# Pricing an Option on a Non-Decreasing Asset Value:

## An Application to Movie Revenue

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**Abstract**

In recent years financial engineers have created instruments that facilitate the efficient transfer of the risk associated with certain forms of entertainment revenues. This paper focuses on one particular instrument, options on streams of movie revenues. These options enable film distributors to manage the risk of a movie, and they offer diversification opportunities for investors. Pricing the options is complicated by the fact that cumulative revenue is zero when the options are first offered, uncertainty is high at the start and quickly diminishes, and the underlying is non-decreasing. We propose a continuous time Gamma process to capture the underlying revenue stream and derive the option price formula. To estimate the parameters of the process before revenue observations are available, we propose a regression method using revenue data from a data base of earlier movies. With the arrival of the revenue observations, we update the initial estimates in a Bayesian framework. The fit of the revenue process and the option price formula to movie data as well as sensitivities of the option price with respect to the parameters are explored. We apply our method to a set of example movies.

**Key Words**: option pricing, gamma process, risk management, movie revenues, non-decreasing process, Bayesian update

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# 1 Introduction

Over the last twenty years, financial engineers have created an impressive array of instruments designed to manage the risk of uncertain market values and cash flows. In this paper, we examine one particular instrument created for the purpose of managing the risk associated with the revenues generated by entertainers. These revenues can be in the form of a movie, a package of movies, a CD, or a tour. Our interest is in the valuation of options on a stream of such revenues. These instruments have several interesting and complex characteristics that pose challenging problems for the derivation of a model and its application.

For example, the investor who is exposed to the risk of the underlying revenue stream would typically sell a call option before any revenue is generated. Consequently, there is no observable underlying at the start of the option's life. The underlying revenue stream is, therefore, somewhat like an initial public offering. An IPO, however, will eventually start trading and produce observable prices, whereas the revenue stream will not trade unless securitized and offered in a public market.

A second complicating factor is that the underlying revenue stream is non-decreasing. Although revenue can theoretically remain constant over a time period, it would typically increase. This characteristic is most unusual compared to underlyings of options on more traditional instruments such as securities, currencies, and commodities, which can increase or decrease in value. Diffusion models are, therefore, inappropriate for valuing options on revenue streams.

When the revenue stream is a movie, another interesting characteristic is that the uncertainty resolves rapidly in the first few weeks after release. Models for the valuation of options on movie revenues must, therefore, accommodate the intense concentration of uncertainty at the start of the option's life.

In this paper, we will focus on options on movie revenues. Movies are a highly visible institution in contemporary society and are of considerable cultural and economic interest. Moreover, there is a large body of statistical information available on movies that will enable us to operationalize the model. The practitioner literature has suggested that there is considerable interest among distributors in such instruments, and they would seem attractive for hedge funds and other investors seeking exposure to areas unrelated to mainstream market sectors.

Because these options are different in so many ways, we will examine the underlying boundary conditions of European and American call and put options to determine minimum values, put-call parity, and early exercise conditions. We will then develop a stochastic process for the revenue stream and use it to derive an option pricing model. As we will show, this model, while resembling a conventional option pricing model, nonetheless requires information not directly observable. Thus, the second part of the paper focuses on developing and applying the econometrics necessary to extract estimates of model parameters from data on movie revenues. We show how to estimate these parameters before the movie is released and how Bayesian analysis can be used to update these estimates as revenues accrue. With these estimates, we are able to examine the characteristics of the model and illustrate how to apply it to actual movies.

The paper is organized as follows. Section 2 provides background information on options on movie revenues. Section 3 examines boundary conditions for options on movie revenues. Section 4 introduces a deterministic model of adoption that forms the basis for the stochastic model developed in Section 5. Section 5 develops option pricing formulae and Section 6 addresses parameter estimation. In Section 7, we price options on our example movie and explore their characteristics as well as the fit of our model numerically. Section 8 con-

cludes.

## 2 Options on Movie Revenue

A movie is an excellent example of a capital investment decision. A studio commits a significant amount of initial funding, acquires the resources (including contracts with the actors, director, and other personnel), produces the movie, and then expends additional resources in promotion and distribution while the movie generates cash flows. The cash flows from box office receipts have an exceptionally short life. For example, one of the most successful movies of all time, *Titanic*, lasted in theaters nine months. In contrast one of the least successful, *Gigli*, lasted only about two weeks. More common time horizons are between 10 and 20 weeks.

Most often, movies have an exponentially declining revenue process. Only in rarer cases, so-called platform releases where the number of initial screens is small, the revenue process has a hump shape (Sawhney and Eliashberg, 1996). Figure 1 shows weekly U.S. box office revenues (bars) and cumulative revenue (lines) for four example movies. *The Lord of the Rings: The Fellowship of the Rings* and *Ocean's 11* are two of the most successful movies released in 2001 and examples for highly advertised wide releases. They had marketing budgets of $40 million and $30 million and were shown on 3,359 and 3,075 initial screens, respectively. The run time in theaters was 36 weeks and 21 weeks. The graph shows only the first 20 and 15 weeks, respectively, since these weeks show by far the largest part of the dynamics. Afterwards, the movies had reached a saturation point. In both cases, the number of theaters decreased after the opening weekend. On the other hand, *Gosford Park*, also released in 2001 and directed by Robert Altman, is an example for a platform release. It had a low budget, $18 million in total and was initially released on 9 screens. Over the

following weeks, the distributor USA Films increased the number of screens to 131, 518, 658..., with a maximum of 918 in the 11th week. The movie lasted 23 weeks in theaters. *The Others* falls in between: Released on 1,678 screens, it reached a maximum of 2,843 theaters in week 6 and was shown for 14 weeks. Figure 1 illustrates that these characteristics have a direct influence on the shape of the revenue process. Wide releases have exponential dynamics in the weekly revenues and approach their maximum adoption faster than platform releases, which reach a maximum in weekly revenues during the first few weeks and then decline slowly.

FIGURE 1 ABOUT HERE.

Of course, most movies also have a second life, generating cash flows through video rentals, video sales, and television rights. These amounts can be substantial. For example, Titanic was released on video in 1999 and has since generated another $300 million from domestic rentals. A third major source of revenue from a movie is box office receipts, video sales, and rentals in non U.S. countries. *Titanic*, for example, has generated about twice as much in box office revenue and almost three times as much in video rentals outside the U.S. as inside the U.S. Foreign cash flows can occur almost simultaneously with U.S. cash flows. We do not consider any of these other revenues in our analysis.

As a risky investment, a movie is characterized by a tremendous concentration of uncertainty during the first few weeks after release. A number of factors determine success or failure of a movie and the process is extremely complex. Expensive movies with well-known stars are sometimes dismal failures, while low-budget movies are sometimes highly successful. [1] Some movies receive ex-

[1] For example, *The Alamo* released in 2004 starring Billy Bob Thornton and Dennis Quaid cost $95 million and earned about $22 million in box office revenues. At the other extreme, *The Blair Witch Project* released in 1999 cost about $35,000 and generated box office receipts of over $140 million.

cellent critical reviews and awards but are commercially disappointing.[2] It is common to observe movies that do well in spite of poor critic reviews. Movies that are commercial failures in the U. S. are oftentimes highly successful in foreign markets.[3] Because of the high degree of uncertainty and the large financial investment, movies are prime candidates for the financial engineering of risk transfer instruments.

One of the first such instruments was a $400 million seven-year Eurobond released in 1992 by The Walt Disney Company. The interest rate was tied to the revenues from a combination of 13 Disney movies released in Europe. The rate was set at 7 for 18 months. Beyond that point, the coupon was set at a formula directly related to revenues from these movies. The total rate would end up being between 3% and 13.5%. Obviously the return on this bond takes on the characteristic of a call option, exercising and paying additional money if target revenue levels are met.

In 1997, Risk magazine (Conway, 1997) reported on the creation of an Entertainment Industry Options Exchange in London. This exchange was conceived as an organized marketplace for trading derivatives based on movies and other entertainment-based revenues. One of the first instruments planned was options based on the album *Perfect World* by pop singer Debbie Bonham. There is no subsequent evidence that this exchange was ever formally operational.

