symmetric under the zero correlation assumption, and then becomes sharply skewed rapidly as we increase the level of correlation.

4 Asset Value Correlations (AVC)

The agencies involved in the formulation of the NPR have mandated two principles that need some discussion. First, the concept of *portfolio invariance*, and second, the use of correlation factors. The first allows each individual position's credit exposure to be calculated without accounting for other risks that might be taken in the remainder of the franchise. The second adjusts capital for the additional risk that arises from correlations. The *first question* that arises is whether the correlation adjustment that is made accounts properly for the sensitivity of the risk measures to correlation changes. As we have seen in Table 2, this certainly appears to be so.

An important *second question* is whether the assumption that low-PD portfolios are more correlated than high-PD portfolios is a valid one [this question is raised for discussion on page 67 of the draft NPR]. In this setting, the analysis appears to be incomplete to some extent. It is useful here to take the viewpoint of doubly-stochastic reduced-form models. In these models, default depends on two stochastic processes, (i) one process driving default probabilities, and (ii) conditional on a PD, another random variable driving the event of default. Correlated default occurs because of either or both of these stochastic processes. Defaults are correlated because firms' PDs are correlated. Defaults may also be correlated even when PDs are independent, if contagion effects exist, and the default of one firm triggers the default of others.

As it turns out, low-PD firms tend to display higher PD correlations than high-PD firms. This is intuitive. Low-PD firms also tend to be larger, and have greater systematic risk, and thus tend to be correlated to each other, whereas firms that are high-PD tend to also have a high degree of idiosyncratic movements in value, and thus high-PD firms will display lower correlation in PDs. This is confirmed in a study by Das, Freed, Geng and Kapadia (2001). Figure 3 contains a reproduction of Table 4 from this paper. It shows that high quality firms have higher PD correlation than low quality firms, across four economic regimes. This confirms and supports the ideas embedded in the NPR.

However, our analysis here is not complete. We need to consider whether defaults might be correlated differently for the second part of the doubly stochastic reduced form model. In other words, are contagion effects more prevalent amongst high-PD firms as opposed to low-PD ones? The presence of contagion (or frailty) effects has been empirically confirmed in Das, Duffie, Kapadia and Saita (2004). Whether these are more prevalent amongst high or low quality firms is an open question that required further empirical analysis. From a historical perspective, the late 1980s were a time when contagion might arguably have been residing in the realm of high-PD firms. But in the early 2000s, the major contagion effects were evidenced amongst fairly

Table 4: Correlations of Defaults Intensities.

The table reports the median correlation ρ_{ij} computed using default intensities from Moody's PDs. The correlation is defined as the correlation between the shocks in each of the regressions,

$$\lambda_i(t) - \lambda_i(t-1) = \epsilon_i(t), \qquad \text{MODEL 1}$$

$$\lambda_i(t) = \alpha_i + \beta_i \lambda_i(t-1) + \tilde{\epsilon}_i(t), \qquad \text{MODEL 2}$$

where $\lambda_i(t)$ is the default intensity of firm i in month t. Correlations are estimated pairwise for each pair of firms in the sample, and the median is reported. Panel A reports the results by credit class. The first line reports the Pearson correlation coefficient. The second line reports the rank correlation coefficient. Panel B reports results by SIC code; to conserve space, only the Pearson correlation coefficient is reported.

Panel A

Group	**Period I**		**Period II**		**Period III**		**Period IV**	
	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2
High Grade	0.36	0.37	0.10	0.10	0.02	0.01	0.37	0.38
	0.33	0.30	0.12	0.11	0.04	0.04	0.34	0.31
Medium Grade	0.22	0.23	0.10	0.10	0.03	0.02	0.24	0.25
	0.25	0.23	0.14	0.12	0.06	0.06	0.27	0.24
Low Grade	0.16	0.16	0.06	0.07	0.02	0.02	0.17	0.17
	0.19	0.17	0.13	0.13	0.05	0.05	0.20	0.18
Not Rated	0.16	0.16	0.05	0.06	0.02	0.02	0.16	0.17
	0.15	0.13	0.07	0.07	0.02	0.02	0.16	0.13

Figure 3: Reproduction of Table 4 from Das, Freed, Geng and Kapadia (2001) showing that low-PD (high quality) firms have higher correlations than high-PD (low quality) firms.

large, well established firms. Additional analysis is required here.

