FDIC Center for Financial Research
Working Paper

No. 2004-02

Capital Adequacy and Basel II

Paul H. Kupiec

Revised December 2004

Federal Deposit Insurance Corporation • Center for Financial Research

Capital Adequacy and Basel II

by

Paul H. Kupiec*
September 2004
Revised December 2004

FDIC Center for Financial Research Working Paper No. 2004-02

ABSTRACT

Using a one common factor Black-Scholes-Merton (BSM) equilibrium model of credit risk, this paper derives unbiased capital allocation rules for portfolios in which idiosyncratic risk is fully diversified. When these rules are compared with the Basel II Internal Ratings Based (IRB) minimum capital requirements for corporate exposures, the comparison shows that the Basel Advanced IRB (A-IRB) approach drastically undercapitalizes portfolio credit risk relative to the supervisory target of a 99.9 percent bank solvency rate. Estimates show that Basel regulations will allow fully compliant A-IRB banks to have default rates that exceed 5 percent. In contrast, the Foundation IRB minimum capital requirements allocate multiple times the capital necessary to achieve the supervisory objective. These results raise a number of important issues including the potential for increased systemic risk as regulatory capital rules promote banking sector consolidation and poor risk management standards in A-IRB banks.

Key words: economic capital, credit risk, Basel II, internal models

JEL Classification: G12, G20, G21, G28

CFR research programs: risk measurement, bank regulatory policy

---

* Associate Director, Division of Insurance and Research, Federal Deposit Insurance Corporation. The views expressed in this paper are those of the author and do not reflect the views of the FDIC. I am grateful to Mark Flannery, Dilip Madan, Haluk Unal, Jack Reidhill, Dan Nuxoll, Wenying Jiangli, and Andy Jobst for comments on an earlier draft of this paper.
Email: pkupiec@fdic.gov; phone 202-898-6768.

Capital Adequacy and Basel II

1. Introduction

Economic capital is the market value of equity in a bank's capital structure. Equity capital functions as a buffer that protects all bank creditors from potential loss. Increases in equity capital raise the probability that a bank will fully perform on its contractual obligations. In practice, economic capital allocations often are estimated using value-at-risk (VaR) techniques that attempt to maximize bank leverage while ensuring that the potential default rate on a bank's outstanding debt is below a maximum target rate selected by management.[1]

Regulatory capital requirements set legally binding minimum capitalization standards for banks. Supervisory measures of capital differ from theoretical measures of economic capital in that the former may include selected classes bank liabilities such as subordinated debt, trust preferred stock, loan loss reserve balances, and other so-called hybrid capital instruments in the bank's loss absorbing buffer. Relative to economic capital, additional components are included in the regulatory measure because regulators and bank managers have different objective functions. A primary objective of bank regulatory policy is to preserve the integrity of banks' core transactions and savings deposits in the event of insolvency while allowing losses to accrue to hybrid forms of bank capital. In contrast, managers must capitalize to ensure that a bank is a going-concern, and to remain a going concern, banks must perform fully on all of their liabilities.

Under the June 2004 Basel II agreements, national supervisory authorities may choose among three alternative minimum regulatory capital frameworks for their internationally active banks. One approach, the standardized approach, sets minimum capital standards using a modified version of the 1988 Basel Capital Accord that links capital

---

[1] Alternatively, the constraint can be described as a minimum bank solvency margin (1 minus the bank's expected default rate).

requirements to external credit ratings. The remaining two approaches, the so-called Foundation (F-IRB) and Advanced (A-IRB) Internal Ratings Based approaches, use mathematical rules to assign minimum capital requirements according to an individual credit's probability of default (PD), loss given default (LGD), maturity, and exposure at default. In the U.S., banking supervisors have determined that Basel II implementation will require so-called core banks to adopt the A-IRB approach, while other banks may petition supervisors for A-IRB capital treatment (so-called opt-in banks). The remaining banks (so-called general banks) will continue using the capital requirements specified in the 1988 Basel Capital Accord.[2]

The Basel II IRB framework has been distilled from a special class of VaR models, the so-called asymptotic single risk factor (ASRF) class, in part because these models generate portfolio-invariant capital requirements that can be used to calibrate IRB capital assignment rules.[3] In designing its prudential standards, the Basel Committee on Banking Supervision (BCBS) has expressed an objective that includes setting minimum regulatory capital requirements at a level consistent with a minimum bank solvency margin of 99.9 percent over a one-year horizon.[4] In practice, the BCBS arrived at the June 2004 IRB framework through a process in which successive IRB calibrations were modified in response to industry commentary and information gleaned from Quantitative Impact Studies (QIS). A series of three QIS studies required banks to estimate the effects of alternative IRB calibrations on their minimum regulatory capital requirements. Consecutive IRB formulations were modified toward a goal of achieving capital neutrality while creating incentives that encouraged banks to adopt the IRB approaches.[5] While the QIS exercises

[2] Core banks are defined as institutions with total banking (and thrift) assets of $250 billion or more or total on-balance-sheet foreign exposure of $10 billion or more. General banks likely will be subject to a modified version of the 1988 Basel Accord, but the potential modifications have yet to be publicly discussed.

[3] See for example, Gordy (2003).

[4] Basel Committee on Banking Supervision, 2004, paragraph 667.

[5] The Basel Committee on Banking Supervision, October 2002, paragraphs 46-47

have provided the BCBS with information on the minimum capital requirements that banks would face under Basel II's alternative approaches given their existing portfolios, the QIS studies do not provide any evidence regarding the minimum solvency margins implicit in the alternative Basel II approaches. To date, no published study has analyzed the rigor of the prudential standards that are set by the June 2004 Basel II IRB model calibrations.

This paper analyzes the minimum solvency standards associated with the Basel II IRB approaches using the Black-Scholes-Merton (BSM) equilibrium model of credit risk. Semi-closed form solutions for the capital allocation problem are derived using the physical and equivalent martingale return distributions for credit portfolios that satisfy the ASRF assumptions—default correlations are driven by a single common factor and idiosyncratic risk is fully-diversified. Since optimal capital allocation rules are derived from a full BSM portfolio model, the capital rules fully anticipate the correlations that arise among individual credit's probability of default and loss given default. The solvency margins implicit in the June 2004 IRB specifications are estimated in a calibration exercise that compares IRB capital requirements to the unbiased capital allocations specified by the BSM ASRF model. The results of the calibration comparison have important regulatory and competitive implications.

A comparison of the alternative capital allocation approaches suggests that, compared to the true capital needed to ensure a regulatory target solvency margin of 99.9 percent, the June 2004 A-IRB approach undercapitalizes banks by more than 80 percent assuming that all bank regulatory capital is Tier 1 (equity) capital. The A-IRB capital shortfall is shown to be consistent with bank default rates that are in excess 5 percent. Since Basel II allows banks to use subordinated debt and other hybrid forms of capital to satisfy minimum regulatory requirements, the potential default rates of fully compliant A-IRB banks could be in excess of 5 percent. If widely adopted, the regulatory standards set by the A-IRB approach will allow a system-wide bank insolvency rate similar in magnitude to the 6.4 percent failure rate experienced by insured U.S. savings and loan institutions during the height of the 1980s crisis.[6] These analysis shows that June 2004 A-IRB approach is seriously flawed. The A-

[6] "History of the Eighties—Lessons for the Future," p. 168.

IRB represents a very poor standard for measuring the quality of bank risk measurement practices.

The calibration analysis indicates that, unlike A-IRB banks, F-IRB banks will be over-capitalized relative to the 99.9 percent target solvency rate. For high quality (low default risk) portfolios, F-IRB capital requirements specify more than seven times the level of capital needed to achieve the regulatory target. F-IRB capital requirements for lower quality (high default risk) credits are overstated by about 170 percent. The overcapitalization owes in part to the F-IRB assumption that LGD is 45 percent, an assumption that overestimates the loss rates on the credits examined in this study. For short-maturity credits, F-IRB capital requirements provide capital relief relative the 8 percent required by the 1988 Basel Accord. For longer maturity credits, the 1988 Basel Accord assigns lower capital requirements. Overall, the analysis demonstrates that the F-IRB prudential standard is much more conservative than the standard that will apply to A-IRB banks.

In designing Basel II, the BCBS calibrated the alternative approaches so that, for a wide range of credits, the A-IRB approach produces the lowest capital requirements. This calibration is designed to encourage banks to transition from the Standardized and F-IRB approaches to the A-IRB approach. Capital savings accorded under the A-IRB are intended to offset the costs associated with developing and operating A-IRB systems and to reflect efficiencies that are presumed to be generated by the A-IRB's more efficient measurement of credit risk and assignment of minimum capital. While the BCBS have succeeded in creating incentives that encourage adoption of the IRB approaches, the large differences in the regulatory capital requirements specified by the alternative Basel II IRB approaches raise important prudential and structural issues.

