Lease Financing, Credit Risk, and Optimal Capital Structure

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Abstract

This paper studies the impact of lease financing for a defaultable firm employing an optimal capital structure. We summarize ours results as follow. First, an increase in lease financing will decrease the optimal leverage ratio of the firm and its bankruptcy probability. Second, low tax rate will first decrease then increase the bankruptcy probability of the firm by increasing lease financing. Third, debt and lease contracts are substitutes, and the substitution ratio increases as debt maturity increases. Lastly, we explain the impact of off-balance sheet leases on the firm's security valuation.

Key words: Leasing; Credit risk; Capital structure
JEL Classification: G12, G32, G33.

1 Introduction

Corporate debt value under default risk has been widely discussed in the finance literature, especially after the boom of the credit derivatives market in recent decades. The existing corporate bond pricing literature focuses on the capital structure of the firm containing only debt and equity. Graham, Lemmon and Schallheim (1998) provide summary statistics of the financing sources of COMPUSTAT firms from 1981 to 1992, and find that 99.9% of the firms in the sample use operating leases, and 52.6% of them use capital leases. However, little is known about the interaction of lease financing, credit risk and the optimal capital structure. Therefore, the ignorance of the lease in capital structure will affect the capital structure decision and the corporate bond prices. We employ the Leland and Toft (1996) model, which endogenously determines the bankruptcy trigger level based on equity holders' value maximizing decision, to further discuss how lease financing affects the firm's optimal capital structure and related properties of corporate debt.

Capital structure theory starting with Modigliani and Miller (1958) concludes that the choice between debt and equity financing is not important on the value of firm. Later, by Jensen and Meckling (1976), financial distress and agency costs are included in firm valuation to modify the hundred percent debt financing concept of earlier work.

Myers and Majluf (1984) is the classical paper under asymmetric information concluding the firm follows a "pecking order" in raising money; finance internally (using retained earnings) first, then with debt, finally sell stock to raise money. Another view in the optimal capital structure literature is the tradeoff theory between tax gains and bankruptcy costs introduced to literature first by Miller (1977). As seen from the literature, capital structure theory has traditionally focused on the optimal levels of debt and equity. However, Myers, Dill and Bautista (1976) examine the firm's lease versus debt financing decision under the Modigliani and Miller environment. In their paper, the tax differential between lessee and lessor is an important reason for existence of lease.

In recent years, many papers have been published on leasing, and most of them are based on tax related incentives for leasing. Smith and Wakeman (1985) offer eight reasons for leasing besides tax motivation. Graham, Lemmon and Schallheim (1998) demonstrate other non tax related factors in addition to tax rate effects, such as financial distress, government regulation and firm size, that affect a firm's decision to lease instead of debt financing. They document a negative relationship between financial leases and tax rates, and positive relationship between debt levels and tax rates.

Sharpe and Nguyen (1995) find that firms facing high cost of external funds can economize on the cost of funding by leasing. Their results suggest that a low rated firm should use more lease financing compared to a highly rated firm after controlling the firm size and other factors. They also find that tax rate and leasing propensity is negatively correlated. Furthermore, based on their results, they suggest that a comprehensive analysis of capital structure should not disregard the role of leasing. Similarly, Krishnan and Moyer (1994) find that leasing reduces bankruptcy costs than borrowing, and it becomes attractive financing option as bankruptcy potential of a firm increases.

The issue of tradeoff between debt and lease has also been widely discussed in the literature, however, the empirical evidence is mixed. Finance theory predicts a negative correlation of debt and lease financing, implying they are substitutes. Ang and Petersen (1984) find that debt and leases are positively correlated, suggesting that they are complements. Hence, they call it a "leasing puzzle". Lewis and Schallheim (1992) also examine the complementarity of debt and lease financing, whereas the recent empirical paper of Beattie, Alan and Thomson (2002) suggest that debt and leases are substitutes. They find £1 of operating lease displaces £0.23 of debt over five years on average. Lasfer and Lewis (1998) show the substitutability of debt and lease contracts driven by taxes for large companies. Yan (2002) finds empirically that the degree of substitutability between debt and leases increases for firms facing more agency problems, or for firms having more redundant tax shields.

In this paper, we describe a continuous time structural framework to discuss the interaction between the use of debt and lease of a firm employing an optimal capital structure. It provides an insight that under the default risk and lease market competition, the firm which uses lease as a financing source will decrease its debt capacity, suggesting that lease and debt are substitutes. Thus, our analysis is related to the literature in explaining the leasing puzzle. We also show that increase in lease financing will decrease the optimal leverage ratio of the firm and its bankruptcy probability. Second, low tax rate will first decrease then increase the bankruptcy probability of the firm by increasing lease financing. Third, the substitution ratio increases as debt maturity increases. Lastly, we explain the impact of off-balance sheet leases on the firm's security valuation that the financial markets can not infer the extent.

The remainder of the paper is organized as follows. Section two describes the process of lease rate determination based on no arbitrage framework, and provides the fair lease rate under competitive lease market. Section three develops the model setting, and proposes a structural model based on Leland and Toft (1996) to express the components of firm value given the firm's use of debt and leases as its financing instruments. Section four derives the endogenous decision rule for the optimal bankruptcy level. Section five proposes a numerical example to illustrate the optimal capital structure, corporate bond properties, and the interaction of lease and debt. Finally, section six concludes the paper.

2 Determining the Lease Rates

A lease is a contractual agrement between two counter parties, the lessor and the lessee, for the use of a certain asset for a specific period of time. For the purposes of this paper we will distinguish between an operating lease and a capital lease. Operating lease[1] is a way of providing off-balance sheet financing listed in footnotes, in which the lessee pays the rent to exchange the economic value of the service provided by the leased asset, and retains the tax benefit, meanwhile, the lessor owns the asset and debits the depreciation expenses.

Off-balance sheet financing is attractive to the firms. In recent years, it is used as a source of financing to keep the pressure on corporate structure off, to show a low leverage ratio and default risk, and to increase firm's equity value. However, the collapse of Enron is the most recent example of the effect of omitting off-balance sheet financing on the capital structure.

During the 1990s, Enron grew rapidly, and moved into areas other than their specialization such as power plant, bandwidth and credit derivative trading among others[2]. Much of their growth involved large initial capital investment that were not expected to generate earnings in the short term. The management team had a couple of options to release this pressure on Enron's balance sheet. Issuing additional debt was unattractive because it would effect Enron's credit ratings. Alternatively, issuing additional equity was also unattractive because short term earnings were not expected. Therefore, they found outside investors, and formed a "Special Purpose Entity" (SPE)[3] primarily to isolate financial risk. One of the ways Enron used SPEs was for synthetic lease transactions. The synthetic lease is an off-balance-sheet financing that is accounted for as an operating lease, but treated for tax purposes as a capital lease. These transactions

[1]We assume a full service lease without any embedded options, in which the lessor promises to maintain and insure the underlying asset.

[2]See http://en.wikipedia.org/wiki/Enron for an extensive list of Enron's products.

[3]A SPE is typically owned by the parent company, but it is required that at least 3% be owned by other investors so that they do not have to consolidate the transaction under Securities and Exchange Commission (SEC) and Financial Accounting Standards Board (FASB) rules.

kept Enron's credit rating still in investment grade level. The following passage from the Wall Street Journal shows how heavily leasing is used:

> "...The scale of these off-balance-sheet obligations – stemming from leases on everything from aircraft to retail stores to factory equipment – can be huge:
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> - US Airways Group Inc., which recently filed for Chapter 11 bankruptcy protection, showed only \$3.15 billion in long-term debt on its most recently audited balance sheet, for 2003, and didn't include the \$7.39 billion in operating-lease commitments it had on its fleet of passenger jets.
> - Drugstore chain Walgreen Co. shows no debt on its balance sheet, but it is responsible for \$19.3 billion of operating-lease payments mainly on stores over the next 25 years.
> - For the companies in the Standard & Poor's 500-stock index, off-balance-sheet operating-lease commitments, as revealed in the footnotes to their financial statements, total \$482 billion.
> - When UAL filed for Chapter 11 bankruptcy protection, it showed \$25.2 billion of assets and \$22.2 billion of liabilities. Not included: \$24.5 billion in noncancellable operating-lease commitments, mostly for aircraft..."
>
> WSJ article (September 22, 2004; Page A1)

This is the shadow area of the capital structure. Even though the leverage ratio is one of the most important measures of a company's creditworthiness, omitting leasing in capital structure will impact the picture of a company's financial health.

In contrast to the operating lease, the capital lease[4] transfers all benefits and risks related to the ownership of asset from lessor to the lessee. The cost of providing service flow does not include the maintenance cost. In IRS regulation, a capital lease is usually classified as a "nontrue lease". Therefore, only the lessee deducts the depreciation expense on the balance sheet and maintains the asset. We will now formalize both types of leases in details and include them in capital structure.

We determine the leasing rate of operating and capital lease contracts by applying the spirit of Brennan and Kraus (1982), McConnell and Schallheim (1983), and Stanton

[4]FASB No. 13 provides that if a particular lease contract meets any of the following four conditions: (1) if there is a transfer of ownership to the lessee at the end of the lease term; (2) if the lease term exceeds 75 percent of the economic life of the asset; (3) if there is an option to purchase the asset at a bargain price at the end of the lease term; (4) if the discounted present value of the lease payments exceeds 90 percent of the fair market value of the asset.

and Wallace (2004) assuming no arbitrage condition. For simplicity, we consider the lease contract without any embedded option[5], and the leased asset value is determined by the present value of its future service flows. Assume the service flow before and after depreciation is given by the following stochastic differential equations:

$$\frac{dS_{BD}(t)}{dS_{BD}(t)} = \mu_S dt + \sigma_S dW_S(t), \quad \text{and} \tag{1}$$

$$\frac{dS_{AD}(t)}{dS_{AD}(t)} = (\mu_S - q)dt + \sigma_S dW_S(t) \tag{2}$$

where drift coefficient (μ_S), volatility (σ_S), and depreciation rate (q) are constant. $W_S(t)$ denotes the standard Brownian motion for the service flow process. We define the operating and capital lease payments in the following subsection.