In the context of contagion, we see that in periods when PDs were high, as in Figure 3, (periods I and IV), default correlations tend to be 2 to 4 times as high as in periods with low correlation. Thus, in downturn scenarios, UL might be based on a correlation level of 0.40 versus normal times, with correlation of 0.10. From Table 2 we can see that UL will change by a factor of 2.7, which would also be the change in economic capital required. Hence, it seems appropriate to allow capital to be dynamically adjusted when in periods of downturn (as best we may know this), rather than build this into capital requirements on a continuous basis as envisaged in Table 2, page 405 of the NPR.

This naturally raises a *third* question of how to detect a down cycle, which is characterized in the industry debate as the "detection of pro-cyclicality". We submit here that this is not as infeasible as it might have been a few years ago. We now have evidence that models for aggregate default intensity correlate strongly with actual default levels. Give this, we might be able to use aggregate PD measures to assess when a regime shift has occurred. Figure 4 is a reproduction of Figure 2 from Das, Duffie, Kapadia and Saita (2004). What is clearly noticeable in the figure is how well the line for aggregate default intensity tracks actual defaults. Hence, we may be able to use various models such as the one by Duffie, Saita, and Wang (2005) for detecting pro-cyclicality.

A *fourth* question that arises is whether the specific assumption of low (high)-PD corresponding to high (low)-credit correlation will distort the amount of economic capital required. Noting from Table 2 that UL (economic capital requirement) is very sensitive to correlation might well result in a high-PD portfolio requiring less economic capital than a low-PD portfolio. Given that a high-PD portfolio attracts a greater regulatory capital requirement, this may not be an issue, until we realize that the total capital required may even be higher for a low-PD portfolio versus a high-PD one. Again, this highlights that correlation assumptions may be tricky, resulting in non-intuitive capital requirements.

5 Loss Given Default

The determination of LGD required for the EL computation is a difficult issue. We are in need of models that allow us to determine a forecast of recovery conditional on default. One such model that makes use of easily available information at a given point of time has been developed by Das and Hanouna (2006). The model is flexible and uses the information in CDS spreads to determine both default probability and

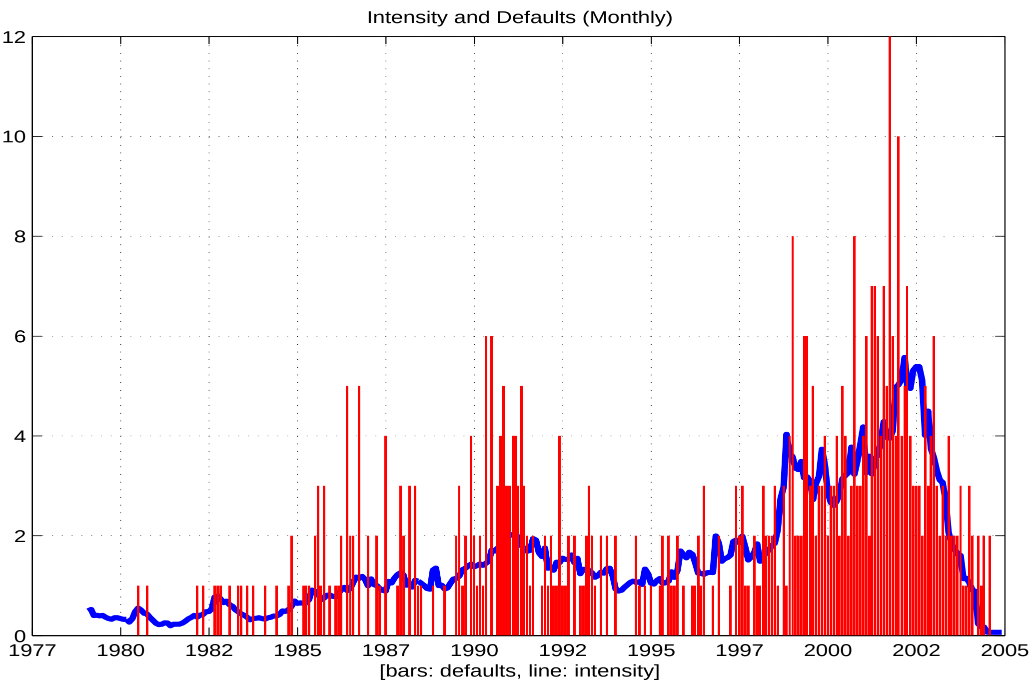

Figure 2. Intensities and Defaults. Aggregate (across firms) of monthly default intensities and number of defaults by month, from 1979-2004. The vertical bars represent the number of defaults, and the line depicts the intensities.