To the extent that banks enjoy safety-net engendered subsidies that are attenuated by minimum regulatory capital requirements, the IRB calibrations engender strong incentives that will encourage banking system assets to migrate toward A-IRB banks in order to maximize the value of the implicit safety net subsidy. Asset migration could be achieved through consolidation or through an increase in the number banks that are granted regulatory approval for the A-IRB approach. Regardless of how asset migration occurs, the capital relief granted under the A-IRB approach may raise long-term prudential concerns. If the minimum leverage requirements of FDICIA's prompt corrective action regulations are relaxed once Basel II is implemented, unless market discipline forces intervene and "require" banks to maintain additional capital buffers, regulations will allow A-IRB banks to operate with substantial reductions in regulatory capital.[7] Should A-IRB capitalization levels approach Basel II regulatory minimums, the associated solvency margin estimates portend a material increase in bank failure rates to levels beyond those experienced in the 1980s banking crisis.

An outline of this paper follows. Section 2 summarizes the general methodology for constructing unbiased economic capital allocations. Section 3 revisits unbiased credit risk capital allocation in the context of the Black-Scholes-Merton (BSM) model. Section 4 derives unbiased portfolio-invariant credit risk capital measures for an ASRF version of the BSM. Section 5 reviews the procedures for setting minimum capital requirements under the Basel IRB approaches. Section 6 discusses the calibration results and Section 7 concludes the paper.

2. UNBIASED BUFFER STOCK CAPITAL FOR CREDIT RISKS

The intuition that underlies the construction of an unbiased economic capital allocation for a portfolio is transparent when considering portfolios composed of long positions in traditional financial assets such as simple loans, bonds, or equities because the value of the investment portfolio cannot become negative. For instruments on which losses have the potential to exceed their initial market value, as they can for example on short positions, futures, derivatives, or other structured products, then economic capital calculations must be modified from the techniques described subsequently because capital structure modifications alone may not be able to ensure that a bank is able to perform on its liabilities. In these circumstances, ensuring a minimum solvency margin may require

[7] Unless prompt corrective action (PCA) minimum capital requirements are relaxed (12 U.S.C. Section 1831), PCA may become the binding regulatory capital constraint on A-IRB banks.

changing portfolio investment shares.[8] For purposes of the analysis that follows, portfolio composition is restricted to include only long positions in fixed income claims that may generate losses that are bounded above by the initial market value of the credit.

Defining an Appropriate Value-at-Risk (VaR) Measure

Let T represent the capital allocation horizon of interest. The purchased asset A, has an initial market value A_0, a time T value of $\tilde{A}_T$, that has an associated cumulative distribution function represented by $\Psi(\tilde{A}_T, A_T)$, and a probability density function represented by $\psi(\tilde{A}_T, A_T)$. Let $\Psi^{-1}(\tilde{A}_T, 1-\alpha)$ represent the inverse of the cumulative density function of $\tilde{A}_T$ evaluated at $1-\alpha$. Define an α coverage VaR measure, $VaR(\alpha)$, $\alpha \in [0,1]$, as,

$$VaR(\alpha) = A_0 - \Psi^{-1}(\tilde{A}_T, 1-\alpha) \tag{1}$$

$VaR(\alpha)$ measures the loss that could be exceeded by at most $(1-\alpha)$ of all potential future value realizations of $\tilde{A}_T$. Note that expression (1) measures value-at-risk relative to the initial market value of the asset. When credit risk losses are bounded above by the initial invested amount, A_0, $\Psi^{-1}(\tilde{A}_T, 1-\alpha)$ is bounded below by 0.

Unbiased Capital Allocation for Credit Risk

Assume, hypothetically, that a bank follows a capital allocation rule that sets equity capital equal to $VaR(.999)$. By definition, there is less than 0.1 percent probability the investment's value will ever post a loss that exceeds the $VaR(.999)$ measure. The amount that must be borrowed to finance this investment asset under this $VaR(.999)$ capital allocation rule is $A_0 - VaR(.999)$. If the bank borrows $A_0 - VaR(.999)$, it must promise to pay back more than $A_0 - VaR(.999)$ if equilibrium interest rates and credit risk compensation are positive. Because the $VaR(.999)$ capital allocation rule ignores the equilibrium returns

[8] See Kupiec (2004) for a discussion of the capital allocation problem in these instances.

that are required by bank creditors, the probability that the bank will default on its funding debt under a $VaR(.999)$ capital allocation rule is greater than 0.1 percent if the bank's debts can only be satisfied by funds raised from selling the purchased asset for $\tilde{A}_T$ at time T.[9]

An unbiased economic capital allocation rule for 0.1 percent target default rate is: set equity capital equal to $VaR(.999)$ plus the interest that will accrue on the bank's borrowings. Alternatively, set the par (maturity) value of the funding debt equal to $VaR(.999)$ and estimate its current equilibrium market value. The difference between the current market value of the purchased asset and the current market value of the associated funding debt is the economic capital needed to fund the investment and satisfy the solvency rate target. This capital allocation rule generalizes to the portfolio context.

In order to estimate the equilibrium interest cost on funding debt, one must go beyond the tools of value-at-risk and employ formal asset pricing models or use empirical approximations to value bank's funding debt. The following section modifies the Black-Scholes-Merton (BSM) model to price the bank's funding debt issue.

3. Unbiased Buffer Stock Capital Allocation in a Black-Scholes-Merton Model

If the risk-free term structure is flat and a firm issues only pure discount debt, and asset values follow geometric Brownian motion, under certain simplifying assumptions[10], Black and Scholes (1973), and independently Merton (1974) (hereafter BSM) established that the market value of a firm's debt issue is equal to the risk free discounted value of the bond's par value, less the market value of a Black-Scholes put option written on the value of the firm's assets. The put option has a maturity identical to the debt issue maturity, and a strike price equal to the par value of the debt. More formally, if A_0 represents the initial

[9] A formal proof is given in Proposition 1 in the Appendix.

[10] There are no taxes, transactions are costless, short sales are possible, trading takes place continuously, if borrowers and savers have access to the debt market on identical risk-adjusted terms, and investors in asset markets act as perfect competitors.

value of the firm's assets, B_0 the bond's initial equilibrium market value, and Par the bond's promised payment at maturity date M, BSM establish,

$$B_0 = Par\ e^{-r_f M} - Put(A_0, Par, M, \sigma), \tag{2}$$

where r_f represents the risk free rate and $Put(A_0, Par, M, \sigma)$ represents the value of a Black-Scholes put option on an asset with an initial value of A_0, a strike price of Par, maturity M, and an instantaneous return volatility of σ.

The default (put) option is a measure of the credit risk of the bond. Merton (1974), Black and Cox (1976), and others show that the model will generalize as to term structure assumptions, coupon payments, default barrier assumptions, and generalized volatility structures. The capital allocation discussion that follows uses the simplest formulation of the BSM model.[11]

Modifying the BSM Model for Credit Risk Capital Allocation

In the original BSM model, the underlying assets exhibit market risk. To examine portfolio credit risk issues, it is necessary to modify the BSM model so that the underlying assets in the bank's portfolio are themselves risky fixed income claims. Consider the case in which a bank's only asset is a risky BSM discount debt issued by an unrelated counterparty. Assume that the bank will fund this bond with its own discount debt and equity issues. In this setting, the bank's funding debt issue can be valued as a compound option.

Let $\tilde{A}_T$ and Par_P represent, respectively, the time T value of the assets that support the discount debt investment and the par value of the bond. Let Par_F represent the par value of the discount bond that is issued to fund the investment. For purposes of simplifying this discussion we restrict attention to the case where the maturity of the bank's funding debt

[11] That is, it assumed that the term structure is flat, asset volatility is constant, the underlying asset pays no dividend or convenience yield, and all debt securities are pure discount issues.

matches the maturity of the BSM asset (both equal to M).[12] The end-of-period cash flows that accrue to the funding debt holders are,

$$Min\left[Min\left(\tilde{A}_M , Par_P\right), Par_F\right] . \tag{3}$$

In BSM model, the firm's underlying assets evolve in value according to geometric Brownian motion.

$$dA = \mu\, A\, dt + \sigma\, A\, dW \tag{4}$$

where dW is a standard Weiner process. If A_0 represents the initial value of the firm's assets, equation (4) implies that the physical probability distribution for the value of the underlying assets at time T is,

$$\tilde{A}_T \sim A_0\, e^{\left(\mu - \frac{\sigma^2}{2}\right)T + \sigma\sqrt{T}\ \tilde{z}} \tag{5}$$

where $\tilde{z}$ is a standard normal random variable.