2.1 Operating Lease Payment

Suppose lessor writes an operating lease contract of t periods with lessee, and the leased asset provides service flow continuously following the stochastic differential equation (1). Similar to Smith and Wakeman (1985), the lessor's net cost to offer the contract equals the economic value of the service flow before depreciation provided by the leased asset minus the tax deductible[6] of the depreciation expense:

$$\widetilde{E}\left(\int_0^t S_{BD}(u)e^{-ru}du\right) - T_c\left[\widetilde{E}\left(\int_0^t S_{BD}(u)e^{-ru}du\right) - \widetilde{E}\left(\int_0^t S_{AD}(u)e^{-ru}du\right)\right] \tag{3}$$

where $\widetilde{E}(\int_0^t S_{BD}(u)e^{-ru}du)$ represents the expected present value of the service flows before depreciation from time 0 to t under the risk neutral measure; similarly, $\widetilde{E}(\int_0^t S_{AD}(u)e^{-ru}du)$ represents the one after depreciation. The difference between these two terms is the depreciation cost of the leased asset. T_c is the corporate tax rate of lessor. In a competitive market, net cost equals the present value of continuous future lease payments from 0 to t represented by $\int_0^t r_o^t e^{-rs}ds$, which is also equal to $r_o^t(\frac{1-e^{-rt}}{r})$.

[5]McConnell and Schallheim (1983), Stanton and Wallace (2004), Grenadier (1995), Grenadier (1996) incorporate many embedded option in lease contract to discuss the fair leasing rate. But, here we only focus on the interaction between lease financing and capital structure; therefore, we ignore those option properties in lease contract.

[6]About the accounting and tax rules for leases, see Graham, Lemmon, Schallheim (1998), Appendix A. Bankruptcy Code §365 governs relationship between a lessee and lessor when the lessee files a bankruptcy proceeding.

r_o^t denotes the rent for a t period operating lease contract. Therefore, the competitive operating lease payment can be represented as[7]:

$$r_o^t = S_0 \left(\frac{e^{(\mu_S - \delta\sigma_S - r - q)t} - 1}{\mu_S - \delta\sigma_S - r - q} \right) \left[(1 - T_c)\, Q + T_c \right] \left(\frac{r}{1 - e^{-rt}} \right) \tag{4}$$

where, $Q = (\frac{e^{(\mu_S - \delta\sigma_S - r)t} - 1}{e^{-(\mu_S - \delta\sigma_S - r - q)t} - 1})(\frac{\mu_S - \delta\sigma_S - r - q}{\mu_S - \delta\sigma_S - r})$, S_0 is the initial value of the service flow process, and δ is the market price of the risk factor implicitly in the service flow process.

2.2 Capital Lease Payment

Suppose the lessor signs a capital lease contract of t periods, and the leased asset provides service flow continuously following the stochastic differential equation (2). The lessor's net cost of providing the lease service is the expected present value of service flow after depreciation, which equals $\widetilde{E}(\int_0^t S_{AD}(u) e^{-ru} du)$. Under the competitive lease market, it is the present value of the lease payments:

$$\widetilde{E} \left(\int_0^t S_{AD}(u) e^{-ru} du \right) = r_c^t \left(\frac{1 - e^{-rt}}{r} \right) \tag{5}$$

where r_c^t denotes the capital lease payment with a t period contract on a leased asset which follows equation (3) to provide the service flow. Thus, the capital lease rent, r_c^t, can be represented as:

$$r_c^t = S_0 \left(\frac{e^{(\mu_S - \delta\sigma_S - r - q)t} - 1}{\mu_S - \delta\sigma_S - r - q} \right) \left(\frac{r}{1 - e^{-rt}} \right). \tag{6}$$

If there is no depreciation, the depreciation effect on the service flow and the tax deductibility of depreciation expense will disappear and Q in the equation becomes one. Therefore, the operating lease rent and the capital lease rent are the same.

3 The Capital Structure Setting

In this section, we provide an extension of the Leland and Toft (1996) structural model to investigate the lease financing on the capital structure of a firm. Let V be the

[7] Proof of this equation is given in Appendix A.

unleveraged asset value of the firm which assumes the following stochastic differential equation with constant volatility:

$$dV(t) = (\mu_V(t) - \delta_V)\, V(t)dt + \sigma_V V(t)dW_V(t), \tag{7}$$

where $\mu_V(t)$ is the instantaneous expected rate of return of asset, δ_V the total dollar payout by the firm per unit of assets, σ_V is the constant volatility of the underlying assets return, and $W_V(t)$ is a standard Brownian motion. Equity holders under Leland and Toft (1996) have the full information of the asset process, and choose a bankruptcy boundary V_B endogenously to maximize their value. The firm defaults on its debt once the firm value crosses the boundary.

Suppose the firm uses debt and leases to finance its productive activities, and in lease financing the firm uses capital leases and operating leases simultaneously. We will examine the composition of the capital structure in the following subsections.

3.1 Lease Contract Value

Consider that the firm (lessee) signs a capital lease contract with a maturity of t periods to lease an asset for production. Firm has the right to use the leased asset, but also has the responsibility to maintain the asset. In accounting standards, the capital lease is treated as a long-term liability and the lessee has to place the value of the leased asset in the balance sheet at the beginning of the contract. For the duration of the contract, firm is responsible for the maintenance, therefore, it debits the depreciation expenses periodically on the balance sheet. Hence, firm can benefit from the depreciation expense. Also, if firm defaults on its debt payment and files for bankruptcy, it stops the payment of the debt claims until the bankruptcy is resolved[8] . Meanwhile, under the petition of the court, firm can still keep using the leased asset only if it can pay the lease payments before the bankruptcy is resolved. The capital lease has the higher priority of claim than the secured debt in case of bankruptcy, which is an advantage to a lessor. Therefore, it can be treated as a risky fixed income instrument with a proportion of the future lease payments used as the recovery rate when bankruptcy occurs.

Similarly, in the case of an operating lease, if firm files for bankruptcy, the court can require that firm continues to make the rental payments to the lessor until the

[8]Refer to Barclay and Smith (1995), or Graham, Lemmon, and Schallheim (1998). In practice perspective, refer to Rosen, A. and Rooney, P. (2002)

bankruptcy process is resolved. In this situation as in a capital lease, the lessor will be allowed to recover some of its future rental payments.

Consider that the firm issues a t maturity of capital and a t maturity of operating lease contract, i.e. $i = c, o$, which continuously pays a constant lease rent of r_i^t. Once the firm crosses the bankruptcy triggering value of V_B, the firm defaults on its debts. Because of the nature of lease contracts, the lessor could recover part of the remaining lease payments even after bankruptcy. Let ρ_i^t denote the fraction of the remaining lease payments, for a t period lease contract i, in which the lessor could receive in the event of bankruptcy. R_i^t denotes the present value of remaining lease payments at bankruptcy time t. Under the risk-neutral valuation, the value of the lease contract, l_i, is given by

$$l_i(V; V_B, t) = \int_0^t e^{-rs} r_i^t \left(1 - F(s; V, V_B)\right) ds + \int_0^t e^{-rs} \rho_i^t R_i^s f(s; V, V_B) ds \tag{8}$$

where $F(s; V, V_B)$ is the cumulative bankruptcy probability up to time s, and $f(s; V, V_B)$ is the instantaneous bankruptcy probability at time s. The first term on the right hand side represents the expected discounted lease payments flow from 0 to t. The second term represents the expected discounted value of remaining lease payments conditional on the occurrence of bankruptcy. R_i^s is the present value of remaining lease payments at the bankruptcy time s calculated by (5). We can now obtain the lease contract value applying the integration by parts in equation (8):

$$\begin{aligned} l_i(V; V_B, t) &= r_i^t \left(\frac{1 - e^{-rt}}{r}\right) - \left(\frac{r_i^t}{-r} F(t) e^{-rt} + \frac{r_i^t}{r} G(t)\right) \\ &\quad + \rho_i^t \frac{r_i^t}{r} \left(G(t) - e^{-rt} F(t)\right). \end{aligned} \tag{9}$$

Definitions of $F(t)$, $G(t)$ are explained in Appendix B. Under endogenous bankruptcy boundary (V_B), the bankruptcy probability is trivially affected by this factor and all contingent claims on the firm's assets are surely affected by the bankruptcy boundary decision.

The value of operating lease contract shares the same features with the value of capital lease contract except the lease rent and the default recovery ratio. Because of the differences in tax treatment and the maintenance responsibility between capital lease and operating lease, the lease rents determined by the competitive market will not be the same. Furthermore, the differences in recovery ratio is determined by the contract provision, the lessee's bargaining power, or the decision of the court when lessee goes into bankruptcy.

3.2 Debt Contract Value

Consider a single bond issue with maturity t, coupon $c(t)$, and principal $p(t)$. Upon bankruptcy, the bondholder recovers a fraction ($\rho_D(t)$) of defaulted net asset value of $\widetilde{V}_B$, where $\widetilde{V}_B$ represents the net asset value after bankruptcy cost minus the present value of lessor's recovery lease payments. We can write the value of risky debt as:

$$\begin{aligned} d(V;V_B,t) &= \int_0^t e^{-rs}c(t)\left(1-F(s;V,V_B)\right)ds + p(t)e^{-rt}\left(1-F(s;V,V_B)\right) \\ &\quad + \int_0^t e^{-rs}\rho_D(t)\widetilde{V}_B f(s;V,V_B)ds. \end{aligned} \tag{10}$$

If there is no bankruptcy, the first term on the right hand side of (10) represents the present value of coupon payments, and the second term represents the present value of principal payment, respectively. The third term represents the present value of the net asset value, which will go to debt holders when bankruptcy occurs. We also can rewrite the equation after integrating by parts as:

$$\begin{aligned} d(V;V_B,t) &= \frac{c(t)}{r}\left(1-e^{-rt}\right) - \left(\frac{c(t)}{r}G(t) - \frac{c(t)}{r}F(t)e^{-rt}\right) + e^{-rt}p(t)\left(1-F(t)\right) \\ &\quad + \int_0^t e^{-rs}\rho_D(t)\widetilde{V}_B f(s;V,V_B)ds. \end{aligned} \tag{11}$$

3.3 Aggregated Value of Lease and Debt Contracts

In order to simplify the analysis, we assume that the firm is under an environment with stationary debt and lease structure. Under this environment, the firm continuously sells a constant amount of new debt issued with maturity T and principal amount of $p = P/T$ per year where P is the total principal amount of the outstanding debts. The same amount of old debt is retired per year and so the debt structure is stationary. Therefore, as long as the firm is still solvent, the total outstanding principal amount is P at any time s, and the firm value has a uniform distribution over maturities in the interval (s, T). The total coupon payment is C per year where debt with principal pays a constant coupon $c = C/T$ and the total annual debt service payment is $C + P/T$. With regards to the recovery rate, we assume equal seniority of all outstanding debt which implies that the sum of all fractional claims ($\rho_D(t)$) for debt of all maturities outstanding equals one and $\rho_D(t) = 1/T$ per year for all t.