Figure 4: Reproduction of Figure 2 from Das, Duffie, Kapadia and Saita (2004). The figure shows the number of defaults as well as the aggregate of default intensity (probability) over time. The two series track each other very well, and hence, the aggregate intensity, based on the model of Duffie, Saita, and Wang (2005) may be used to detect pro-cyclicality.

recovery. The model may also be applied on average to sector spreads if need be to obtain a coarser estimate of recovery rates that may be more amenable to regulatory use.

The brief outline of the Das and Hanouna (2006) algorithm is as follows (full details are available in their paper). In a reduced-form default model, we may use CDS spreads to extract the term structure of forward default probabilities (λ), making some assumption about recovery rates (ϕ, or $LGD = 1 - \phi$). If we do not know ϕ, we may identify it by specifying a functional relationship between ϕ and probability of default, and Das and Hanouna develop a fast algorithm that exploits the relationship between ϕ and λ, and recovers both in a fully identified model.

Firms that choose to adopt the A-IRB approach may use this framework to determine the recovery rates for individual names in their portfolios.

6 Contagious Interaction of PD and LGD

When the Das and Hanouna (2006) algorithm is applied to 3,130 firms over the period 2000-2002, and λ and ϕ is aggregated for all firms (equally-weighted), we get a strong inverse relationship between default probabilities and recoveries, complementing the findings of Altman, Brady, Resti and Sironi (2004). Figure 5 reproduces a plot (Fig 5) from the Das-Hanouna paper.

They also examined the correlation of default probability and recovery in the cross-section of firms within each month. A strong negative correlation is found and their Figure 6 is reproduced here in Figure 6.

From 2000 to 2002, as default rate rose, recovery rates fell, *and* the correlation between the two became more negative. The relevant implications of these results for Basel II are that (a) the correlation between PD and LGD is important in the application of the $EL = PD \times LGD \times EAD \times M$ equation. (b) We know from Das, Freed, Geng and Kapadia (2001) that when PDs rise, their correlations increase, and from Das, Duffie, Kapadia and Saita (2004) that there is additional correlation of the contagion form even after conditioning on PDs. Now, from Das and Hanouna (2006), we see that recovery rates become increasingly negatively correlated with PDs as PDs rise, resulting in correlated LGDs (another form of contagion not recognized earlier). Hence, not only do defaults cluster, but when they do, LGDs cluster as well. This has important implications for capital adequacy, and if anything should bias capital requirements higher. Further, the systemic risk implications are also of concern.

Therefore, we now have three different sources of correlation to deal with in the framework of reduced form models. First, there is the correlation between default