Equilibrium absence of arbitrage conditions impose restrictions on these asset's drift rate, $\mu = r_f + \lambda\sigma,$ where λ is the market price of risk. If $dA^{\eta} = \left(\mu - \lambda\sigma\right)A^{\eta}\,dt + A^{\eta}\sigma\, dz$ is defined as the "risk neutralized" process under the equivalent martingale measure, the underlying end-of-period asset value distribution under the equivalent martingale measure, $\tilde{A}_T^{\eta}$, is,

$$\tilde{A}_T^{\eta} \sim A_0\, e^{\left(r_f - \frac{\sigma^2}{2}\right)T + \sigma\sqrt{T}\ \tilde{z}} \tag{6}$$

The initial equilibrium market value of the bank's discount bond issue is the discounted (at the risk free rate) expected value of the end-of-period funding debt cash flows taken with respect to the probability distribution for $\tilde{A}_M^{\eta}$. In the held-to-maturity (HTM) case,

[12] Kupiec (2004) discusses the case where maturity of the funding debt and the investment differ.

when the maturity of the investment and the bank's funding debt match (both equal to M),
the initial market value of the bank's funding debt is,

$$e^{-r_f M} E^{\eta}\left[Min\left[Min\left(\tilde{A}_M, Par_P\right), Par_F\right]\right] \tag{7}$$

The notation $E^{\eta}[\ ]$ denotes the expected value operator with respect to the probability density for $\tilde{A}_M^{\eta}$. [13]

Unbiased Buffer Stock Capital

Assume that the bank is investing in a BSM risky discount bond of maturity M. At maturity, the payoff of the bank's purchased bond is given by $Min\left[Par_P, \tilde{A}_M\right]$. Let $\Phi(x)$ represent the cumulative standard normal distribution function evaluated at x, and let $\Phi^{-1}(\alpha)$ represent the inverse of this function for $\alpha \in [0,1]$. Using the general notation defined in Section 2, the upper bound on the par (maturity) value of the funding debt that can be issued under the target solvency constraint is,

$\Psi^{-1}\left(\tilde{A}_T, 1-\alpha\right) = A_0\, e^{\left[\mu - \frac{\sigma^2}{2}\right]M + \sigma\sqrt{M}\,\Phi^{-1}(1-\alpha)}$. The initial market value of this funding debt issue is given by,

$$e^{-r_f M} E^{\eta}\left[Min\left[Min\left(\tilde{A}_M, Par_P\right), A_0\, e^{\left[\mu - \frac{\sigma^2}{2}\right]M + \sigma\sqrt{M}\,\Phi^{-1}(1-\alpha)}\right]\right]. \tag{8}$$

Equation (8) implies that the initial equity allocation consistent with the target solvency rate α is,

$$B_0 - e^{-r_f M} E^{\eta}\left[Min\left[Min\left(\tilde{A}_M, Par_P\right), A_0\, e^{\left[\mu - \frac{\sigma^2}{2}\right]M + \sigma\sqrt{M}\,\Phi^{-1}(1-\alpha)}\right]\right]. \tag{9}$$

[13] Kupiec (2004) derives the pricing expression for the funding debt in the mark-to-market setting when the bank's funding debt matures before the investment.

In the so-called mark-to-market (MTM) setting when $T \le M$, Kupiec (2004) shows that the unbiased economic capital allocation is[14],

$$B_0 - e^{-r_f T} E^{\eta}\left[Min\left[\left(Par_P\, e^{-r_f(M-T)} - Put\left(\tilde{A}_T, Par_P, M-T, \sigma\right)\right), A_0\, e^{\left[\mu - \frac{\sigma^2}{2}\right]M + \sigma\sqrt{M}\Phi^{-1}(1-\alpha)} \right]\right] \quad (10)$$

Portfolio Capital

To generalize capital allocation program to the portfolio setting, define Par_{iP}, to be the maturity value of the i[th] discount debt instrument in a bank's investment portfolio and let $\tilde{A}_{iT}$ represent the market value of the assets that support this bond. Assume all credits mature at date *M*. The end-of-period value of the investment portfolio is ${}_P\tilde{A}_M = \sum_{\forall i} Min\left(\tilde{A}_{iM}, Par_{iP}\right)$. ${}_P\tilde{A}_M$ has a cumulative distribution function represented by $\Psi\left({}_P\tilde{A}_M, {}_P A_M\right)$. The payoff on the bank's funding debt is,

$$Min\left[\sum_{\forall i} Min\left(\tilde{A}_{iM}, Par_i\right), Par_F \right] = Min\left[{}_P\tilde{A}_M, Par_F^P\right]. \quad (11)$$

When the bank's target default rate, $(1-\alpha)$, $VaR(\alpha)$, is given by,

$$VaR(\alpha) = \sum_{\forall i} B_{i0} - \Psi^{-1}\left({}_P\tilde{A}_M, 1-\alpha\right). \quad (12)$$

---

[14] In many situations, $Par_P > A_0\, e^{\left[\mu - \frac{\sigma^2}{2}\right]M + \sigma\sqrt{M}\Phi^{-1}(1-\alpha)}$, and expression (9) simplifies to,

$$B_0 - E^{\eta}\left[Min\left[\tilde{A}_M, A_0\, e^{\left[\mu - \frac{\sigma^2}{2}\right]M + \sigma\sqrt{M}\Phi^{-1}(1-\alpha)} \right]\right] e^{-r_f M}.$$

The maximum par value of the funding debt consistent with the target solvency margin is $Par_F^P = \Psi^{-1}\left({}_P\tilde{A}_M, 1-\alpha\right)$, and the equity capital necessary to satisfy the minimum solvency requirement is,

$$\sum_{\forall i} B_{i0} - E^{\bar{\eta}}\left[Min\left[{}_P\tilde{A}_M, \Psi^{-1}\left({}_P\tilde{A}_M, 1-\alpha\right)\right]\right]e^{-r_f M} \tag{13}$$

The notation $E^{\bar{\eta}}[\;]$ represents the expectation taken with respect to the risk neutralized multivariate distribution of asset prices which support the bonds in the investment portfolio.

Expression (13) is the economic capital allocation rule in the so-called held-to-maturity (HTM) case, when the maturity of the investments and the bank's funding debt are matched. In the MTM case, when the maturity of the funding debt (T) is less than the maturity (M) of the bonds in the investment portfolio, an unbiased economic capital allocation that sets a solvency margin α is,

$$\sum_{\forall i} B_{i0} - E^{\bar{\eta}}\left[Min\left[\sum_{\forall i}\left(Par_i\, e^{-r_f(M-T)} - Put\left(\tilde{A}_{iT}, Par_i, M-T, \sigma_i\right)\right), \Psi^{-1}\left({}_P\tilde{A}_T, 1-\alpha\right)\right]\right]e^{-r_f T} \tag{14}$$

In general, expressions (13) and (14) require the evaluation of high order integrals that do not have closed-form solutions. The next section considers portfolio capital allocation under the ASRF assumptions which reduce the complexity of the capital calculations.

4. Unbiased Capital Allocation in an Asymptotic Single Factor BSM Model

The BSM framework can accommodate any number of factors in the underlying specification for asset price dynamics. The capital allocation calculations can be simplified if a portfolio is well-diversified and asset values are driven by a single common factor in addition to individual idiosyncratic factors.

Let dW_M represents a standard Wiener process common in all asset price dynamics, and dW_i represents an independent standard Weiner process idiosyncratic to the price dynamics of asset i. Assume that asset price dynamics are given by,

$$dA = \mu A dt + \sigma_M A\, dW_M + \sigma_i A\, dW_i, \tag{15}$$

$$dW_i\, dW_j = \rho_{ij} = 0, \quad \forall i, j.$$
$$dW_i\, dW_M = \rho_{im} = 0, \forall i.$$

Under these dynamics, asset prices are log normally distributed,

$$\tilde{A}_{iT} = A_{i0}\, e^{\left[r_f + \lambda \sigma_M - \frac{1}{2}\left(\sigma_M^2 + \sigma_i^2\right)\right]T + \left(\sigma_M \tilde{z}_M + \sigma_i \tilde{z}_i\right)\sqrt{T}}, \tag{16}$$

and $\tilde{z}_M$ and $\tilde{z}_i$ are independent standard normal random variables. Under the equivalent martingale change of measure, asset values at time T are distributed,

$$\tilde{A}_{iT}^{\eta} = A_{i0}\, e^{\left[r_f - \frac{1}{2}\left(\sigma_M^2 + \sigma_i^2\right)\right]T + \left(\sigma_M \tilde{z}_M + \sigma_i \tilde{z}_i\right)\sqrt{T}}. \tag{17}$$

Correlations between geometric asset returns are,

$$Corr\left[\frac{1}{T}\ln\left(\frac{\tilde{A}_{it}}{A_{i0}}\right), \frac{1}{T}\ln\left(\frac{\tilde{A}_{jt}}{A_{j0}}\right)\right] = \frac{\sigma_M^2}{\left(\sigma_M^2 + {\sigma_i}^2\right)^{\frac{1}{2}}\left(\sigma_M^2 + {\sigma_j}^2\right)^{\frac{1}{2}}}, \forall i, j. \tag{18}$$

If the model is further specialized so that the volatilities of assets' idiosyncratic factors are assumed identical, $\sigma_i = \sigma_j = \overline{\sigma}, \forall i, j,$ the pair-wise asset return correlations are,

$$\rho = Corr\left[\frac{1}{T}\ln\left(\frac{\tilde{A}_{it}}{A_{i0}}\right), \frac{1}{T}\ln\left(\frac{\tilde{A}_{jt}}{A_{j0}}\right)\right] = \frac{\sigma_M^2}{\sigma_M^2 + \overline{\sigma}^2} \quad \forall i, j. \tag{19}$$

Portfolio Invariant Buffer Stock Capital

In the single common factor case, the calculations necessary to estimate a portfolio capital allocation can be simplified using properties of the end-of-period investment portfolio return distribution. The investment return distribution differs according to the capital allocation horizon and the maturity of the credits in the portfolio. The process for setting an unbiased equity capital allocation is conceptually the same regardless of the capital allocation horizon, but we will observe industry practice and treat MTM and HTM as separate cases in part because these horizons have different investment return distributions.