Also in 1997 pop singer David Bowie released $55 million of bonds with coupons tied to the revenue from some of his albums. These instruments became known as Bowie Bonds, but they were not successful for investors, however, and were downgraded to near junk status in 2004.[4]

[2]For example, in 1995 *Bravehart* generated only about $76 million in revenue in the U.S., just slightly above its cost of $72 million. The movie received 10 Oscar nominations and won five, including Best Picture.

[3]For example, in 2004 the movie *Troy*, costing $185 million, earned only $133 million in U.S. box office receipts but has grossed about $350 million outside the U.S., placing it in the top 50 movies of all time.

[4]In August, 2005, the Wall Street Journal (Richardson, 2005) noted that instruments

In 2004 Risk (Patel, 2004) describes the efforts of an American company called Center-Group to create an electronic market for new derivative instruments based on box office receipts. No further evidence exists of whether this market has been formally created. The article also references the securitization of revenues of such movie studios such as Vivendi and Dreamworks SKG. It also notes, however, that some defaults have occurred in previous securitizations.

Virtual derivatives on movies can be traded on the Hollywood Stock Exchange (www.hsx.com), a subsidiary of the American bond trading firm Cantor Fitzgerald. This exchange was created in 1996, and all trading is based on fictional money. Stocks and options on movies can be purchased and sold. The exchange also offers "bonds" on actors and actresses in which value is accrued based on revenues generated by their movies. The exchange creates value by selling the data to the movie industry, where it could presumably be used to assess the public's interest in movies and performers.

While it is not clear that derivatives on revenues from movies, music recordings, and tours have been widespread, there would appear to be great potential for such instruments. Americans spend more than $13 billion a year on movies and there has been an increasing interest in the accumulation and analysis of movie revenue data (Lippman, 2005) with no less than five web sites devoted to this subject. Not only is the entertainment industry in need of risk management techniques, but claims on these instruments could be particularly attractive to investors because of diversification potential. Hedge funds and other institutional investors would seem to be an ideal market, but individual investors might well become interested, as evidenced by the over one million participants as claimed by the Hollywood Stock Exchange.

Although numerous types of derivatives are potentially viable for entertain-

similar to Bowie bonds are expected to stage a comeback, stimulated by growth in sales of digital downloads and cellphone ringtones.

ment based products, our focus will be on options on movie revenue. As briefly mentioned in the introduction, these options have several unique features that must be addressed in building pricing models. These features pose interesting and unusual challenges for pricing these types of claims. An added benefit of this research is that modeling the revenue stream would be beneficial for constructing securitized equity shares. We confine our research to European and American options. Exotic variations will undoubtedly be quite interesting, but we leave that subject to future research.

# 3 Boundary Conditions for Options on Movie Revenues

We begin by examining the basic pricing results that can be developed without specifying a stochastic process for movie revenues. We start by defining the underlying revenue stream as a random value $R(s)$, which represents revenue accrued over the period 0 to $s$. Although a discrete-time model could be used here, we let revenue accrue continuously. Hence,

$$R(s) = \int_0^s dR. \tag{3.1}$$

initialized at $R(0) = 0$. The value at time $s$ of the stream received at time $T$ is specified as

$$V(s) = e^{-r(T-s)}\mathbb{E}R(T), \tag{3.2}$$

where $r$ is an appropriate discount rate. At time $s$, some of the revenue stream will have been accumulated. That is

$$R(T) = \int_0^s dR + \int_s^T dR. \tag{3.3}$$

In that case, the known portion clearly has a present value of $R(s)\mathrm{e}^{-r(T-s)}$. Define the present value of the unknown portion as $\Omega(s,T)$. Thus,

$$V(s) = R(s)e^{-r(T-s)} + \Omega(s,T). \tag{3.4}$$

At expiration, a European call worth is worth $C(T) = \max\{0, R(T) - K\}$, while a European put is worth $P(T) = \max\{0, K - R(T)\}$. As with standard European options, lower strikes for calls and higher strikes for puts are more valuable.

Minimum values can be established by creating combinations of options, the underlying revenue stream, and risk-free bonds. We assume frictionless trading in these instruments is possible. For calls, hold the revenue stream, borrow $Ke^{-r(T-s)}$, and sell the call. The payoffs are $R(T) - K < 0$ if $R(T) \leq K$ and 0 otherwise. To prevent arbitrage, a non-positive payoff must guarantee a non-positive initial value. Hence, the call has the same minimum value as for standard European calls: $\max\{0, V(s) - Ke^{-r(T-s)}\}$. For puts, hold the revenue stream, borrow $K$, and buy a put. The payoffs are 0 if $R(T) \leq K$ and $R(T) - K > 0$ otherwise. For a non-negative payoff, we must have a non-negative current value, which leads to the minimum value of $\max\{0, Ke^{-r(T-s)} - V(s)\}$.

Put-call parity is similarly established by holding the revenue stream, buying a put, selling a call, and borrowing $K$. This position provides a zero payoff for certain, so its current value must be zero. Put-call parity is, therefore, the same as for standard options

$$P(s,T) + V(s) = C(s,T) + Ke^{-r(T-s)}. \tag{3.5}$$

American options on revenue have some interesting properties. In-the-money American calls can be exercised for a current value of $R(s) - K$. Such

calls are worth a minimum of $V(s) - Ke^{-r(t-s)}$. No early exercise will occur if $V(s) - Ke^{-r(T-s)} > R(s) - K$. Recall that $V(s) = R(s)e^{-r(T-s)} + \Omega(s,T)$. Thus, we have the condition as

$$R(s)e^{-r(T-s)} + \Omega(s,T) - Ke^{-r(T-s)} > R(s) - K \tag{3.6}$$

which means that

$$(R(s) - K)\left(1 - e^{-r(T-s)}\right) < \Omega(s,T) \tag{3.7}$$

This condition means that the present value of the remaining revenue stream exceeds the interest on the payoff from early exercise. This condition will not always be met. Hence, early exercise of calls can occur.

Conventional American puts on revenue are trivial instruments. If out-of-the-money, they can never move in-the-money. If in-the-money, they can only move less in-the-money or out-of-the-money. Hence they are immediately exercised if in-the-money and worthless if not. The Bermuda put exercisable at $\tau \leq t \leq T$ is straightforward. It will be exercised at $\tau$ if in-the-money and will be dead at $\tau$ if out-of-the-money. Its payoff and hence its value is therefore equal to that of a standard European put maturing at $\tau$.

# 4 A Deterministic Model for the Adoption of an Innovation

Modeling the process by which movie revenue has been generated has been the subject of a number of papers in the economics and marketing literature. The most recent work includes the models of Sawhney and Eliashberg (1996), Ravid (1999), Simonoff and Sparrow (2000), Corts (2001), Elberse and Eliashberg

(2003), Goetzmann, Pons-Sanz, and Ravid (2004), and Filson (2005). Most of these models focus on forecasting revenue as a function of the characteristics of the movie and certain time series properties. The predominant structure is to forecast revenue after the release of the movie using information from the first few weeks.

For the purpose of pricing options on movie revenue, we require a tractable and simple model for the evolution of the revenue series. Such a model should contain a deterministic component plus a volatility factor. Moreover, we must be able to model revenue before the movie is released.

In this study, we will use the Bass (1969) model as the deterministic baseline model for the revenue process. Bass assumes that there are two forces influencing the adoption of an innovation. One force is independent of the previous number of adopters (innovations) and the other force is positively influenced by the previous number of adopters (imitations). Consider first an individual adopter. The probability of an event (adoption) in the interval $[t,\, t + dt]$, given that the event has not occurred previously, is called the hazard function, $h(t)$. The Bass hazard model is

$$h(t) = \frac{f(t)}{1 - F(t)} = p + qF(t), \tag{4.1}$$

where $f(t)$ is the density function of the time of adoption and $F(t)$ is the cumulative density up to time $t$. The structure of the model is determined by $p$ and $q$. The parameter $q$ must be non-negative and $p$ must be positive. Both parameters must be finite if the density function is to be non-degenerate.

