THE WELFARE COST OF BANK CAPITAL REQUIREMENTS

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Abstract

This paper measures the welfare cost of bank capital requirements and finds that it is surprisingly large. I present a simple framework which embeds the role of liquidity creating banks in an otherwise standard general equilibrium growth model. A capital requirement plays a role, as it limits the moral hazard on the part of banks that arises due to the presence of a deposit insurance scheme. However, this capital requirement is also costly because it reduces the ability of banks to create liquidity. A key result is that equilibrium asset returns reveal the strength of households' preferences for liquidity and this allows me to derive a simple formula for the welfare cost of capital requirements that is a function only of observable variables. Using U.S. data, the welfare cost of current capital adequacy regulation is found to be equivalent to a permanent loss in consumption of between 0.1 to nearly 1 percent.

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This paper asks, and provides an answer to, the following question: How large are the welfare costs of bank capital requirements? While there are a number of papers on the theoretical *benefits* of capital adequacy regulation, based on limiting the moral hazard involved with deposit insurance[2] or externalities associated with bank failures, much less is known about whether there are also costs involved with imposing restrictions on the capital structure of banks. But if there are only benefits to capital requirements, why not raise them to 100 percent and require all bank assets to be financed with equity? Clearly, to determine the optimal level of capital requirements, the question of their social cost must be addressed.

In this paper, I argue that capital adequacy regulation may have an important cost because it reduces the ability of banks to create liquidity by accepting deposits. After all, a capital requirement limits the fraction of bank assets that can be financed by issuing deposit-type liabilities. The paper's main contribution is to build a framework to analyze the resulting welfare cost, to derive a simple formula for its magnitude and to use that formula to measure the welfare cost.

The framework embeds the role of liquidity creating banks in an otherwise standard general equilibrium growth model and models the value of their liquidity creation in a flexible way. The welfare cost of capital requirements depends crucially on households' preferences for the liquidity created by banks. A key insight from the model is that equilibrium asset returns reveal the strength of these preferences for liquidity and this allows us to quantify the welfare cost without imposing restrictive assumptions on preferences. Furthermore, the analysis shows that capital requirements can affect capital accumulation and the aggregate level of bank assets. The formula for the welfare cost derived here takes these general equilibrium feedbacks into account.

The model also incorporates a rationale for the existence of capital adequacy regulation, based on a moral hazard problem created by deposit insurance. A capital requirement is helpful in limiting this moral hazard problem, but only in conjunction with bank supervision. This gives rise to a tradeoff between the level of the capital requirement and the cost of supervision. The resulting welfare benefit of the capital requirement is characterized. With the help of some additional assumptions, a separate section of the paper quantifies this benefit as well, and compares it to the welfare cost in order to examine whether capital requirements in the U.S. are currently too high or too low.

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[2] See, for example, Giammarino, Lewis, and Sappington (1993) and Dewatripont and Tirole (1994). Hellman et al. (2000) and Allen and Gale (2003) offer a more skeptical view. Diamond and Dybvig (1983) is often viewed as a theoretical justification for deposit insurance.

In many countries, including the U.S., capital adequacy regulation is based on the Basel Accords. In response to perceived shortcomings in the original Accord, practitioners have added more and more detailed refinements, culminating in the soon-to-be implemented Basel 2. One significant change is the increased attention to bank supervision, formalized in the so-called Pillar 2 of the new Accord, which gives supervisors a range of new instruments. In the language of Basel 2, the model in this paper sheds light on the relation between Pillar 1, the formal capital adequacy rules, and Pillar 2, and shows how this relation gives rise to a tradeoff between the two Pillars.

At the same time, in designing the new rules, regulators have attempted to keep the required ratio of capital to risk-weighted assets for a typical bank approximately the same. But is the 8% of the original Basel Accord a good number for the total risk-based capital ratio? This question remains unaddressed.

If we find that the welfare cost of capital requirements is trivial, this could be an argument for creating a simple, robust system of capital adequacy regulation, with low compliance and supervision costs, but with relatively high capital ratios so as to make bank failure a sufficiently infrequent event. On the other hand, if we find a high welfare cost of capital requirements, this could be an argument for lowering them, by either accepting a higher chance of bank failure, or by designing a more risk-sensitive system with the associated increased supervision and compliance costs, which seems to be the trend in practice.

This paper is related to recent work by Diamond and Rajan (2000) and Gorton and Winton (2000), who also show capital requirements may have an important social cost because they reduce the ability of banks to create liquidity. Unfortunately, the models in these papers do not easily lend themselves to quantification of this cost, which is the main goal of this paper. Except for the banking sector, the model presented here is closely related to Sidrauski (1967). In using asset prices to learn about preferences, the methodology follows Alvarez and Jermann (2004).

The rest of the paper is organized as follows. Section 1 presents the model and analyzes agents' decision problems, as well as equilibrium outcomes. Section 2 derives a formula for the welfare cost of capital requirements. Section 3 uses this formula to measure the cost. Section 4 analyzes an extension of the model that incorporates intermediation frictions and the following section presents additional welfare cost measurements, in part based on this extension. The welfare benefit of capital requirements is discussed and measured in section 6. Section 7 addresses the effect of the capital requirement on economic activity. The final section concludes.

1. The model

The most important respect in which the model deviates from the standard growth model is that households have a need for liquidity, and that certain agents, called banks, are able to create financial assets, called deposits, which provide liquidity services. Since a central goal of the model is to provide a framework not just for illustrating, but for actually measuring the welfare cost of capital requirements, it is important to model the preferences for liquidity in a way that is not too restrictive. As much as possible, we would like the data to provide the answer, not the specific modeling choices. To that end, I follow Sidrauski (1967) in adopting the modeling device of putting liquidity services in the utility function.[3] This has two disadvantages and one advantage.

One disadvantage is that it does not further our understanding of *why* households like liquid assets, but this is not the topic of this paper, so this concern, in and of itself, can be dismissed. It is of course important to know that the Sidrauski modeling device is consistent with a range of more specialized, and arguably 'deeper', micro-foundations. As shown by Feenstra (1986), a variety of models of liquidity demand, such as those with a Baumol-Tobin transaction technology, are functionally equivalent to problems with 'money (or deposits)-in-the utility function'. In that equivalence, the latter is simply a derived utility function. Therefore, unless we impose restrictions on that derived utility function, all results will hold for any of those more primitive models.

A second disadvantage is that if one needs to specify a particular functional form for the utility function, one is on loose grounds. For example, is the marginal utility of consumption increasing or decreasing in deposits?

Fortunately – and this is the advantage of this approach – there is no need to make unpalatable assumptions of this kind. I will show that it is possible to derive a first-order approximation of the welfare cost of the capital requirement without making *any* assumptions on the functional form of the utility function, beyond the standard assumptions that it is increasing and concave. A trade-off involved with modeling liquidity in this flexible way, and embedding it in a general equilibrium analysis, is that the modeling of the banks' assets is not rich enough to incorporate many of the details of *risk-based* capital requirements.

The environment and the agents' decision problems

Time is discrete and there are infinitely many periods. The economy consists of households, banks, (nonfinancial) firms, and a government or

[3] Lucas (2000) uses the framework of the Sidruaski model to measure the welfare cost of inflation.

regulatory agent. Households own both the banks and the nonfinancial firms. These firms combine capital and labor to produce the single good which households consume. I now discuss the assumptions for each of these agents, and analyze their decision problems in turn. For the reader's convenience, figure 1 displays a timeline of the model.

<u>Households</u>: There is a continuum of identical households with mass one. Households are infinitely lived dynasties and value consumption and liquidity services. Households can obtain these liquidity services by allocating some of their wealth to bank deposits, an asset created by banks for this purpose. As mentioned, the liquidity services of bank deposits are modeled by assuming that the household's utility function is increasing in the amount of deposits.

Besides holding bank deposits, denoted d_t, households can store their wealth by buying and selling shares, or equity, e_t. They supply a fixed quantity of labor, normalized to one, for a wage, w_t. Taxes are lump-sum and equal to T_t. There is no aggregate uncertainty, so the representative household's problem is one of perfect foresight:

$$\max_{\{c_t, d_t, e_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t, d_t)$$

$$\text{s.t.} \quad d_{t+1} + e_{t+1} + c_t = w_t 1 + R_t^D d_t + R_t^E e_t - T_t$$

$$\lim_{T \to \infty} \left(\prod_{s=0}^{T} R_s^E \right)^{-1} (d_T + e_T) \geq 0$$

$$d_0 + e_0 \equiv a_0 \quad \text{given}$$

where c_t is consumption in period t, R_t^D is the return on bank deposits, R_t^E is the return on (bank or firm) equity, and β is the subjective discount factor. The returns R_t^D and R_t^E, and the wage are determined competitively, so the household takes these as given. The same applies for the taxes. There is no distinction between bank and nonbank equity, since, in the absence of risk, they are perfect substitutes for the household and will thus also command the same return. The second constraint is a no-Ponzi game condition, the third an initial condition.

The utility function is assumed to be concave, at least once continuously differentiable on $\mathbb{R}_{++}^2$, increasing in both arguments, and strictly increasing in consumption:

$$u_c(c,d) \equiv \partial u(c,d)/\partial c > 0 \quad \text{and} \quad u_d(c,d) \equiv \partial u(c,d)/\partial d \geq 0$$

The first-order conditions to the household's problem are:

(c) $$u_c(c_t, d_t) = \lambda_t$$

(d) $$u_d(c_t, d_t) + \lambda_t R_t^D - \beta^{-1}\lambda_{t-1} = 0$$

(e) $$\lambda_t R_t^E - \beta^{-1}\lambda_{t-1} = 0$$

where λ_t is the Lagrange multiplier associated with the intertemporal budget constraint. Rewriting this,

$$R_t^E = \beta^{-1} u_c(c_{t-1}, d_{t-1}) / u_c(c_t, d_t) \tag{1}$$

$$u_d(c_t, d_t) / u_c(c_t, d_t) = R_t^E - R_t^D \tag{2}$$

Equation (1), which determines the return on equity, is the standard Euler equation for the intertemporal consumption-saving choice in a deterministic setting, with one difference: the marginal utility of consumption may depend on the level of deposits. Equation (2) states that the marginal utility of the liquidity services provided by deposits, expressed in units of the consumption good, should equal the spread between the return on equity and the return on bank deposits. This spread is the opportunity cost of holding deposits rather than equity. If $u_d(c,d) > 0$, then the return on equity will be higher than the return on deposits to compensate for the fact that equity does not provide any liquidity services.

Banks: There is a continuum of banks with mass one, which make loans to nonfinancial firms and finance these loans by accepting deposits from households and issuing equity. The ability of banks to create liquidity through deposit contracts is their defining feature. Banks last for one period[4] and every period new banks are set up with free entry into banking. The balance sheet, and the notation, for the representative bank during period t is:

Assets		**Liabilities**	
L_t	Loans	D_t	Deposits
		E_t	Bank Equity

Banks are subject to regulation, as well as supervision, by the government. One form of regulation is deposit insurance. If a bank fails, the government (through a deposit insurance fund) ensures that no depositor suffers a loss as a consequence of this failure. That is, all deposits are fully insured. Equity holders, as residual claimants, are left with nothing in the event of failure. The rationale for the

[4] This is without loss of generality, since there are no adjustment costs, nor any agency problems between banks and the other optimizing agents, households and firms.

deposit insurance is left unmodeled. However, it has been argued that deposit insurance improves the ability of banks to create liquidity.[5]

Secondly, banks face a capital requirement, which requires them to have a minimum amount of equity as a fraction of (risk-weighted) assets. Since loans are the only type of asset in this model, the capital requirement simply states that equity needs be at least a fraction γ of loans for a bank to be able to operate:

$$E_t \geq \gamma L_t$$

For the moment, the capital requirement is merely assumed. It will later be shown how it can be socially desirable to have such a requirement, as it mitigates the moral hazard problem that arises due to the presence of deposit insurance.

The bank can make safe or risky loans to nonfinancial firms, described below. Riskless[6] loans yield a rate of return R_t^L, which is determined competitively in equilibrium, so each bank takes it as given. Thus, a bank that lends out L_t units of the good at the beginning of the period will receive nonrandom total return of $R_t^L L_t$ units at the end of the period. For now, it is assumed that there are no transaction costs involved in making loans and accepting deposits. Section 4 will analyze an extension of the model with costly financial intermediation.

The presence of deposit insurance creates a moral hazard problem: the bank has an incentive to engage in excessive risk-taking. Since this is the justification for the capital requirement, I introduce a way for the bank to engage in excessive risk-taking by assuming that the bank has the option of artificially raising the riskiness of its assets. Specifically, by directing a fraction of its lending to a different set of nonfinancial firms with a risky technology,[7] the bank can create a loan portfolio with riskiness σ_t that pays off $R_t^L + \sigma_t \varepsilon_t$, where ε_t is a bank-specific shock with positive variance and negative mean, equal to $-\xi$ ($\xi \geq 0$). Thus, the expected return of the loan portfolio is decreasing in its risk. It is in this sense that risk-taking is excessive: absent a moral hazard problem due to deposit insurance, the bank would always prefer $\sigma_t = 0$. While the bank can choose the riskiness of its loans, it is assumed that bank supervision imposes an upper bound on the choice: $\sigma_t \in [0, \bar{\sigma}]$. This will be explained more fully in the discussion of the government.

In the main text I will work with the following example distribution for ε:

[5] Diamond and Dybvig (1983) provide a model of panic based bank runs, which can be seen as a rationale for deposit insurance.

[6] The assertion that the bank can make riskless loans is a consequence of the technology of the nonfinancial firms, as detailed below.

[7] Again, the technology will be described below and will be consistent with the rates of return assumptions made here.

$$\varepsilon_t = \begin{cases} 1 & \text{with probability } 0.5 \\ -(1+2\xi) & \text{with probability } 0.5 \end{cases} \tag{3}$$

This very special example distribution is used purely for expositional reasons. As shown in the appendix C.1 to section 6, all the results in this paper hold for an arbitrary distribution of ε with bounded support and nonpositive mean.

I am now in a position to state the bank's problem. The objective of the bank is to maximize shareholder value by deciding how many loans to make, how much risk to take on, and how to finance its assets with equity and deposits. Although the decision on how much equity to issue will be endogenized, it is convenient to first analyze the sub-problem of maximizing shareholder value right after the equity has been issued and the bank has raised E_t in equity at the beginning of period t. At that point the value of the bank's equity is: [8]

$$\begin{aligned} V^B(E) = \max_{L,D,\sigma} \mathbb{E}_\varepsilon \left[\left((R^L + \sigma\varepsilon)L - R^D D \right)^+ \right] / R^E \\ \text{s.t.} \quad L = E + D \\ E \geq \gamma L \\ \sigma \in [0, \bar{\sigma}] \end{aligned} \tag{4}$$

The notation $(x)^+$ stands for $\max(x,0)$ and $\mathbb{E}$ is the expectations operator. The first constraint is the balance sheet identity, the second is the capital requirement, and the third bounds σ.

The term $(R^L + \sigma\varepsilon)L - R^D D$ is the bank's net cash flow at the end of the period. It consists of interest income from loans, minus any possible charge-offs on the loans, and minus the interest owed to depositors. If the net cash-flow is positive, shareholders are paid this full amount in dividends.[9] If the net cash flow is negative, the bank fails and the deposit insurance fund must cover the difference in order to indemnify depositors, as limited liability of shareholders rules out negative dividends. Shareholders receive zero in this event, so dividends equal $\left((R^L + \sigma\varepsilon)L - R^D D \right)^+$.