Held-to-Maturity (HTM) Return

The *T*-year rate of return on a BSM risky bond that is held to maturity is,

$$\tilde{M}_{iT} = \frac{1}{B_{i0}}\left(Min\left(\tilde{A}_{iT}, Par_i\right)\right) - 1. \tag{20}$$

For bonds or loans with conventional levels of credit risk, $\tilde{M}_{iT}$ is bounded in the interval $[-1, a]$, where *a* is a finite constant. In most applications, *a* typically is less than 1. When return realizations are in the range, $-1 < M_{iT} < 0$, M_{iT} represents the loss rate on the bond held to maturity. When, $0 < M_{iT} < \frac{Par_i}{B_{i0}} - 1$, the bond has defaulted on its promised payment terms, but the bond has still generates a positive return. A fully performing bond posts a return equal to $\frac{Par_i}{B_{i0}} - 1 < a$ which is finite (typically $a < 1$).

The physical rate of return distribution (20) has an associated equivalent martingale distribution,

$$\tilde{M}^{\eta}_{iT} = \frac{1}{B_{i0}}\left(Min\left(\tilde{A}^{\eta}{}_{iT}, Par_i\right)\right) - 1. \tag{21}$$

By construction, expressions (20) and (21) have identical support.

Asymptotic Portfolio Return Distribution

The *T*-period return on a portfolio of *n* risky individual credits, ${}_P\tilde{M}_T$, is

$${}_P\tilde{M}_T \equiv \frac{\sum_{i=1}^{n} \tilde{M}_{iT} B_{i0}}{\sum_{i=1}^{n} B_{i0}} \tag{22}$$

Conditional on a realization of the common market factor, $\tilde{z}_m = z_M$, the conditional portfolio return, ${}_P\tilde{M}_T \big| \tilde{z}_M = z_M = {}_P\tilde{M}_T \big| z_M$, is,

$${}_P\tilde{M}_T \big| z_M \equiv \frac{\sum_{i=1}^{n} \tilde{M}_{iT} \big| z_M \cdot B_{i0}}{\sum_{i=1}^{n} B_{i0}} \tag{23}$$

Under the single common factor assumption for asset price dynamics, $\Psi\left(\tilde{M}_{iT} \big| z_M\right)$ is independent of $\Psi\left(\tilde{M}_{jT} \big| z_M\right)$ for $\forall\, i \neq j.$[15]

Consider a portfolio composed of equal investments in individual bonds that have identical *ex ante* credit risk profiles. That is, assume that the bonds in the portfolio are identical regarding their par value $\{Par_i = Par_j, \forall\, i, j\}$, maturity $\{T\}$, and volatility

[15] Independence in this non-gaussian setting requires that an observation of the return to bond j be uninformative regarding the conditional distribution function for bond i, $\Pr\left(\tilde{M}_{iT} \mid z_M\right) < a = \Pr(\tilde{M}_{it} \mid z_M) < a$ *given that* $\tilde{M}_{jt} = M_{jt}, \forall a, i \neq j.$ This condition is satisfied under the single common factor model assumption.

characteristics, $\{\sigma_i = \sigma_j = \overline{\sigma}, \forall\, i, j\}$. The ASRF assumptions imply that the bonds will have conditional returns that are independent and identically distributed with a finite mean. As the number of bonds in portfolio, n, grows without bound, the Strong Law of Large Numbers requires,

$$\underset{n\to\infty}{Lim}\left[{}_P\tilde{M}_T \,\middle|\, z_M\right] = \underset{n\to\infty}{Lim}\left[\frac{\sum_{i=1}^{n} \tilde{M}_{iT} \,\middle|\, z_M}{n}\right] \xrightarrow[a.s]{} E\left[\psi\left(\tilde{M}_{iT} \,\middle|\, z_M\right)\right] \tag{24}$$

where $\psi\left(\tilde{M}_{iT} \middle| z_M\right)$ represents the conditional density function of ${}_P\tilde{M}_T \middle| z_M$. The notation *a.s.* indicates "almost sure" convergence (convergence with probability one).

The conditional expected return can be written,

$$\begin{aligned} \underset{n\to\infty}{Lim}\left[{}_P\tilde{M}_T \,\middle|\, z_M\right] = &\left(\frac{Par_i}{B_{i0}}\right)\left[1-\Phi\left(z_{iT}^{*}\left(z_M\right)\right)\right] \\ &+\left(\frac{Q(z_M)}{B_{i0}}\right)\left[1-\Phi\left(-z_{iT}^{*}\left(z_M\right)+\gamma_{iT}\right)\right]-1 \end{aligned} \tag{25}$$

where,

$$\mu_{iT}\left(z_M\right) = \ln\left[A_{i0}\right] + \left[r_f + \lambda\sigma_M - \frac{1}{2}\left(\sigma_M^2 + \sigma_i^2\right)\right]T + z_M\sigma_M\sqrt{T} \tag{26}$$

$$\gamma_{iT} = \sigma_i\sqrt{T} \tag{27}$$

$$z_{iT}^{*}\left(z_M\right) = \frac{\ln[Par_i] - \mu_{iT}\left(z_M\right)}{\gamma_{iT}} \tag{28}$$

$$Q(z_M) = e^{\mu_{iT}\left(z_M\right)+\frac{\gamma_{iT}^2}{2}} \tag{29}$$

The asymptotic limit of the conditional portfolio value (expression (25)) defines the unconditional portfolio return distribution $\Psi\left(\tilde{M}_{iT} \middle| z_M\right)$ as an implicit function of z_M,

$$\Pr\left[{}_P\tilde{M}_T < \begin{pmatrix} \left(\dfrac{Par_i}{B_{i0}}\right)\left[1-\Phi\left(z_{iT}^*(z_M)\right)\right] \\ +\left(\dfrac{Q(z_M)}{B_{i0}}\right)\left[1-\Phi\left(-z_{iT}^*(z_M)+\gamma_{iT}\right)\right] \end{pmatrix} - 1 \right] = \Phi(z_M),\ z_M \in (-\infty,\infty)$$

(30)

The unconditional distribution for the investment portfolio's end-of-period T value is,

$$\Pr\left[\sum_{i=1}^{n} B_{i0}\left({}_P\tilde{M}_T + 1\right) < \left(\frac{\sum_{i=1}^{n} B_{i0}}{B_{i0}} \right) \begin{pmatrix} Par_i\left[1-\Phi\left(z_{iT}^*(z_M)\right)\right] \\ + Q(z_M)\left[1-\Phi\left(-z_{iT}^*(z_M)+\gamma_{iT}\right)\right] \end{pmatrix} \right] \quad (31)$$
$$= \Phi(z_M),\ z_M \in (-\infty,\infty)$$

Similar reasoning may be used to derive the equivalent martingale portfolio return distribution. Using the definitions,

$$\mu_{iT}^{\eta}(z_M) = \ln[A_{i0}] + \left[r_f - \frac{1}{2}\left(\sigma_M^2 + \sigma_i^2\right)\right]T + z_M\sigma_M\sqrt{T} \quad (32)$$

$$z_{iT}^{\eta *}(z_M) = \frac{\ln[Par_i] - \mu^{\eta}{}_{iT}(z_M)}{\gamma_{iT}} \quad (33)$$

$$Q^{\eta}(z_M) = e^{\mu_{iT}^{\eta}(z_M) + \frac{\gamma_{iT}^2}{2}} \quad (34)$$

the T-period equivalent martingale unconditional return distribution, $\Psi\left(\tilde{M}_{iT}^{\eta} \middle| z_M\right)$, can be written,

$$\Pr\left[{}_P\tilde{M}_T^{\eta} < \left(\begin{array}{l} \left(\dfrac{Par_i}{B_{i0}}\right)\left[1-\Phi\left(z_{iT}^{\eta*}(z_M)\right)\right] \\ +\left(\dfrac{Q^{\eta}(z_M)}{B_{i0}}\right)\left[1-\Phi\left(-z_{iT}^{\eta*}(z_M)+\gamma_{iT}\right)\right] \end{array} \right) - 1 \right] = \Phi(z_M), \tag{35}$$

$$z_M \in (-\infty, \infty)$$

The unconditional equivalent martingale distribution for the portfolio's time-*T* value is,

$$\Pr\left[\sum_{i=1}^{n} B_{i0}\left({}_P\tilde{M}_T^{\eta}+1\right) < \left(\left(\frac{\sum_{i=1}^{n} B_{i0}}{B_{i0}} \right) \left(\begin{array}{l} Par_i \left[1-\Phi\left(z_{iT}^{\eta*}(z_M)\right)\right] \\ +Q^{\eta}(z_M)\left[1-\Phi\left(-z_{iT}^{\eta*}(z_M)+\gamma_{iT}\right)\right] \end{array} \right) \right) \right] \tag{36}$$

$$= \Phi(z_M), \; z_M \in (-\infty, \infty)$$

Unbiased Portfolio Buffer Stock Capital for HTM Capital Allocation Horizon

The maximum par value of a funding debt issue that is consistent with a target solvency rate of α is ${}_{HTM}Par_F^P = \Psi^{-1}\left(\sum_{i=1}^{n} B_{i0}\left({}_P\tilde{M}_T+1\right), 1-\alpha\right)$. Under the ASRF BSM model assumptions, for a portfolio of bonds with identical *ex ante* credit risk characteristics, the optimal par value of funding debt can be determined by setting $z_M = \Phi^{-1}(1-\alpha)$ and using expression (30) to solve for the end-of-horizon portfolio critical value,

$${}_{HTM}Par_F^P = \left(\frac{\sum_{i=1}^{n} B_{i0}}{B_{i0}} \right) \left(\begin{array}{l} Par_i \left[1-\Phi\left(z_{iT}^{*}\left(\Phi^{-1}(1-\alpha)\right)\right)\right] \\ +Q(z_M)\left[1-\Phi\left(-z_{iT}^{*}\left(\Phi^{-1}(1-\alpha)\right)+\gamma_{iT}\right)\right] \end{array} \right) \tag{37}$$