The first force in the adoption process, $p$, has been called the coefficient of innovation. It is the decision to adopt independent of the actions of others. Bass calls the second force, $q$, the coefficient of imitation. This coefficient is related to the cumulative probability of adoption up until time $t$. Lekvall and Wahlbin (1973) have referred to these coefficients, respectively, as the external

and internal influence in the adoption process. The solution of the ordinary differential equation (4.1) with the initial condition $F(0) = 0$ is

$$F(t) = \frac{1 - e^{-(p+q)t}}{1 + \frac{q}{p} e^{-(p+q)t}}. \tag{4.2}$$

The density function for the time of adoption is then

$$f(t) = \frac{dF(t)}{dt} = \frac{(p+q)^2 \, p e^{-(p+q)t}}{\left(p + q e^{-(p+q)t}\right)^2}, \; 0 < t < \infty. \tag{4.3}$$

If the potential population is $m$, fixed, the total number $n(t)$ of adoptions up to time $t$ under the deterministic model is $n(t) = mF(t)$ and the rate of adoptions is $mf(t)$. The same solution can be obtained by defining $F(t) = n(t)/m$ as the fraction of individuals who have adopted by time $t$. From (4.1) the differential equation for $n(t)$ is given by

$$\frac{dn(t)}{dt} = (m - n(t)) \left( p + q \frac{n(t)}{m} \right), \tag{4.4}$$

$$n(t) = m \left( \frac{1 - e^{-(p+q)t}}{1 + \frac{q}{p} e^{-(p+q)t}} \right) = mF(t). \tag{4.5}$$

From equation (4.4) it is clear that the rate of adoptions is proportional to the remaining non-adopters. The coefficient of proportionality is $p + qn(t)/m$.

# 5 A Stochastic Revenue Model

The gamma distribution possesses desirable properties for modeling non-decreasing revenue since increments are non-negative and, under appropriate choice of parameters, reproduces under addition. Unlike Geometric Brownian Motion, zero initial values do not cause problems. Consistent with the traditional innovation literature, we construct a gamma process so that the time varying mean

is equal to that of the deterministic Bass model. This section describes the properties of the density function and proposes a Gamma process as a revenue model. [5]

## 5.1 The Gamma Density

The density function of the gamma variate $x$ is given by

$$f_X(x) = \frac{x^{\alpha-1}e^{-x/\beta}}{\beta^\alpha \Gamma_\alpha}, \ x \geq 0, \ \alpha > 0, \ \beta > 0, \tag{5.1}$$

where $\alpha$ and $\beta$ are parameters. Two useful results for option pricing are

$$\int_d^\infty f_X(x)dx = \frac{\Gamma_\alpha\left(\frac{d}{\beta}\right)}{\Gamma_\alpha} \tag{5.2}$$

and

$$\int_d^\infty x f_X(x)dx = \frac{\alpha\beta\Gamma_{\alpha+1}\left(\frac{d}{\beta}\right)}{\Gamma_{\alpha+1}} = \frac{\beta\Gamma_{\alpha+1}\left(\frac{d}{\beta}\right)}{\Gamma_\alpha}, \tag{5.3}$$

where $\Gamma_\alpha = \int_0^\infty t^{\alpha-1}e^{-t}dt$ is the gamma function and $\Gamma_\alpha(d) = \int_d^\infty t^{\alpha-1}e^{-t}dt$ is the "upper" incomplete gamma function. Also note that in terms of the cumulative gamma distribution, $F_X(d) = 1 - \Gamma_\alpha(d/\beta)/\Gamma_\alpha$.

The moment generating function of $X$ is

$$M_X(t) = (1 - \beta t)^{-\alpha}, \tag{5.4}$$

giving $E(X) = \alpha\beta$ and $Var\,(X) = \alpha\beta^2$. Furthermore, if $U = \sum_{i=1}^n X_i$ is a sum of independent gamma random variables with parameters $\alpha_i$ and $\beta$, then $U$ is

[5] We will assume that the underlying follows a Gamma process. Many other studies incorporate the Gamma process into the volatility term. See for example, Carr et al. (2006), Heston (1993), Madan, Carr, and Chang (1998), Madan and Milne (1991), Madan and Seneta (1990). Other applications of the Gamma process in financial modeling can be found in Todorov and Tauchen (2005) and Shaliastovich and Tauchen (2005). Wenocur (1989) proposes the Gamma process as a reliability model.

also gamma distributed with moment generating function

$$M_U(t) = (1 - \beta t)^{-\left(\sum_{i=1}^{n} \alpha_i\right)}. \tag{5.5}$$

## 5.2 Imbedding the Bass Model in a Gamma Process

To model random deviations from the expected revenue given by the Bass model, we specify a process whose increments $N(t)-N(s)$, $t > s$, are gamma distributed with mean equal to $m(F(t) - F(s))$ and variance $m(F(t) - F(s))\beta$, where $\beta$ is the main volatility parameter.[6]

Consider the probability space $(\mathbb{R}_+, \mathcal{F}_t, \mathbb{P})$ on which there is defined a gamma process $N(t)$ in continuous time. That is, $N(t)$ has independent increments and the density of $\Delta N := N(t + h) - N(t)$ is given by

$$f_{\Delta N}(x) = \frac{x^{\alpha-1} e^{-x/b}}{b^{\alpha} \Gamma_{\alpha}}, 0 < x < \infty,$$

where $\alpha = \mu^2 h/\nu$ and $b = \nu/\mu$. Then, $\mathbb{E}(\Delta N) = \mu h$,$\mathrm{Var}(\Delta N) = \nu h$ and we can write symbolically for the limit $h \to 0$:

$$\mathbb{E}dN = \mu dt,$$

$$\mathrm{Var}(dN) = \nu dt.$$

The density $f_N(\cdot)$ gives rise to the probability measure $\mathbb{P}$, given by

$$\mathbb{P}(N(t) \in dx) = \frac{x^{t\alpha-1}\, e^{-x/b}}{b^{\alpha t}\, \Gamma_{t\alpha}}\, dx.$$

The filtration $\mathcal{F}_t$ is given by the sigma fields generated by $N(t)$, $\mathcal{F}_t := \sigma(N(s),\ s \in$

[6]Note that we use the symbol $N(t)$ for the adoption process instead of $n(t)$ used in the Bass model in Section 4. $N(t)$ captures $n(t)$ in the first moment, $EN(t) = n(t)$, but also contains a random element. Therefore, $N(t)$ does not satisfy the differential equation (4.4) but $EN(t)$ does.

$[0, t])$. The process $N(t)$ is a special case of a Lévy process (Bertoin, 1996 p 73; Tsilevich, Vershik, and Yor, 2001). The Lévy measure $\Pi$ is given by

$$\Pi(dx) = \frac{\mu^2}{\nu} x^{-1} e^{-\mu x/\nu} dx.$$

To capture the Bass model in the first moment, we define $\mu$ and $\nu$ such that for $s < t$ they satisfy

$$\mathbb{E}\Delta N = \mu(t-s) := m(F(t) - F(s)),$$
$$\mathrm{Var}\Delta N = \nu(t-s) := \beta m(F(t) - F(s)).$$

where $F(t)$ is the Bass model given by equation (4.2). Therefore, as $s \to t$, $\mu dt = m dF(t)$ and $\nu dt = \beta m dF(t)$ so that our model can be expressed as

$$dN(t) \sim \Gamma\left(\frac{m dF(t)}{\beta}, \beta\right). \tag{5.6}$$

For arbitrary increments of length $t - s$

$$N(t) - N(s) \sim \Gamma\left(\alpha(s,t), \beta\right), \tag{5.7}$$

where $\alpha(s,t) := m(F(t) - F(s))/\beta$. Consistent with the rapid resolution of uncertainty in movie revenues, the variance likewise diminishes rapidly with $t - s$.