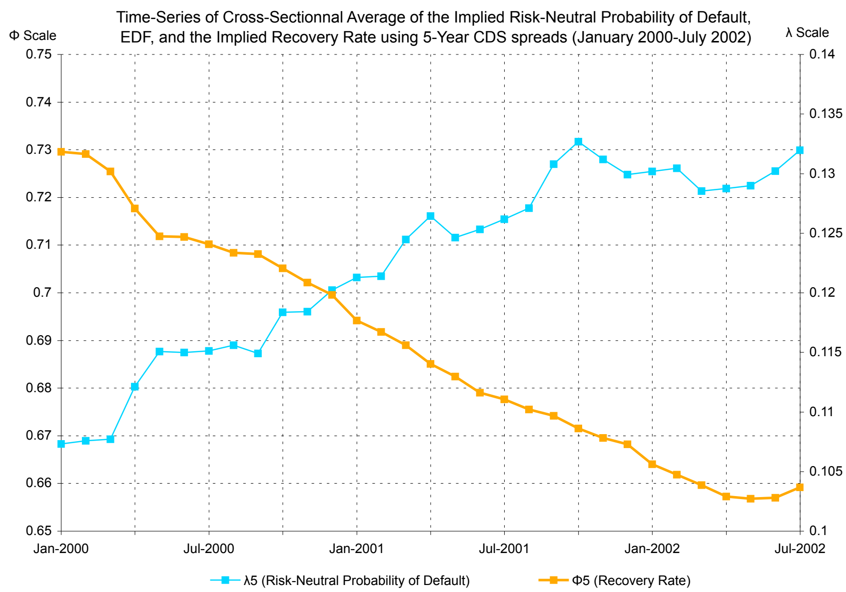

Figure 5: Cross-sectional average of the implied recovery rate (ϕ) and the corresponding risk-neutral probability (λ) for all firms in the sample for the period from January 2000 to July 2002. For each month and each firm, we computed the implied recovery rate using the Merton-based algorithm for each half-year forward period upto 5 years. The plot above uses the implied forward recovery rates for the last period, which proxies for the asymptotic recovery rate in the model. Recovery rates are implied using information from both, the equity and credit default swap markets. Full details of the algorithm used are presented in section 5.2. The recovery rate time series slopes downwards, and the default probability series slopes upwards, clearly evidencing the negative correlation between the two.

Figure 5: Default and recovery rates over time, from Das and Hanouna (2006) - "Implied Recovery"

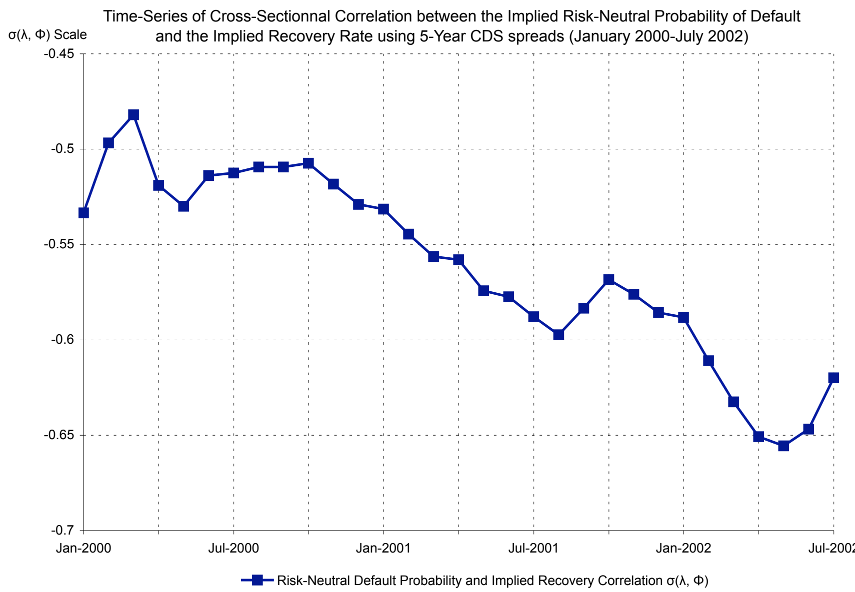

Figure 6: Cross-sectional correlation of the implied recovery rate (ϕ) and the corresponding risk-neutral probability (λ) for all firms in the sample for the period from January 2000 to July 2002. For each month and each firm, we computed the implied recovery rate and default probability using the Merton-based iterative algorithm for each half-year forward period upto 5 years. The plot above uses the implied forward recovery rates for the last period, which proxies for the asymptotic recovery rate in the model. Full details of the algorithm used are presented in section 5.2. Correlations between recovery and default rates are computed in the cross-section for each month. The correlations become more negative (increases in absolute sign) as default risk in the economy increases.

Figure 6: Correlation of default and recovery rates over time, from Das and Hanouna (2006) - “Implied Recovery”

probabilities of various counterparties. Second, the correlation between PD and LGD as just shown to be negative, and therefore, resulting in greater capital requirements. Third, is the contagion effect, where the incidence of defaults triggers more defaults.

And finally, in addition to these sources of correlation within the realm of credit risk, there is the interaction of market risk and credit risk as well. The sign of this correlation tends to be adverse as well. When market risk increases, the three credit correlations are also higher.