To solve for the market value of the funding debt, it is necessary to solve for $(1-\hat{\alpha})$, the probability that the funding debt defaults under the investment portfolio's equivalent martingale distribution,

$$
\begin{aligned}
&(1-\hat{\alpha})\ \text{э}\ {}_{HTM}Par_F^P = \\
&\left(\frac{\sum_{i=1}^{n} B_{i0}}{B_{i0}}\right)\begin{pmatrix} Par_i\ \left[1-\Phi\left(z_{iT}^{\eta *}\left(\Phi^{-1}(1-\hat{\alpha})\right)\right)\right] \\ \qquad + Q^{\eta}(z_M)\ \left[1-\Phi\left(-z_{iT}^{\eta *}\left(\Phi^{-1}(1-\hat{\alpha})\right)+\gamma_{iT}\right)\right]\end{pmatrix}
\end{aligned} \tag{38}
$$

Under the single common factor assumption, the critical value of the equivalent martingale market factor is,

$$
\hat{z}_M = \Phi^{-1}(1-\alpha)+\frac{\lambda}{\sqrt{T}} \tag{39}
$$

and the risk neutral probability that the funding debt will default is,

$$
1-\hat{\alpha} = \Phi(\hat{z}_M) \tag{40}
$$

The initial market value of the funding issue, B_{F0}^P, is the expected value of its discounted (at r_f) terminal cash flows where the expectation is taken with respect to the equivalent martingale measure,

$$
B_{F0}^P = \left(\frac{e^{-r_f T}\sum_{i=1}^{n} B_{i0}}{B_{i0}}\right)\begin{pmatrix} \int_{-\infty}^{\hat{z}_M}\left[Par_i\ \left[1-\Phi\left(z_{iT}^{\eta *}(z_M)\right)\right]\right]\phi(z_M)\,dz_M \\ + \int_{-\infty}^{\hat{z}_M}\left[Q^{\eta}(z_M)\left[1-\Phi\left(-z_{iT}^{\eta *}(z_M)+\gamma_{iT}\right)\right]\right]\phi(z_M)\,dz_M \\ + \left({}_{HTM}\,Par_F^P\right)\left[1-\Phi(\hat{z}_M)\right] \end{pmatrix} \tag{41}
$$

The economic capital allocation required to fund the portfolio under the constraint that the funding debt has at most a default probability of α is, $\sum_{i=1}^{n} B_{i0} - B_{F0}^{P}$. Expressed as a proportion of the portfolio's initial market value, the required economic capital allocation is,

$$1-\left(\frac{e^{-r_f T}}{B_{i0}}\right)\left(\begin{array}{l}\int_{-\infty}^{\hat{z}_M}\left[Par_i\left[1-\Phi\left(z_{iT}^{\eta*}(z_M)\right)\right]\right]\phi(z_M)\,dz_M \\ +\int_{-\infty}^{\hat{z}_M}\left[Q^{\eta}(z_M)\left[1-\Phi\left(-z_{iT}^{\eta*}(z_M)+\gamma_{iT}\right)\right]\right]\phi(z_M)\,dz_M \\ +\left({}_{HTM}\,Par_F^{P}\right)\left[1-\Phi\left(\hat{z}_M\right)\right]\end{array}\right) \tag{42}$$

The expression for an unbiased economic capital allocation in the mark-to-market case is derived in the Appendix.

5. Minimum Capital Requirements Under the Basel II IRB Approaches

The June 2004 formula for calculating A-IRB capital requirements, K, for corporate credits is,[16]

$$K=\left[LGD\times\Phi\left[\frac{1}{\sqrt{1-R}}\Phi^{-1}(PD)+\sqrt{\frac{R}{1-R}}\Phi^{-1}(.999)\right]-PD\times LGD\right]\left(\frac{1+(M-2.5)b}{1-1.5b}\right) \tag{43}$$

where, $R=0.12\left(\frac{1-e^{-50PD}}{1-e^{-50}}\right)+0.24\left(1-\frac{1-e^{-50PD}}{1-e^{-50}}\right)$, $b=\left(0.11852-.05478\,Ln(PD)\right)^2$,

PD is a credit's probability of default expressed as a percentage, LGD is a credit's expected loss given default expressed as a percentage, M is the credit's maturity in measured in years, and K represents the percentage capital requirement per dollar of exposure. If for any credit,

[16] See BCBS, June 2004, page 60.

K<0, regulatory capital requirements are set to zero. F-IRB capital requirements are calculated by using the A-IRB capital requirement formula with LGD set at 45 percent.

6. Implied Bank Default Risk under the Basel II IRB Approaches

This section compares unbiased economic capital allocations prescribed by the BSM model to the minimum capital requirements that are set by the Basle II IRB approaches. Simple inspection indicates that expressions (42) and (43) are not equivalent capital allocation rules. These expressions are complex and the quantitative differences in the assigned capital allocations are not immediately apparent. In the analysis that follows, portfolio capital requirements are calculated using the ASRF BSM model and the Basel IRB approaches for portfolios with a wide range of risk characteristics. The assumptions regarding asset price dynamics that are maintained throughout the analysis appear in Table 1. All individual credits are assumed to have identical firm specific risk factor volatilities of 20 percent. The common factor has a volatility of 10 percent and the market price of risk is set at 10 percent. The risk free rate is 5 percent. The market and firm specific factor volatilities imply an underlying geometric asset return correlation of 20 percent.[17]

In this analysis, all credits in an asymptotic portfolio are assumed to have the same initial value, and all share an identical *ex ante* credit risk profile that is determined by the par value and maturity of the credit. For a given maturity, the par values of individual credits are altered to change the credit risk characteristics of a portfolio.

---

[17] When the bond PDs and LGDs in Table 2 are input into the A-IRB approach, the correlation parameter, R, ranges from 13.6 (par value 70) to 22.7 percent (par value 55).

Table 1: Calibration Assumptions

risk free rate	$r_f = .05$
market price of risk	$\lambda = .10$
market factor volatility	$\sigma_M = .10$
Firm specific volatility	$\overline{\sigma}_i = .20$
initial market value of assets	$A_0 = 100$
correlation between asset returns	$\rho = .20$

Consistent with Basel II requirements, the BSM analysis focuses on a one-year capital allocation horizon. Capital requirements are examined for two alternative investment portfolio maturities—one year (the HTM case, expression (42)), and three years (the MTM case, expression (A19) in the Appendix). In each case, BSM unbiased capital allocations are compared to the minimum capital requirements set by the Basel IRB rules.

Under the ASRF assumptions, the physical probability that a BSM bond defaults is, $PD = \Phi\left(z_i^{df}\right)$ where,

$$z_i^{df} = \left(\frac{Log\left(Par_i\right) - Log\left(A_{i0}\right) - \left(r_f + \lambda\sigma_M - \frac{\sigma_M^2 + \overline{\sigma}_i^2}{2} \right) T}{\sqrt{T}\sqrt{\sigma_M^2 + \overline{\sigma}_i^2}} \right) \tag{44}$$

The expected value of the bond's payoff, given that it defaults is,

$$E\left[Min\left(A_{iT}, Par_i\right) \mid A_{iT} < Par_i\right] = \frac{1}{\Phi\left(z_i^{df}\right)} \int_{-\infty}^{z_i^{df}} A_{i0} e^{\left(r_f + \lambda\sigma_M - \frac{1}{2}\left(\sigma_M^2 + \sigma_i^2\right)\right)T + \sqrt{T\left(\sigma_M^2 + \sigma_i^2\right)}z} \phi(z)dz \,. \qquad (45)$$

A bond's LGD measured from initial market value is,

$$LGD = 1 - \frac{1}{B_{i0}\Phi\left(z_i^{df}\right)} \int_{-\infty}^{z_i^{df}} A_{i0} e^{\left(r_f + \lambda\sigma_M - \frac{1}{2}\left(\sigma_M^2 + \sigma_i^2\right)\right)T + \sqrt{T\left(\sigma_M^2 + \sigma_i^2\right)}z} \phi(z)dz \,. \qquad (46)$$

This LGD measure does not discount the expected value of the terminal payoff in default.

The analysis includes 16 portfolios of one- and three-year credits. The credit risk characteristics of the one-year maturity portfolios are reported in Table 2. Individual credit PDs range from 23 basis points—for a bond with par values of 55, to 3.99 percent for a bond with a par value of 70. The LGD characteristics (measured from initial market value) range from 1.40 percent to 3.28 percent. Low LGDs (high recovery rates) are a signature characteristic of the BSM model and the LGDs of the bonds examined in this analysis are modest relative to the observed default loss history on corporate bonds.[18] The A-IRB capital allocation rule explicitly accounts for loss given default, so *a priori*, there is no reason to expect that any specific set of loss given default values may compromise the performance of the A-IRB approach.[19]

[18] Some industry credit risk models include a stochastic default barrier such as in the Black and Cox (1976) model to increase the LGD relative to a basic BSM model and thereby improve correspondence with observed market data.