# 6 Option Value

European options are issued by the firm on cumulative revenue and are cash-settled at expiration. We assume a continuous time CAPM world with constant investment opportunity set and with revenue uncorrelated with the market

(Merton, 1972).[7] Thus, the risk is diversifiable and not priced. Under this setup, the call option pricing formula at $s$ for an option expiring at $T$ can be obtained under the physical measure and is

$$\begin{aligned} C(s,T) &= ae^{-r(T-s)}\mathbb{E}\left[\max\{0, N(s)+U(s,T)-K\right] \\ &= ae^{-r(T-s)}\int_d^{\infty}\left(N(s)+U(s,T)-K\right)f_U\left(u\right)du \end{aligned} \tag{6.1}$$

where $U(s,T) := N(T) - N(s)$, $a$ is average ticket price and $d = \max\{0, K - N(s)\}$. Because of the properties of the Gamma distribution, call value can be written

$$\begin{aligned} &C(s,T) \\ &= ae^{-r(T-s)}\left(\left(N(s)-K\right)\frac{\Gamma_{\alpha(s,T)}(\frac{d}{\beta})}{\Gamma_{\alpha(s,T)}} + \frac{\left(m\left(F(T)-F(s)\right)\Gamma_{\alpha(s,T)+1}\left(\frac{d}{\beta}\right)\right.}{\Gamma_{\alpha(s,T)+1}}\right) \\ &= ae^{-r(T-s)}\left(\left(N(s)-K\right)\frac{\Gamma_{\alpha(s,T)}(\frac{d}{\beta})}{\Gamma_{\alpha(s,T)}} + \frac{\beta\Gamma_{\alpha(s,T)+1}\left(\frac{d}{\beta}\right)}{\Gamma_{\alpha(s,T)}}\right). \end{aligned} \tag{6.2}$$

where $\alpha(s,T) = m\left(F(T)-F(s)\right)/\beta$.

Boundary conditions are satisfied since $s = T$ is sufficient for $U(s,T) = 0$ and equation (6.2) is the mathematical equivalent of equation (6.1). Specifically, note in the first part of equation (6.2) that $C(T,T) = a\left(N(T)-K\right)$ when the option is in-the-money. When the option is out-of-the money $C(T,T) = 0$ since $\lim_{s\to T}\Gamma_{\alpha(s,T)}(\frac{d}{\beta})/\Gamma_{\alpha(s,T)} = \Gamma_0(\frac{d}{\beta})/\Gamma_0 = \int_{d/\beta}^{\infty}\frac{e^{-t}}{t}dt/\int_0^{\infty}\frac{e^{-t}}{t}dt = 0$.

Similarly, the value of an European put is given by put-call parity or com-

[7] Using our data set (Section 7.1), we constructed an index for weekly movie returns during 1998–2000. Comparing this index to the weekly returns from the S&P500, we found no significant correlation in bivariate Granger-causality tests. The details of the test are available from the authors upon request.

puted directly as

$$\begin{aligned} P(s,T) &= ae^{-r(T-s)}\mathbb{E}\left[\max\{0, K - N(s) - u(s,T)\right] \\ &= ae^{-r(T-s)}\int_0^d \left(K - N(s) - u(s,T)\right) f_U\left(u\right) du \\ &= ae^{-r(T-s)}\Bigg(\left(1 - \frac{\Gamma_{\alpha(s,T)}\left(\frac{d}{\beta}\right)}{\Gamma_{\alpha(s,T)}}\right)(K - N(s)) \\ &\quad + \frac{\beta\Gamma_{\alpha(s,T)+1}\left(\frac{d}{\beta}\right)}{\Gamma_{\alpha(s,T)}} - \alpha(s,T)\beta\Bigg). \end{aligned} \tag{6.3}$$

**Irreversibly in-the-money:** An option on revenue can be irreversibly in-the-money, meaning that at a given time during its life, cumulative revenue can already exceed the exercise price, guaranteeing that the option will expire in the money. This result occurs when $N(s) > K$ in which case $d = 0$ so the option price becomes

$$C(s,T) = ae^{-r(T-s)}\left(N(s) + m\left(F(T) - F(s)\right) - K\right). \tag{6.4}$$

Note that volatility does not matter if the option is irreversibly in-the-money. This is a little-known attribute of options, which can also be seen in the binomial model for standard European options when states can be identified in which there is zero probability of the option expiring out-of-the money.

**Probability of finishing in-the-money:** If the option is not irreversibly in-the-money, it is out-of-the-money. One value of interest is the probability of the option finishing in-the-money. Since our model prices under the physical measure, we write

$$\mathbb{P}(N(t) > K) = \mathbb{P}(u(s,T) > K - N(s)) = \frac{\Gamma_{\alpha(s,T)}\left(\frac{d}{\beta}\right)}{\Gamma_{\alpha(s,T)}} \tag{6.5}$$

If $N(s) > K$, the option is irreversibly in-the-money and $d = 0$. Thus, the probability of finishing in-the-money is one.

**Valuing the revenue stream:** Since we assume that the revenue stream has no systematic risk, the value at time $s$ of a cumulative stream to be realized at time $T$ is

$$\begin{aligned} V(s,T) &= ae^{-r(T-s)}\left(\mathbb{E}_s\left(N(s)+u(s,T)\right)\right) \\ &= ae^{-r(T-s)}\left(N(s)+m\left(F(T)-F(s)\right)\right) \end{aligned} \tag{6.6}$$

where $aN(s)$ is revenue already received and $am\left(F(T)-F(s)\right)$ is the expected remaining revenue.

# 7 Parameter Estimation

In estimating model (5.6) we need methods to construct initial estimates and updates of the parameter vector

$$\theta := (m, p, q, \beta).$$

Our solution to this problem proceeds as follows: 1) First, we estimate the parameters of a data base of movies by maximum likelihood (Section 7.2). 2) Then, we regress the estimated parameters on movie characteristics like genre, rating, etc. (Section 7.3). 3) For a set of example movies, we use the movie characteristics and regression parameters to estimate the initial parameter vector. 4) We update the parameter vectors for the example movies as real time data becomes available in a Bayesian framework. Since we are dealing with a non-linear model with non-Gaussian errors, we use a Markov-Chain Monte Carlo (MCMC) setup, because the Kalman filter does not apply.

We are mostly dealing with wide releases (see Section 2), so we set $q = 0$ in the Bass model. The estimation of a fully specified Bass model poses substantial econometric problems as described in Boswijk and Franses (2005). In particular, for consistent estimation, observations beyond the hump are needed. Therefore, we resort to the simpler form.[8]

## 7.1 Data

Our data set consists of the 100 most successful movies for each of the years 1998, 1999, and 2000 in terms of U.S. box office revenues. The data are obtained from `www.the-numbers.com` and the Internet Movie Database (`www.imdb.com`). We require the sample period to cover the entire life span of the movie. That is, we do not use movies released prior to 1998 but still playing in 1998 and we also do not use movies still playing in early 2001. This requirement leaves us with a sample of 244 movies.

## 7.2 Maximum Likelihood Estimation

Maximum likelihood estimation of the revenue model in equation (5.7) is straightforward. Given a time series of box office revenue $R(t)$, $t = 1, \ldots, T$, we divide the series by the average ticket price $a$ to obtain the adoption series $N(t)$, $t = 1, \ldots, T$. We use weekly data, so estimates of parameters $p$ and $\beta$ are thus in units per week.

The likelihood function for the parameter vector $\theta = (m,\, p,\, \beta)$ is given by

$$L(\theta|\{N(t)\}_{t=1,\ldots,T}) = \prod_{t=1}^{T} \frac{1}{\beta^{\alpha(t-1,t)}\Gamma(\alpha(t-1,t))} u(t)^{\alpha(t-1,t)-1} e^{u(t)/\beta}, \qquad (7.1)$$

where $\alpha(t-1,t) = m(F(t) - F(t-1))/\beta$ and $u(t) = N(t) - N(t-1)$, $N(0) = 0$.

[8] Kreider and Weinberg (1998) have also modeled movie revenues with an exponential function.

FIGURE 2 ABOUT HERE.

Figure 2 shows the histograms of the maximum likelihood estimates and their logs for the movies in our data base.