At the beginning of period t shareholders discount the value of dividends, which are paid at the end of that period, by their opportunity cost of holding this particular bank's equity. This opportunity cost is R^E, the market rate of return on equity. Because dividends are either not subject to risk, or, if $\sigma > 0$, their risk is perfectly diversifiable, shareholders do not price the bank's risk.[10]

First, I characterize the choice of σ conditional on L and D. Note that

[8] In what follows, time subscripts will be used only where necessary to avoid confusion.
[9] There are no retained earnings since the bank lasts for one period only.
[10] Hence, the treatment of R^E as nonstochastic in the household problem is still correct, since, even if banks are risky, households would not leave any such risk undiversified.

$$\mathbb{E}_{\varepsilon}\left[\left((R^L+\sigma\varepsilon)L-R^D D\right)^+\right]$$
$$=\begin{cases}(R^L-\sigma\xi)L-R^D D & \text{if } (R^L-\sigma(1+2\xi))L-R^D D\geq 0\\ 0.5((R^L+\sigma)L-R^D D) & \text{otherwise}\end{cases}$$

Expected dividends are thus strictly decreasing in σ for low values of σ and strictly increasing in σ for sufficiently high values of σ. [11] The reason is that for high values of σ, if the bank suffers a negative shock, there is not enough equity to absorb the loss and the excess loss is covered by the deposit insurance fund. Increasing risk further at this point increases the payoff to shareholders in the good state ($\varepsilon = 1$) without lowering it in the bad state. In other words, the value of the put option associated with the deposit insurance fund increases with σ. In contrast, when σ is low, the value of this put option is zero and shareholders fully take into account the reduction in net present value that occurs when risk is increased.

Because expected dividends are a convex function of σ, there are only two values to consider for the optimal choice of riskiness: $\sigma = 0$ or $\sigma = \bar{\sigma}$. It is easy to show that

$$\begin{aligned}&\sigma = 0 \text{ iff } \bar{\sigma}\leq R^L-R^D(D/L)\\ &\sigma=\bar{\sigma} \text{ otherwise}^{12}\end{aligned} \tag{5}$$

Because $E = L - D \geq \gamma L$, the following is a sufficient condition for $\sigma = 0$:

$$\bar{\sigma}\leq R^L-R^D(1-\gamma) \tag{6}$$

This is also a necessary condition when the capital requirement is binding. From now on, unless explicitly stated otherwise, it is assumed that (6) holds.

The bank's sub-problem in (4) for shareholder value now simplifies to:

$$\begin{aligned}V^B(E) &= \max_{L,D}\left(R^L L-R^D D\right)/R^E\\ \text{s.t.}\quad & L-D=E\\ & E-\gamma L\geq 0\end{aligned}$$

The first-order conditions are easily simplified to

$$R^L-R^D=\gamma R^E\chi$$

[11] Note that there is *no* discontinuity at $(R^L-\sigma(1+2\xi))L-R^D D=0$.

[12] When $\bar{\sigma}=R^L-R^D(D/L)$, the bank is indifferent between the two choices. For convenience, it is assumed that the bank chooses $\sigma = 0$ in that case.

where χ is the Kuhn-Tucker multiplier associated with the capital requirement: $\chi \geq 0$ and $\chi(E - \gamma L) = 0$. The existence of a (finite) solution requires $R^L \geq R^D$. Under that condition, the solution is

$$V^B(E) = \left(R^L + (\gamma^{-1} - 1)(R^L - R^D)\right) E \Big/ R^E \tag{7}$$

The capital requirement binds if and only if $R^L > R^D$. The interpretation is straightforward: an extra unit of equity can be lent out at the rate R^L. In addition, the extra unit of capital allows the bank to make $(\gamma^{-1} - 1)$ *additional* loans and finance those with deposits, without violating the capital requirement, which requires $L \leq \gamma^{-1} E$. If $R^L > R^D$, the second option has value, and the capital requirement will be binding, otherwise not.

I can now turn to the bank's decision on how much equity to raise. The pre-issue value of the bank is $V^B(E) - E$. The bank maximizes this value when choosing *E*. Using (7), the first-order condition to that problem is:[13]

$$R^E = R^L + (\gamma^{-1} - 1)(R^L - R^D)$$

It is helpful to rewrite this as

$$R^L = \gamma R^E + (1 - \gamma) R^D \tag{8}$$

This has the interpretation of a zero-profit condition: for a bank with a binding capital requirement, one unit of lending is financed by γ in equity and $(1 - \gamma)$ in deposits. Thus, competition will equalize the rate of return to lending to the similarly weighted average of the required rates of return of equity and deposits, which is what is stated in (8). (The case of a nonbinding requirement will be clear in a moment.)

We have already established that $R^D \leq R^L$ is necessary for a solution to exist and that the capital requirement binds if and only if $R^D < R^L$. Hence, two cases are possible:

1. If $R^D = R^L = R^E$, the capital requirement is slack.

2. If $R^D < R^L < R^E$, the capital requirement is binding, so $E = \gamma L$.

In either case, $V^B(E) - E = 0$.

[13] As is common in problems with constant returns to scale, the first-order condition, rather than fully determining the agent's choice, has the interpretation of a necessary condition for the existence of a finite solution. If $R^E < (>) R^L + (\gamma^{-1} - 1)(R^L - R^D)$, then *E* tends to plus (minus) infinity. If the first-order condition holds, *E* is indeterminate, and thus so is the scale of the bank.

Note that the sufficient condition for $\sigma = 0$ to be optimal, given in (6), is seen to be equivalent to

$$\bar{\sigma} \leq \gamma R^E \tag{9}$$

Again, this condition is also necessary if the capital requirement is binding.

Firms: Nonfinancial firms cannot create liquidity through deposits. They can, however, buy goods to use them as capital, which can be combined with labor input, to produce output of the good. Capital is purchased at the beginning of the period. To finance their capital stock, firms can issue equity to households, borrow from banks, or some combination of both. The firm's balance sheet, and notation, for period t is:

Assets		**Liabilities**	
K_t	Physical Capital	L_t	Loans
		E_t^F	Firm Equity

Firms can employ a riskless or a risky production technology.[14] The riskless technology is standard. Output in period t is $F(K_t, H_t)$, where H_t is hours of labor input and $F(\ )$ is a well-behaved production function exhibiting constant returns to scale. A fraction δ of the capital stock depreciates during the period. There are no adjustment costs and firms last for one period.[15] Each period, there is a continuum of firms with mass normalized to one, so each firm takes prices as given.

As in the analysis of the bank's problem, it is convenient to start with the firm's decision problem right after it has raised E_t^F in equity. At that point the value of the firm to its shareholders is[16]

$$V^F(E^F) = \max_{K,H} \Big(F(K,H) + (1-\delta)K - wH - R^L(K - E^F) \Big) / R^E$$

Here I have substituted out loans using the balance sheet identity. The first-order conditions for the choices of labor and capital are standard:

[14] It would be straightforward to let all firms have risky production, and therefore make all individual loans risky, even while keeping $\sigma = 0$ as feasible for banks, as long as the production shocks are sufficiently imperfectly correlated across firms, so that the risk is perfectly diversifiable by lending to many firms. Excessive risk taking would then correspond to not diversifying this risk. (ξ would equal zero in this case.)

[15] The absence of adjustment costs and agency problems implies that this is without loss of generality. One can think of ongoing firms as repurchasing their capital stock each period.

[16] Note that the absence of arbitrage opportunities implies that nonfinancial firms have to offer shareholders the same return on equity as banks, since there is no aggregate risk.

$$(H) \qquad F_H(K,H) = w \tag{10}$$

$$(K) \qquad F_K(K,H) + (1-\delta) = R^L \tag{11}$$

These optimality conditions, together with the constant returns to scale assumption, imply that the solution for the firm's shareholder value is:

$$V^F(E^F) = R^L E^F / R^E$$

The pre-issue value of the firm is $V^F(E^F) - E^F$. It is assumed that equity cannot be negative: $E^F \geq 0$. Subject to that constraint, the firm chooses E^F to maximize its pre-issue value. The first-order condition to this problem is:

$$(E^F) \qquad R^L / R^E = 1 - \mu, \;\; \mu \geq 0, \;\; \mu E^F = 0 \tag{12}$$

where μ is the Kuhn-Tucker multiplier associated with the constraint that firm equity cannot be less than zero. A finite solution thus requires $R^E \geq R^L$.

If $R^E > R^L$, then $E^F = 0$, so $K = L$. In other words, if bank loans are cheaper than equity finance, the firm chooses to use only bank loans to finance its capital. If $R^E = R^L$, the firm's financial structure is not determined by individual optimality. In either case economic profits, $V^F(E^F) - E^F$, equal zero. As the discussion of the economy's equilibrium will make clear, which case applies will depend on whether households' demand for liquidity is satiated or not.

In addition to this riskless technology, firms can also employ a risky technology. Risky firms provide a vehicle for banks to make the kind of risky loans described in the subsection on banks. Although these firms thus provide a rationale for the existence of capital regulation, as mentioned, I will usually focus on the case that the capital requirement is sufficiently high, according to condition (9), to prevent banks from engaging in excessive risk taking. These risky firms will then not exist in equilibrium. For this reason analysis of these firms is left for appendix A. Here I simply state the assumptions regarding the risky technology.

Risky firms differ in that their output is subject to an additive shock that is proportional to their capital stock: output is $F(K,H) + \sigma_{RF}\varepsilon K$, where ε is the same negative mean, idiosyncratic shock as defined in (3) and σ_{RF} is a parameter ($\sigma_{RF} \geq \bar{\sigma}$). The choice of technology is observable to all parties to a financial contract with the firm, as is the value of ε when realized. The appendix shows how the optimal loan contract with such a firm allows a bank to create a loan portfolio with riskiness σ by directing a fraction σ / σ_{RF} of lending to a risky firm.

<u>Government</u>: The government manages the deposit insurance fund, sets a capital requirement $\gamma \in [0,1)$ and conducts bank supervision. The purpose of bank

supervision is not only to enforce the capital requirement, but also to monitor excessive risk taking by banks, σ. Supervisors can to some degree detect such behavior and stop any bank that is 'caught' attempting to take on excessive risk in order to protect the deposit insurance fund. It seems reasonable to assume that a small amount of risk taking is harder to detect than a large amount. The largest level of risk-taking that is still just undetectable is $\bar{\sigma}$. $\bar{\sigma}$ is assumed to be a decreasing function of the resources spent on bank supervision:

$$\bar{\sigma} = S(T) \quad \text{with} \quad S'(\bullet) \leq 0 \ \text{ and } \ 0 < S \leq \sigma_{RF}$$

where *T*, a choice variable for the government, is the part of tax revenue spent on bank supervision.[17] The interpretation is that, as more resources are devoted to bank supervision, banks are less able to engage in excessive risk taking without being detected.

The assumption of *imperfect* observability of excessive risk taking is important. If regulators could perfectly observe each bank's riskiness, they could simply adjust each bank's deposit insurance premium so as to make the bank pay for the ex ante expected loss to the deposit insurance fund, thus eliminating any moral hazard. Or they could set each bank's capital requirement as an increasing function of its riskiness, in such a way as to ensure that the bank always internalizes all the risks. But such perfect observability is simply not realistic, so a moral hazard problem does exist.

Not allowing σ to exceed $\bar{\sigma}$ can be interpreted as a risk-based capital requirement or a risk-based deposit insurance premium, but one based on *observable* risk. Under that interpretation, regulators deter detectable excessive risk taking (as they should) by imposing a sufficiently high capital requirement or a sufficiently high deposit insurance premium when such excessive risk taking is detected. As long as it is sufficiently high, the precise value of the capital requirement or premium when $\sigma > \bar{\sigma}$ is not important, as it will never be implemented in equilibrium.[18]

The government has a balanced budget. Lump-sum taxes are set at

$$T_t = T + 0.5\left(X_t + 1_{\{X_t > 0\}} \psi D_t\right) \tag{13}$$

[17] As in the standard growth model with government spending and lump sum taxes, if *T* is set too high, no equilibrium with positive consumption exists. I assume that *T* is sufficiently low so that a steady state equilibrium with positive consumption exists. Appendix D will make precise what 'sufficiently low' means for a particular functional form of the utility function, introduced in section 7.

[18] I have assumed that the bank pays a deposit insurance premium equal to zero when $\sigma < \bar{\sigma}$. In the model, this is the actuarially fair deposit insurance premium when regulation is successful in deterring excessive risk taking (i.e. when (9) holds – the case I focus on). It also happens to be the deposit insurance premium that virtually all U.S. banks currently pay.

with

$$X_t = \left(R_t^D D_t - (R_t^L - \sigma_t(1+2\xi))L_t\right)^+ \tag{14}$$

If $X_t > 0$, X_t is the loss to deposit insurance fund due a bank failure, and 0.5 is the mass of banks that fails if $X_t > 0$. In addition, there may be a deadweight cost of resolving bank failures, equal to $\psi \geq 0$ per unit of deposits in failed banks. If (9) holds, we know that $\sigma_t = 0$ and in that case taxes are simply: $T_t = T$.

General Equilibrium

Given a government policy γ and T, an equilibrium is defined as a path of consumption, capital, deposits, equity holdings, bank loans and financial returns, for t = 0,1,2,…, such that:

1. Households, banks and firms all solve their maximization problems, described above, with $\bar{\sigma} = S(T)$ and taxes set according to (13)-(14);
2. All markets clear, i.e.

$$d_t = D_t$$
$$e_t = E_t + E_t^F$$
$$L_t = K_t - E_t^F$$
$$H_t = 1$$

and

$$F(K_t,1) - \xi\sigma_t L_t + (1-\delta)K_t = c_t + K_{t+1} + T + 1_{\{X_t>0\}}(\psi/2)D_t$$

I focus on the case that (9) holds: $S(T) \leq \gamma R_t^E$. Government policy can accomplish this by setting γ and/or T sufficiently high. In that case, $\sigma_t = 0$, $X_t = 0$ and $T_t = T$. I will first describe the resulting allocation and then provide an explanation.

By combining the market clearing conditions and equations (1), (2), (8), (10), (11), and (12), it is possible to characterize the equilibrium in terms of a system in (K_t, c_t) with R_t^E and d_t as auxiliary variables:

$$K_t = F(K_{t-1},1) + (1-\delta)K_{t-1} - c_{t-1} - T \tag{15}$$

$$\beta^{-1}(u_c(c_{t-1},d_{t-1})/u_c(c_t,d_t)) = R_t^E \tag{16}$$

$$F_K(K_t,1) + 1 - \delta = R_t^L = R_t^E - (1-\gamma)\frac{u_d(c_t,d_t)}{u_c(c_t,d_t)} \tag{17}$$

where d_t is determined according to one of the following two cases:

1. If $u_d(c_t,(1-\gamma)K_t)=0$, the capital requirement is not binding and

$$u_d(c_t,d_t)=0, \quad \text{with } d_t \le (1-\gamma)K_t \tag{18}$$

2. If $u_d(c_t,(1-\gamma)K_t)>0$, the capital requirement is binding and

$$d_t=(1-\gamma)K_t \tag{19}$$

Remark: In case 2, $R_t^L < R_t^E$, so $E_t^F=0$ (by (12)), $L_t = K_t$, and $e_t = E_t = \gamma K_t$. In case 1, which requires that demand for liquidity be satiated at $d=(1-\gamma)K_t$, d_t, E_t, E_t^F, e_t and L_t are not uniquely determined. Note that if $u_d(c_t,d_t)=0$, $u_c(c_t,d_t)$ does not depend on d_t. In both cases, remaining variables are determined through (2) and (10) with $H_t=1$.