[19] Paragraph 407 of the Basel Committee on Banking Supervision (2004) discusses the minimum requirements for A-IRB bank LGD treatment. Basel II guidelines do not impose a lower bound on the LGDs that banks can use in the A-IRB approach.

The results for one-year bond portfolios are reported in Table 3 and plotted in Figure 1. The results show that capital requirements generated under the A-IRB capital rule are far smaller than the capital needed to achieve the regulatory target default rate of 0.1 percent. The true capital needed to achieve the 99.9 percent target solvency rate is almost 5 times larger than those set by the A-IRB Approach.

Table 2: Credit Risk Characteristics of 1-Year Credits

par value	initial market value	probability of default in percent	expected value given default	in percent: loss given default from initial value	loss given default from par value	yield to maturity
55	52.31	0.23	51.58	1.40	6.22	5.142
56	53.26	0.30	52.45	1.53	6.35	5.145
57	54.2	0.38	53.31	1.64	6.47	5.166
58	55.15	0.48	54.17	1.78	6.60	5.168
59	56.1	0.59	55.03	1.91	6.73	5.169
60	57.04	0.73	55.88	2.03	6.87	5.189
61	57.98	0.90	56.73	2.16	7.00	5.209
62	58.92	1.09	57.57	2.29	7.14	5.227
63	59.86	1.31	58.41	2.42	7.28	5.246
64	60.8	1.57	59.25	2.55	7.43	5.263
65	61.73	1.86	60.08	2.68	7.57	5.297
66	62.66	2.20	60.90	2.80	7.72	5.330
67	63.59	2.57	61.73	2.93	7.87	5.362
68	64.51	3.00	62.54	3.05	8.03	5.410
69	65.43	3.47	63.35	3.17	8.18	5.456
70	66.34	3.99	64.16	3.28	8.34	5.517

The BSM capital allocation rule can be inverted to recover the A-IRB approach's implied probability of default and the final column of Table 3 reports these values as solvency margin estimates. On one-year credits, the solvency margin set by the A-IRB rule depends on the risk attributes of the credit. For the credits examined in this analysis, the A-IRB solvency margins range from 97.5 percent to 93.4 percent. Solvency margins decline as credit risk increases and portfolios with only modest credit risk have solvency margins less than 95 percent.

No published study has documented the accuracy of the June 2004 Basel II IRB model calibrations or estimated the bank solvency standard that they engender. Notwithstanding the BCBS's goal of creating a prudential standard consistent with a 0.1 percent bank default rate, the Basel II Quantitative Impact Studies (QIS) and subsequent IRB model calibration adjustments have not focused on producing an IRB calibration consistent with any specific target solvency margin. Rather, QIS results have been reflected in updated IRB calibrations that have been designed to create incentives to promote A-IRB adoption, to achieve a measure of intuitive consistency among IRB credit classes (corporate, retail, etc) and to accomplish the aforementioned goals without creating a set of capital rules that will materially alter the regulatory capital requirements of an "average" internationally active bank. The analysis in this paper suggests that the protracted Basel II development process has produced IRB calibrations that are heavily skewed toward capital relief for A-IRB banks.[20]

[20] Some may object because the analysis ignores operational risk capital requirements. It should be noted that, by construction, there is no operational risk in this calibration exercise. Operational risk capital is not a buffer to compensate for poorly designed credit risk regulatory capital rule.

Table 3: Alternative Capital Allocation Recommendations

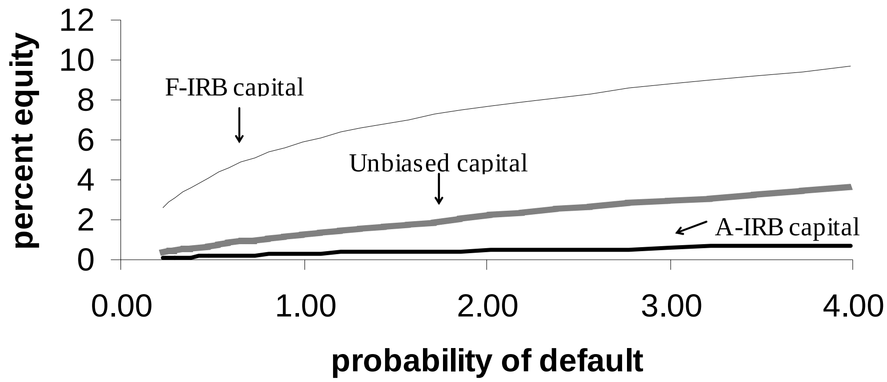
		capital requirements in percent of initial value for 1-year maturity credits			
par value	probability of default in percent	99.9 percent unbiased BSM portfolio capital	June 2004 Foundation IRB capital requirement	June 2004 Advanced IRB capital requirement	approximate solvency margin under A-IRB
55	0.23	0.396	2.630	0.082	97.5
56	0.30	0.487	3.106	0.105	97.5
57	0.38	0.593	3.576	0.130	97.4
58	0.48	0.715	4.081	0.160	97.4
59	0.59	0.854	4.557	0.194	97.3
60	0.73	1.011	5.071	0.230	97.1
61	0.90	1.187	5.595	0.268	96.9
62	1.09	1.384	6.083	0.309	96.7
63	1.31	1.601	6.557	0.353	96.4
64	1.57	1.839	7.027	0.399	96.1
65	1.86	2.098	7.470	0.444	95.7
66	2.20	2.379	7.917	0.493	95.3
67	2.57	2.681	8.343	0.544	94.9
68	3.00	3.005	8.788	0.595	94.4
69	3.47	3.348	9.236	0.651	93.9
70	3.99	3.712	9.702	0.708	93.4

The implicit default rates that are set under the A-IRB approach are approximately equal to the failure rate experienced by U. S. savings and loan institutions during the height of the 1980s S&L crisis. In 1988, the failure rate among insured savings and loans was 6.4 percent.[21] Over the 1980-1994 period, the annual compound average default rate of banks insured by the Federal Deposit Insurance Corporation was less than 1.2 percent, and even in

[21] "History of the Eighties—Lessons for the Future," p. 168.

the worst year (1988) of a period that has been characterized as a "banking crisis," the default rate on FDIC insured banks never exceed 2 percent.[22]

Figure 1: June 2004 IRB Capital Requirements



The Foundation IRB approach uses the A-IRB capital allocation rule (expression (43)) assuming that LGD is 45 percent for all credits. As Figure 1 shows, this LGD assumption dramatically increases capital requirements over the A-IRB Approach. For the portfolios examined in this analysis, the F-IRB will set capital requirements that are many times larger than are needed to achieve the regulatory target default rate. Other things equal, these F-IRB banks will face a maximum default rate that is far less than the 0.1 percent regulatory target.

The economic capital allocation comparison is repeated for longer maturity investment portfolios. BSM economic capital allocations on 3-year investment portfolios are calculated using the MTM BSM capital allocation rule derived in the Appendix (expression

[22] See , "History of the Eighties—Lessons for the Future," p. 479.

(A19)). The credit risk characteristics of the individual portfolio credits are reported in Table 4. Note that for many of these credits, expected values given default exceed the initial market value of the credit. If, in practice, banks are allowed to use these implied LGD values in the A-IRB approach, then portfolios for which $LGD \leq 0$ will not require any regulatory capital.

Table 5 reports alternative capital recommendations for portfolios of 3-year bonds. The results show that F-IRB capital requirements will be set much higher than is needed to achieve the regulatory target default rate. For all portfolios considered, the F-IRB approach will require substantially more capital on long-maturity investment portfolios than would be required by the BSM economic capital allocation rule. For high quality (low credit risk) portfolios, the F-IRB capital requirements are almost 5 times larger than unbiased capital requirements; for lower quality credits, F-IRB capital is about 3 times the amount needed to achieve the target solvency margin. For all the credits examined, the F-IRB Approach requires more than the 8 percent capital that is required under the 1988 Basel Accord. The final column of Table 5 reports estimates of the solvency margin that is set by the A-IRB approach. When the A-IRB capital requirement is 0, the solvency margin estimates (to 5 decimal places) represent the probability of fully performing on the 1-year funding debt, when the entire portfolio initial market value is financed with debt.[23] The results indicate that the probability of default allowed under the A-IRB will exceed 60 percent for long-

[23] Conceptually, a portfolio of 3-year bonds is purchased and funded entirely with debt. After a year, the portfolio is sold at fair market value. The solvency margin is an estimate of the probability that the proceeds will fully discharge the principle and interest on the funding debt issue. The implied default rate is 1 minus the solvency margin.

maturity credit portfolios. The results call into question the prudential benefit associated with the A-IRB maturity adjustment factor.