### 7.3 Estimation Prior to any Revenue Data

If there are no revenue and thereby adoption data available, the adoption process must be forecast using historical data from previously released movies. Variables with forecast power discussed in the context of movies are, to name a few, genre, rating, country of release, budget, presence or absence of stars, number of screens, sexual or violent content, and sequel (Jones and Ritz 1991, Sawhney and Eliashberg 1996, Neelamegham and Chintagunta 1999).

Here, we use a regression model where the logs of the maximum likelihood estimates of the parameters of the movies in a database are regressed on dummy variables for genre, rating, scenes of violent or sexual content, sequel, and on the non-dummy variables number of initial screens and the log of the budget. The estimation equations are, thus,

$$\log \hat{\theta}_i = \log \begin{bmatrix} \hat{m}_i \\ \hat{p}_i \\ \hat{\beta}_i \end{bmatrix} = \begin{bmatrix} \xi_i b^m + \eta_i^m \\ \xi_i b^p + \eta_i^p \\ \xi_i b^\beta + \eta_i^\beta \end{bmatrix}, \; i = 1, \ldots, N, \tag{7.2}$$

where the $(1 \times K)$ regressor vector $\xi_i$ for $i$ fixed consists of $K$ explanatory variables, $b^{(\cdot)}$ are parameter vectors, $N$ is the number of movies in the database, and $\eta_i^{(\cdot)}$ is white noise with mean zero and variance $\sigma_i^2$. We assume that there is no covariance across parameters and across movies. Plugging in the characteristics $\xi$ of the movie for which the initial adoption is to be estimated yields a point estimate $\hat{\theta}_0$ for $\theta$ at time $t = 0$, that is, before revenue data are available.

Table 1 reports the regression parameter estimates from the regression of the maximum likelihood parameter estimates on the movie characteristics. Since the genre "Drama" is by far the most common, it is replaced by a constant. The genre results thus reflect the marginal effect of a movie not being a drama. The results show that the genre influenced only the estimates of the attenuation parameter $p$ significantly for the movies in the sample. The genres Suspense/Thriller and Action/Adventure have a significantly steeper decline (higher $p$) in revenues than other genres. Scenes of violent content increased the potential adoption $m$ and decreased attenuation $p$ significantly, an overall positive effect on revenues. The most significant influences are the number of initial screens and the budget. A higher number of initial screens increases the market potential and reduces volatility. It also leads to a sharper drop-off in revenues (higher $p$), this probably being a technical effect since a higher number of initial screens is likely to be followed by a sharp drop in the number of screens. A higher budget increases the market potential and leads to a slower decline in revenues but increases volatility. The rating has apparently no significant influence. Sequels have a relatively higher market potential.

TABLE 1 ABOUT HERE.

## 7.4 Update Scheme for Incoming Revenue Observations

Since the uncertainty in the revenue stream resolves rapidly during the first weeks after the release, it is advisable to update the initial estimates $\hat{\theta}_0$ as the revenue observations come in. Since the process is non-Gaussian, a non-linear Kalman filter approach is not feasible. Therefore, we employ a full Bayesian update approach and evaluate the involved integrals by MCMC simulation.

Exploring the statistical properties of the estimated parameters from the

database movies as shown in Figure 2, we find that a log-normal distribution describes the empirical distributions of the estimates of $m$ and $\beta$ best. For $p$, on the other hand, a normal distribution seems appropriate. Table 2 reports the estimated parameters of the prior distributions from a fit of the 244 estimated adoption parameters.

TABLE 2 ABOUT HERE.

Both the normal and log-normal distributions are determined by a location parameter $\mu$ and a scale parameter $\sigma$. The interpretation of the location parameter $\mu$ as the expected value of the (log of the) parameter $\theta_i$ allows us to plug in the initial estimate $\hat{\theta}_{0,i}$ obtained from the regression model and then update the initial estimate by convoluting with the likelihood equation (7.1) as revenue observations arrive. For example, the likelihood for the first adoption observation $N(1)$, obtained by dividing the first revenue observation $R(1)$ by the average ticket price $a$, is given by

$$f(N(1) - N(0)|\theta) = \frac{1}{\beta^{\alpha(1)}\,\Gamma(\alpha(1))}\, u(1)^{\alpha(1)-1}\, e^{\frac{u(1)}{\beta}}, \tag{7.3}$$

where $\alpha(1) = m(F(1) - F(0))/\beta$, $u(1) = N(1) - N(0)$, and $N(0) = 0$. The likelihood for the first two adoption observations is given by

$$f(N(2) - N(1),\, N(1) - N(0)|\theta) = f(N(2) - N(1)|\theta)\, f(N(1) - N(0)|\theta), \tag{7.4}$$

and so forth.

In the Bayesian update scheme, the marginal posterior distribution of the updated parameter vector after the first adoption observation is obtained by integrating over the product of the prior distribution and the likelihood equation (7.3). For the second observation, we consider the product of the prior distri-

bution with equation (7.4), and so forth. The expected value of the marginal posterior distribution then serves as the point estimate of the updated parameter vector. For example, for the adoption parameter $m$, the marginal posterior density of the update given the first adoption observation is given by

$$f(m|N(1) - N(0), \hat{\theta}_0) = \int_p \int_\beta f(N(1) - N(0)|\theta) f(\theta) dp\, d\beta, \tag{7.5}$$

where $f(\theta)$ is the full (multivariate) prior distribution. Evaluating this integral by numerical integration is infeasible since we have estimates of the marginal prior distributions only. In particular, the covariance of the parameters, here $p$ and $\beta$, is unknown. Therefore, we employ MCMC simulation (Liu, 2001 and Gilks et.al., 1996) to evaluate equation (7.5).

# 8 Applications and Characteristics of Revenue Options

In this section, we will first explore how well our proposed revenue process (5.6) fits the data from our sample of movies. Then, we will apply the option pricing method to the three movies *The Fellowship of the Ring*, *Ocean's 11*, and *The Others* of Figure 1. These movies were all released in 2001, one year after our sample period ends. We do not apply the method to *Gosford Park*, since this is a platform release. Finally, we will explore the sensitivities of the option price formula to changes in the parameters (the "Greeks").

## 8.1 Data Fit

In order to evaluate the model fit, we use real revenue data from the $i = 1, \ldots, 244$ movies in our sample. We compute the discounted payoffs $e^{-rT} \times$

$\max\{0, R_i(T) - K_{i,j}\}$ for an array of call options with strike set at multiples of the movie budget. Since the budgets are different for all movies, we standardize the budget to \$100 million by multiplying the revenue by 100 million/budget$_i$. Note that the option price formula is homogeneous under scalar multiplication, just like the Black Scholes formula.[9] We set the grid of strikes to $K_j = x_j 100$ million. The $x_j$'s span an equidistant grid between 0.2 and 7. These multiples span most of the movies in our sample. Then, we average the discounted payoffs over the movies $i$ and plot the average against the different strike multiples $x_j$ (Figure 3).

FIGURE 3 ABOUT HERE.

Note that this operation involves real data from the movies only. We compare the discounted payoffs to the option prices (6.2) implied by our model and using the maximum likelihood estimated parameters. If the model (5.6), the option price formula (6.2), *and* the maximum likelihood estimator are appropriate, the two resulting lines should be close to each other. Figure 3 shows that the difference between the two lines is two orders of magnitude smaller than the two lines, i.e. less than one percent. The plot also shows convex sensitivity of the option price with respect to the call price, same as in the Black Scholes framework.

## 8.2 Application of the Update

We study the three movies *The Fellowship of the Ring*, *Ocean's 11*, and *The Others* from Figure 1 as prototype movies. Table 3 shows the updating procedure. Using the characteristics of the movies in the regression equation, we obtain

[9]The option price is given by $e^{-rT} E \max\{0, S - K\}$ for any asset $S$. The function $\max\{0, S - K\}$ is homogeneous and $E$ is a linear operator, therefore the option price is homogeneous.

the initial estimates $\hat{\theta}_0$ of the parameter vectors reported in the first row. We update the estimate as the observations of the weekly revenue data come in by evaluating (7.5) by MCMC. The priors are chosen according to Table 2, where the means are set to the initial estimate $\hat{\theta}_0$. The initial values of the Markov chain are also set to $\hat{\theta}_0$. Note how the update picks up under- or overestimations in particular of $m$ and $\beta$ very quickly and corrects them in the right direction in the first few weeks. Also note how much of the uncertainty resolves after about five to six weeks for all three movies: the parameter estimates become stable and begin to move towards the maximum likelihood estimate reported in the last row of the tables.