Two results are key to understanding how the equilibrium differs from the standard growth model: First, utility maximization by households implies that the pecuniary return on deposits is lower than the return on equity by a spread equal to the marginal value of deposits' liquidity services expressed in units of consumption, $u_d(c,d)/u_c(c,d)$ (see equation (2)). Second, the zero-profit condition for banking implies that the rate on bank loans is the weighted average of the required returns on equity and deposits (equation (8)).

In the first case, the level of deposits is so high that the marginal value of liquidity provision is zero, so the equity-deposit spread is also zero. As a result, the capital requirement is not binding, and deposits, equity and bank loans all command the same return. Equity and loans, and equity and deposits, are perfect substitutes for firms and banks, respectively, so these financial quantities are not uniquely determined in equilibrium. On the real side, there is no material difference in this case with a standard growth model (with government spending and lump-sum taxes). Banks channel funds from households to firms, but they do not really matter for capital accumulation, since their special ability to create liquidity has no marginal value.

In the second, more interesting case, the demand for liquidity is not satiated and the marginal value of liquidity services is positive. As a result, the spread between the return on equity and deposits also exceeds zero and the capital requirement is now binding, since banks want to fund their assets as much as possible with the cheaper deposits. Because of perfect competition, banks fully pass on the lower cost of funding their loans to their borrowers. Thus, the loan rate declines, though only by $(1-\gamma)(u_d/u_c)$, as banks still have to finance a fraction γ of their lending with equity. Nonfinancial firms now choose to finance all their capital stock with these cheaper bank loans, rather than equity.

Because banks pass on the low pecuniary return on deposits to firms, in steady state the capital stock is higher than without any preference for liquidity.[19] An important related result is that the steady state level of the capital stock is generally *not* invariant to the level of the capital requirement. Section 7 will explore this further. As will be shown there, an increase in γ can increase or decrease the steady state capital stock, depending on the interest elasticity of the demand for liquidity.

2. The welfare cost of the capital requirement

The strategy for quantifying the welfare cost of the capital requirement is as follows. First, I present a constrained social planner's problem. The qualification 'constrained' means that the social planner's problem shall respect the capital requirement and devote the same level of resources to bank supervision. This will ensure that the allocation that solves the planner's problem is incentive compatible for the banks. Rather than solve for the first-best allocation, this planner's problem is designed to replicate the decentralized equilibrium described above. Next, I show that the allocation associated with the planner's problem is indeed identical to the decentralized equilibrium's allocation. Finally, I exploit this equivalence to derive analytically a simple formula that provides a first-order approximation of the welfare cost of increasing the capital requirement.

The social planner's problem

Define the following constrained social planner's problem:

$$\begin{aligned} V_0(\theta) = \max_{\{c_t, d_t, K_{t+1}\}_{t=0}^{\infty}} & \sum_{t=0}^{\infty} \beta^t u(c_t, d_t) \\ \text{s.t.} \quad & F(K_t, 1) + (1-\delta)K_t = c_t + K_{t+1} + T \\ & (1-\gamma)K_t - d_t \geq 0 \end{aligned} \tag{20}$$

where $\theta = (\gamma, T, \delta, \beta, K_0)$. The first constraint is the social resource constraint for $\sigma = 0$;[20] the second constraint rewrites the capital requirement, imposing $E_t^F = 0$. The first-order conditions to this problem are:

[19] See equation (17) and note that $R_t^E = \beta^{-1}$ in steady state (see (16)) and $F_{KK} < 0$. 'No preference for liquidity' refers to the special case that, $u(c,d) = \tilde{u}(c)$ for all *c* and *d*, and for some function $\tilde{u}$. All other functions and parameters are kept the same in the comparison.

[20] The absence of excessive risk taking is simply part of the definition of the planner's problem.

(c) $\quad u_c(c_t, d_t) = \lambda_t^{sp}$

(d) $\quad u_d(c_t, d_t) = \chi_t^{sp}$

(K) $\quad \lambda_t^{sp}[F_K(K_t,1) + 1 - \delta] - \beta^{-1}\lambda_{t-1}^{sp} + \chi_t^{sp}(1-\gamma) = 0$

where λ_t^{sp} and χ_t^{sp} are the Lagrange multipliers on the social resource constraint and the capital requirement, respectively. Combining these first-order conditions yields:

$$F_K(K_t,1) + 1 - \delta = \left(\frac{\beta^{-1}u_c(c_{t-1}, d_{t-1})}{u_c(c_t, d_t)}\right) - (1-\gamma)\left(\frac{u_d(c_t, d_t)}{u_c(c_t, d_t)}\right) \tag{21}$$

In addition, the complementary slackness conditions $d_t \le (1-\gamma)K_t$, $\chi_t^{sp} \ge 0$ and $\chi_t^{sp}((1-\gamma)K_t - d_t) = 0$, combined with the first-order condition with respect to deposits, and the concavity of u (so $u_{dd}(c,d) \le 0$), imply:

$$\text{if } u_d(c_t, (1-\gamma)K_t) = 0 \text{ then } d_t \le (1-\gamma)K_t, \text{ with } u_d(c_t, d_t) = 0; \tag{22}$$

$$\text{if } u_d(c_t, (1-\gamma)K_t) > 0 \text{ then } d_t = (1-\gamma)K_t \tag{23}$$

Combining equations (21), (22) and (23) with the social resource constraint (the first constraint to the problem in (20)), it is apparent that the allocations of K_t, c_t and d_t are identical to those of the decentralized equilibrium summarized above in equations (15) through (19). Equation (22) corresponds to an equilibrium with a nonbinding capital requirement ('case 1', equation (18)), while equation (23) corresponds to the case of a binding capital requirement ('case 2', equation (19)).

Hence, ***the constrained social planner's problem replicates the decentralized equilibrium*** when $\sigma = 0$ in the decentralized equilibrium. As a result, if $\sigma = 0$, *welfare in the decentralized equilibrium is equal to* $V_0(\theta)$, the value of the objective function to the constrained social planner's problem.

A formula for the marginal welfare cost

The equivalence of the constrained social planner's problem and the decentralized equilibrium can be used to measure the marginal effect on welfare of a change in the capital requirement in the following way. Call the current period period 0. Assume government policy is such that (9) holds: $S(T) \le \gamma R_t^E$ for all $t \ge 0$, so that $\sigma_t = 0$ in the decentralized equilibrium. That is, the capital requirement and supervision spending are jointly sufficiently high to be successful in preventing excessive risk taking. Starting from this situation, I compute the

marginal effect on welfare of raising γ, without altering *T*, using the envelope theorem, as follows:

$$\frac{\partial V_0(\theta)}{\partial \gamma} = -\sum_{t=0}^{\infty} \beta^t \chi_t^{sp} K_t = -\sum_{t=0}^{\infty} \beta^t u_d(c_t, d_t) K_t$$

The last equality follows from the first-order condition (*d*) to the social planner's problem. Since the allocations of c_t, d_t and K_t are identical to those in the decentralized equilibrium, I can use the decentralized equilibrium values to evaluate the right hand side of this equation. Moreover, in the decentralized equilibrium, we have, using (2),

$$\begin{aligned} u_d(c_t, d_t) K_t &= u_c(c_t, d_t)(R_t^E - R_t^D) K_t \\ &= u_c(c_t, d_t)(R_t^E - R_t^D) d_t / (1-\gamma) \end{aligned}$$

where the last equality follows from the fact that $d_t = (1-\gamma)K_t$ if the capital requirement binds, and that $(R_t^E - R_t^D) = 0$ if it does not bind.

I compare this to the welfare effect of a permanent change in consumption by a factor $(1+\nu)$. Starting from the initial equilibrium, the effect on welfare of changing consumption from c_t to $(1+\nu)c_t$, for all *t*, equals, to a first-order approximation, $\Sigma_{t=0}^{\infty}(\beta^t u_c(c_t, d_t) c_t)\nu$.

Next, assume that the economy is in steady state in period 0. Then the first-order approximation of the welfare effect of an increase in γ by $\Delta\gamma$ simplifies as follows:

$$\frac{\partial V_0(\theta)}{\partial \gamma} \Delta\gamma = -\frac{u_d(c_0, d_0) K_0}{1-\beta} \Delta\gamma = -\frac{u_c(c_0, d_0)(R_0^E - R_0^D) d_0}{(1-\beta)(1-\gamma)} \Delta\gamma$$

Similarly, if period 0 is a steady state,

$$\left(\sum_{t=0}^{\infty} \beta^t u_c(c_t, d_t) c_t\right) \nu = \frac{u_c(c_0, d_0) c_0}{1-\beta} \nu \tag{24}$$

Equating the right hand sides of these last two equations, yields the following result.[21]

[21] Equate the right hand sides of these two equations to solve for ν and multiply the result by minus 1 to get the welfare-equivalent *loss* in consumption. The formula in the proposition omits time (0) subscripts for readability.

Proposition 1 *Assume that the economy is in steady state in the current period and that (9) holds. Consider permanently increasing* γ *by* $\Delta\gamma$*. A first-order approximation to the resulting welfare loss, expressed as the welfare-equivalent permanent relative loss in consumption, is* $\nu\Delta\gamma$*, where*

$$\nu = \frac{d}{c}\left(R^E - R^D\right)(1-\gamma)^{-1} \tag{25}$$

The above formula is empirically implementable. Remarkably, it does not rely on any assumptions about the functional form of preferences, beyond the standard assumptions of monotonicity, differentiability and concavity. Instead, the formula relies on asset returns to reveal the strength of the household's preference for liquidity. An unnecessary increase in the capital requirement reduces welfare by reducing the ability of banks to issue deposit-type liabilities.

The first factor in the formula for the welfare loss concerns the importance of deposits in the economy. The second is the spread between the return on bank equity and the pecuniary return to deposits. This spread equals the amount of consumption households are willing to forgo in order to enjoy the liquidity services of one additional unit of deposits. Finally, $(1-\gamma)^{-1}\Delta\gamma$ is the relative change in deposits as a result of changing the capital requirement by $\Delta\gamma$ for a given level of bank assets. The formula is valid even if the capital requirement happens not to bind. In that case, $R^E - R^D$ is zero, so the welfare cost is also zero.

Note that, while the proposition assumes that the economy is initially in steady state, the welfare loss takes into account, to a first-order approximation, all the gains or losses associated with the transition to a new steady state upon changing the capital requirement. (Recall that the steady state capital stock depends on γ.) That is, simply comparing the welfare levels of different steady states associated with different values of γ yields a different (and wrong) answer because it does not take into account the welfare effects of the transition between the steady states. Interestingly, the formula can also be 'derived' by incorrectly (!) assuming that the equilibrium levels of the capital stock and bank assets are invariant to changes in γ. [22] The fact that this is true is a manifestation of the envelope theorem: these quantities are constrained optimal in the sense of the social planner's problem, so their response to a change in γ has only a second-order effect on welfare. Of course, one would not have known this before going through the entire exercise.

[22] This is not inconsistent with the previous statements: the welfare effect of the transition to a new steady state cancels with the difference in welfare between the two steady states that is due to the different level of the steady state capital stock rather than γ alone.

It may still seem surprising that no assumptions were needed on functional form the utility function. After all, to use the Sidrauski model to measure the welfare cost of inflation, as Lucas (2000) does, one needs to know the interest elasticity of money demand, which amounts to requiring more knowledge of the utility function. The difference is that, while money in the Sidrauski model is created by a non-optimizing monopolist (the government), in this model the supply of liquidity is created by competitive banks. This additional structure in the model means we have some extra information on the welfare effect of the change in the quantity of deposits.

The methodology used here may be of independent interest. Recapitulating, the steps are: (1) guess a constrained social planner's problem, intended to mimic the decentralized economy (rather than solve for the first-best); (2) verify that it replicates the decentralized equilibrium; and (3) differentiate the value of the planner's problem with respect to the policy parameter, and exploit the equivalence with the decentralized allocation, to obtain an analytic expression for the welfare effect. This method will generally only yield a first-order approximation and it may not always be possible to guess a workable planner's problem. Nonetheless, when it works, this methodology has a number of advantages compared with a brute-force numerical approach. First, it is simpler. Second, the analytic expression may yield insight into the result. Third, with a brute-force numerical approach all functional forms (e.g. the utility function) and parameters need to be specified. As can be seen from the formula in the proposition, the informational requirements here are much weaker.

The optimal capital requirement

The rationale for capital adequacy regulation in the model is its role, joint with bank supervision, in preventing excessive risk taking. If bank supervision is imperfect ($S(T) > 0$ for all T) and preventing excessive risk taking is socially optimal, then the optimal capital requirement will be strictly positive. In the model, preventing excessive risk taking is socially optimal if either its direct cost, ξ, or its indirect cost due to costly resolution of bank failures, ψ, is sufficiently large. In contrast, if both these costs are small the social optimum is to have a zero capital requirement and accept the result that half the banks will fail. The formula for the welfare cost of the capital requirement is still valid in this case, but it expresses a cost that is to be avoided, rather than to be compared to the benefit of the capital requirement. Having said that, the case for preventing excessive risk taking may be stronger than the model suggests, as the model (to economize on notation) lacks one potential cost of bank failures: it implicitly assumes that the liquidity services of deposits in failed banks are identical to those of solvent

banks, which seems hard to believe. For the remainder of this section, I will focus on the case that preventing excessive risk taking is socially optimal (due to high ψ and/or ξ), so that there exists a rationale for capital adequacy regulation.

Under that hypothesis, in steady state, the capital requirement that maximizes welfare is defined by:

$$\max_{T,\gamma} V_0(\theta)$$
$$\text{s.t.} \quad \gamma\beta^{-1} \geq S(T)$$

The constraint is the incentive compatibility condition (9), with $R^E = \beta^{-1}$ in steady state. The first-order conditions to this problem imply

$$\frac{\partial V_0(\theta)}{\partial \gamma} + \left(\frac{\partial V_0(\theta)}{\partial T}\right)\frac{dT}{d\gamma}\bigg|_{S(T)=\gamma\beta^{-1}} = 0$$

Evaluating this in steady state yields

$$c\nu(1) = -\frac{dT}{d\gamma}\bigg|_{S(T)=\gamma\beta^{-1}} = \frac{-1}{\beta S'(T)} \tag{26}$$

That is, the marginal welfare cost of the capital requirement (in units of the good per period) should equal its marginal benefit in reducing bank supervision and compliance costs, given the incentive compatibility constraint. Making the reasonable assumption that there are diminishing returns to bank supervision, so that $S'' > 0$, a larger welfare cost demands higher supervision expenditures (a larger *T*) and thus a lower capital requirement. Measuring the marginal benefit of the capital requirement will require some additional information on the supervision technology *S*. This is not true for the marginal welfare cost, and I now turn to quantifying this cost.