7 Non-Gaussian Distributions

Much of the regulatory framework for Basel II implementation is based on the single risk factor Gaussian framework. By using different joint distributions, we may assess the impact of incorrectly adopting the Gaussian model. In Das and Geng (2004) different copulas were applied to the PDs from Moodys over a fourteen year period (1987-2000). The Gaussian copula with normal marginal distributions was found to be inferior to the Clayton copula model with double exponential marginals. Figure 7 shows the cumulative loss distributions for four copulas assessed in their paper. Also, the tail of the Clayton copula loss distribution is seen to be much fatter than that of the student T. Hence, if anything the Gaussian model will understate the amount of capital required to be maintained.

8 Does accounting for regimes increase or decrease regulatory capital?

As we have seen, credit risk can vary substantially, both in the level of risk and in credit correlations across economic regimes. Correct maintenance of capital in a regimes-based model comes with complications, as we will now see.

With a time horizon of VaR for a year, a confusing issue arises, in that capital requirements account for more risk than is necessary. Consider the following thought experiment - say we are currently in a low risk regime in the economy. Also assume that we can, within a reasonable time (say 1 month) make substantive changes to the portfolio to mitigate risk. Then, accounting for a possible regime shift that might occur in one year, where a bad regime is feasible, will result in keeping more capital than is necessary because the probability of switching to the bad regime before the portfolio can be immunized is over-stated. On the other hand, if we are in a risky regime, but there is a likely switch into a low risk one, will result in keeping less

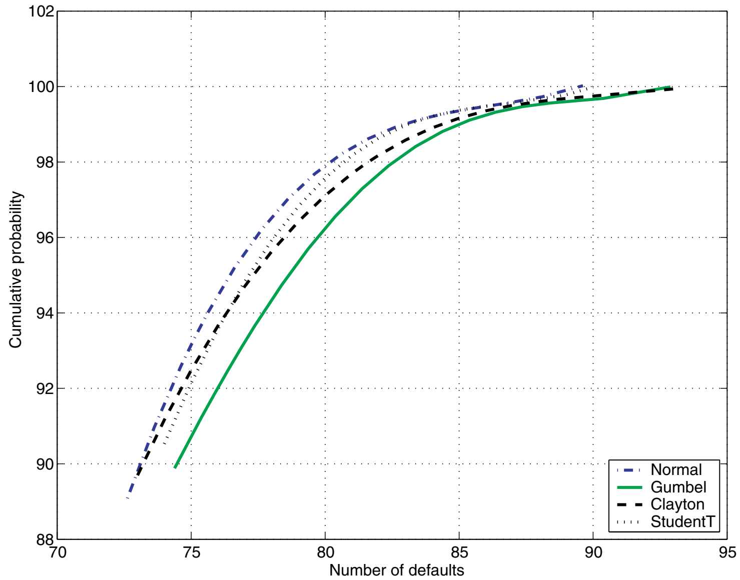

Figure 5 Comparing copula tail loss distributions: this figure presents plots of the tail loss distributions for four copulas, when the marginal distribution is normal. The *x*-axis shows the number of losses out of more than 600 issuers, and the *y*-axis depicts the percentiles of the loss distribution. The simulation runs over a horizon of 5 years and accounts for regime shifts as well. The copulas used are: normal, Gumbel, Clayton, Student's *t*.