In addition, the firm signs the lease contract with maturity T continuously, and it will expire at maturity if the firm is still solvent. Suppose that the total lease payments of outstanding capital lease contracts per year is Ω_c which is equal to w_1 units of capital lease rent with maturity T, and the total lease rent of outstanding operating lease contract per year is Ω_o which is equal to w_2 units of operating lease rent with maturity T. Therefore, following the same logic as the stationary debt structure, the capital lease contract with maturity T pays a constant rent $r_c^T = \Omega c/T$ per year under the stationary capital lease structure. The sum of all fractional claims ρ_c for capital lease contract of all maturities outstanding is equal to ρ_c and $\rho_c^t = \rho_c/T$. Similarly, the operating lease contract with maturity T pays a constant rent $r_o^T = \Omega o/T$ per year under the stationary operating lease structure. The sum of all fractional claims ρ_o^t for capital lease contract of all maturities outstanding is equal to ρ_o and $\rho_o^t = \rho_o/T$. Based on these assumptions, we can determine the value of all outstanding bonds and leases. The value of all outstanding capital, $L_c(V; V_B, T)$, and operating leases, $L_o(V; V_B, T)$, can be determined by integrating equation (10) over the period T for $i = c, o$:

$$\begin{aligned} L_i(V; V_B, T) &= \int_0^T l_i(V; V_B, t)dt \\ &= \frac{\Omega_i}{r} - \frac{\Omega_i}{r}\left(\frac{1-e^{-rT}}{rT} - \left(1 - \frac{\rho_i}{T}\right) I(T)\right) - \frac{\Omega_i}{r}\left(1 - \frac{\rho_i}{T}\right) J(T). \end{aligned} \tag{12}$$

Definition of $I(T)$, $J(T)$ are explained in Appendix B. Besides, under the assumptions given above, we can think that the outstanding capital lease at anytime is w_1 units of capital lease with maturity T. Hence, the depreciation expense associated with capital lease is given by[9]:

$$Dep_c = \frac{w_1\left(\widetilde{E}\left(\int_0^T S_{BD}(u)e^{-ru}du\right) - \widetilde{E}\left(\int_0^T S_{AD}(u)e^{-ru}du\right)\right)}{T}. \tag{13}$$

We can rewrite the equation (11) using integration by parts for the last integral, and using the definition of $\tilde{V_B}$ as:

$$\begin{aligned} d(V; V_B, T) &= \frac{c(t)}{r}\left(1 - e^{-rt}\right) - \left(\frac{c(t)}{r}G(t) - \frac{c(t)}{r}F(t)e^{-rt}\right) + e^{-rt}p(t)\left(1 - F(t)\right) \\ &\quad + \rho_D(t)\left((1-\alpha)V_B G(t)\right) \\ &\quad - \rho_D(t)\left(\rho_c(t)\left(\frac{\Omega c}{r}\right) + \rho_o(t)\left(\frac{\Omega o}{r}\right)\right)\left(G(t) - e^{-rt}F(t)\right). \end{aligned}$$

[9]For simplicity, we assume the leased asset is linearly depreciated. We can apply other depreciation methods dependent on the accounting practice.

Let $D(V; V_B, t)$ denote the total value of debt, when debt of maturity T is issued. Integrating all outstanding debt, we can have the debt value over period T as below:

$$\begin{aligned} D(V; V_B, T) &= \int_0^T d(V; V_B, t)dt \\ &= \frac{C}{r} + \left(P - \frac{C}{r}\right)\left(\frac{1-e^{-rT}}{rT} - I(T)\right) + \left((1-\alpha)V_B - \frac{C}{r}\right)J(T) \\ &\quad - \left(\left(\frac{\rho_C}{T}\right)\left(\frac{\Omega_C}{r}\right) + \left(\frac{\rho_o}{T}\right)\left(\frac{\Omega_0}{r}\right)\right)(J(T) - I(T)). \end{aligned} \tag{14}$$

The third term on the right hand side represents the debt value loss because of its junior claim priority, and this term is exactly the recovery value of the operating lease contract and capital lease contract when the lessee firm bankrupts. In most term structure literature, like Merton (1974), Black and Cox (1976), Brennan and Schwartz (1978) and Leland and Toft (1996), they assume that debt is the only instrument in a firm's liability. In our framework, we consider the leasing finance that is not mentioned in previous literature in determining debt value. From equation (14), we find that the debt value will be affected by the lease contract through the amount of outstanding lease contracts and the fraction of remaining payments the lessors can recover. Therefore, all relevant debt properties that include the yield-spread, duration, and convexity will also be affected by lease financing.

4 Determining the Endogenous Default Boundary

In traditional capital structure trade-off theory, the firm has to trade off between tax benefit and bankruptcy cost of debt financing. In our setting, we incorporate the lease financing into the capital structure decision. Therefore, the benefit of tax deductibility is composed of interest rate expense of debt, lease payment, and depreciation expense associated with capital lease. On the other hand, bankruptcy cost is composed of all related cost associated with bankruptcy procedure. Following Leland (1994), the total market value of the firm can be represented as the unleveraged firm value plus the tax benefit of debt and lease financing minus the bankruptcy cost during the observation period. The relation can be represented as:

$$v(V; V_B) = V + t_c\left(\frac{C}{r} + \frac{\Omega_o}{r} + \frac{\Omega_c}{r} + \frac{Dep_c}{r}\right)\left(1 - \left(\frac{V_B}{V}\right)^x\right) - \alpha V_B\left(\frac{V_B}{V}\right)^x \tag{15}$$

where $v(V;V_B)$ is the market value of the firm, V is the unleveraged firm value, Dep_c is the deprecation expense associated with capital lease, t_c is the corporate tax of the lessee, and $x = a + z$, where a and z are defined in Appendix B. The second term in equation (15) represents the tax benefits associated with interest rate expense, lease payments, and depreciation expense given the firm is not in bankruptcy yet. The third term is the bankruptcy cost. Therefore, equation (15) is consistent with the traditional capital structure theory, which takes tax deductibility as a positive effect to firm value, and the bankruptcy cost as a negative effect to firm value. In addition, our setting also considers lease financing and default probability to enrich the flexibility of the previous models.

To determine the bankruptcy boundary, we solve the equation to find out the endogenous bankruptcy boundary, V_B, in this model. Let

$$\left.\frac{\partial E(V;V_B,T)}{\partial V}\right|_{V=V_B} = 0$$

by solving this equation, we can get the endogenous bankruptcy boundary given by:

$$V_B^* = \frac{\left(\frac{\Omega_o}{r} + \frac{\Omega_c}{r}\right)(K_1 - K_2) - K_3 - \left(P - \frac{C}{r}\right)K_1 - \left(\frac{C}{r}\right)K_2}{1 + \alpha x - (1-\alpha)K_2} \quad (16)$$

where, the definition of K_1, K_2, K_3 are listed in Appendix B. Here, if we suppose lease payments are equal to 0, we can obtain the same formula as the one defined in Leland and Toft (1996) equation (11). Hence, if we include the lease in the capital structure decision, the amount and the recovery rate of lease will affect the bankruptcy boundary of the firm.

The inclusion of lease financing can decrease the bankruptcy boundary through the default recovery rate of the lease contract, the tax-deductibility of lease payments, and depreciation expense associated with capital lease. It can increase the bankruptcy boundary through the perpetual lease payments. In addition, the lease rent is determined by the interaction of the lessor and the lessee through the tax deductibility of depreciation expense and the maintenance expenditure. Therefore, lease market condition also affects the lessee's capital structure decision.

5 Numerical Analysis

In this section, we will conduct a numerical analysis of the model we set up in previous section. Our aim is to shed a light on the lease financing, and explain some of the issues discussed in literature.

We assume the firm not only uses debt, but also uses lease contracts to finance its investment. According to the liquidation rule of the bankruptcy law, the lease financing has more flexibility and higher claim priority than debt. Therefore, the existence of the lease will affect the value of debt and relevant financial decisions specifically under the default risk. We will also illustrate the "debt-lease substitutability", which has been discussed in the finance literature for many years, but has never been discussed under the endogenous default and continuous-time setting. Hence, we follow Lewis and Schallheim (1992) to illustrate this issue in our setting. We follow the steps of Leland and Toft (1996) to conduct the numerical analysis.

First, we set the initial asset value equal \$100 and determine the issuing coupon and principle value assuming bond is issued at par[10]. Later, we adopt the parameters provided in Leland and Toft (1996) as our base case[11]. In terms of lease parameters, we employ the parameters estimated in Stanton and Wallace (2004). In the literature, there is no complete and general empirical study about the lease valuation and relevant parameter estimation except Stanton and Wallace (2004). The lease dataset used in their paper is relatively complete compared to previous research, but the dataset focusses only on real estate leases. In our numerical examples, we assume the volatility of the service flow process (σ_S) equals to 4.8%, the risk-adjusted return ($\mu_S - \delta\sigma_S$) equal to 2.01%[12].

We will now conduct the numerical analysis to examine how lease financing affects the optimal leverage ratio of a firm, the bond pricing and the yield to maturity. Further, we will assume different lease bundles and examine how these bundles affect the components of a firm. Finally, we will examine the "debt-lease substitutability" issue, and affects of taxes on this ratio.