TABLE 3 ABOUT HERE.

This procedure is fairly flexible and allows the use of other priors. For example, for *Ocean's 12*, released in 2004, one could use the maximum likelihood parameters of *Ocean's 11* as priors instead of the regression results. Similar adaptations could be made for the two sequels of *The Lord of the Rings*. This structure is also suited to handle certain platform releases. Consider, for example, *The Life Aquatic with Steve Zissou*, released in 2004, directed by Wes Anderson. It was released for two weeks on two initial screens only. Then the distributor Buena Vista decided for a wide release on 1,105 screens. Buena Vista released a comparable movie by Wes Anderson, *The Royal Tenenbaums*, in 2001. It was a similarly experimental release with only five screens in the first week. Now, if during the experimental phase, *The Life Aquatic with Steve Zissou* performed per theater as well as *The Royal Tenenbaums*, and it did, the maximum likelihood estimator of the latter could be used as prior for the former for the wide release after the experimental phase.

## 8.3 A Simple Calculation Example

Consider *The Others* for a calculation example. Assume the distributor Miramax wishes to sell a call option on this movie and that the option is sold the instant before the movie is released. Let the option expire in six weeks. At the time of release, the continuously compounded risk-free rate was about 3.6%, based on the U. S. Treasury bill secondary market rate. Plugging in the characteristics (1,678 initial screens, production budget \$17 million plus \$10 million in advertising) into the regression equation (7.2), we obtain the initial estimate $\hat{m}_0 = 1.0109\text{e}7$, $\hat{p}_0 = 0.2654$, $\hat{\beta}_0 = 92312$.

The average ticket price in 2001 is given as $a = \$5.65$. Hence, the forecast $a\hat{m}_0$ of peak revenue is about \$57 million. Given the production budget and the \$10 million spent on advertising and distribution, the movie is forecast to be profitable by U.S. box office receipts alone, and indeed it was, with a total revenue take of about \$96 million.

Let us set the exercise price at the total budget of \$27 million. Inserting these parameters into the option pricing formula gives a call option value of \$18,419,498.

Hence, if the distributor sold options on the entire position after six weeks, it would collect revenue of \$18,419,498 up front. At expiration, six weeks later, total revenue was \$73,422,887 implying attendance of 12,995,201. Thus, the option would have expired with a payoff

$$\max\{0;\ \$73,422,887 - \$27,000,000\} = \$46,422,887.$$

After the remaining eight weeks of the movie, the distributor would then be left with net revenue of \$96,471,845 - \$46,422,887 = \$50,048,958. Given production and distribution costs of \$27,000,000 and adding the option premium, Miramax would have realized a profit of about \$41.5 million. Of course, it might not sell

options on the entire movie proceeds. In addition, some of the proceeds go to the theaters, largely to cover their costs.

If the holder of the option wished to sell the option during the six weeks lifetime, the updated parameters from Table 3 would be used to evaluate the option. The call was irreversibly in the money from week two on, with \$32 million in cumulative revenue in week two. The sequence of call option prices according to the realizations of cumulative revenue and the parameter updates is presented in Table 4

## 8.4 Option Price Update for the Prototype Movies

We repeat this computational exercise for each of the three movies and calculate the call option prices for a grid of strikes similar to Figure 3. The expiries are set at week 20 for *The Lord of the Rings*, week 15 for *Ocean's 11*, and week 14 for *The Others*. The grid of strikes is set at multiples $x_j$ of the budget, as in Figure 3. We then plot the option prices $C(t, T)$ for selected $t$ against the strike multiples $x_j$.

Figures 4 through 6 show the graphs. The option price profiles all approach the payoff with elapsed time. For *The Lord of the Rings*, the approach is strictly from below, for the other two movies, the line switches to the top of the payoff profile. As a benchmark, each graph also shows the option price $C(0, T)_{MLE}$ computed from the maximum likelihood estimator. This uses all revenue data, which is not available at time $t = 0$.

FIGURE 4, 5, 6 ABOUT HERE.

## 8.5 Sensitivities

In this section, we will study the sensitivities of the option price formula (6.2) with respect to the interest rate $r$, and with respect to the parameters $m$, $p$, $\beta$ of the revenue generating Gamma process (5.6). The sensitivity with respect to the strike price can be seen in Figure 3. The derivative has convex shape, as in standard options. The derivatives of (5.6) can be obtained analytically but the expressions are long and cumbersome and offer little additional insight over simple numeric experiments. We therefore present the latter here.

FIGURE 7 ABOUT HERE.

We will study the sensitivities in the context of the option pricing example presented in Section 8.3. Figure 7 shows the dependence of the option price on variations in the different parameters. An increase in the risk free rate $r$ leads to a decrease in the call option price, contrary to the Black-Scholes situation. The reason is that we price under the physical measure. The expected payoff does not depend on the interest rate and thus a higher interest rate only increases the denominator of the discount factor. An increase in the adoption $m$ implies a higher option price, since it translates directly into higher revenues. The sensitivity with respect to the attenuation $p$ reflects the two-sided dependency of the revenue process on $p$: While a low $p$ means that the revenue process starts at a relatively low point, it also means that it decreases more slowly. On the other hand, a large $p$ starts the revenue process high but also leads to a quick drop-off. The higher the volatility parameter $\beta$, the higher the option price and this in a convex way. The reason is that high volatility increases the chance that the option moves irreversibly into the money. Since there are no negative increments, it will stay in the money, so high volatility is unambiguously beneficial for the holder of the call.

# 9 Summary and Conclusions

In this paper we have examined the pricing of options on a non-decreasing underlying. These instruments have surfaced in the form of contingent claims on the revenue streams of movies, concert tours, and music recordings. The options are typically sold before any revenue is generated; hence, at that time there is no observable value of the underlying. Moreover, the non-decreasing revenue stream poses significant challenges to modeling the stochastic evolution of the underlying.

We develop a stochastic model based on the notion of the Bass model that an individual's decision to purchase the product is driven by two factors: the systematic influence of others who have already purchased the product and an idiosyncratic effect independent of the actions of others. To capture the stochastic component, we embed the Bass model in a gamma process. We assume and test empirically that this stochastic process is uncorrelated with the market factor. We derive boundary conditions, put-call parity, results for early exercise of American options, and of course option pricing equations.

Implementing the model poses significant challenges because we have no observations on the target movie at time zero. Our approach to this problem is to obtain an initial estimate of the parameter vector using a historical database of movies. We update the initial estimate as the revenue observations become available in a Bayesian framework.

Hollywood has shown considerable ingenuity in partnering with Wall Street to offer these instruments, as well as securitized equity claims on movie revenues. The results of our paper can be applied to the pricing of these types of options but could also be useful for other possible structures. For example, the owner of an oil field might sell a call option on oil revenues where the exercise price is the production cost. The value of such an option would be driven by two

factors, a non-decreasing but stochastic stream of output and a stochastic price. Options on revenue streams would seem to be a natural component of real option theory, where claims are often a function of revenues rather than traded and easily valued assets. In addition, the modeling of non-decreasing asset streams is of great interest in the field of credit derivatives, where payoffs are based on the cumulative credit losses of a debt portfolio. Our gamma process could be applicable to this type of problem.