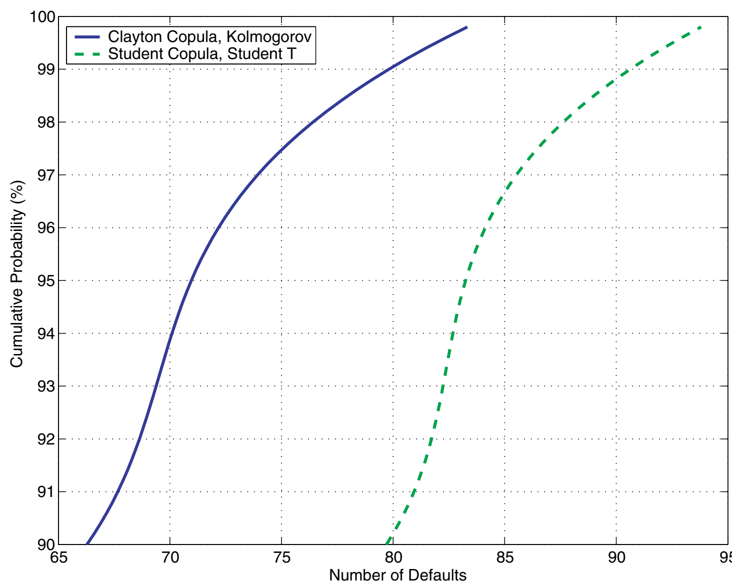

Figure 6 Comparing copula tail loss distributions: in this figure we plot the tail loss distributions for two models, the best fitting one and the worst. The best fit model combines the Clayton copula, and marginal distributions based on the Kolmogorov criterion. The worst fit copula combines the Student's *t* copula with Student's *t* marginals. The simulation runs over a horizon of 5 years and accounts for regime shifts as well.

Figure 7: Plots of loss distributions reproduced from the paper by Das and Geng (2004), showing the substantial increase is joint loss likelihood from the best-fitting Clayton copula.

capital than is currently necessary, unless there is infinite access to capital. Therefore, sometime we keep too much capital and at other times too little. In any case, we always keep incorrect amounts of capital.

See Gore (2006) for a discussion that relates to this issue in the context of retail banking risk. What the discussion suggests is that it might be best to use horizons appropriate for each business segment. Segments with low liquidity and longer times to restructure will attract more capital, which also correctly accounts for the liquidity risk in the product line. On the other hand, businesses that engage in liquid transactions, will naturally be more manageable and the liquidity effect will be small, resulting in keeping lesser amounts of capital. A one-horizon fits all approach clearly has its problems, and in particular, complicates keeping correct capital in regime switching environments.

9 Merton's 1997 Model

Merton (1977) showed that risk-based capital per dollar of liabilities for a financial or depository institution was the same as a put option on the bank's assets A with a strike price of the liabilities L plus interest thereon, i.e. Le^{rT}, where T is the maturity of the liabilities. This liability insurance is equal to risk-based capital C.

$$C = N(d_2) - \frac{A}{L} N(d_1)$$

where

$$d_1 = \frac{\ln(L/A) - 0.5\sigma^2 T}{\sigma\sqrt{T}}, \quad d_2 = d_1 + \sigma\sqrt{T}$$

Might it be possible to simply compute and report the Merton model capital required using this simple formula directly at the firm level? Implementation would be undertaken exactly in the same way as is done with the Merton (1974) model in various market implementations. We might think of this as a "top-down" approach to capital requirements. This is easily reported and also provides another point of comparison with the more detailed "bottom-up" approach. For amplification of this idea, see Merton and Perold (1993).

10 Regulatory safeguards

10.1 Floors on capital reductions

The BIS press release of 10th July, 2002, states - "More fundamentally, the Committee is proposing to alter the structure of the minimum floor capital requirements in the revised Accord. Under the new approach, there will be a single capital floor for the first two years following implementation of the new Accord. This floor will be based on calculations using the rules of the existing Accord. Beginning year-end 2006 and during the first year following implementation, IRB capital requirements for credit risk together with operational risk capital charges cannot fall below 90% of the current minimum required, and in the second year, the minimum will be 80% of this level. Should problems emerge during this period, the Committee will seek to take appropriate measures to address them, and, in particular, will be prepared to keep the floor in place beyond 2008 if necessary."

First, this has implications for the incentives to implement the new IRB based capital requirements, as there is a floor on the benefit that might be attained from moving to the IRB standard. Banks that are likely to have only a small reduction from moving to IRB will find that the benefits from capital requirement reductions might indeed be overwhelmed by the costs of implementing the new Basel II standard.

A *second* effect applies to banks that will experience large reductions in risk capital were they to use the new IRB approach. Such banks will inevitably be disappointed with the floors being placed on capital. In any case, when banks eventually suspend computing Basel I approaches, then the basis for minimum capital floors will need to be revisited.