[10]Because of parameters chosen, the debt value may not match the principal exactly. In this case, we choose the coupon rate which minimizes the difference between debt value and principle as the coupon rate for the numerical illustration.

[11]About the source and selection criteria of these parameters, see Leland and Toft (1996).

[12]The parameters we adopted here are the mean of the risk-adjusted growth rate and the mean of spot rent volatility which are listed in Table 2 of Stanton and Wallace (2004).

5.1 Optimal Leverage

Figure (1) plots the firm value versus the leverage ratio, which is defined by the firm's debt value over the total value, for different initial service flows assuming the firm uses 1.0 unit of operating lease and 0.2 unit of capital lease. The firm issues debt with maturities 5 years and 20 years. The lease bundle we adopted is based on Graham, Lemmon and Schallheim (1998)[13]. We observe that increase in debt maturity will increase optimal leverage ratio of the firm. Whereas, increase in lease financing will decrease the optimal leverage ratio for all debt maturities while increasing the firm value.

If firm starts to incorporate lease contracts in capital structure, it can benefit from the tax shield. As the leverage ratio increases, the default probability will increase, the tax shield benefit will decrease and the bankruptcy loss will increase. If firm increases its lease financing, the tax benefit will increase, and the effect caused by the increase of default probability will be more profound compared with lower lease-use case. Hence, from Figure (1), we can observe that no matter what the maturity of the debt, the maximized firm value increases and optimal leverage ratio decreases, as lease financing increases.

5.2 Price and Yield of Corporate Bond

Figure (4) plots the bond value per $100 principal and yield to maturity of the firm's outstanding bond as a function of maturity at different levels of leverage and service flow. We suppose firm uses 30%, 50%, and 70% leverage ratios to observe the change of bond value and yield to maturity. The upper two panels are bond value versus bond maturity under different initial service flows, and the two bottom panels are yield to maturity versus bond maturity under different initial service flows.

In the bond value panels, we observe that when the initial service flow is equal to 0, the patterns of bond price are very similar to the pattern presented in Leland and Toft (1996). The "about-to-be redeemed bond" converges to par because of the basic property of the bond. The bond value with maturity between 0 to T may be greater or less than par, and have the sigmoid shape. The sigmoid effect is more significant when the firm uses more lease, and the bond maturity is longer. Specially, when firm uses a 5 unit initial lease under 70% leverage, the bond value is below par most of the bond

[13]They provide a summary statistic of large proportion of the companies in COMPUSTAT from 1981-1992. They find the capital lease over total firm value is about 1.6%, and operating lease over total firm value is about 8% on average. The ratio of operating lease over capital lease is about 5.

maturity. It shows that if firm is highly leveraged and using high level of lease financing, it will lead to a seriously reduced bond value because the increase in leasing increases bankruptcy probability. When time approaches to maturity, bankruptcy probability is getting smaller; therefore, in a typical case, the bond price will go above par value even in the sigmoid situation. But, if the firm is highly leveraged and the leasing amount is large, the bond value reduction caused by the high default probability may be stronger than the bond value increase caused by the default probability decrease due to its approaching maturity.

The bottom panels of Figure (4) are about the yield to maturity in different bond maturities. When bond maturity is less than five years (the bottom left panel), the yield to maturity increases while the firm increases its lease financing. In addition, the yield to maturity is also higher when the firm is highly leveraged. The mark up for the yield to maturity is to compensate the default risk due to leverage. If firm is highly leveraged and increases its lease financing, the yield to maturity shows a different pattern from others. The yield to maturity is upward sloping in this case. One explanation of this is due to the fast convergence of the bond price to par when it approaches to its maturity. The upper right panel also shows a fast convergence of bond value to par from the area below the par as T is approaching 0 when firm's leverage ratio equals 70% and initial service flow equals 5.

5.3 Cumulative Bankruptcy Probability

Increase in lease financing can provide a tax shield, and decrease the bankruptcy trigger level. In this section, we will show the effect of leasing in bankruptcy probability. The cumulative bankruptcy probability of the firm over the interval $(0, s]$ is defined as in Leland and Toft (1996):

$$N\left(\frac{-b-\lambda s}{\sigma\sqrt{s}}\right) + e^{\frac{-2\lambda b}{\sigma^2}} N\left(\frac{-b+\lambda s}{\sigma\sqrt{s}}\right) \tag{17}$$

where $\lambda = \mu(s) - \delta_V - 0.5\sigma_V^2$, and we assume $\mu(s)$, the total rate of return, is fixed at 15% per year, and the risk premium is 7.5% over the default-free interest rate.

Figure (5) shows cumulative bankruptcy probabilities under different initial service flows over time. As time goes by, the cumulative bankruptcy probability increases and impact of increase is stronger for longer debt maturities. Increase in lease financing decreases the cumulative bankruptcy probability due to the tax-shield benefit. We can confirm the trade-off relationship in equation (15).

The inclusion of lease contract in capital structure can increase the bankruptcy trigger level, because it increases the cash outflow. On the other hand, the inclusion of lease can also decrease the bankruptcy trigger level, because of the tax-shield benefit. In our case, the positive effect of tax shield on the bankruptcy trigger level is much higher than the negative effect brought by the lease payments. Therefore, the inclusion of lease contract in capital structure decreases the bankruptcy trigger level; furthermore, it decreases the cumulative bankruptcy probability.

5.4 The Tradeoff between Lease and Debt

The issue of substitutability or complementarity between debt and lease contract has been discussed in the finance literature for many years, but empirical research findings are mixed, and the theoretical research concerning the topic is rare. Specifically, previous research only focused on the "true lease" which is equivalent to the operating lease, and mentioned capital leases only in passing. Additionally, all researchers except Lewis and Schallheim (1992) excluded the optimal capital structure decision from the model setting. Therefore, this paper provides theoretical insights about the lessee's default risk and endogenous capital structure decision.

Figure (2) plots the debt value of the firm versus the initial service flow given firm is optimally leveraged under different maturities. We observe that firm's optimal debt value decreases with the use of leasing, and increases with the maturity of the debt. Based on this observation, we can say that the leases and debt are substitutes in terms of amounts. This observation is consistent with the findings of Figure (1). If firm uses more leasing, optimal leverage ratio will decrease based on Figure (1). Therefore, the outstanding debt is less, so does the total debt value.

In order to further discuss the effect of leasing on debt value, we plot the value of the debt versus the leverage ratio under different initial service flow. In Figure (3), we observe that maximal debt value, called debt capacity, decreases with the use of leasing in each maturity scenario and the debt capacity increases with maturity. Hence, the debt value decreases with the use of lease in a optimally-leveraged case and also in terms of debt capacity. This is consistent with the empirical results of Beattie, Goodacre, Thomson (2000) and Yan (2002) who show that debt and leases are substitutes. We also conduct the numerical analysis with different bundles of lease to check if the combination of leases affects our result. In Table 1, Panel A is the numerical result of the base case which we used in Figure (1), Figure (2), and Figure (3). We suppose that total units of leasing is fixed at 1.2 units. Panel B is the numerical result of the lease bundle which fully uses operating lease, and Panel C is the numerical result of the lease bundle which fully uses capital lease. In these three bundles, we can observe that the debt value still decreases with the lease usage for each maturity. Although the results in these three panels do not differ too much, we can observe the firm value in Panel C leads in these 3 panels as lease usage increases.

In an operating lease, the deprecation cost associated with the leased asset is reflected in the lease rent through the competition of the lease market and the tax rate of the lessor. Therefore, the tax benefit of the deprecation cost associated with the leased asset is determined by the tax status of the lessee and the lessor. In Table (1), the tax rate assumptions for the lessor and the lessee in every case are the same. The capital lease user benefits from the tax deductibility of the leased asset's deprecation expense endogenously, but the operating lease user benefits the tax deductibility of leased asset's depreciation expense exogenously through the competition of the leasing market. Hence, in Table (1), the results show that a capital lease provides a more efficient inner channel to utilize the tax benefit of depreciation expense than an operating lease given the same tax assumptions.

Table (2) illustrates the comparative statistics of bankruptcy boundary, tax rate, and lease relationships calculated from equation (16). Lessor tax rate is kept constant at 30% level while lessee's tax rate changes. Table (2) shows that low tax rate will first decrease and then increase bankruptcy probability as lease financing increases. Whereas, keeping the tax rate relatively high or on average, bankruptcy probability will decrease as lease financing increases. Magnitude of negative effect of bankruptcy probability versus lease amount decreases as maturity goes up. We have also examine the relationship of tax rate versus leasing in Table (3). We find that increase in tax rate will decrease both the operating and capital lease ratios.

Table (4) summarizes the lease-to-debt substitution ratio based on the equation, $DR_{NL} = DR_L + \alpha LR_L$ as in Ang and Peterson (1984). DR_{NL} is the debt ratio of a non-lease firm, DR_L is the corresponding debt ratio of a similar firm which uses lease, LR_L is the lease ratio of the latter, and α is the debt-to-lease ratio. We have redefined α in terms of lease-to-debt substitution ratio. α values of exactly one corresponds the perfect substitution of lease and debt financing. If leasing and debt are substitutes, but not on a dollar for dollar basis, then the the debt to lease displacement ratio should be strictly positive. A negative debt to lease displacement ratio indicates that leasing and debt are complementary. As seen from the table, they are not perfect substitutes, but the substitution ratio increases with more lease financing. It is even higher for long term debt than short term debt, and roughly six times more for operating lease compare to the capital lease. The results also imply that on average 1 dollar of leasing displaces approximately 13.52 cents for the debt maturing at 6 months, and 74.04 cents for the debt maturing at 20 years.

6 Conclusion

We employ the Leland and Toft (1996) structural model and incorporate lease financing in the firm's capital structure. This helps us explain some of the incentives for leasing, empirically analyzed in literature, namely, debt capacity, agency cost, and taxation.

First, the inclusion of leasing significantly decreases the optimal leverage ratio and the debt capacity, suggesting that the lease and debt are substitutes. We also adapted the equation used in Ang and Peterson (1984) to examine the substitution effect in details, and find the effect is higher for long term debt than short term debt, supporting the argument of long term financial leases are nearly equivalent of debt financing. Second, we find that the cumulative default probability can be reduced by the inclusion of lease financing in capital structure. Third, we find that increase in tax rate will decrease the lease financing, also documented in Graham, Lemmon, and Schallheim (1998). We also document that low tax rate will first decrease then increase the bankruptcy probability of the firm by increasing lease financing.