3. Measurement of the welfare cost

The main result so far is an expression for the welfare cost of a bank capital requirement. The expression lends itself to a calculation of this cost based on data. For this purpose, I employ annual aggregate balance sheet and income statement data for all FDIC-Insured Commercial Banks in the United States. These data are obtained from the FDIC's Historical Statistics on Banking (HSOB) and are based on regulatory filings.

In mapping the theory to the data, some choices need to be made. For deposits, $D\ (=d)$, the HSOB's Total Deposits is used. The net return on deposits $(R^D - 1)$ is calculated as Interest on Total Deposits divided by Total Deposits.[23] For consumption, c, I use personal consumption expenditures from the NIPA. As a measure of the capital requirement γ the empirical counterpart of E/L is used.[24] This is computed as Total Equity Capital plus Subordinated Notes divided by Total Assets. Subordinated Notes are included because subordinated debt counts, within certain limits, towards regulatory tier 2 capital. Total Equity Capital plus Subordinated Notes does not exactly correspond to total capital in the sense of the Basle Accord, on which current capital adequacy regulation in the US (and many other countries) is based. However, data on total capital in the sense of the Basle Accord is only available starting in 1996 and it seems more important to be able to use a longer time span, especially since the formula for the marginal welfare cost in (25) is not very sensitive to the measurement of γ.

An alternative would have been to use the actual regulatory numbers for the capital requirement (either 0.08 for total capital based on the Basle Accord or 0.10 based on the FDICIA, the CAMELS ratings and the Gramm-Leach-Bliley Act). However, both the data and theory[25] suggest that the vast majority of banks hold a buffer of equity above the regulatory minimum so as to lower the risk of an adverse shock leading to capital inadequacy. Since the model abstracts from this buffer stock behavior by assuming away any shocks, one would want to include this buffer in the measurement of γ as it is due to the capital adequacy regulation in the first place.[26] There is little reason to expect the buffer itself would change dramatically in response to a change in the regulatory minimum capital ratio. In any case, as mentioned, the point is not quantitatively very important. For example, as we change the measure of γ from an unreasonably low value, say, 0.04 to an unreasonably high value, say 0.15, holding constant the other measurements, the estimated marginal welfare cost increases only by a factor 1.13 $(=(1-0.15)^{-1}/(1-0.04)^{-1})$.

Finally, a measure of the required return on (bank) equity is needed. Since the model abstracts from aggregate risk, a risk-adjusted measure is needed. To avoid the difficulties inherent in measuring the (ex ante) risk premium on regular equity,[27] the measure I use is the average return on subordinated bank debt. The

[23] All data are nominal. While the model is real, using nominal data consistently is fine, because the formula for the welfare cost in (25) contains only ratios of quantities and spreads of returns.
[24] This may seem incorrect if the capital requirement is not binding. However, if that is the case, the model implies that $R^E = R^D$, so the welfare cost is zero regardless of how γ is measured.
[25] See Van den Heuvel (2004) for a quantitative model.
[26] In addition these ratios apply to risk weighted assets and off-balance sheet items, considerations from which the model also abstracts.
[27] For example, the historical average excess return on bank equity would imply a high premium, but does this equal the *ex ante* expected premium? In addition, depending on what interest rate is

reason for this choice is that (a) subordinated debt counts towards regulatory equity capital, albeit within certain limits, and (b) defaults on this type of debt have historically been very rare, so the debt is not very risky. As a measure for $(R^E - 1)$, the net return on subordinated debt is calculated as Interest on Subordinated Notes and Debentures divided by Subordinated Notes and Debentures.[28]

The limits on the use of subordinated debt for regulatory purposes imply that this is a conservative measure for the risk-adjusted required return on bank equity. First, because it is regarded as an inferior form of equity, subordinated debt can count only towards tier 2 capital. Second, and more importantly, the amount of subordinated debt is limited to 50 percent of the bank's tier 1 capital. What this means is that if the tier 1 capital ratio is close to binding, subordinated debt can count for at most approximately 25 percent of total capital. Since banks may use subordinated debt to meet their capital requirements only up to these limits (and they do not have to use it), it is possible that for many banks the required return on subordinated debt is lower than the risk-adjusted return on regular equity.

To measure the welfare cost using the derived formula I compute long run averages for the deposit consumption ratio, for the spread between the return on subordinated debt and deposits, and for the capital asset ratio. The Basle Accord and the FDIC Improvement Act enacting it were not fully implemented until January 1, 1993. For this reason, the sample period is set at 1993-2002. [29]

For 1993-2002 the mean deposit to consumption ratio is 0.61, the average net returns on deposits and subordinated debts are, respectively, 3.08% and 6.26%, so the average spread is 3.18%, and the mean capital asset ratio is 0.10. Hence, applying (25), a first-order approximation to the welfare cost of raising the capital requirement by $\Delta\gamma$ is:

$$\begin{aligned} \nu\Delta\gamma &= (d/c)\times(R^E - R^D)\times(1-\gamma)^{-1}\Delta\gamma \\ &= 0.61 \times 0.0318 \times (1-0.1)^{-1}\Delta\gamma = 0.0216\Delta\gamma \end{aligned}$$

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used to measure the excess return on equity, this approach runs the risk of contaminating the measured risk premium with a liquidity premium, which one would definitely want to avoid in the present context. If on the other hand one takes a model based measure of the ex ante risk premium based on 'reasonable' standard preferences, one would likely get a much lower measure of this premium. (This is the well known equity premium puzzle.)

[28] Part of the HSOB's Subordinated Debt and Debentures does not qualify as regulatory tier 2 capital. However, cross-checking with the Reports on Condition and Income ('call reports') item RCFD5610 indicates that the difference is minimal after 1992.

[29] As shown in section 5, using a longer sample (1986-2002) yields very similar results.

To interpret this number, consider the welfare cost of the current level of the capital requirement, $\gamma = 0.1$, compared to a zero capital requirement ($\gamma = 0$).[30] This welfare cost is equivalent to a permanent loss in consumption of

$$\nu \times 0.1 = 0.0216 \times 0.1 \times 100\% = 0.216\%.$$

This is not, in my view, a trivial welfare cost. Some well-known estimates on the welfare costs of business cycles or the welfare gains of implementing the optimal monetary policy rule (taking as given average inflation) are much smaller.

Here is another way to interpret this number. Consider lowering the effective capital requirement by 1 percentage point (to 0.086). And suppose regulators can keep the probability of bank failure as low as it is today despite this change by spending more on bank supervision and imposing higher compliance costs. If the total cost of keeping the probability of bank failure the same is less than $\nu \times 0.01 \times c_{2002} = 0.0216 \times 0.01 \times 7376$ = 1.6 billion $ per year, then this ought to be done: lowering the capital requirement would be welfare improving in this way.[31] If not, the capital requirement ought to be increased.

It should be pointed out that this estimate is conservative in the sense that, as mentioned, the true spread between the required return on equity and deposits may be higher than the one measured here due to the limits on the use of subordinated debt for regulatory purposes. Section 5 will provide some alternative measures of the spread and associated estimates of the welfare cost.

A different objection one might have to the above calculation of the welfare cost is that it does not take into account any resource costs that banks incur in servicing deposits or making loans. The former include the costs of ATM networks, part of the cost of maintaining a network of branches, etc. The latter include the costs of screening loan applications, collecting payments, as well as part of the cost of maintaining a branch network. These costs are not trivial. For the period 1993-2002 net noninterest costs of U.S. banks have averaged 1.29% of total assets. The next section will address this concern by incorporating into the model resource costs associated with accepting deposits and/or making loans. Section 5 will use the results of the model to show how this affects the measured welfare cost of the capital requirement.

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[30] This is, of course, a gross cost which ultimately needs to be compared to the benefit of a 10% capital requirement in reducing the risk of bank failures or in economizing on supervision cost. The number can also be interpreted as (a first-order approximation to) the cost of a new regulation that increases γ by 10 percentage points (doubling the effective capital requirement, from 0.1 to 0.2) without any change in bank supervision.

[31] Of course, in reality taxation is not lump-sum but usually distortionary, so one would want to make some allowance for that.

4. Costly financial intermediation

This section extends the model to allow for resource costs associated with servicing deposits and/or making loans. More precisely, it is now assumed that a bank with D in deposits and L in loans pays a cost $g(D,L)$ to service those financial contracts. The cost enters negatively in the calculation of the bank's net cash flow.

I make the following assumptions on g: $g(D,L)$ is nonnegative, twice continuously differentiable, increasing in its first argument, strictly increasing in its second argument, convex, and homogenous of degree 1, i.e. it exhibits constant returns to scale. Note that linear costs are included as a special case.

For the rest, the model is the same as presented in section 1. It is, however, convenient to impose the following additional assumption on the utility function:

$$\lim_{d \downarrow 0} \frac{u_d(c,d)}{u_c(c,d)} > \frac{g(1-\gamma,1)}{1-\gamma} \text{ for all } c > 0 \tag{27}$$

This 'weak Inada' condition[32] is imposed only to streamline the analysis of the equilibrium. If it fails to hold, there is an additional –empirically irrelevant– case to consider in which banks do not exist in equilibrium because the cost of intermediation is too high relative to the marginal value of liquidity, regardless of how scarce liquidity is. If that case applies, the model in any case closely resembles a standard growth model.[33] The above assumption is sufficient, but not necessary, to rule out this empirically uninteresting case.

The introduction of the cost of intermediation has a direct effect only on the bank's decision problem. The rest of this section analyses the bank's decisions in the presence of g, and then moves on to describe how the equilibrium changes.

Banks under costly financial intermediation

With costly intermediation, the bank's net cash flow is equal to $(R^L + \sigma\varepsilon)L - R^D D - g(D,L)$. With this modification, the bank's problem can still be analyzed in much the same way as in section 1. To avoid repetition this is left for an appendix. Appendix B.1 proves the following results.

The bank's choice still involves zero or maximum excessive risk. Zero risk is optimal under the same condition (9) as with costless intermediation: $\bar{\sigma} \le \gamma R^E$.

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[32] $\lim_{d \downarrow 0} u_d(c,d) = \infty$ is a sufficient condition for the assumption to hold.

[33] Obviously, without banks, the welfare cost of increasing the capital requirement would be zero. The elements in the formula for the welfare cost, derived below, would be unobservable.

With costly financial intermediation, the capital requirement always binds, so $E = \gamma L$. (Briefly, a nonbinding capital requirement would imply that the bank is willing to finance the marginal loan with 100 percent equity, but with $g_L > 0$ that loan would have to be strictly more expensive than equity, so no firm would want to take the loan – see the appendix.) As a result, condition (9) is necessary as well as sufficient for $\sigma = 0$.

Moreover, the zero profit condition for the bank is now:

$$R^L = \gamma R^E + (1-\gamma)R^D + g(1-\gamma, 1) \tag{28}$$

Under costless financial intermediation, the rate on bank loans was simply equal to the appropriately weighted average of the returns on equity and deposits. Now, the return on bank loans also includes the term $g(1-\gamma,1)$ - the resource cost of lending one additional unit and servicing $1-\gamma$ additional units of deposits to finance the loan. As constant returns to scale of *g* implies that the marginal cost of increasing the scale equals the average cost, the bank still has zero economic profits: $V^B(E) = E$.

General equilibrium with costly intermediation

As mentioned, for households and nonfinancial firms, the analysis is exactly the same as in section 1. For the government, a small change is necessary. Taking into account the effect of intermediation costs, the loss to deposit insurance fund due to a bank failure is now:

$$X_t = \left(R_t^D D_t - (R_t^L - \sigma_t(1+2\xi))L_t - g(D_t, L_t)\right)^+ \tag{29}$$

With this change, the government's budget constraint is still given by (13).

The definition of equilibrium is the same as in the first section, except that, of course, the bank's maximization problem referred to in the definition is now the one presented in this section, (29) replaces (14), and the market clearing condition for the goods market is altered to:

$$F(K_t,1) - \xi\sigma_t L_t + (1-\delta)K_t = c_t + K_{t+1} + g(D_t, L_t) + T + 1_{\{X_t>0\}}(\psi/2)D_t$$

Again, I focus on the case that (9) holds: $S(T) \le \gamma R_t^E$, so that $\sigma_t = 0$, $X_t = 0$ and $T_t = T$. As in section 1, it is possible to characterize the equilibrium as a dynamic system. Details are shown in Appendix B.2. Not surprisingly, the equilibrium behavior is in many ways similar to the case of costless intermediation. There are, however, two noteworthy differences.

First, the spread between the required return on equity and the rate on bank loans is lowered by cost of intermediation. Specifically, this spread equals:

$$R_t^E - R_t^L = \Delta(c_t, d_t) \equiv (1-\gamma)\frac{u_d(c_t, d_t)}{u_c(c_t, d_t)} - g(1-\gamma, 1)$$

Banks still pass on the low pecuniary return on deposits to firms, but now have to be compensated for the resource cost of intermediation, $g(1-\gamma,1)$. As a result, the cost of intermediation counteracts the stimulus to capital accumulation that stems from the liquidity creating role of banks.

Second, the equilibrium may be characterized by firms relying on a mix of bank and equity finance.[34] This will happen if the cost of intermediation is high relative to households' preferences for liquidity. To see this, note that the equity-loan spread implied by pure bank finance ($L_t = K_t$) is $\Delta(c_t, (1-\gamma)K_t)$ (since the capital requirement binds). Now, if this implied spread is positive, pure bank finance is in fact an equilibrium: bank loans are cheaper than equity for firms, so firms rely exclusively on bank loans, as was the case with costless intermediation.

However, if the cost of financial intermediation is high relative to the value of liquidity services, it is possible that $\Delta(c_t, (1-\gamma)K_t) < 0$. In that case, pure bank finance would imply that equity is cheaper than bank loans, which is not an equilibrium. Instead, firms will use both equity and bank loans, in such proportion that, in equilibrium, their costs are exactly equal: $R_t^L = R_t^E$. This determines size of the banking sector, through the condition $\Delta(c_t, (1-\gamma)L_t) = 0$. To sum up, two cases are possible:

(a) If $\Delta(c_t, (1-\gamma)K_t) \geq 0$, firms rely solely on bank loans and $d_t = (1-\gamma)K_t$.

(b) If $\Delta(c_t, (1-\gamma)K_t) < 0$, firms use a mix of equity and bank loans and $\Delta(c_t, d_t) = 0$.

(In each case, the capital requirement binds, so $L_t = d_t/(1-\gamma)$.)

It is interesting to note that in case (b) the steady state capital stock is invariant to the capital requirement γ, in contrast to the result under costless financial intermediation.[35] In fact, in this case the steady state capital stock is no different from its value in the standard growth model. In case (a), however, it is easy to verify that such an invariance result fails to hold, as in section 1. In either case, welfare is affected by the capital requirement, as I will now show.

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[34] This was a possibility with costless intermediation also – in the case of a nonbinding capital requirement –, but it was due purely to indeterminacy of the financial quantities.