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Table 1: Regression results for equation (7.2) for 244 movies.

| | $\log(m)$ | $\log(p)$ | $\log(\beta)$ |
|---|---|---|---|
| **Genre** | | | |
| Constant | 10.80*** (0.80) | -0.17 (0.27) | 4.78*** (1.29) |
| Comedy | -0.01 (0.12) | 0.08 (0.09) | 0.12 (0.19) |
| Romantic Comedy | 0.31* (0.17) | -0.21 (0.13) | 0.06 (0.28) |
| Suspense / Thriller | -0.23 (0.17) | 0.23* (0.13) | -0.06 (0.28) |
| Action / Adventure | -0.20 (0.14) | 0.20* (0.11) | -0.24 (0.23) |
| Science Fiction | -0.03 (0.21) | 0.20 (0.16) | 0.04 (0.33) |
| Horror | -0.30 (0.18) | 0.21 (0.14) | -0.03 (0.29) |
| Animated | -0.12 (0.24) | -0.03 (0.18) | 0.40 (0.38) |
| Initial Screens | 1e-4* (6e-5) | 4e-4*** (4e-5) | -5e-4*** (1e-4) |
| log(Budget) | 0.31*** (.05) | -0.12*** (0.04) | 0.45*** (0.08) |
| **Rating** | | | |
| PG | -0.27 (0.26) | 0.05 (0.20) | -0.48 (0.42) |
| PG-13 | -0.23 (0.27) | 0.28 (0.21) | -0.69 (0.44) |
| R | -0.33 (0.29) | 0.41* (0.21) | -0.76 (0.45) |
| Sex | -0.09 (0.09) | 0.04 (0.07) | -0.16 (0.14) |
| Violence | 0.20** (0.10) | -0.21*** (0.08) | 0.20 (0.16) |
| Sequel | 0.33* (0.14) | -0.17 (0.11) | 0.11 (0.48) |
| $R^2$ | 0.34 | 0.40 | 0.25 |

***, **, * denote significance at the two-sided 99%, 95%, and 90% significance level, respectively.

Table 2: Estimated parameters of the prior distributions.

| | $\hat{m}$ | $\hat{p}$ | $\hat{\beta}$ |
|---|---|---|---|
| | log-normal | normal | log-normal |
| $\mu$ | 16.1514 | 0.3788 | 10.8144 |
| $\sigma$ | 0.7095 | 0.1436 | 1.0828 |

Table 3: Updating the parameter estimates for *The Fellowship of the Rings*, *Ocean's Eleven*, and *The Others*.

| | *The Fellowship* | | | *Ocean's 11* | | | *The Others* | | |
|---|---|---|---|---|---|---|---|---|---|
| | $\hat{m}$ | $\hat{p}$ | $\hat{\beta}$ | $\hat{m}$ | $\hat{p}$ | $\hat{\beta}$ | $\hat{m}$ | $\hat{p}$ | $\hat{\beta}$ |
| $\hat{\theta}_0$ | 2.3439e7 | 0.4006 | 106518 | 2.2346e7 | 0.3603 | 118013 | 1.0109e7 | 0.2654 | 92312 |
| update 1 | 3.6908e7 | 0.7900 | 284935 | 2.3271e7 | 0.6700 | 172701 | 9.7506e6 | 0.5852 | 131455 |
| update 2 | 3.3473e7 | 0.6281 | 2460561 | 2.9258e7 | 0.4776 | 310862 | 1.1989e7 | 0.4947 | 349189 |
| update 3 | 3.7361e7 | 0.5595 | 2111715 | 3.4842e7 | 0.3568 | 348281 | 1.3707e7 | 0.3981 | 372646 |
| update 4 | 4.1595e7 | 0.4877 | 1709611 | 3.7433e7 | 0.3139 | 656265 | 1.5518e7 | 0.3256 | 402052 |
| update 5 | 4.5696e7 | 0.4260 | 1424676 | 4.0009e7 | 0.2795 | 486442 | 1.7229e7 | 0.2774 | 301539 |
| update 6 | 4.8812e7 | 0.4001 | 1166370 | 4.0930e7 | 0.2673 | 408726 | 1.8465e7 | 0.2498 | 229362 |
| update 7 | 5.1581e7 | 0.3815 | 940020 | 4.1966e7 | 0.2576 | 339646 | 2.0002e7 | 0.2216 | 186457 |
| update 8 | 5.3312e7 | 0.3737 | 814861 | 4.1045e7 | 0.2673 | 341085 | 2.1466e7 | 0.1988 | 162932 |
| update 9 | 5.4644e7 | 0.3520 | 761602 | 4.0585e7 | 0.2754 | 326693 | 2.2101e7 | 0.1907 | 133393 |
| update 10 | 5.5762e7 | 0.3390 | 700355 | 4.0139e7 | 0.2850 | 325254 | 2.2094e7 | 0.1917 | 120696 |
| update 11 | 5.6481e7 | 0.3245 | 665736 | 3.9764e7 | 0.2930 | 323815 | 2.1499e7 | 0.2005 | 127713 |
| update 12 | 5.7227e7 | 0.3126 | 625792 | 3.9347e7 | 0.3021 | 339646 | 2.0891e7 | 0.2111 | 141012 |
| update 13 | 5.7573e7 | 0.3004 | 612477 | 3.9203e7 | 0.3079 | 335328 | 2.0824e7 | 0.2141 | 134463 |
| update 14 | 5.7892e7 | 0.2874 | 609814 | 3.9232e7 | 0.3096 | 325254 | 2.0624e7 | 0.2196 | 135265 |
| update 15 | 5.7892e7 | 0.2720 | 617803 | 3.9419e7 | 0.3098 | 302227 | | | |
| update 16 | 5.8079e7 | 0.2602 | 620466 | | | | | | |
| update 17 | 5.8079e7 | 0.2510 | 609814 | | | | | | |
| update 18 | 5.8292e7 | 0.2434 | 601826 | | | | | | |
| update 19 | 5.8691e7 | 0.2371 | 580522 | | | | | | |
| update 20 | 5.8611e7 | 0.2302 | 575196 | | | | | | |
| MLE | 6.2246e7 | 0.2425 | 537915 | 3.6797e7 | 0.3144 | 267687 | 2.0224e7 | 0.2200 | 108226 |

Table 4: Update of the call option price for *The Others* using the updated parameter estimates ("updated") and the initial estimates $\hat{\theta}_0$.

| week | option price | updated | $\hat{\theta}_0$ |
|---|---|---|---|
| 0 | $C(0,6)$ | 18,419,498 | 18,419,498 |
| 1 | $C(1,6)$ | 16,074,244 | 19,205,766 |
| 2 | $C(2,6)$ | 26,797,650 | 27,065,755 |
| 3 | $C(3,6)$ | 35,425,833 | 33,219,443 |
| 4 | $C(4,6)$ | 44,303,025 | 41,036,539 |
| 5 | $C(5,6)$ | 46,374,986 | 44,017,188 |
| payoff | $C(6,6)$ | 46,422,887 | 46,422,887 |

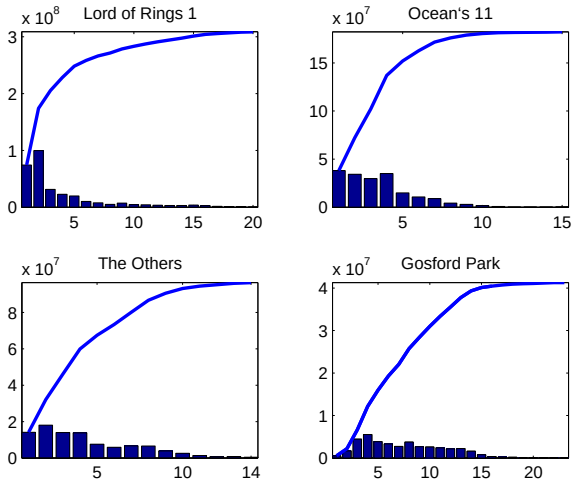


Figure 1: Weekly and cumulative U.S. box office revenue for four example movies.

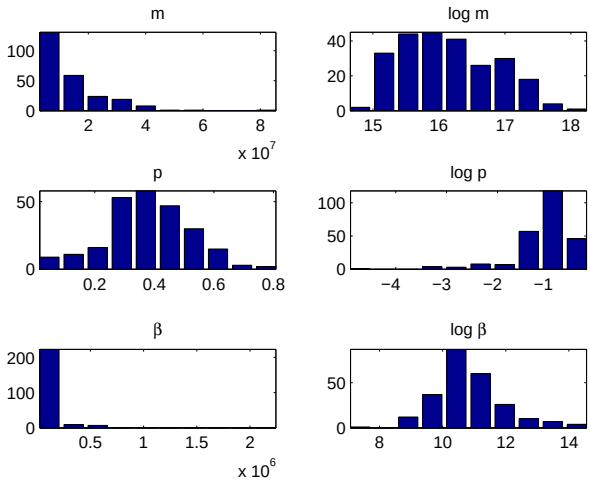


Figure 2: Histograms of the estimated parameters and their logs for 244 movies.