Lastly, we examine the bond value and its yield to maturity under different maturity terms. The bond value is seriously reduced when firm is highly leveraged and uses a high level of lease financing. The yield to maturity is also enhanced because the bond price in this case greatly reverses when the bond is approaching to its maturity.

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A Appendix

By applying Ito's lemma to the service flow process in equation (2.1), we can derive:

$$S_{BD}(t) = S_0 e^{(\mu_S - \frac{1}{2}\sigma_S^2)t + \sigma_S W_S(t)} \tag{A-1}$$

where S_0 is the initial service flow of the leased asset, and $W(t)$ is the standard Brownian motion at time t under the true measure. Because the leased asset may not be very liquid or easily traded, in order to obtain the expected present value of the discounted service flow, we employ the market price risk adjustment to derive the service flow process under the risk-neutral measure:

$$S_{BD}(t) = S_0 e^{(\mu_S - \delta\sigma_S - \frac{1}{2}\sigma_S^2)t + \sigma_S \widetilde{W}_S(t)} \tag{A-2}$$

where δ is the market price of risk, and $\widetilde{W}(t)$ is the standard Brownian motion under risk-neutral measure. Using the equation (1.19), the present value of future service flow is:

$$\begin{aligned} \widetilde{E}\left(\int_0^t S_{BD}(u)e^{-ru}du\right) &= \widetilde{E}\left(\int_0^t S_0 e^{(\mu_S - \delta\sigma_S - \frac{1}{2}\sigma_S^2)u + \sigma_S \widetilde{W}_S(u)} e^{-ru}du\right) \\ &= S_0 \widetilde{E}\left(\int_0^t e^{(\mu_S - \delta\sigma_S - \frac{1}{2}\sigma_S^2)u + \sigma_S \widetilde{W}_S(u)} e^{-ru}du\right). \end{aligned} \tag{A-3}$$

Furthermore, by applying the Fubini's Theorem (refer to Arnold (1974)),

$$\begin{aligned} \widetilde{E}\left(\int_0^t e^{(\mu_S - \delta\sigma_S - \frac{1}{2}\sigma_S^2)u + \sigma_S \widetilde{W}_S(u)} e^{-ru}du\right) &= \int_\Omega \int_0^t e^{(\mu_S - \delta\sigma_S - \frac{1}{2}\sigma_S^2)u + \sigma_S \widetilde{W}_S(u)} e^{-ru}du d\widetilde{P} \\ &= \int_0^t e^{(\mu_S - \delta\sigma_S - \frac{1}{2}\sigma_S^2 - r)u} \left(\int_\Omega e^{\sigma_S \widetilde{W}_S(u)} d\widetilde{P}\right) du \\ &= \int_0^t e^{(\mu_S - \delta\sigma_S - \frac{1}{2}\sigma_S^2 - r)u} \widetilde{E}\left(e^{\sigma_S \widetilde{W}_S(u)}\right) du \end{aligned} \tag{A-4}$$

where Ω is the sample space of the random variable $\widetilde{W}(u)$, and $\widetilde{E}(e^{\sigma_S \widetilde{W}_S(u)})$ is the moment generating function of the Brownian motion $\widetilde{W_S}(u)$, which is equal to $e^{\frac{u}{2}\sigma_S^2}$. Therefore, the expected present value of the service flow from 0 to t can be written as:

$$\begin{aligned} \widetilde{E}\left(\int_0^t S_{BD}(u)e^{-ru}du\right) &= \frac{S(0)}{(\mu_S - \delta\sigma_S - r)}\left(e^{(\mu_S - \delta\sigma_S - r)t} - 1\right) \\ & \text{if } \ (\mu_S - \delta\sigma_S - r) \neq 0. \end{aligned} \tag{A-5}$$

By applying the same technique to the after-depreciation case, we can obtain a similar result:

$$\widetilde{E}\left(\int_0^t S_{AD}(u)e^{-ru}du\right) = \frac{S(0)}{(\mu_S - \delta\sigma_S - q - r)}\left(e^{(\mu_S-\delta\sigma_S-q-r)t} - 1\right)$$
$$\text{if} \quad (\mu_S - \delta\sigma_S - q - r) \neq 0 \qquad \text{(A-6)}$$

Plugging equation (A.5) and (A.6) into equation (2.3) and make that equal to $r_o^t\left(\frac{1-e^{-rt}}{r}\right)$, we can obtain the expression for r_o^t in equation (2.4).

B Appendix

B.1 Definitions of $G(t)$, $F(t)$ and relevant equations

$$G(t) = \int_0^t e^{-rs} f(s; V, V_B) ds$$

and by Leland and Toft (1996), we can represent $G(t)$, $F(t)$ as:

$$\begin{aligned} F(t, V, V_B) &= N(h_1(t)) + \left(\frac{V}{V_B}\right)^{-2a} N(h_2(t)) \\ G(t, V, V_B) &= \left(\frac{V}{V_B}\right)^{-a+z} N(q_1(t)) + \left(\frac{V}{V_B}\right)^{-a-z} N(q_2(t)) \end{aligned}$$

$N(\cdot)$ is the cumulative standard normal distribution, and

$$q_1(t) = \frac{-b - z\sigma^2 t}{\sigma\sqrt{t}}; \qquad q_2(t) = \frac{-b + z\sigma^2 t}{\sigma\sqrt{t}}$$

$$h_1(t) = \frac{-b - a\sigma^2 t}{\sigma\sqrt{t}}; \qquad h_2(t) = \frac{-b + a\sigma^2 t}{\sigma\sqrt{t}}$$

$$a = \frac{r - \delta - \left(\frac{\sigma^2}{2}\right)}{\sigma^2}; \qquad b = \ln\left(\frac{V}{V_B}\right); \qquad z = \frac{\left((a\sigma^2)^2 + 2r\sigma^2\right)^{0.5}}{\sigma^2}$$

B.2 Definitions of $I(t)$, $J(t)$ and relevant equations

$$I(T) = \frac{1}{T}\int_0^T e^{-rt} F(t) dt; \qquad J(T) = \frac{1}{T}\int_0^T G(t) dt$$

In Leland and Toft (1996), they show that $I(t)$, $J(t)$ also can be represented as:

$$\begin{aligned} I(T) &= \frac{1}{rT}\left(G(T) - e^{-rT}F(T)\right) \\ J(T) &= \frac{1}{z\sigma\sqrt{T}}\left(-\left(\frac{V}{V_B}\right)^{-a+z} N(q_1(T))\, q_1(T) + \left(\frac{V}{V_B}\right)^{-a-z} N(q_2(T))\, q_2(T)\right) \end{aligned}$$

It is important to note that the value of $J(T)$ is greater than the value of $I(T)$, and we can observe that from the definition of these two functions.

B.3 Definitions of K_1, K_2, K_3

$$\begin{aligned}
K_1 &= \left(\frac{\partial I(T)}{\partial V}\Big|_{V=V_B}\right)V_B \\
&= \frac{1}{rT}\left((-a+z)N(-z\sigma\sqrt{T}) + \frac{n(-z\sigma\sqrt{T})}{\sigma\sqrt{T}} + (-a-z)N(z\sigma\sqrt{T}) + \frac{n(z\sigma\sqrt{T})}{\sigma\sqrt{T}}\right) \\
&\quad -\frac{e^{-rt}}{rT}\left(\frac{n(-a\sigma\sqrt{T})}{\sigma\sqrt{T}} - 2aN(a\sigma\sqrt{T}) - \frac{n(z\sigma\sqrt{T})}{\sigma\sqrt{T}}\right)
\end{aligned}$$

$$\begin{aligned}
K_2 &= \left(\frac{\partial J(T)}{\partial V}\Big|_{V=V_B}\right)V_B \\
&= (-a+z)N(-z\sigma\sqrt{T}) - \frac{n(-z\sigma\sqrt{T})}{\sigma\sqrt{T}} + N(-z\sigma\sqrt{T})\frac{1}{z\sigma^2 T} + (-a-z)N(z\sigma\sqrt{T}) \\
&\quad -n(z\sigma\sqrt{T})\frac{1}{\sigma\sqrt{T}} - N(z\sigma\sqrt{T})\frac{1}{z\sigma^2 T}
\end{aligned}$$

$$K_3 = (C + \Omega_0 + \Omega_c + Dep_c)\left(\frac{t_c}{r}\right)x$$

where $N(\cdot)$ is the cumulative standard normal distribution, and $n(\cdot)$ is the probability density function of standard normal distribution. Other variables are defined in the main context of this paper.