[35] The reason is that $R_t^L = R_t^E$ in case (b), and the latter is simply β^{-1} in steady state. With the marginal product of capital equal to R_t^L, this pins down the steady state capital stock

A formula for the marginal welfare cost for the case of costly intermediation

The strategy for deriving an expression for the welfare cost is the same as in section 2. Again, to avoid repetition the details are left to the appendix. First, in part B.3, a constrained social planner's problem is presented, with two differences from the one in section 2: it includes the intermediation cost g in the social resource constraint and it allows for the possibility that firm equity is strictly positive ($L_t < K_t$). Next, it is shown that this planner's problem replicates the decentralized equilibrium with costly intermediation when σ = 0 in that equilibrium (i.e. when $S(T) \leq \gamma R_t^E$). Finally, this equivalence is exploited to obtain a formula for the marginal effect on welfare of a change in the capital requirement. Going through the same steps as in section 2, part B.4 proves the following proposition:

Proposition 2 *Assume that the economy is in steady state and that (9) holds. Consider permanently increasing γ by $\Delta\gamma$. With costly financial intermediation, a first-order approximation to the resulting welfare loss, expressed as the welfare-equivalent permanent relative loss in consumption, is $\nu^{ci}\Delta\gamma$, where*

$$\nu^{ci} = \frac{d}{c}\left(R^E - R^D - g_D(d,L)\right)(1-\gamma)^{-1} \tag{30}$$

This formula differs from the one derived for costless intermediation in that it subtracts the marginal resource cost of servicing deposit contracts from the spread between the returns on equity and deposits. The intuition for this change is straightforward: If liquidity creation is costly, then, even in the absence of a binding capital requirement, this creates a spread between the returns on equity and deposits, as banks need to be compensated for this cost. It is only to the extent that the spread exceeds the marginal resource cost of deposits that a scarcity of deposits due to the binding capital requirement is revealed. Only then is there a welfare effect at the margin.

It is worth emphasizing that the formula is valid whether the equilibrium is characterized by pure bank finance or by mixed bank and equity finance. In addition, if g = 0, the formula specializes to the one derived in proposition 1 for costless intermediation. How one might measure, or bound, the marginal cost of deposits is addressed in the next section.

5. Measurement of the welfare cost, part 2

Arguably the most conservative way of measuring the new term, $g_D(D,L)$, in the expression for the welfare cost (30) is to calculate upper and lower bounds based only on assumptions already made, namely that the cost function *g* is nondecreasing and exhibits constant returns to scale. These imply:[36]

$$0 \le g_D(D,L) \le \frac{g(D,L)}{D}$$

Setting $g_D = 0$ yields the same result as with costless financial intermediation. Section 3 found that $\nu = 0.0216$ in this case. To implement the upper bound, *g* is measured as net noninterest cost (Total Noninterest Expense minus Total Noninterest Income). The average ratio of net noninterest cost to deposits for 1993-2002 is 0.0187 (i.e. 1.87 percent). With this upper bound for g_D we get the following *lower* bound for the welfare cost:

$$\nu^{ci}\Delta\gamma \ge 0.61 \times (0.0318 - 0.0187) \times (1 - 0.096)^{-1}\Delta\gamma = 0.0089\Delta\gamma$$

According to this method, the welfare loss of the current effective capital requirement ($\gamma = 0.1$) is then somewhere between 0.089% and 0.216% of consumption (permanently). Naturally, recognizing that financial intermediation is costly leads to a somewhat lower estimate of the marginal welfare cost.

As mentioned, using subordinated debt to measure the required return on equity is likely a conservative way to measure the welfare cost. We can measure the spread in an alternative way by rewriting the bank's zero profit condition (28), as follows:

$$R^E - R^D = \gamma^{-1}\left(R^L - R^D - g(1-\gamma,1)\right)$$

Using the constant returns to scale of *g* and the result that the capital requirement binds, this yields an alternative, theoretically equivalent, way of measuring the welfare cost:

$$\nu^{ci} = \frac{d}{c}\left(\frac{1}{\gamma}\left(R^L - R^D - \frac{g(D,L)}{L}\right) - g_D(D,L)\right)(1-\gamma)^{-1} \qquad (31)$$

To implement this, I use two alternative quantities for measuring loans *L*: Total Loans and Total Assets, as it is possible to regard securities owned by banks as bank loans in another form. When using Total Loans, the net return on loans

[36] The assumption that $g_L > 0$ actually implies that the second inequality is strict.

$(R^L - 1)$ is calculated as (Total Interest Income on Loans minus the Provision for Loan Lease Losses) divided by Total Loans. The Provision for Loan Lease Losses represents "the amount needed to make the allowance for loan and lease losses adequate to absorb expected loan and lease losses, based upon management's evaluation of the bank's current loan and lease portfolio" (HSOB). That is, it captures the decline in the value of loans due to an increase in expected default losses. When using Total Assets, $(R^L - 1)$ is computed as (Total Interest Income minus the Provision for Loan Lease Losses plus Securities Gains/Losses) divided by Total Assets. All other variables are measured in the same way as in section 3.

I also consider two sample periods for computing the historical averages: the post-Basel/FDICIA period 1993-2002, as before, and 1986-2002. While the second sample periods includes the regulatory changes associated with Basel and the FDICIA (which increased the effective capital requirement somewhat), it has the advantage that it is longer. The reason for not going back further in time, is that regulation Q, which placed some restrictions on banks' setting of deposit rates, was not fully phased out until January 1, 1986. Incidentally, in section 3, I chose not to use the 86-02 sample with subordinated debt, because before the Basel Accord its use in satisfying capital adequacy rules was rather limited.[37]

The returns on loans and assets are likely to contain a risk premium, albeit a much smaller one than for equity, so using them without risk adjustment may well result in an upwardly biased measurement of the welfare cost. I will first present results using unadjusted returns and then construct risk-adjusted measures and use those.

For the longer sample period (1986-2002), the unadjusted return on Total Assets averages 6.75%. After deducting noninterest cost (1.51% of total assets) this exceeds the return on deposits (3.96%) by 128 basis points. As a result, using (31), the first-order approximation to welfare cost of the current effective capital requirement ($\gamma = 0.1$) is found to be equivalent to a permanent cut in consumption of between 0.91% and 1.05%, depending on the cost share of deposits. This is considerably higher, by about a factor 5, than the previous estimates, which used subordinated debt.

For comparability, table 1 displays the results of the various strategies for measuring the welfare cost. The numbers are the permanent loss in consumption, in percent, that is welfare equivalent to $\Delta\gamma = 0.1$. As can be seen in the fourth row of the table, using Total Loans rather than Total Assets results in an estimate of the welfare cost that is even a little higher, ranging from 1.17% to 1.32%. The reason is that the return on total loans is, at 8.07%, exceeds the return on total

[37] Nonetheless, as can be seen in table 1, the results are quite similar when using 86-02 instead of 93-02 with subordinated debt.

TABLE 1. WELFARE COST OF THE CURRENT EFFECTIVE CAPITAL REQUIREMENT ($\nu^{ci} \times 0.1$ in percent).

Sample:	**1986 - 2002**		**1993 – 2002**	
	$g_D = 0$	$g_D = g/D$	$g_D = 0$	$g_D = g/D$
Subordinated Debt	0.21	0.06	0.22	0.09
Total Assets	1.05	0.91	1.19	1.07
Risk adjusted	*0.82*	*0.67*	*1.16*	*1.03*
Total Loans	1.32	1.17	1.56	1.43
Risk adjusted	*0.96*	*0.82*	*1.46*	*1.33*

Notes: first-order approximation to the welfare loss associated with $\Delta\gamma = 0.1$, expressed as the welfare-equivalent percent permanent loss in consumption. The first row implements (30), all other rows are based on (31).

assets. Using the 1993-2002 sample also results in slightly higher measurements of the cost in all cases.

While the estimates using subordinated debt should be considered conservative, there is, as mentioned, a concern that these new results overstate the welfare cost, because in reality the returns on total assets or loans likely contain a nontrivial risk premium, which is absent in the model. To examine to what extent this accounts for the higher estimates, I perform a crude, back-of-the-envelope risk adjustment, as follows.

The historical standard deviation of the spread between loans and deposits, net of noninterest cost, i.e. of $R_t^L - R_t^D - g(D_t, L_t)/L_t$, is 0.57% for total assets, or 0.86% for total loans.[38] Treating R_t^D as a risk free rate, the resulting Sharpe ratio of $R_t^L - g_t / L_t$ is about 2 in each case. Compared to the Sharpe ratio of the US stock market, approximately 0.5 for the S&P500, this seems very high. To view the average excess return $R_t^L - g_t / L_t - R_t^D$ as purely a risk premium, one must believe that bank loans are about 4 times as risky per unit standard deviation as the stock market. That does not seem very plausible to me. To the extent that one regards the equity premium as a puzzle, it is then an even greater puzzle to explain this 'banking premium', if it is purely a reward for risk.

Thus, borrowing by accepting (FDIC-insured) deposits and making bank loans with the proceeds is a strategy with very favorable risk-return properties. However, it is not actually possible to execute this strategy exactly if the capital

[38] These numbers are for 1986-2002; for 1993-2002 they are 0.10% and 0.28%, respectively.

requirement binds – some of the loans must be financed with equity. And this is precisely the point: to the extent that the capital requirement binds, the high Sharpe ratio of bank loans relative to deposits is not such a puzzle, because the model predicts a positive 'banking premium' even in the absence of risk.

This discussion suggests a very simple way of risk-adjusting the return on loans. Assume the market price of risk equals the Sharpe ratio of the stock market, roughly 0.5 annually. In addition, assume that all the variation in the excess return on loans is risk.[39] Under these assumptions, the risk premium in $R_t^L - g_t / L_t - R_t^D$ is $0.5 \times 0.57\% = 0.29\%$ for total assets, or $0.5 \times 0.86\% = 0.43\%$ for total loans. Deducting this risk premium from the spread lowers the measured welfare cost moderately. For the longer sample, the risk-adjusted estimates for the welfare cost of the current effective capital requirement ($\gamma = 0.1$) range from 0.67 to 0.96 percent (see table 1).

These numbers are similar to the estimated welfare cost of permanently increasing inflation from zero to 10 percent, as measured by Lucas (2000). They are also close to the low end of the range of estimates of the welfare gain of eliminating capital income taxation, when taking into account the welfare effects of the transition to a new steady state (see, e.g., Lucas 1990).

Admittedly, the risk adjustment is outside the model and the calculation is rather crude. Nonetheless, it suggests that the welfare cost of capital requirements may well be underestimated considerably by using subordinated debt as a proxy for the required return on equity, though the difference is almost certainly less than an order of magnitude. Despite the limited remaining uncertainty, taken together, these results suggest a fairly large welfare cost of bank capital adequacy regulation.

6. Are capital requirements too high or too low?

What does the sizable welfare cost imply for optimal bank regulation? It is important to realize that the welfare cost measured here is a *gross* cost.[40] It needs to be compared with the benefit of capital requirements in reducing the cost of bank supervision and compliance. As discussed in section 2, capital adequacy regulation and bank supervision are jointly needed to prevent socially undesirable excessive risk taking, but there exists a trade-off between the two: with a higher

[39] E.g., the CAPM holds and both the stock market and bank loans have a beta equal to one.
[40] It has been argued that a similar point applies to standard estimates of the welfare cost of inflation: increasing inflation results in more seignorage, so that taxes can be lowered while keeping government revenue constant. If taxes are distortionary, then the *net* welfare cost of higher inflation is lower. Lucas (2000) finds that the resulting adjustment is small, however.

capital requirement it is easier for bank supervisors to prevent excessive risk taking by banks, so that spending on bank supervision can be lowered. It is conceivable that the resulting marginal welfare benefit (derived in equation (26)) is as large as the marginal welfare cost, in which case current regulations are in fact optimal. This section attempts to measure this marginal welfare benefit in order to find out whether current capital requirements are too high, too low or about right.

Before moving on to this exercise, it is worth emphasizing that if one believes that deposit insurance does *not* create a moral hazard problem (perhaps because bank risk is perfectly and costlessly observable), or that deposit insurance is undesirable to start with (and there are no other externalities associated with bank failures), then capital requirements have no benefits at all. In that case, the welfare cost measured above is a net cost as well as a gross cost and a sizable welfare gain can be attained by lowering the capital requirement to zero.

This paper has instead adopted the view that there exists a rationale for a capital requirement. Its marginal welfare benefit (*mwb*) is simply the incentive compatible reduction in supervision cost per unit change in the capital requirement, which, in steady state, is given by the right hand side of equation (26) (in units of the good per period). As this expression is applicable only if the incentive compatibility constraint binds ($S(T) = \gamma\beta^{-1}$), it can be rewritten as: [41]

$$mwb = -\left.\frac{dT}{d\gamma}\right|_{S(T)=\gamma\beta^{-1}} = \frac{1}{\alpha_S \gamma} \tag{32}$$

where α_S denotes the semi-elasticity of S with respect to T: $\alpha_S \equiv -S'(T)/S(T)$. Recall that T is spending on bank supervision and S is the 'supervision technology' mapping T into $\bar{\sigma}$, the maximum undetectable level of risk taking ($S' \leq 0$), so α_S is a measure of the marginal effectiveness of supervision spending. Ceteris paribus, a higher value of α_S implies a lower marginal welfare benefit of the capital requirement, because supervision is a very effective alternative way to limit excessive risk taking by banks.

In light of the very low rate of bank failures in the U.S. since the implementation of the Basel Accord,[42] it seems appropriate to regard the current

[41] As this expression depends on the particular form of the incentive compatibility constraint, $S(T) \leq \gamma R^E$, the reader may be concerned that the form of this constraint in turn depends on the rather special example distribution of the bank-specific shock for excessive risk, ε, given in equation (3). This is not the case, however: Appendix C.1 shows that, with an appropriate normalization, this incentive compatibility constraint is valid for any distribution of ε that has a nonpositive mean and bounded support $[\underline{\varepsilon}, \bar{\varepsilon}]$, with $\underline{\varepsilon} < 0 < \bar{\varepsilon}$.

[42] During 1993-2002 about 0.1% of FDIC insured commercial failed on average per year and these failing banks represented about 0.02% of aggregate bank assets.

U.S. regulatory and supervisory regime as largely successful in preventing excessive risk taking, so that the incentive compatibility constraint is in fact satisfied, though not necessarily binding. Note that if the incentive compatibility constraint is merely satisfied but not binding (i.e. if $S(T) < \gamma\beta^{-1}$), then the marginal welfare benefit of the capital requirement equals zero: at the margin γ could be lowered without any increase in supervision spending, thus improving welfare, if the marginal welfare cost is strictly positive.[43]

To evaluate the expression for the marginal welfare benefit shown in (32), the marginal effectiveness of supervision spending, α_S, needs to be measured. To that end, I perform a calibration exercise, which is detailed in Appendix C.2. Under the assumption that the incentive compatibility constraint currently binds, the result is that $\alpha_S = 1.6$. (Otherwise, the exercise yields $\alpha_S > 1.6$.) Inserting this into (32), with $\gamma = 0.1$, yields a marginal welfare benefit of 6.2 *billion $ per year*.