Discussion

The use of the BSM model as a benchmark of comparison merits discussion because of the model's well-known empirical shortcomings. Econometric studies suggest that, on average, the BSM model overprices corporate bonds (underestimates required bond yields). Empirical evidence indicates that the BSM bias is related to maturity and credit quality.[24] BSM overpricing errors are more severe on short-term high quality credits. In the context of this capital calibration exercise, the observed pattern of bias implies that the BSM portfolio capital allocation model will understate the true amount of capital that is required to support a credit risky portfolio under a given solvency margin constraint because the bank's funding debt issue is likely to be significantly overpriced by the BSM model. Recognizing the shortcomings of the BSM model, true economic capital allocations are likely larger than the estimates in this paper suggest and true A-IRB capital shortfalls are likely more severe than indicated.

The calibration analysis shows that there are serious shortcomings in the June 2004 IRB model calibrations. A-IRB banks are granted substantial regulatory capital relief on all the portfolios that have been examined. While the BCBS had intended to include capital benefits to encourage banks to migrate to the A-IRB approach, the incentives that have been provided would substantially reduce prudential standards even for a bank that is perfectly

[24] See for example, Jones, Mason, and Rosenfeld (1984), Ogden (1987), or Eom, Helwege and Huang (2004).

diversified. Bank default rates under A-IRB capital requirements may exceed 5 percent for a bank holding an asymptotically diversified portfolio. In contrast, the F-IRB Approach overcapitalizes portfolios, and for long-maturity credits, the F-IRB requires capital in excess of 1988 Basel Accord.

The substantial capital relief granted under the A-IRB Approach will likely encourage banks to petition their supervisors for A-IRB approval provided the fixed costs associated with A-IRB adoption are not prohibitive for smaller banks. Absent liberal regulatory approval policies or large A-IRB fixed costs, strong economic incentives are in place to encourage industry consolidation into institutions that gain A-IRB regulatory approval. The calibration analysis suggests that the migration of assets into A-IRB banks could substantially increase systemic risk in the financial system as banks that fully meet A-IRB minimum capital requirements may have unacceptably high default rates (in excess of 5 percent) unless PCA minimum leverage regulations prohibit realization of the full capital relief granted by the A-IRB Approach.

Table 4: Credit Risk Characteristics of Individual 3-year Credits

par value	initial market value	probability of default in percent	expected value given default	loss given default from initial value in percent	loss given default from par value in percent	yield to maturity in percent
55	47.07	3.48	47.61	-1.14	13.44	5.33
56	47.89	3.85	48.37	-1.00	13.62	5.35
57	48.71	4.25	49.13	-0.86	13.80	5.38
58	49.53	4.67	49.89	-0.73	13.98	5.40
59	50.34	5.12	50.64	-0.60	14.16	5.43
60	51.15	5.59	51.39	-0.48	14.35	5.46
61	51.95	6.09	52.14	-0.36	14.53	5.50
62	52.75	6.61	52.88	-0.25	14.71	5.53
63	53.54	7.16	53.61	-0.14	14.90	5.57
64	54.33	7.73	54.35	-0.04	15.09	5.61
65	55.11	8.33	55.07	0.06	15.27	5.66
66	55.88	8.95	55.80	0.15	15.46	5.70
67	56.65	9.59	56.52	0.24	15.65	5.75
68	57.42	10.26	57.23	0.32	15.84	5.80
69	58.17	10.95	57.94	0.40	16.03	5.86
70	58.92	11.66	58.65	0.47	16.22	5.91

7. Conclusions

Compared to unbiased capital requirements for an ASRF BSM portfolio, the Basel II A-IRB approach substantially understates the capital that is required to achieve the regulatory target of 99.9 percent bank solvency rate. Estimates suggest that banks with default rates in excess of 5 percent could meet the minimum risk-based regulatory capital requirements promulgated by the June 2004 the A-IRB Approach.

Table 5: Alternative Capital Allocations for Long-Term Credits

		capital requirements in percent of initial value for 3-year maturity credits			
par value	probability of default in percent	99.9 percent unbiased BSM portfolio capital	Foundation IRB capital requirement	Advanced IRB capital requirement*	approximate solvency margin under A-IRB
55	3.48	2.111	9.894	0	2.54
56	3.85	2.294	10.360	0	2.32
57	4.25	2.483	10.715	0	2.11
58	4.67	2.681	11.077	0	1.95
59	5.12	2.885	11.456	0	1.79
60	5.59	3.096	11.841	0	1.65
61	6.09	3.315	12.239	0	1.52
62	6.61	3.539	12.642	0	1.41
63	7.16	3.769	13.055	0	1.30
64	7.73	4.005	13.467	0	1.21
65	8.33	4.246	13.883	0.019	13.84
66	8.95	4.493	14.294	0.049	23.17
67	9.59	4.743	14.697	0.079	29.00
68	10.26	4.999	15.097	0.108	33.09
69	10.95	5.258	15.485	0.137	36.04
70	11.66	5.521	15.861	0.164	38.23

* When loss given default from initial market value is negative, A-IRB regulatory capital is 0.

The calibration results reported in this paper suggest that, as the IRB alternatives are currently calibrated, Basel II will result in alternative regulatory capital regimes which promulgate markedly different prudential standards. Under these regulatory alternatives, banks that adopt the A-IRB approach will gain substantial regulatory capital relief without a commensurate reduction in their potential risk profile. In contrast, F-IRB and general banks will face a much stricter prudential standard. As it is currently calibrated, the Basel II system will create strong economic incentives for banking system assets to migrate into A-IRB banks. Since the analysis suggests that A-IRB banks potentially carry higher default risk absent safety net support, the migration of banking system assets toward A-IRB regulatory capital treatment is unlikely to enhance financial stability. Given the prudential weaknesses associated with the A-IRB approach, the adoption of Basel II in its current form will not promote better risk management practices in banks or reduce systemic risk in the international banking system.

Appendix

Proposition 1

Should a bank set its equity capital equal to a $VaR(\alpha)$ *measure of its investment portfolio's future potential value, when interest rates are positive and investors require positive compensation for credit risk, other things held constant, the probability that the bank will be insolvent at the end of the capital allocation horizon is greater than* $(1-\alpha)$.

Proof: Let the amount that the bank must repay on its funding debt be represented by $A_0 - VaR^{A_0}(.999) + \zeta = \Psi^{-1}\left(\tilde{A}_T, .001\right) + \zeta$, for some $\zeta > 0$. Assuming that $\Psi^{-1}(\tilde{A}_T, 1-\alpha)$ is continuous and monotonically decreasing in $\alpha \in [0,1]$, there exists a unique $0 < \hat{\alpha} < .999$, such that $\Psi^{-1}\left(\tilde{A}_T, 1-\hat{\alpha}\right) = \Psi^{-1}\left(\tilde{A}_T, .001\right) + \zeta$. Thus, $\hat{\alpha} > .001$, which implies that the probability that the time T value of investment asset is insufficient to discharge the principal and interest on the funding debt exceeds 0.1 percent.

MTM Capital Allocation under the Asymptotic Single Common Factor Model

The *T*-year return on an *M*-year BSM risky bond, with $T < M$ (a mark-to-market (MTM) return) is,

$$\tilde{U}_{iT} = \left(\frac{Par_i}{B_{i0}} e^{-r_f(M-T)} - \frac{Put\left(\tilde{A}_{iT}, Par_i \sigma, M-T\right)}{B_{i0}} \right) - 1 . \tag{A1}$$

For bonds or loans with conventional levels of credit risk, $\tilde{U}_{iT}$ is bounded in the interval $[-1, a]$, where a is a finite constant. Unlike the HTM return case, there is no return interpretation regarding default in this holding period because a BSM bond is a discount instrument that can only default at maturity.

Conditional on a realization of the market factor, $\tilde{z}_M = z_M$, the individual MTM bond returns are independent. If the bonds in the portfolio are identical regarding: par value $\{Par_i = Par_j, \forall\, i, j\}$, maturity T, and volatility characteristics, as the number of bonds in portfolio, n, grows without bound, the Strong Law of Large Numbers requires,

$$\lim_{n\to\infty}\left[{}_P\tilde{U}_T \mid z_M\right] = \lim_{n\to\infty}\left[\frac{\sum_{i=1}^{n}\tilde{U}_{iT}\,\big|\, z_M}{n}\right] \xrightarrow[a.s]{} E\left[\psi\left(\tilde{U}_{iT}\,\big|\, z_M\right)\right] \tag{A2}$$