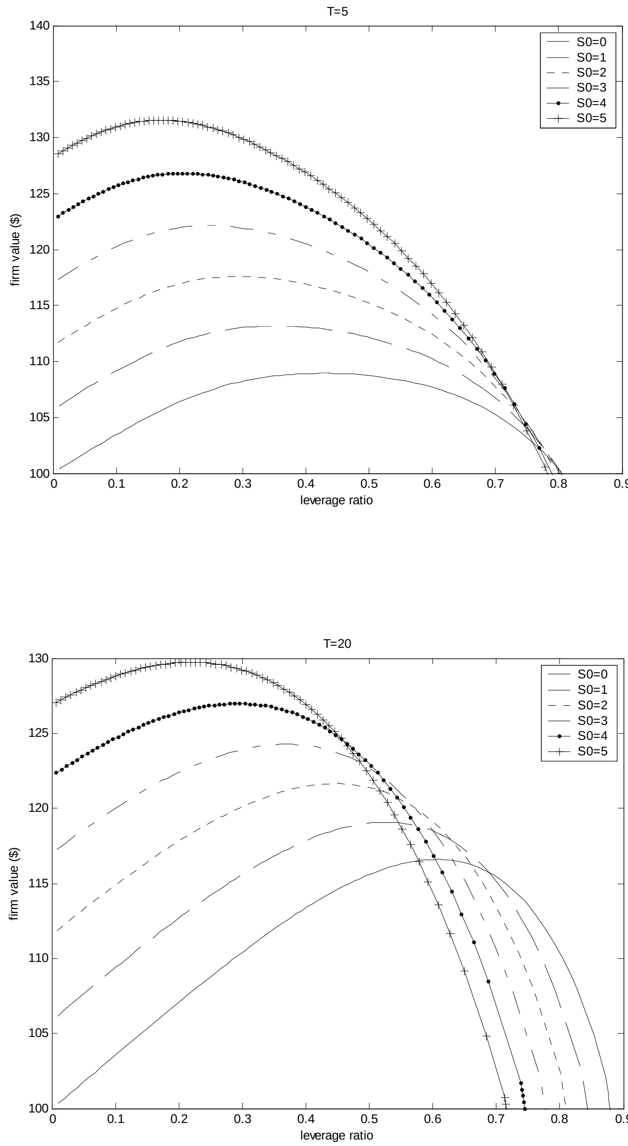


Figure 1: Firm value versus leverage ratio under different initial service flow scenarios. The value of the firm is plotted as a function of leverage ratio for firms issuing debt with maturity T equal to 5 years (upper graph), and 20 years (bottom graph). For each maturity T, six initial service flow value are applied: S0=0, 1, 2, 3, 4, and 5. In this figure, we assumed that the risk free interest rate r =7.5%, the volatility of the firm's assets σ_V =20%, the bankruptcy costs α =50%, the lessee's corporate tax rate t_C =35%, the lessor's corporate tax rate T_C =30%, the firm's payout rate δ_V =7%, the drift term of service flow process μ_S =6%, market price of risk for service flow process δ =0.83, σ_S =4.8%, the depreciation rate for service flow q =5%, the recovery rate for capital lease ρ_C =50%, and the recovery rate for operating lease ρ_O =50%, w_1 =0.2, and w_2 =1.

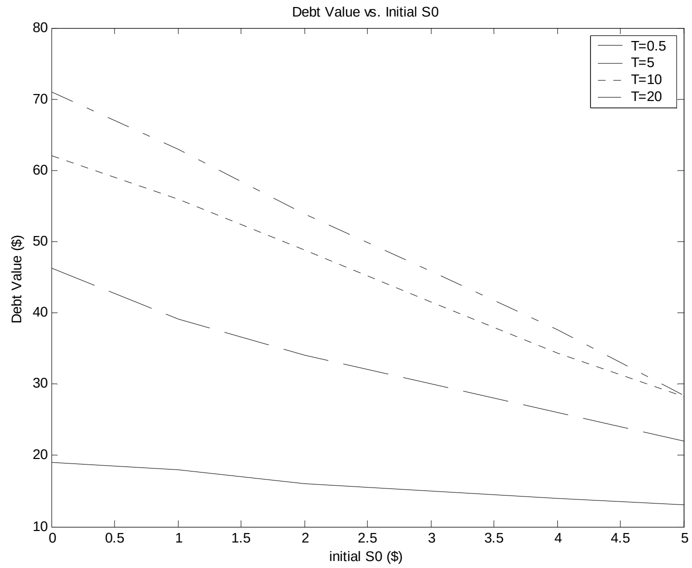
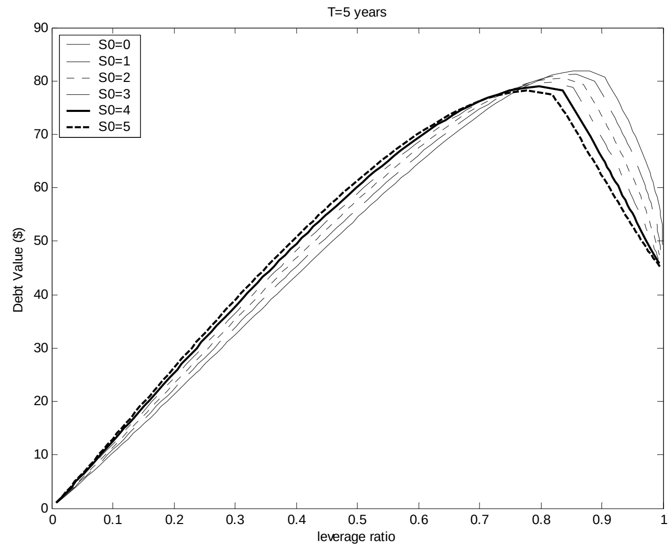


Figure 2: Optimally-levered debt value versus initial service flow value under different maturities. The debt value of the firm under optimal leverage is plotted as a function of initial service flow for firms issuing debt with maturity T equal to 0.5 years, 5 years, 10 years, and 20 years. In this figure, we assumed that the risk free interest rate r =7.5%, the volatility of the firm's assets σ_V =20%, the bankruptcy costs α =50%, the lessee's corporate tax rate t_C =35%, the lessor's corporate tax rate T_C =30%, the firm's payout rate δ_V =7%, the drift term of service flow process μ_S =6%, market price of risk for service flow process δ =0.83, σ_S =4.8%, the depreciation rate for service flow q =5%, the recovery rate for capital lease ρ_C =50%, and the recovery rate for operating lease ρ_O =50%, w_1 =0.2, and w_2 =1.



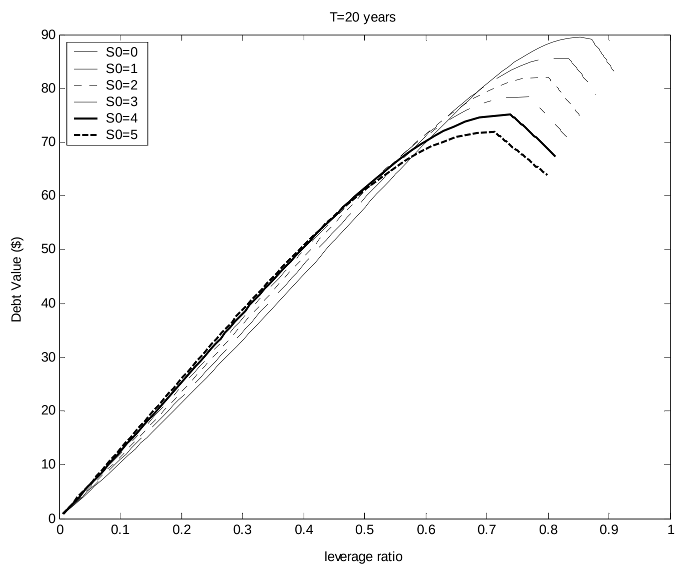


Figure 3: Debt value versus leverage ratio under different initial service flow value. The debt value of the firm is plotted as a function of leverage ratio under different initial service flows (S0) which equal to 0, 1, 2, 3, 4, and 5. In this figure, we assumed that the risk free interest rate r =7.5%, the volatility of the firm's assets σ_V =20%, the bankruptcy costs α =50%, the lessee's corporate tax rate t_C =35%, the lessor's corporate tax rate T_C =30%, the firm's payout rate δ_V =7%, the drift term of service flow process μ_S =6%, market price of risk for service flow process δ =0.83, σ_S =4.8%, the depreciation rate for service flow q =5%, the recovery rate for capital lease ρ_C =50%, and the recovery rate for operating lease ρ_O =50%, w_1 =0.2, and w_2 =1.