The key ingredient to this measurement of α_S is the level of bank supervision spending, which is $1.4 billion.[44] However, this number does not include the banks' cost of compliance with the supervisory process. If we include compliance cost and assume it is *twice* bank supervision spending, we have T = $4.2 billion. Using that, the calibration results in a lower value for α_S ($\alpha_S \geq 0.53$) and the estimated marginal welfare benefit rises to $18.7 billion per year. [45] [46]

As mentioned, this marginal welfare benefit must be compared to the marginal welfare cost of capital requirements, in units of the good per year, cv^{ci} (see equation (26)). Using the estimates for the welfare cost based on subordinated debt, as well as the risk-adjusted ones using total assets and loans, the marginal welfare cost cv^{ci} *ranges from $57 billion to $603 billion per year, with a mean of $375 billion.*[47] Even using the lowest number, which is based on subordinated debt and attributes *all* net noninterest cost to servicing deposits, it appears that the marginal welfare cost substantially exceeds the marginal welfare

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[43] This is of course a local calculus of variation argument. I will return to this case below - see footnote 46.

[44] This is total spending on bank supervision by the OCC, the FDIC, the Federal Reserve System, and state agencies in 1999 (Hawke (2000)).

[45] One can also argue that $1.4 billion is an *over*estimate of *T*, since a part of this amount is spent on examining for compliance with regulations that are only tangentially related to promoting the 'safety and soundness' of the banking system. To name just a few examples from among many: the Fair Lending Act, Right to Financial Privacy Act, National Flood Insurance Act, etc. Using a lower estimate for *T* would result in a lower estimate for the marginal welfare benefit.

[46] As mentioned, if $S(T) < \gamma\beta^{-1}$, the marginal welfare benefit is zero. If *T* were lowered to $T' < T$, where $S(T') \equiv \gamma\beta^{-1}$, the *mwb* would again equal $1/(\alpha_S\gamma)$. As $\alpha_S > 1.6$ (or > 0.53) if $S(T) < \gamma\beta^{-1}$, we would have a *mwb* of *less* than 6.2 (or 19) billion $ at *T'* .

[47] See table 1. This uses preferred sample periods (1993-2002 for sub-debt and 1986-2002 for total assets and total loans) and 1999 consumption, which is $6283 billion, for comparability with the marginal welfare benefit. For example, the lower bound of the range is based on sub-debt, $g_D = g/D$ and sample 93-02: $v^{ci} \times 0.1 = 0.09\%$, so $cv^{ci} = 6283 \times 0.0009 \times (1/0.1) = 57$.

benefit. The conclusion is that capital requirements are currently set *too high*: welfare can be raised by lowering them and the welfare gains of doing so are potentially large.

It is true that the calculation of the marginal welfare benefit relies on some additional assumptions. It is certainly possible to obtain other and perhaps better measures of this quantity. Nonetheless, I would be surprised if, based on $1.4 billion in supervision spending, one would find a number in the hundred billion-plus range. To regard the present regulatory environment as optimal, one has to believe that there is something magical about the current 10% capital ratio; that a slight decrease in this ratio will lead to a sudden and large increase in the number of bank failures from the near zero rate today, *and* that this increase is not preventable by, say, a doubling or tripling of supervision spending.

What should happen to bank supervision spending? This is not clear. If the incentive compatibility constraint currently binds, i.e. if $S(T) = \gamma\beta^{-1}$, then the conclusion that γ should be lowered immediately implies that supervision spending, *T*, should be increased to compensate (as $S' < 0$). However, if the constraint does not bind, such an implication is not warranted. In that case, it may well be optimal to lower both the capital requirement and bank supervision cost.

7. The effect of the capital requirement on income

As mentioned in section 1, the steady state capital stock, and thus income, is generally not invariant to changes in the capital requirement,[48] in stark contrast with the famed superneutrality result of the Sidrauski (1967) model.[49] This section shows that raising γ can increase or lower the steady state capital stock, depending on the interest elasticity of liquidity demand. As an example, consider the case where financial intermediation is costless and suppose that

$$u(c,d) = \tilde{u}\left(\phi\left(c,d\right)\right) \text{ and } \phi\left(c,d\right) = \left\{c^{(\eta-1)/\eta} + ad^{(\eta-1)/\eta}\right\}^{\eta/(\eta-1)}$$

with $a,\eta > 0$, $\tilde{u}' > 0$ and $\tilde{u}'' < 0$. Using (2), with this utility function the demand for deposits is given by

[48] As mentioned, such an invariance result does hold for 'case (b)' with costly intermediation.
[49] In the Sidrauski model, the rate of inflation (which is what determines the rate of return on monetary assets) has no impact on the steady state capital stock. The reason is that in that model money is created by a monopolistic entity, the government, which does not in any way use the revenues from liquidity creation (seignorage) to lower the marginal cost of funding investment. In this paper, liquidity is created by competitive banks which do pass on the lower cost of funding to firms.

$$d_t = a^{\eta} c_t (R_t^E - R_t^D)^{-\eta}.$$

Hence, η has the interpretation of the interest elasticity of the demand for deposits. It is straightforward to show that with this specification, *the steady state level of the capital stock, K*, is increasing (decreasing) in* γ *if* $0 < \eta < 1$ *(*$\eta > 1$*)*. A proof can be found in Appendix D.

The intuition for this result is as follows. Firms set the marginal product of capital equal to the rate on bank loans, which in turn equals

$$R^L = R^E - (1-\gamma)(R^E - R^D)$$

(see (8)). In steady state, $R^E = \beta^{-1}$. An increase in γ has two effects on R^L: one is to force banks to rely more on equity finance, which is more expensive than deposits ($R^E - R^D > 0$). This effect, which is explicit in the above equation, increases R^L. The second effect is a general equilibrium feedback. The fact that bank must rely less on deposits makes them more scarce to households, which increases the spread $R^E - R^D$. This second effect lowers the competitive rate on bank loans. If the interest elasticity of the demand for deposits is low ($0 < \eta < 1$), a large increase in the spread will be necessary to convince households to make do with fewer deposits, and the second effect will dominate. In that case, R^L falls and the steady state level of capital thus rises, otherwise not.

8. Conclusion

This paper has developed a framework for measuring the welfare cost of bank capital requirements. Such requirements can be socially costly because they reduce banks' ability to create liquidity in equilibrium. Using U.S. data, I have measured this cost in a variety of ways. According to the most conservative estimates, the welfare cost of a 10 percentage point increase in capital requirements is equivalent to a permanent loss in consumption of 0.1 to 0.2 percent. The other measurements find a cost equal to slightly less than 1 percent of consumption. This is a fairly large welfare cost.

Moreover, it is much larger than the estimates of the benefit of capital requirements in reducing the cost of bank supervision, which is the other tool regulators possess to limit the moral hazard problem associated with deposit insurance. Though the measurement of the welfare benefit requires stronger assumptions, it thus appears that capital requirements are currently too high.

Regulators face an important trade-off between, on the one hand, keeping the effective capital requirement ratio as low as possible and, on the other hand,

limiting the supervision and compliance cost associated with capital adequacy regulation, all the while keeping the probability of bank failure acceptably low. It is thus not obvious that the current trend towards a more complex regulatory regime is wrong. But the stated goal of keeping capital ratios at about the same level for the average bank is not justified.

APPENDIX A. RISKY FIRMS

Since there is no asymmetric information between the bank and the firm, the optimal financial contract will have the repayment depend on the realization of the shock ε. Let $\tilde{R}^L(\varepsilon)$ denote the contractual loan repayment rate as a function of the shock. Profits of a risky firm are:

$$\tilde{\pi}^F = F(K,H) + \sigma_{RF}\varepsilon K + (1-\delta)K - wH - \tilde{R}^L(\varepsilon)K$$

(It is straightforward to verify that no household is willing to provide the risky firm with equity.) For ease of exposition define

$$f(K) \equiv \max_H F(K,H) + (1-\delta)K - wH$$

One of the results in the main text is that, in an equilibrium in which riskless firms exist, they have zero profits and indeterminate scale, so that

$$f(K) = R^L K$$

where R^L is the equilibrium riskless loan rate. Hence, given an optimal choice for H, profits of the risky firm equal

$$\tilde{\pi}^F = R^L K + \sigma_{RF}\varepsilon K - \tilde{R}^L(\varepsilon)K$$

Limited liability of the owners implies $\tilde{\pi}^F \geq 0$ in each state. Hence,

$$\tilde{R}^L(\varepsilon) \leq R^L + \sigma_{RF}\varepsilon$$

The right hand side is the most the bank can charge in each state without violating limited liability. Suppose the loan rate equals this upper bound in each state. Then,

$$\mathbb{E}_\varepsilon[\tilde{R}^L(\varepsilon)] = R^L - \xi\sigma_{RF} < R^L$$

(recall that $\mathbb{E}_\varepsilon[\varepsilon] = -\xi \leq 0$). Since this is still a worse expected return than for a nonrisky loan, the risky firm cannot hope to get better terms, so that, in fact, if any lending to risky firms occurs,

$$\tilde{R}^L(\varepsilon) = R^L + \sigma_{RF}\varepsilon$$

With this loan contract, the risky firm has zero (expected) profits, so its participation constraint is satisfied. Its scale is not determined by individual firm optimality. As mentioned in the main text, this implies that a bank can create a portfolio of riskiness σ by directing a fraction σ/σ_{RF} of its lending to one risky firm.[50] Finally, it is easily verified that labor demand satisfies the same first-order condition (10) as for a nonrisky firm.

[50] The model implies that a bank that engages in excessive risk taking will want to maximize risk while minimizing the reduction in net present value. With uncorrelated firm-level shocks, this requires lending to a single risky firm. It would be straightforward to modify the model to be consistent with a single bank lending to multiple risky firms. E.g., one could assume that there is a 'double continuum' of firms, indexed by $(i,j) \in [0,1]\times[0,1]$, and that, firm (i,j)'s output, if it is risky, is subject to a 'sectoral' or 'lender-specific' shock ε_i, where ε_i is uncorrelated across i.

APPENDIX B. COSTLY FINANCIAL INTERMEDIATION

B.1. The bank's problem. The value of the bank to its shareholders right after the bank has raised E in equity at the beginning of the period is now:

$$V^B(E) = \max_{L,D,\sigma} \mathbb{E}_\varepsilon \left[\left((R^L + \sigma\varepsilon)L - R^D D - g(D,L) \right)^+ \right] / R^E \qquad (33)$$
$$\text{s.t.} \quad L = E + D, \quad E \geq \gamma L, \quad \sigma \in [0, \bar{\sigma}]$$

The only difference with (4) is the presence of the resource cost $g(D,L)$.

First, a similar argument as in section 1 can be used to characterize the choice of σ conditional on L and D. Expected dividends are

$$\mathbb{E}_\varepsilon \left[\left((R^L + \sigma\varepsilon)L - R^D D - g(D,L) \right)^+ \right]$$
$$= \begin{cases} (R^L - \sigma\xi)L - R^D D - g(D,L) & \text{if } (R^L - \sigma(1+2\xi))L - R^D D - g(D,L) \geq 0 \\ 0.5((R^L + \sigma)L - R^D D - g(D,L)) & \text{otherwise} \end{cases}$$

As before, expected dividends are a piecewise linear, convex function of σ, so the optimal choice of riskiness is at a boundary of the feasible set $[0, \bar{\sigma}]$. By evaluating expected dividends under $\sigma = 0$ versus $\sigma = \bar{\sigma}$, and using the constant returns to scale of g, it is easy to verify that

$$\sigma = 0 \text{ iff } \bar{\sigma} \leq R^L - R^D(D/L) - g(D/L, 1)$$
$$\sigma = \bar{\sigma} \text{ otherwise} \qquad (34)$$

Again, for convenience it is assumed that the bank chooses $\sigma = 0$ when bank is indifferent between the two choices at $\bar{\sigma} = R^L - R^D(D/L) - g(D/L, 1)$. Because $E = L - D \geq \gamma L$, a sufficient condition for $\sigma = 0$ is:

$$\bar{\sigma} \leq R^L - R^D(1-\gamma) - g(1-\gamma, 1) \qquad (35)$$

This is also a necessary condition if $E = \gamma L$, i.e. if the capital requirement is binding. If (35) holds, the bank's sub-problem in (33) simplifies to:

$$V^B(E) = \max_L \left(R^L L - R^D(L-E) - g(L-E, L) \right) / R^E$$
$$\text{s.t.} \quad E - \gamma L \geq 0$$

where the balance sheet identity, $D = L - E$, has been substituted into the objective function. While this sub-problem is straightforward to solve, it economizes on algebra to characterize the solution as part of an analysis of the bank's full problem, which includes the choice on how much equity to raise. In choosing E, the bank maximizes its pre-issue value, $V^B(E) - E$:

$$\pi^B = \max_E \left(V^B(E) - E \right)$$
$$= \max_{E,L} \left(R^L L - R^D(L-E) - g(L-E, L) \right) \Big/ R^E - E$$
$$\text{s.t.} \quad E - \gamma L \geq 0$$

The first-order conditions are:

(L) $\quad R^L - R^D - g_D(D,L) - g_L(D,L) = \gamma R^E \chi$

(E) $\quad R^D + g_D(D,L) = (1-\chi)R^E$

where χ is the Kuhn-Tucker multiplier associated with the capital requirement: $\chi \geq 0$ and $\chi(E-\gamma L) = 0$. There are two cases to consider:

If $\chi = 0$, i.e. if the capital requirement is slack, we have

$$R^E = R^D + g_D(D,L)$$
$$R^L = R^E + g_L(D,L)$$

The second requirement entails $R^L > R^E$, since $g_L(D,L) > 0$ by assumption. However, by equation (12), a requirement for a finite solution to the firm's problem is that $R^L \leq R^E$. Intuitively, if $R^L > R^E$, firms would strictly prefer equity finance to loans, there would be no demand for bank loans,[51] and banks would not exist. Hence, if banks exist in equilibrium, *with costly intermediation, the capital requirement always binds.*

If the capital requirement binds ($\chi > 0$), $E = \gamma L$ and the first-order conditions yield

$$R^L = \gamma R^E + (1-\gamma)R^D + (1-\gamma)g_D(D,L) + g_L(D,L)$$

Since $D = (1-\gamma)L$ in this case, and using the fact that the partial derivatives of g are homogenous of degree zero as well as Euler's theorem, this simplifies to

$$R^L = \gamma R^E + (1-\gamma)R^D + g(1-\gamma,1)$$

Note that the condition for $\sigma = 0$, (35), is equivalent to (9): $\bar{\sigma} \leq \gamma R^E$. Since the capital requirement binds, this condition is necessary and sufficient.

B.2. General Equilibrium. Combining the market clearing conditions with $\sigma_t = 0$, $T_t = T$ and equations (1), (2), (28), (10), (11) and (12), it is possible to characterize the equilibrium in terms of a system in (K_t, c_t) with R_t^E and d_t as auxiliary variables:

$$K_t = F(K_{t-1},1) + (1-\delta)K_{t-1} - c_{t-1} - g(1-\gamma,1)d_{t-1}/(1-\gamma) - T \tag{36}$$

$$\beta^{-1}(u_c(c_{t-1},d_{t-1})/u_c(c_t,d_t)) = R_t^E \tag{37}$$

$$F_K(K_t,1) + 1 - \delta = R_t^L = R_t^E - \Delta(c_t,d_t) \tag{38}$$

with $\Delta(c,d) \equiv (1-\gamma)\dfrac{u_d(c,d)}{u_c(c,d)} - g(1-\gamma,1)$ and where d_t is determined as follows:

(a) If $\Delta(c_t,(1-\gamma)K_t) > 0$, [$R_t^L < R_t^E$, so $E_t^F = 0$, $L_t = K_t$ and] $d_t = (1-\gamma)K_t$ (39)

(b) If $\Delta(c_t,(1-\gamma)K_t) \leq 0$, [$R_t^L = R_t^E$, $E_t^F \geq 0$, $L_t \leq K_t$ and] $\Delta(c_t,d_t) = 0$ (40)

Remark: For case (b), assumption (27) guarantees that there exists a d_t between 0 and $(1-\gamma)K_t$ such that (40) holds. In both cases, remaining variables are determined through (2) and (10) with $H_t = 1$.