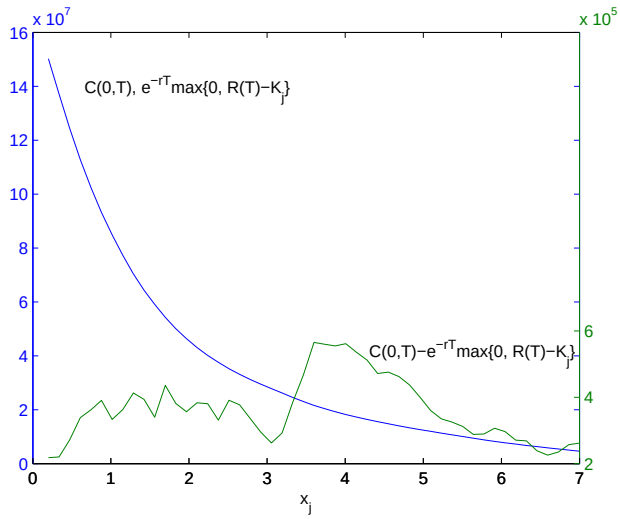


Figure 3: The smooth curve shows call payoffs $e^{-rT}\max\{0, R_i(T) - K_{i,j}\}$ from the movies in our data base, averaged over the $i = 1, \ldots, 244$ movies. The strike prices $K_{i,j} = x_j \text{Budget}_i$ are set at multiples $x_j$ of the movie budget; the $x_j = 0.2 + 0.09j$, $j = 0, \ldots, 30$, are plotted on the x-axis. Most of the movies in our sample earned between 0.2 and 7 times their budget. The smooth curve also shows the call option prices $C_{i,j}(0,T)$ given by (6.2) and using the maximum likelihood estimators, averaged over the $i = 1, \ldots, 244$ movies. The two lines collapse, indicating a good fit of our model to the real movie data. The error $C_{i,j}(0,T) - e^{-rT}\max\{0, R_i(T) - K_{i,j}\}$, averaged over $i = 1, \ldots, 244$, is two orders of magnitude smaller and vanishes at the left-hand y-axis. It is therefore plotted on the bottom scaled at the smaller y-axis on the right.

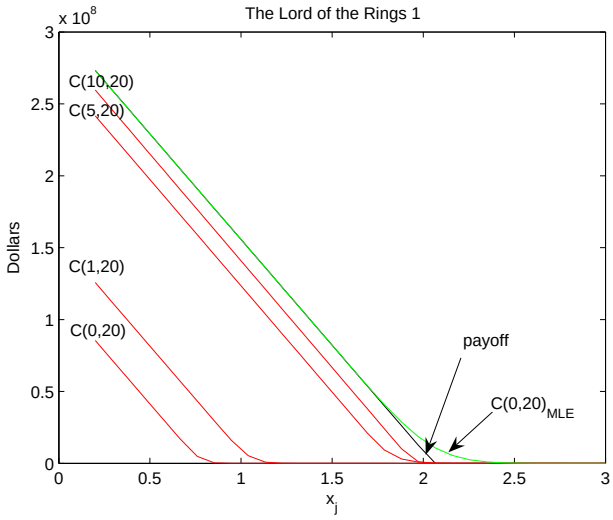


Figure 4: Option price update for *The Lord of the Rings*. Selected option price profiles are shown. $C(0,20)$ is computed from the initial estimate $\hat{\theta}_0$, the $C(t,20)$ are computed from the cumulative revenue at $t$ and the updated parameter estimates from (7.5). $C(0,20)_{MLE}$ gives the MLE estimate of the call at time $t=0$ (using data that is not available at $t=0$.)

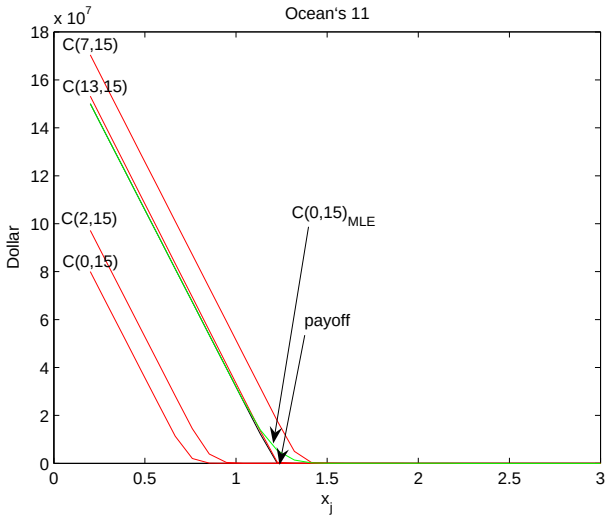


Figure 5: Option price update for *Ocean's 11*. Selected option price profiles are shown. $C(0, 15)$ is computed from the initial estimate $\hat{\theta}_0$, the $C(t, 15)$ are computed from the cumulative revenue at $t$ and the updated parameter estimates from (7.5). $C(0, 15)_{MLE}$ gives the MLE estimate of the call at time $t = 0$ (using data that is not available at $t = 0$.)

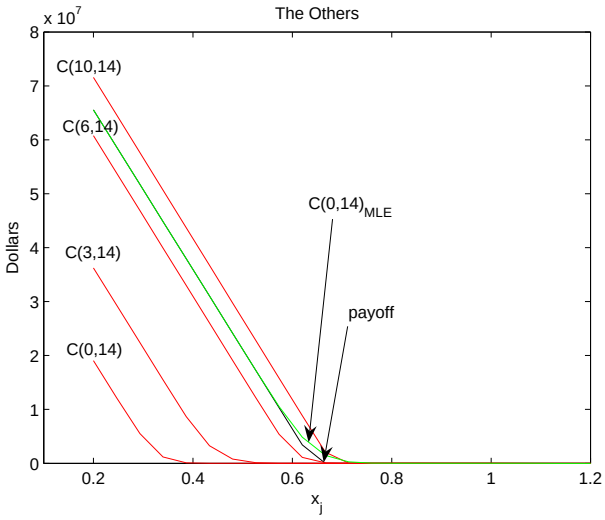


Figure 6: Option price update for *The Others*. Selected option price profiles are shown. $C(0,14)$ is computed from the initial estimate $\hat{\theta}_0$, the $C(t,14)$ are computed from the cumulative revenue at $t$ and the updated parameter estimates from (7.5). $C(0,14)_{MLE}$ gives the MLE estimate of the call at time $t=0$ (using data that is not available at $t=0$.)

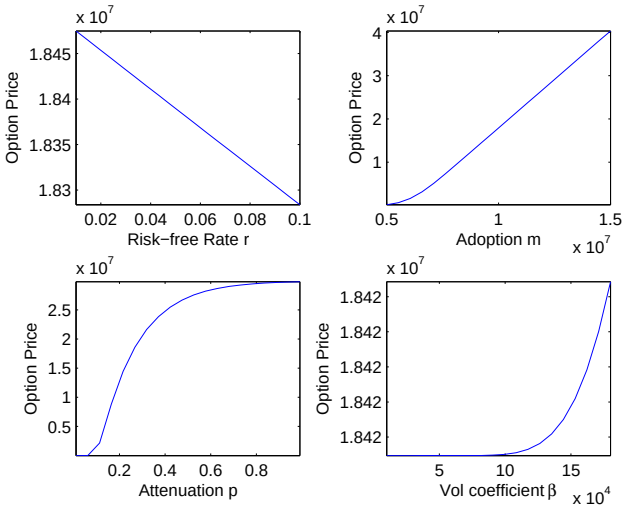


Figure 7: Sensitivities of the option price formula (6.2) with respect to the risk free rate $r$ and the parameters ($m$, $p$, $\beta$) of the revenue model (5.6).