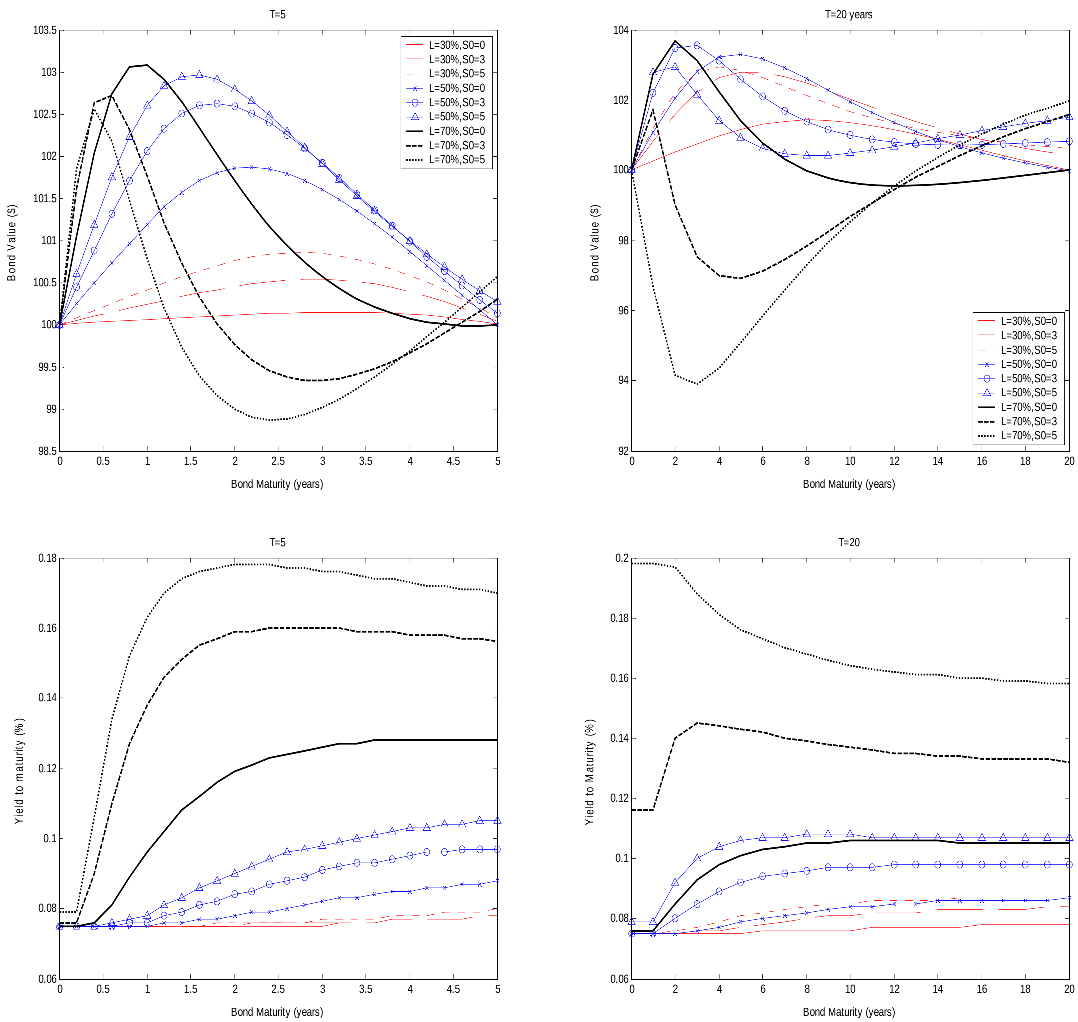


Figure 4: Bond value versus Bond Maturity and Yield to Maturity versus Bond Maturity under different initial service flow value. The bond value of the firm is plotted as a function of bond maturity under different initial service flows (S0) which equal to 0, 1, 2, 3, 4, and 5. The yield to maturity of the bond is plotted as function of Bond Maturity under different initial service flows (S0) which equal to 0, 1, 2, 3, 4, and 5. In this figure, we assumed that the bond value is per$100 face value, and the risk free interest rate r =7.5%, the volatility of the firm's assets σ_V =20%, the bankruptcy costs α =50%, the lessee's corporate tax rate t_C =35%, the lessor's corporate tax rate T_C =30%, the firm's payout rate δ_V =7%, the drift term of service flow process μ_S =6%, market price of risk for service flow process δ =0.83, σ_S =4.8%, the depreciation rate for service flow q =5%, the recovery rate for capital lease ρ_C =50%, and the recovery rate for operating lease ρ_O =50%, w_1 =0.2, and w_2 =1.

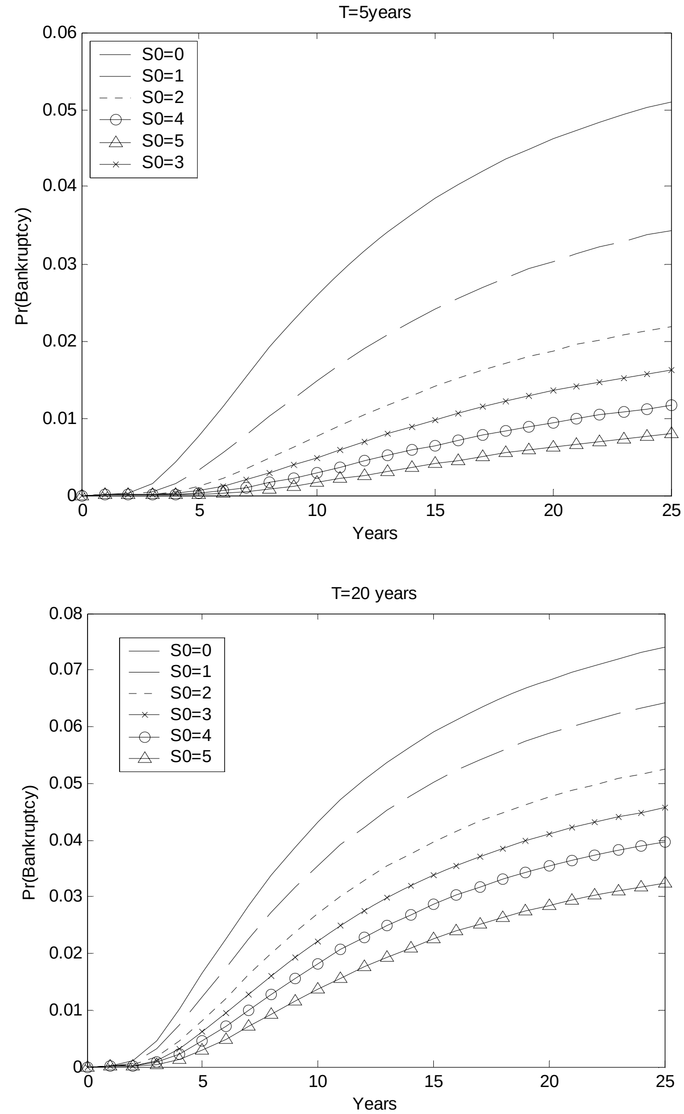


Figure 5: Cumulative Default probability over years under different initial service flows. The default probability of the firm is plotted as function passage years under different initial service flows (S0) which equal to 0, 1, 2, 3, 4, and 5. In this figure, we assumed that the risk free interest rate r =7.5%, the volatility of the firm's assets σ_V =20%, the mean total rate of return $\mu(t)$ =15%, a risk premium of 7.5% over the default-free interest rate, the bankruptcy costs α =50%, the lessee's corporate tax rate t_C =35%, the lessor's corporate tax rate T_C =30%, the firm's payout rate δ_V =7%, the drift term of service flow process μ_S =6%, market price of risk for service flow process δ =0.83, σ_S =4.8%, the depreciation rate for service flow q =5%, the recovery rate for capital lease ρ_C =50%, and the recovery rate for operating lease ρ_O =50%, w_1 =0.2, and w_2 =1.

Table 1: Characteristics of optimally levered firm under different initial lease service flow value and different lease bundle. In the following three panels, we assumed that the risk free interest rate r =7.5%, the volatility of the firm's assets σ_V =20%, the bankruptcy costs α =50%, the lessee's corporate tax rate t_C =35%, the lessor's corporate tax rate T_C =30%, the firm's payout rate δ_V =7%, the drift term of service flow process μ_S =6%, market price of risk for service flow process δ =0.83, σ_S =4.8%, the depreciation rate for service flow q =5%, the recovery rate for capital lease ρ_C =50%, and the recovery rate for operating lease ρ_O =50%.

Panel A: Lease bundle (w_1 =0.2； w_2 =1)

Initial service flow (S0) (Dollars)	**Optimal leverage** (%)	**Operating lease value/Firm Value** (%)	**Capital lease value/Firm Value** (%)	**Total annual coupon** (Dollars)	**Firm value** (Dollars)	**Total Debt** (Dollars)	**Bankruptcy trigger** (Dollars)
			Maturity T= 6 months				
0	18.25%	0.00%	0.00%	1.42	104.12	19.00	27.22
1	16.48%	0.23%	0.04%	1.35	109.21	18.00	25.16
2	13.98%	0.43%	0.09%	1.20	114.42	16.00	21.67
3	12.53%	0.62%	0.12%	1.12	119.72	15.00	19.61
4	11.19%	0.79%	0.16%	1.05	125.10	14.00	17.55
5	9.96%	0.95%	0.19%	0.98	130.55	13.00	15.49
			Maturity T= 5 years				
0	42.49%	0.00%	0.00%	3.70	108.94	46.29	39.35
1	34.58%	1.98%	0.36%	3.02	113.16	39.13	34.17
2	28.97%	3.82%	0.70%	2.59	117.58	34.06	30.76
3	24.59%	5.52%	1.02%	2.27	122.14	30.03	28.23
4	20.52%	7.08%	1.30%	1.96	126.81	26.02	25.71
5	16.73%	8.54%	1.57%	1.65	131.56	22.01	23.20
			Maturity T= 10 years				
0	55.76%	0.00%	0.00%	5.74	113.09	63.06	44.86
1	48.03%	3.39%	0.58%	4.88	116.42	55.92	41.54
2	40.65%	6.65%	1.14%	4.11	119.90	48.73	38.38
3	33.63%	9.75%	1.67%	3.40	123.52	41.54	35.32
4	27.00%	12.67%	2.17%	2.74	127.28	34.36	32.37
5	21.53%	15.42%	2.64%	2.21	131.16	28.24	30.20
			Maturity T= 20 years				
0	60.85%	0.00%	0.00%	6.60	116.60	70.95	43.96
1	52.90%	5.21%	0.79%	5.71	119.08	62.99	42.12
2	44.33%	10.37%	1.58%	4.74	121.64	53.93	39.68
3	36.85%	15.38%	2.34%	3.94	124.27	45.80	38.07
4	29.62%	20.24%	3.08%	3.15	126.96	37.61	36.54
5	21.85%	25.03%	3.80%	2.30	129.72	28.35	34.43

Panel B: Lease bundle (w_1 =0 ; w_2 =1.2)

Initial service flow (S0) (Dollars)	**Optimal leverage** (%)	**Operating lease value/Firm Value** (%)	**Capital lease value/Firm Value** (%)	**Total annual coupon** (Dollars)	**Firm value** (Dollars)	**Total Debt** (Dollars)	**Bankruptcy trigger** (Dollars)
			Maturity T= 6 months				
0	18.25%	0.00%	0.00%	1.42	104.12	19.00	27.22
1	16.48%	0.27%	0.00%	1.35	109.21	18.00	25.16
2	13.98%	0.52%	0.00%	1.20	114.41	16.00	21.67
3	12.53%	0.74%	0.00%	1.12	119.71	15.00	19.61
4	11.19%	0.95%	0.00%	1.05	125.09	14.00	17.56
5	9.96%	1.14%	0.00%	0.98	130.53	13.00	15.50
			Maturity T= 5 years				
0	42.49%	0.00%	0.00%	3.70	108.94	46.29	39.35
1	35.49%	2.38%	0.00%	3.11	113.13	40.15	35.08
2	29.85%	4.58%	0.00%	2.68	117.52	35.08	31.73
3	24.61%	6.62%	0.00%	2.27	122.05	30.04	28.39
4	20.54%	8.51%	0.00%	1.96	126.68	26.02	25.92
5	16.75%	10.26%	0.00%	1.65	131.40	22.01	23.46
			Maturity T= 10 years				
0	55.76%	0.00%	0.00%	5.74	113.09	63.06	44.86
1	48.06%	4.07%	0.00%	4.89	116.36	55.92	41.68
2	40.69%	7.98%	0.00%	4.12	119.78	48.74	38.64
3	33.69%	11.70%	0.00%	3.41	123.35	41.55	35.72
4	27.87%	15.21%	0.00%	2.85	127.04	35.41	33.62
5	20.80%	18.55%	0.00%	2.13	130.86	27.22	30.12
			Maturity T= 20 years				
0	60.85%	0.00%	0.00%	6.60	116.60	70.95	43.96
1	52.09%	6.28%	0.00%	5.59	119.00	61.99	41.66
2	44.40%	12.42%	0.00%	4.76	121.47	53.93	40.24
3	36.11%	18.48%	0.00%	3.84	124.00	44.77	38.20
4	28.08%	24.39%	0.00%	2.96	126.60	35.55	36.27
5	20.33%	30.14%	0.00%	2.11	129.27	26.28	34.44