[51] Technically, the demand for bank loans would be minus infinity.

B.3. Equivalence of the constrained social planner's problem. Recall that $\theta = (\gamma, T, \delta, \beta, K_0)$. Define the following constrained social planner's problem:

$$V_0^{ci}(\theta) = \max_{\{c_t, d_t, L_t, K_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t, d_t)$$
$$\text{s.t.} \quad F(K_t, 1) + (1-\delta)K_t = c_t + K_{t+1} + g(d_t, L_t) + T$$
$$(1-\gamma)L_t \geq d_t, \qquad K_t \geq L_t$$

The Lagrangian and the first-order conditions to this problem are:

$$\ell_0^{ci}(\theta) = \max_{\{c_t, d_t, L_t, K_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \{u(c_t, d_t) + \lambda_t^{sp}[F(K_t, 1) + (1-\delta)K_t - c_t - g(d_t, L_t) - K_{t+1} - T]$$
$$+ \chi_t^{sp}[(1-\gamma)L_t - d_t] + \mu_t^{sp}[K_t - L_t]\}$$

(c) $\qquad u_c(c_t, d_t) = \lambda_t^{sp}$

(d) $\qquad u_d(c_t, d_t) = \lambda_t^{sp} g_D(d_t, L_t) + \chi_t^{sp}$

(L) $\qquad (1-\gamma)\chi_t^{sp} = \lambda_t^{sp} g_L(d_t, L_t) + \mu_t^{sp}$

(K) $\qquad \lambda_t^{sp}[F_K(K_t, 1) + 1 - \delta] - \beta^{-1}\lambda_{t-1}^{sp} + \mu_t^{sp} = 0$

with $\chi_t^{sp} \geq 0$, $\chi_t^{sp}[(1-\gamma)K_t - d_t] = 0$, $\mu_t^{sp} \geq 0$ and $\mu_t^{sp}[K_t - L_t] = 0$. Since $g_L > 0$, the first-order conditions with respect to consumption and loans imply $\chi_t^{sp} > 0$, so that the 'capital requirement' binds: $d_t = (1-\gamma)L_t$. There are thus only two cases to consider:

(a). If $\mu_t^{sp} > 0$, then $K_t = L_t$ and, since $\chi_t^{sp} > 0$, $d_t = (1-\gamma)L_t = (1-\gamma)K_t$. Rewriting the first-order condition with respect to *K*,

$$F_K(K_t, 1) + 1 - \delta = \beta^{-1}\lambda_{t-1}^{sp} / \lambda_t^{sp} - \mu_t^{sp} / \lambda_t^{sp}$$
$$= \frac{\beta^{-1} u_c(c_{t-1}, d_{t-1})}{u_c(c_t, d_t)} - \left\{(1-\gamma)\frac{u_d(c_t, d_t)}{u_c(c_t, d_t)} - g(1-\gamma, 1)\right\}$$

where the last equality follows the first-order conditions with respect to *d* and *L* and the homogeneity of *g*. Since the term in curly brackets equals $\mu_t^{sp} / \lambda_t^{sp}$, it must be strictly positive. As $d_t = (1-\gamma)K_t$ here, this case thus requires that $\Delta(c_t, (1-\gamma)K_t) > 0$. (Recall that $\Delta(c, d) \equiv (1-\gamma)\, u_d(c, d) / u_c(c, d) - g(1-\gamma, 1)$.)

(b). If $\mu_t^{sp} = 0$, the first-order conditions yield:

$$F_K(K_t, 1) + 1 - \delta = \beta^{-1} u_c(c_{t-1}, d_{t-1}) / u_c(c_t, d_t)$$

$$u_d(c_t, d_t) = u_c(c_t, d_t) g(1-\gamma, 1) / (1-\gamma)$$

The second equation is obtained by combining the first-order conditions with respect to *d* and *L*. As case (a) requires $\Delta(c_t, (1-\gamma)K_t) > 0$, case (b) must apply if $\Delta(c_t, (1-\gamma)K_t) \leq 0$. If $\Delta(c_t, (1-\gamma)K_t) \leq 0$, then by assumption (27) there exists a strictly positive $d_t \leq (1-\gamma)K_t$ satisfying this second equation.

Combining the above equations, including the social resource constraint and the binding capital requirement, it is apparent that the allocations of K_t, c_t and d_t are identical

to those of the decentralized equilibrium summarized above in equations (36) through (40). Case (a) corresponds to the case of pure bank finance (also termed 'case (a)' in part B.2), while case (b) corresponds to firms relying on both equity and bank finance (again, same label above). Hence, *this constrained social planner's problem replicates the decentralized equilibrium* when $\sigma = 0$ in the decentralized equilibrium and financial intermediation is costly. As a consequence, welfare in that equilibrium equals $V_0^{ci}(\theta)$.

B.4. Proof of Proposition 2. By assumption (9) holds: $S(T) \leq \gamma R_t^E$ for all $t \geq 0$, so $\sigma_t = 0$ in the decentralized equilibrium. The marginal effect on welfare of raising γ, without altering *T,* is now:

$$\frac{\partial V_0^{ci}(\theta)}{\partial \gamma} = \frac{\partial \ell_0^{ci}(\theta)}{\partial \gamma} = -\sum_{t=0}^{\infty} \beta^t \chi_t^{sp} L_t$$

where it is recalled that χ_t^{sp} is the Kuhn Tucker multiplier on the capital requirement of the social planner's problem. Using the first-order conditions (*d*) and (*c*) to the social planner's problem above,

$$\chi_t^{sp} L_t = \left(u_d(c_t, d_t) - u_c(c_t, d_t) g_D(d_t, L_t)\right) L_t$$

Since the allocations of K_t, c_t and d_t are identical to those of the decentralized equilibrium, I can use the decentralized equilibrium values for the variables on the right hand side of this equation. Moreover, in the decentralized equilibrium, we have, using (2) , and the result that the capital requirement binds, so $d_t = (1-\gamma)L_t$,

$$\left(u_d(c_t, d_t) - u_c(c_t, d_t) g_D(d_t, L_t)\right) L_t = u_c(c_t, d_t)\left(R_t^E - R_t^D - g_D(d_t, L_t)\right)(1-\gamma)^{-1} d_t$$

Next, I use the assumption that the economy is in steady state in period 0. Then the first-order approximation of the welfare effect of an increase in γ by $\Delta\gamma$ simplifies as follows:

$$\frac{\partial V_0^{ci}(\theta)}{\partial \gamma} \Delta\gamma = -\frac{\chi_0^{sp} L_0}{1-\beta} \Delta\gamma = -\frac{u_c(c_0, d_0)(R_0^E - R_0^D - g_D(d_0, L_0)) d_0}{(1-\beta)(1-\gamma)} \Delta\gamma$$

Again, it is useful to compare this to the welfare effect of a permanent change in consumption by a factor $(1+\nu)$, given to a first-order approximation in equation (24) if period 0 is a steady state. Equating the two effects and solving for ν immediately yields Proposition 2.

APPENDIX C. EXCESSIVE RISK AND SUPERVISION

C.1. General distribution of ε.[52] This part shows that the results in the main text do not depend on the specific distribution of ε, the shock to the return on risky loans, assumed in (3). Assumption (3) is generalized to the following:

[52] This part can be read as early as following equation (5) in section 1. The version of the model presented in that section simply has $g = 0$.

Assumption: ε has a cumulative distribution function F_ε with bounded support $[\underline{\varepsilon},\overline{\varepsilon}]$, with $-\infty < \underline{\varepsilon} < 0 < \overline{\varepsilon} < \infty$. (Formally, $\underline{\varepsilon} = \sup\{x \in \mathbb{R} \mid F_\varepsilon(x) = 0\}$ and $\overline{\varepsilon} = \inf\{x \in \mathbb{R} \mid F_\varepsilon(x) = 1\}$.) The mean of ε is equal to $-\xi$ ($\xi \geq 0$).

The assumption that the mean of ε is negative is maintained, because the shock is meant to correspond to excessive risk taking. Note that F_ε need not be continuous, so (3) is a special case.

For convenience, define $r \equiv R^L - R^D D/L - g(D/L,1) > 0$, so that dividends equal $((r+\sigma\varepsilon)L)^+$. Let $\hat{\varepsilon} \equiv -r/\sigma$, so that $r + \sigma\hat{\varepsilon} = 0$. Expected dividends are:[53]

$$\begin{aligned}\mathbb{E}_\varepsilon\left[\left((r+\sigma\varepsilon)L\right)^+\right] &= \int_{\hat{\varepsilon}}^{\overline{\varepsilon}} (r+\sigma\varepsilon)L dF_\varepsilon(\varepsilon) \\ &= \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} (r+\sigma\varepsilon)L dF_\varepsilon(\varepsilon) - \int_{\underline{\varepsilon}}^{\hat{\varepsilon}} (r+\sigma\varepsilon)L dF_\varepsilon(\varepsilon) \\ &= (r-\sigma\xi)L + \sigma L\int_{\underline{\varepsilon}}^{\hat{\varepsilon}} (\hat{\varepsilon}-\varepsilon) dF_\varepsilon(\varepsilon)\end{aligned}$$

where the last equality uses the definition of $\hat{\varepsilon}$. If $\hat{\varepsilon} \leq \underline{\varepsilon}$, the integral in the last line equals zero and the expected net cash flow decreases linearly in σ. If $\hat{\varepsilon} > \underline{\varepsilon}$, the last term is strictly positive and increasing in σ, as $\hat{\varepsilon}$ is increasing in σ. It is straightforward to show that it is also convex in σ (a proof is included at the end of this part), so that expected dividends are convex in σ. Therefore, there are two candidates for the optimal choice for riskiness: 0 and $\overline{\sigma}$. By evaluating the two cases it is easy to verify that $\sigma = 0$ if and only if

$$\int_{\underline{\varepsilon}}^{\hat{\varepsilon}} (\hat{\varepsilon}-\varepsilon) dF_\varepsilon(\varepsilon) \leq \xi, \text{ for } \hat{\varepsilon} = -r/\overline{\sigma}.$$

Define $j(x) \equiv \int_{\underline{\varepsilon}}^{x} (x-\varepsilon) dF_\varepsilon(\varepsilon)$ and let ϕ_ε be defined by $j(\phi_\varepsilon) \equiv \xi$. Note that ϕ_ε exists, is unique and satisfies $\underline{\varepsilon} \leq \phi_\varepsilon < 0$.[54] Restating the above condition, we have that $\sigma = 0$ if and only if

$$-r/\overline{\sigma} \leq \phi_\varepsilon$$

We can always rescale ε by a factor $\lambda > 0$ and, at the same time, rescale σ_{RF} and $\overline{\sigma}$ by $1/\lambda$. With an appropriate choice of λ, ϕ_ε can be normalized to $\phi_\varepsilon = -1$. (Formally, for $\lambda \in \mathbb{R}$, let $F_{\lambda\varepsilon}$ be the c.d.f. of $\lambda\varepsilon$: $F_{\lambda\varepsilon}(x) \equiv F_\varepsilon(x/\lambda)$, for all $x \in \mathbb{R}$. Then, for any $\lambda > 0$, $(\sigma_{RF}/\lambda, F_{\lambda\varepsilon})$ presents exactly the same risky technology as $(\sigma_{RF}, F_\varepsilon)$ and $(\overline{\sigma}/\lambda, F_{\lambda\varepsilon})$ presents exactly the same opportunities to the bank as $(\overline{\sigma}, F_\varepsilon)$.[55] Rescaling ε

[53] With slight abuse of notation intended to avoid clutter, if there is probability mass at $\underline{\varepsilon}$ (i.e if $F_\varepsilon(\underline{\varepsilon}) > 0$), then all integrals below of the form $\int_{\underline{\varepsilon}}^{b} h(\varepsilon) dF_\varepsilon(\varepsilon)$ should be read as $\int_{\underline{\varepsilon}-\iota}^{b} h(\varepsilon) dF_\varepsilon(\varepsilon)$ where ι is an (arbitrary) strictly positive number.

[54] This follows from the fact that $j(x)$ is continuous and increasing in x, equals zero when $x = \underline{\varepsilon}$ and strictly exceeds $\xi \geq 0$ when $x = 0$ (by the definition of ξ and the assumption $\overline{\varepsilon} > 0$).

[55] Recall that σ/σ_{RF} is the fraction of loans made to a single risky firm and $\overline{\sigma}$ is the supervisory upper bound on σ. Rescaling ε by λ and σ_{RF} by $1/\lambda$ implies rescaling $\overline{\sigma}$ by $1/\lambda$.

by λ in this way results in $\mathbb{E}[\lambda\varepsilon] = -\lambda\xi$ and $\phi_{\lambda\varepsilon} = \lambda\phi_\varepsilon$. [56] Hence, by setting $\lambda = -1/\phi_\varepsilon > 0$ we get the desired normalization.) With that normalization, we have

$$\sigma = 0 \text{ iff } \bar{\sigma} \le r = R^L - R^D(D/L) - g(D/L,1)$$
$$\sigma = \bar{\sigma} \text{ otherwise},$$

the same result as (34) in Appendix B (which specializes to (5) in section 1 for the case that $g = 0$). The rest of the analysis is the same as before.[57] Hence, in particular, condition (9): $\bar{\sigma} \le \gamma R^E$, is a sufficient condition for no excessive risk taking, and is also necessary when the capital requirement is binding, as claimed.

Finally, the definition of ϕ_ε implies that, if $\xi = 0$, $\phi_\varepsilon = \underline{\varepsilon}$. The normalization $\phi_\varepsilon = -1$, then implies $\underline{\varepsilon} = -1$. By continuity, if $\xi \approx 0$, $\underline{\varepsilon} \approx -1$.

Proof that expected dividends are convex in σ. It remains to be shown that[58]

$$h(\sigma) \equiv \sigma \int_{\underline{\varepsilon}}^{\hat{\varepsilon}(\sigma)} (\hat{\varepsilon}(\sigma) - \varepsilon) dF_\varepsilon, \quad \text{with} \quad \hat{\varepsilon}(\sigma) = -r/\sigma$$

is convex.