Third, there are many points of tension between the old and new requirements. One might easily imagine circumstances where the risk weights lead to banks that have diversified their portfolios effectively using modern quantitative methods are disappointed when their lower risk levels are not rewarded by an actual reduction in capital required when they hit the floor. This might therefore, disincentivize the introduction of modern risk management methods. The floor requirement also penalizes banks that take active measures to reduce the risk of their franchises, even as they move towards the new IRB approach.

Fourth, in any case, it is unclear as to what the guidelines are for the national supervisor to assess the performance of banks so as to release them from the floor capital requirement at the end of the initial three-year period.

Fifth, banks will keep regulatory capital for meeting EL and also economic capital for further risk. But because we are moving to the IRB approach, the amount of capital to be maintained becomes much more variable given changes in the underlying variables that drive risk even when the portfolio composition does not change. Hence, there is an aspect of the floor requirement that is surely useful, in that it smoothes out fluctuations in capital since a bank already at the floor would not need to keep a reserve buffer given that it was already holding excess capital.

It is clear from the regulator's point of view that the transitional floor requirements are a way of implementing the Basel II framework in a "controlled" environment. Hence, one should not be too critical of the idea. Yet, we do need to take with a pinch of salt the alacrity with which regulators profess they will review their guidelines and remain flexible on changing the norms if they feel that there is a material reduction in capital requirements, failing which the banks would be exposed to unnecessary hardship as a consequence of the transitional floor requirements. Clearly, regulators would like banks to hold more low-risk assets, which did not occur under Basel I guidelines. Given this, the floor capital requirement does not point incentives in this direction.

10.2 Maintenance of minimum leverage requirement

Rules also stipulate a minimum leverage ratio, defined as Tier 1 Capital divided by the adjusted quarterly average Total Assets, after adjustments. The leverage ratio required is a minimum of 3-4% (tier 1 capital divided by average total consolidated assets. Average total consolidated assets equals quarterly average assets from a bank's most recent Call Report less goodwill and other intangible assets). Banking organizations must maintain a leverage capital ratio of at least five percent to be classified as well-capitalized.

This is over and above the Tier 1 capital ratio of 4% (tier 1 capital divided by risk-weighted assets) and a Total Capital ratio of 8% (the sum of tier 1 and tier 2 capital divided by risk-weighted assets). A well-capitalized institution maintains capital ratios 2% higher than the required guidelines.

The minimum leverage ratio does not account for off balance sheet assets and is likely to become increasingly redundant. One envisages a gradual phase-out of this measure.

How does one include leverage from off balance-sheet positions such as in derivatives? For example a long position in a call option may be transferred from off balance-sheet to on balance-sheet before computing the leverage ratio. This may be done by recognizing that the option is decomposable into a long position in equity and a short position in a loan. The equity position may then be added to the denominator of the ratio. Such decompositions are non-trivial across a large portfolio but will eventually enable us to establish correctly what leverage representations are especially in the case of an institutional environment in which derivatives are playing an increasing role.

11 A Proposal for Market Discipline

One of the pillars of the new Basel II accord is that of market discipline. A simple approach that may be added to the NPR is that banks also report their "distance-to-default" (DTD) as per the model of Merton (1974). All banks would then be required to maintain a minimum DTD, and if this fell below the acceptable levels, then the banks would need to recapitalize in order to comply.

Regulatory involvement would require the setting up of this level of DTD. We note that one single level of DTD can apply to all banks, as the DTD is a volatility and leverage adjusted measure, which accommodates differences across banks. Because it is a normalized measure, it is possible to equalize competitive differences across banks and is therefore a possibly useful approach. It is also based on market information and allows risk management of the banking system to be tied to the risk preferences of investors as well.

Regulators may use historical data on defaulted banks to assess the levels of DTD that are "critical" given the target failure rates the FDIC is willing to accept.

Regulators will also need to adjust the DTD limit for economic regime. Given that downturn regimes are characterized by increased default correlations, clearly DTD limits need to be changed so that contagion is not permitted to take root in the banking system. The ample research that now exists on correlated default may now be brought to bear in such a study.

One might conjecture here that such a measure is more transparent and also consistent with the risk management approaches that banks are more comfortable with.

One caveat here that also applies to the computation of minimum leverage ratios is that the application of the Merton (1974) model requires the correct amount of debt on the balance sheet and in order to assess this off balance-sheet items will also have to be correctly factored into the analysis.

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