Panel C: Lease bundle (w_1 =1.2 ; w_2 =0)

Initial service flow (S0) (Dollars)	**Optimal leverage** (%)	**Operating lease value/Firm Value** (%)	**Capital lease value/Firm Value** (%)	**Total annual coupon** (Dollars)	**Firm value** (Dollars)	**Total Debt** (Dollars)	**Bankruptcy trigger** (Dollars)
			Maturity T= 6 months				
0	18.25%	0.00%	0.00%	1.42	104.12	19.00	27.22
1	16.48%	0.00%	0.27%	1.35	109.22	18.00	25.15
2	13.98%	0.00%	0.51%	1.20	114.45	16.00	21.65
3	12.52%	0.00%	0.74%	1.12	119.77	15.00	19.58
4	11.18%	0.00%	0.94%	1.05	125.17	14.00	17.52
5	9.95%	0.00%	1.13%	0.98	130.64	13.00	15.45
			Maturity T= 5 years				
0	42.49%	0.00%	0.00%	3.70	108.94	46.29	39.35
1	36.33%	0.00%	2.19%	3.20	113.31	41.17	35.63
2	29.75%	0.00%	4.21%	2.67	117.89	35.07	31.09
3	25.31%	0.00%	6.07%	2.35	122.61	31.03	28.30
4	21.20%	0.00%	7.78%	2.03	127.45	27.02	25.52
5	18.14%	0.00%	9.37%	1.80	132.37	24.01	23.59
			Maturity T= 10 years				
0	55.76%	0.00%	0.00%	5.74	113.09	63.06	44.86
1	48.79%	0.00%	3.47%	4.98	116.71	56.94	41.66
2	42.14%	0.00%	6.78%	4.29	120.47	50.77	38.58
3	35.85%	0.00%	9.92%	3.66	124.39	44.60	35.60
4	29.92%	0.00%	12.88%	3.08	128.44	38.43	32.71
5	25.12%	0.00%	15.63%	2.63	132.62	33.31	30.62
			Maturity T= 20 years				
0	60.85%	0.00%	0.00%	6.60	116.60	70.95	43.96
1	53.55%	0.00%	4.76%	5.77	119.52	64.00	41.39
2	48.14%	0.00%	9.34%	5.25	122.52	58.98	40.43
3	41.32%	0.00%	13.87%	4.50	125.60	51.90	38.08
4	34.77%	0.00%	18.26%	3.79	128.75	44.77	35.86
5	29.27%	0.00%	22.43%	3.21	131.98	38.64	34.38

Table 2: $\partial V_b^*/\partial S_0$ as function of lessee's tax rate and initial service flow. Panel A, B are results for $\partial V_b^*/\partial S_0$ with maturity equal to 5 years, 20years. In this table, we assume that the risk free interest rate r =7.5%, the volatility of the firm's assets σ_V =20%, the bankruptcy costs α =50%, lessor's corporate tax rate T_C =30%, the firm's payout rate δ_V =7%, the drift term of service flow process μ_S =6%, market price of risk for service flow process δ =0.83, σ_S =4.8%, the depreciation rate for service flow q =5%, the recovery rate for capital lease ρ_C =50%, and the recovery rate for operating lease ρ_O =50%, w_1 =0.2, and w_2 =1

Panel A: T=5 years

Lessee's tax rate	S0=1	S0=2	S0=3	S0=4	S0=5
tc=5%	-0.89	0.10	2.06	3.05	3.05
tc=20%	-1.71	-1.70	-0.78	-1.70	-0.78
tc=35%	-4.27	-2.53	-2.52	-1.66	-2.51
tc=50%	-12.44	-14.20	-5.67	-3.32	-2.54
tc=65%	-8.89	-12.03	-5.09	12.60	-7.56
tc=80%	-5.34	-9.86	-4.52	11.27	-6.82

Panel B: T=20 years

Lessee's tax rate	S0=1	S0=2	S0=3	S0=4	S0=5
tc=5%	-0.35	5.47	6.80	52.64	3.50
tc=20%	-1.43	-1.34	-1.94	-1.18	4.70
tc=35%	-2.47	-1.61	-2.16	-1.41	-1.96
tc=50%	-2.13	-2.62	-1.85	-2.34	-2.23
tc=65%	-1.03	-1.44	-0.91	-1.32	-1.27
tc=80%	0.07	-0.25	0.02	-0.30	-0.31

Table 3 : Characteristics of optimally levered firm under different lessee's tax rate and initial service flow. In these four panels contained in this table, we assumed that the risk free interest rate r =7.5%, the volatility of the firm's assets σ_V =20%, the bankruptcy costs α =50%, , lessor's corporate tax rate T_C =30%, the firm's payout rate δ_V =7%, the drift term of service flow process μ_S =6%, market price of risk for service flow process δ =0.83, σ_S =4.8%, the depreciation rate for service flow q =5%, the recovery rate for capital lease ρ_C =50%, and the recovery rate for operating lease ρ_O =50%.

initial service flow (S0) (Dollars)	**Optimal leverage** (%)	**Operating lease value/Firm Value** (%)	**Capital lease value/Firm Value** (%)	**bankruptcy trigger** (Dollars)
Panel A: Maturity T= 20 years, lessee's tax rate=35%				
0	60.85%	0.00%	0.00%	43.96
1	52.90%	5.21%	0.79%	42.12
2	44.33%	10.37%	1.58%	39.68
3	36.85%	15.38%	2.34%	38.07
4	29.62%	20.24%	3.08%	36.54
5	21.85%	25.03%	3.80%	34.43
Panel B: Maturity T= 20 years, lessee's tax rate=5%				
0	19.97%	0.00%	0.00%	14.84
1	10.91%	6.75%	1.03%	14.47
2	1.94%	13.45%	2.04%	14.12
3	0.88%	20.01%	3.04%	19.60
4	1.78%	26.37%	4.01%	26.40
5	64.72%	20.99%	3.19%	79.04
Panel C: Maturity T= 5 years, lessee's tax rate=35%				
0	42.49%	0.00%	0.00%	39.35
1	34.58%	1.98%	0.36%	34.17
2	28.97%	3.82%	0.70%	30.76
3	24.59%	5.52%	1.02%	28.23
4	20.52%	7.08%	1.30%	25.71
5	16.73%	8.54%	1.57%	23.20
Panel D: Maturity T= 5 years, lessee's tax rate=5%				
0	12.95%	0.00%	0.00%	12.79
1	8.91%	2.22%	0.41%	11.90
2	4.92%	4.42%	0.81%	11.02
3	1.96%	6.59%	1.21%	11.11
4	0.97%	8.73%	1.61%	13.18
5	0.97%	10.86%	2.00%	16.23

Table 4：Characteristics of optimally levered firm under different initial lease service flow value and corresponding lease-debt substitution ratios. In this table, we assumed the same coefficients used in Table (1), and calculated the lease-to-debt substitution ratios based on the following Ang and Peterson (1984)'s equation:

$$DR_{NL} = DR_L + \alpha LR_L \,.$$

DR_{NL} is the debt ratio of a non-lease firm, DR_L is the corresponding debt ratio of a similar firm which uses lease, LR_L is the lease ratio of the latter, and α is the debt-to-lease ratio. In the following table, we have redefined α in terms of lease-to-debt ratio.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
initial service flow (S0)	**debt ratio without lease (DRNL)**	**debt ratio with lease (DRL)**	**lease ratio with operating lease (LRLO)**	**lease ratio with capital lease (LRLC)**	**operating lease-debt substitution ratio (α_o) = (4)/((2)-(3))**	**capital lease-debt substitution ratio (α_c) = (5)/((2)-(3))**	**lease-debt substitution ratio (α) =((4)+(5))/((2)-(3))**
(Dollars)	(%)	(%)	(%)	(%)	(%)	(%)	(%)
Maturity T= 6 months							
1	18.25%	16.48%	0.23%	0.04%	12.81%	2.54%	15.35%
2	18.25%	13.98%	0.43%	0.09%	10.13%	2.01%	12.14%
3	18.25%	12.53%	0.62%	0.12%	10.83%	2.15%	12.98%
4	18.25%	11.19%	0.79%	0.16%	11.20%	2.22%	13.42%
5	18.25%	9.96%	0.95%	0.19%	11.42%	2.26%	13.68%
Maturity T= 5 years							
1	42.49%	34.58%	1.98%	0.36%	25.05%	4.61%	29.66%
2	42.49%	28.97%	3.82%	0.70%	28.24%	5.20%	33.44%
3	42.49%	24.59%	5.52%	1.02%	30.81%	5.67%	36.48%
4	42.49%	20.52%	7.08%	1.30%	32.24%	5.93%	38.17%
5	42.49%	16.73%	8.54%	1.57%	33.13%	6.10%	39.23%
Maturity T= 10 years							
1	54.86%	48.03%	3.39%	0.58%	49.67%	8.49%	58.16%
2	54.86%	40.65%	6.65%	1.14%	46.77%	8.00%	54.77%
3	54.86%	33.63%	9.75%	1.67%	45.92%	7.85%	53.77%
4	54.86%	27.00%	12.67%	2.17%	45.50%	7.78%	53.28%
5	54.86%	21.53%	15.42%	2.64%	46.26%	7.91%	54.17%
Maturity T= 20 years							
1	60.85%	52.90%	5.21%	0.79%	65.50%	9.95%	75.46%
2	60.85%	44.33%	10.37%	1.58%	62.79%	9.54%	72.34%
3	60.85%	36.85%	15.38%	2.34%	64.08%	9.74%	73.82%
4	60.85%	29.62%	20.24%	3.08%	64.81%	9.85%	74.66%
5	60.85%	21.85%	25.03%	3.80%	64.17%	9.75%	73.92%