Proof: Let $\sigma_1 < \sigma_2$ and, for $\lambda \in (0,1)$, define $\sigma_\lambda \equiv \lambda\sigma_1 + (1-\lambda)\sigma_2$. Let $\hat{\varepsilon}_i = \hat{\varepsilon}(\sigma_i)$, for $i = 1, 2, \lambda$. Note that $\hat{\varepsilon}_1 < \hat{\varepsilon}_\lambda < \hat{\varepsilon}_2$.

$$\begin{aligned}
h(\sigma_\lambda) &= \lambda\sigma_1 \left\{ \int_{\underline{\varepsilon}}^{\hat{\varepsilon}_1} (\hat{\varepsilon}_\lambda - \varepsilon) dF_\varepsilon + \int_{\hat{\varepsilon}_1}^{\hat{\varepsilon}_\lambda} (\hat{\varepsilon}_\lambda - \varepsilon) dF_\varepsilon \right\} + \\
&\quad (1-\lambda)\sigma_2 \left\{ \int_{\underline{\varepsilon}}^{\hat{\varepsilon}_2} (\hat{\varepsilon}_\lambda - \varepsilon) dF_\varepsilon - \int_{\hat{\varepsilon}_\lambda}^{\hat{\varepsilon}_2} (\hat{\varepsilon}_\lambda - \varepsilon) dF_\varepsilon \right\} \\
&= \lambda\sigma_1 \left\{ \int_{\underline{\varepsilon}}^{\hat{\varepsilon}_1} (\hat{\varepsilon}_1 - \varepsilon) dF_\varepsilon + (\hat{\varepsilon}_\lambda - \hat{\varepsilon}_1) F_\varepsilon(\hat{\varepsilon}_1) + \int_{\hat{\varepsilon}_1}^{\hat{\varepsilon}_\lambda} (\hat{\varepsilon}_\lambda - \varepsilon) dF_\varepsilon \right\} + \\
&\quad (1-\lambda)\sigma_2 \left\{ \int_{\underline{\varepsilon}}^{\hat{\varepsilon}_2} (\hat{\varepsilon}_2 - \varepsilon) dF_\varepsilon + (\hat{\varepsilon}_\lambda - \hat{\varepsilon}_2) F_\varepsilon(\hat{\varepsilon}_2) + \int_{\hat{\varepsilon}_\lambda}^{\hat{\varepsilon}_2} (\varepsilon - \hat{\varepsilon}_\lambda) dF_\varepsilon \right\} \\
&\le \lambda h(\sigma_1) + \lambda\sigma_1 \left\{ (\hat{\varepsilon}_\lambda - \hat{\varepsilon}_1) F_\varepsilon(\hat{\varepsilon}_1) + \int_{\hat{\varepsilon}_1}^{\hat{\varepsilon}_\lambda} (\hat{\varepsilon}_\lambda - \hat{\varepsilon}_1) dF_\varepsilon \right\} + \\
&\quad (1-\lambda) h(\sigma_2) + (1-\lambda)\sigma_2 \left\{ (\hat{\varepsilon}_\lambda - \hat{\varepsilon}_2) F_\varepsilon(\hat{\varepsilon}_2) + \int_{\hat{\varepsilon}_\lambda}^{\hat{\varepsilon}_2} (\hat{\varepsilon}_2 - \hat{\varepsilon}_\lambda) dF_\varepsilon \right\} \\
&= \lambda h(\sigma_1) + (1-\lambda) h(\sigma_2) + F_\varepsilon(\hat{\varepsilon}_\lambda) \left(\lambda\sigma_1(\hat{\varepsilon}_\lambda - \hat{\varepsilon}_1) + (1-\lambda)\sigma_2(\hat{\varepsilon}_\lambda - \hat{\varepsilon}_2) \right) \\
&= \lambda h(\sigma_1) + (1-\lambda) h(\sigma_2)
\end{aligned}$$

where the last step follows from $\sigma_i \hat{\varepsilon}_i = -r$ for $i = 1, 2$ and λ. Hence, $h(\sigma_\lambda) \le \lambda h(\sigma_1) + (1-\lambda) h(\sigma_2)$, so $h(\sigma)$ is convex.

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[56] $\phi_{\lambda\varepsilon} = \lambda\phi_\varepsilon$ since $\int_{\lambda\underline{\varepsilon}}^{\lambda\phi_\varepsilon} (\lambda\phi_\varepsilon - \varepsilon') dF_{\lambda\varepsilon}(\varepsilon') = \int_{\lambda\underline{\varepsilon}}^{\lambda\phi_\varepsilon} (\lambda\phi_\varepsilon - \varepsilon') dF_\varepsilon(\varepsilon'/\lambda) = \int_{\underline{\varepsilon}}^{\phi_\varepsilon} (\lambda\phi_\varepsilon - \lambda\varepsilon) dF_\varepsilon(\varepsilon) = \lambda\xi$, where the second-to-last step performs a change of variable ($\varepsilon = \varepsilon'/\lambda$).

[57] Only the last term in equation (13) (and (29)), the deposit insurance scheme's cost, needs to be modified in an obvious fashion. However, this term equals zero in any case in the incentive-compatible equilibrium ($\sigma = 0$).

[58] For readability, in what follows the argument ε in $dF_\varepsilon(\varepsilon)$ is omitted.

C.2. Calibration of α_S. To measure the marginal effectiveness of bank supervision, α_S, I make two assumptions:[59]

(i) α_S is constant, i.e. $S(T) = S(0)e^{-\alpha_S T}$, with $\alpha_S \geq 0$. One can think of this assumption as a log-linear approximation of *S*: $\ln S(T) = \ln S(0) - \alpha_S T$. A log-linear specification has the virtue of satisfying all the assumptions made on *S*, in particular that it is positive for all *T*.

(ii) I assume that in the absence of bank supervision there is a positive probability that a bank that engages in maximum excessive risk taking, will realize a (near) total loss, as a fraction of the bank's deposit liabilities, for the deposit insurance fund. Or, equivalently, in the absence of bank supervision, there is a positive probability, however small, that the realized gross return to lending is (close to) zero if the bank has taken on maximum excessive risk.

To see the equivalence in (ii), note that, without supervision, the loss to the deposit insurance fund from a single bank which engages in excessive risk taking, i.e. $\sigma = S(0)$, and realizes the worst possible return, $\underline{\varepsilon}$, is:

$$R^D D - (R^L + S(0)\underline{\varepsilon})L + g(D,L)$$

According to assumption (ii), this should be approximately equal to $R^D D$. This requires

$$R^L - g(D,L)/L + S(0)\underline{\varepsilon} \approx 0$$

In other words, with zero supervision and maximum risk taking, there is some (arbitrarily small, but strictly positive) probability of realizing a near zero gross return on the bank's loan portfolio, net of noninterest cost. Given that the probability can be arbitrarily small, and given the opportunities for banks that currently exists for taking on more risk, e.g. through derivatives, this does not strike me as a very strong assumption.

This condition can be used to infer $S(0)$, which will be useful. With the example distribution in (3), $\underline{\varepsilon} = -(1+2\xi)$. Recall that ξ is the absolute value of the negative mean of ε. Thus, ξ is the direct loss in net present value due to excessive risk per unit of bank risk, σ. In modern financial markets, there exist excellent opportunities for banks to take on more risk, not only by making risky loans, but also through the trading book and through various off-balance sheet activities. It is thus hard to believe that ξ is a large number in reality. I therefore set $\xi \approx 0$. For the example distribution, this implies $\underline{\varepsilon} \approx -1$. The same is true for the general distribution analyzed in the first part of this appendix: as shown there, with the correct normalization, $\underline{\varepsilon} \approx -1$ when $\xi \approx 0$. Hence,

$$S(0) \approx R^L - g(D,L)/L$$

The right hand side is larger than 1 (based on the data) and less than β^{-1} (the model's steady state return on equity). The difference is trivial for our purposes, so I simply set $S(0) = 1$. Thus, using assumption (i), $S(T) = e^{-\alpha_S T}$. The observation that U.S. regulation is currently incentive compatible implies that this is weakly less than $\gamma\beta^{-1}$. Hence,

[59] For clarity, these assumptions are not needed or used for any of the results on the welfare *cost* of capital requirements, nor for any of the measurements thereof.

$$\alpha_S \geq -\ln(\gamma\beta^{-1})/T\ ,$$

with equality if $S(T) = \gamma\beta^{-1}$.

Using γ = 0.1 and $\beta^{-1} = 1.06$, we have, for T = \$1.4 billion, $\alpha_S \geq 1.6$ per billion dollars, and for T = \$4.2 billion, $\alpha_S \geq 0.53$. (See main text for details on these measurements of T). Both inequalities are equalities if the incentive compatibility constraint currently binds.

APPENDIX D. EFFECT OF THE CAPITAL REQUIREMENT ON INCOME

First, I state explicitly a convenient assumption that bounds taxes from above in order to guarantee existence of a steady state equilibrium with positive consumption for the assumed utility function (see footnote 17). Define $\hat{K}$ by writing

$$F_K(\hat{K},1) + 1 - \delta \equiv \beta^{-1} \tag{41}$$

Note that $\hat{K}$ exists and is unique. $\hat{K}$ is the steady state level of the capital stock under the assumption $a = 0$ (i.e. $u(c,d) = \tilde{u}(c)$, so that the model is not materially different from the standard growth model). I assume that

$$T < F_H(\hat{K},1)1 \tag{42}$$

That is, taxes are lower than aggregate wage income at $\hat{K}$.[60]

With a > 0, $u_d(c,d) > 0$ everywhere, so the capital stock is determined by the system of equations (15)-(17) and (19), which for a steady state and the assumed functional form of $u(\ )$ simplifies to

$$F_K(K^*,1) + 1 - \delta = \beta^{-1} - (1-\gamma)a\left(\frac{c^*}{(1-\gamma)K^*}\right)^{1/\eta} \quad \text{and}$$

$$F(K^*,1) = \delta K^* + c^* + T$$

where starred variables denote steady state levels. Equivalently,

$$F_K(K^*,1) + 1 - \delta = \beta^{-1} - a(1-\gamma)^{(\eta-1)/\eta}\Psi(K^*)^{1/\eta} \tag{43}$$

with

$$\Psi(K) \equiv (F(K,1) - T)/K - \delta$$

Since $c^*/K^* = \Psi(K^*)$, $c^* \geq 0 \Leftrightarrow \Psi(K^*) \geq 0 \Leftrightarrow K^* \in [\underline{K}, \overline{K}]$, where $\underline{K}, \overline{K}$ are the two solutions to $F(K,1) - \delta K = T$.[61]

Using assumption (42), the definition of $\hat{K}$ (41), and Euler's theorem, it is straightforward to show that $\Psi(\hat{K}) > 0$, so that $\hat{K} \in (\underline{K}, \overline{K})$. Moreover, again using Euler's theorem,

$$\Psi'(K) = (T - F_H(K,1))/K^2$$

[60] Recall that H = 1 in equilibrium. The assumption is sufficient but not necessary.

[61] The two roots exist since $\Psi(\hat{K}) > 0$, as explained in the next paragraph, and because F satisfies the Inada conditions.

Define $\tilde{K}$ by writing $F_H(\tilde{K},1) \equiv T$. Note that $\tilde{K}$ exists and is unique, $\tilde{K} < \hat{K}$ by (42) and $\Psi'(K) > (<) 0$ iff $K < (>) \tilde{K}$. The last fact also implies that $\tilde{K} > \underline{K}$.

Since $\tilde{K} < \hat{K} < \bar{K}$, as we let K increase from $\tilde{K}$ to $\bar{K}$, the left-hand side of (43), $F_K(K,1) + 1 - \delta$, drops, continuously and monotonically, from a value strictly greater than β^{-1} to a value strictly less than β^{-1}. Again, as we let K increase from $\tilde{K}$ to $\bar{K}$, the right-hand side of (43), $\beta^{-1} - (1-\gamma) a \Psi(K)^{1/\eta}$, rises, continuously and monotonically, from a value strictly less than β^{-1} to exactly β^{-1} (as $\Psi(\bar{K}) \equiv 0$). Hence, there is exactly one K^* in $[\tilde{K}, \bar{K}]$ satisfying (43). By a similar argument it is easy to show that there is no K^* in $[\underline{K}, \tilde{K})$ satisfying (43). Hence, *there exists a unique steady state level of the capital stock*. It is interesting to note that its marginal product is less than β^{-1}, so K^* exceeds $\hat{K}$, the steady state level of capital without liquidity preference.

Total differentiation of equation (43) with respect to K^* and γ yields:

$$\frac{dK^*}{d\gamma} = \frac{(a/\eta)(\eta-1)(1-\gamma)^{-1/\eta} \Psi(K^*)^{1/\eta}}{F_{KK}(K^*,1) + (a/\eta)(1-\gamma)^{(\eta-1)/\eta} \Psi(K^*)^{(1-\eta)/\eta} \Psi'(K^*)}$$

$\tilde{K} < K^* < \bar{K}$ implies that $\Psi(K^*) > 0$ and $\Psi'(K^*) < 0$. Since, in addition, $F_{KK}(K^*,1) < 0$,

$$\text{sign}\left(dK^* / d\gamma\right) = \text{sign}(1-\eta).$$

QED.

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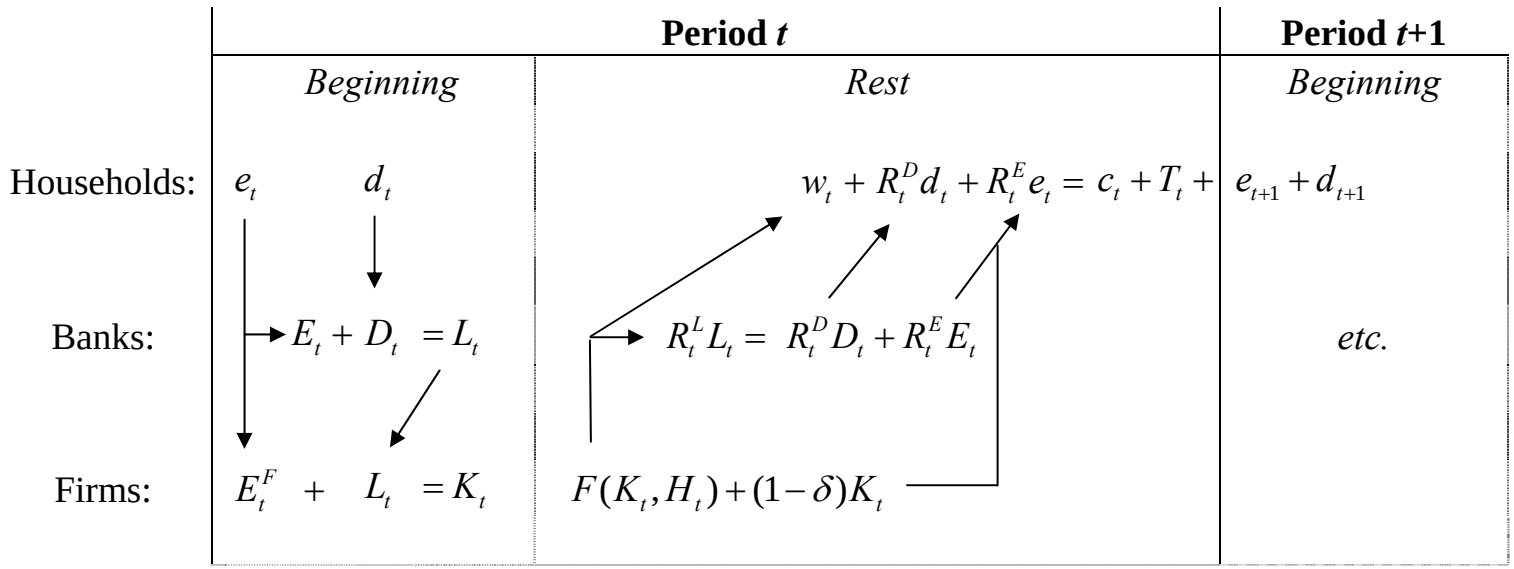
Figure 1. Timeline.



Arrows represent financial flows and wage payments.