Unlike the HTM case, the conditional MTM expected bond return does not have a closed form solution. The conditional expected value can be expressed as

$$\lim_{n\to\infty}\left[{}_P\tilde{U}_T \mid z_M\right] = \left(\frac{Par_i}{B_{i0}}e^{-r_f\,(M-T)} - \frac{\int_{-\infty}^{\infty} Put(\tilde{A}_{iT} \mid z_M, Par_i, \sigma, M-T)\,\phi(z_i)\,dz_i}{B_{i0}}\right) - 1 \tag{A3}$$

where,

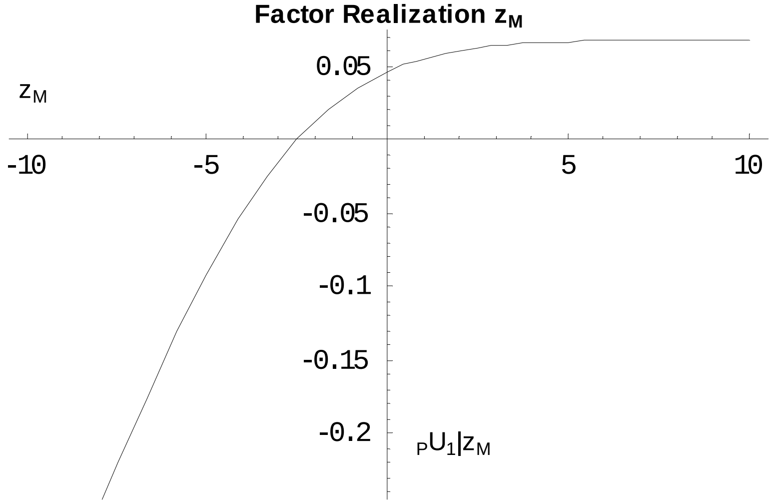
$$\left(\tilde{A}_{iT} \mid z_M\right) = A_{i0}\; e^{\left[r_f + \lambda\sigma_M - \frac{1}{2}\left(\sigma_M^2 + \sigma_i^2\right)\right]T + \sigma_M z_M \sqrt{T} + \sigma_i \tilde{z}_i \sqrt{T}} \tag{A4}$$

$$\tilde{d}_{1T}(z_M) = \frac{\ln\left(\frac{(A_{iT} \mid z_M)}{Par_i}\right) + \left(r_f + \frac{\sigma_M^2 + \sigma_i^2}{2}\right)(M-T)}{\sqrt{\sigma_i^2 + \sigma_M^2}\,\sqrt{M-T}} \tag{A5}$$

$$\tilde{d}_{2T}(z_M) = \tilde{d}_{1T}(z_M) - \sqrt{\sigma_M^2 + \sigma_i^2}\,\sqrt{M-T} \tag{A6}$$

$$\begin{aligned} &Put(\tilde{A}_{iT} \mid z_M, Par_i, \sigma, M-T) \\ &\qquad = Par_i\, e^{-r_f\,(M-T)}\, \Phi(-\tilde{d}_{2T}(z_M)) - (\tilde{A}_{it} \mid z_M)\, \Phi(-\tilde{d}_{1T}(z_M)) \end{aligned} \tag{A7}$$

Figure A1: Portfolio Expected Return Conditional on Market Factor Realization z_M



One year conditional expected return on a portfolio of 3-year bonds with par values of 65. Assets supporting the bonds have initial values of 100, market volatility of .10, specific risk volatility of .20, and a market price of risk of .10.

The conditional expected return must be evaluated numerically. Figure A1 plots the 1-year conditional expected portfolio return for a particular illustrative example of an asymptotic bond portfolio composed of 3-year bonds with par values of 65. These bonds have underlying asset dynamics consistent with Table 1.

Using the expression for the MTM asymptotic conditional expected return, the unconditional asymptotic portfolio return distribution can be written,

$$\Pr\left[{}_P\tilde{U}_T < \left(\frac{Par_i}{B_{i0}} e^{-r_f\,(M-T)} - \frac{1}{B_{i0}}\left(\int_{-\infty}^{\infty} Put(\tilde{A}_{iT} \mid z_M, Par_i, \sigma, M-T)\,\phi(z_i)\,dz_i\right)\right) - 1\right] = \Phi(z_M),\ z_M \in (-\infty, \infty) \qquad \text{(A8)}$$

The asymptotic portfolio's end-of-period value is distributed as,

$$\Pr\left[\sum_{i=1}^{n} B_{i0}\left({}_P\tilde{U}_T+1\right)<\sum_{i=1}^{n} B_{i0}\left(\frac{Par_i}{B_{i0}}e^{-r_f(M-T)}-\frac{1}{B_{i0}}\left(\int_{-\infty}^{\infty} Put(\tilde{A}_{iT} \mid z_M, Par_i, \sigma, M-T)\,\phi(z_i)dz_i\right)\right)\right] \quad \text{(A9)}$$

$$=\Phi(z_M),\ z_M \in (-\infty,\infty)$$

The equivalent martingale MTM return distribution can be expressed as,

$$\lim_{n\to\infty}\left[{}_P\tilde{U}_T^{\eta} \mid z_M\right]=\left(\frac{Par_i}{B_{i0}}\,e^{-r_f\,(M-T)}-\frac{\int_{-\infty}^{\infty} Put(\tilde{A}_{iT}^{\eta} \mid z_M, par, \sigma, M-T)\,\phi(z_i)dz_i}{B_{i0}}\right)-1 \quad \text{(A10)}$$

where,

$$\left(\tilde{A}_{iT}^{\eta} \mid z_M\right)=A_{i0}\ e^{\left[r_f-\frac{1}{2}\left(\sigma_M^2+\sigma_i^2\right)\right]T+\sigma_M z_M\sqrt{T}+\sigma_i \tilde{z}\sqrt{T}} \quad \text{(A11)}$$

$$\tilde{d}_{1T}(z_M)=\frac{\ln\left(\frac{\left(A_{iT}^{\eta} \mid z_M\right)}{Par_i}\right)+\left(r_f+\frac{\sigma_M^2+\sigma_i^2}{2}\right)(M-T)}{\sqrt{\sigma_i^2+\sigma_M^2}\sqrt{M-T}} \quad \text{(A12)}$$

$$\tilde{d}_{2T}(z_M)=\tilde{d}_{1T}(z_M)-\sqrt{\sigma_M^2+\sigma_i^2}\,\sqrt{M-T} \quad \text{(A13)}$$

$$\begin{aligned}&Put(\tilde{A}_{iT}^{\eta} \mid z_M, Par_i, \sigma, M-T)\\ &\quad= Par_i\ e^{-r_f(M-T)}\,\Phi(-\tilde{d}_{2T}(z_M))-(\tilde{A}_{iT}^{\eta} \mid z_M)\,\Phi(-\tilde{d}_{1T}(z_M))\end{aligned} \quad \text{(A14)}$$

Using expression (A10), the end-of-period unconditional equivalent martingale portfolio value distribution can be written,

$$\Pr\left[\sum_{i=1}^{n} B_{i0}\left({}_P\tilde{U}_T^{\eta}+1\right)<\left(\sum_{i=1}^{n} B_{i0}\left(\begin{aligned}&\frac{Par_i}{B_{i0}}\,e^{-r_f\,(M-T)}\\ &-\frac{1}{B_{i0}}\left(\int_{-\infty}^{\infty} Put(\tilde{A}_{iT}^{\eta} \mid z_M, Par_i, \sigma, M-T)\,\phi(z_i)dz_i\right)\end{aligned}\right)\right)\right]=\Phi(z_M)$$

$$z_M \in (-\infty,\infty) \quad \text{(A15)}$$

The optimal par value of funding debt, ${}_{MTM}\,Par_F^P$ by setting $z_M = \Phi^{-1}(1-\alpha)$ and using expression (A9) to solve for the end-of-horizon portfolio critical value,

$$
{}_{MYM}\,Par_F^P = \sum_{i=1}^{n} B_{i0} \left(\begin{array}{l} \dfrac{Par_i}{B_{i0}} e^{-r_f\,(M-T)} \\ -\dfrac{1}{B_{i0}} \left(\displaystyle\int_{-\infty}^{\infty} Put\left[\left(\tilde{A}_{iT} \mid z_M = \Phi^{-1}(1-\alpha)\right), Par_i, \sigma, M-T\right] \phi(z_i) dz_i \right) \end{array} \right)
$$
(A16)

The initial market value of the funding debt is determined by first solving for the equivalent martingale solvency margin on the funding debt, $\hat{\alpha}$,

$$
\hat{\alpha} \ni {}_{MYM}Par_F^P = \sum_{i=1}^{n} B_{i0} \left(\begin{array}{l} \dfrac{Par_i}{B_{i0}} e^{-r_f\,(M-T)} \\ -\dfrac{1}{B_{i0}} \left(\displaystyle\int_{-\infty}^{\infty} Put\left[\left(\tilde{A}_{iT}^{\eta} \mid z_M = \Phi^{-1}(1-\hat{\alpha})\right), Par_i, \sigma, M-T\right] \phi(z_i) dz_i \right) \end{array} \right)
$$
(A17)

The critical value of the equivalent martingale market factor is, $\hat{z}_M = \Phi^{-1}(1-\alpha) + \dfrac{\lambda}{\sqrt{T}}$

and the risk neutral probability of funding debt default is, $1-\hat{\alpha} = \Phi(\hat{z}_M)$

Using $\hat{\alpha}$ to determine the limit of integration when taking the expected discounted value of the funding debt's maturity cash flows under the equivalent martingale measure, the initial equilibrium market value of the funding debt, ${}_{MTM}\,B_{F0}^P$, is,

$$
{}_{MTM}\,B_{F0}^{P} = e^{-r_f T}\left({}_{MYM}\,Par_F^P\right)\hat{\alpha} + e^{-r_f M}\left(\frac{Par_i \sum_{i=1}^{n} B_{i0}}{B_{i0}}\right)(1-\hat{\alpha})
$$

$$
- e^{-r_f T}\left(\frac{\sum_{i=1}^{n} B_{i0}}{B_{i0}}\right)\int_{-\infty}^{\Phi^{-1}\left(1-\hat{\hat{\alpha}}\right)}\left(\int_{-\infty}^{\infty} Put\left[\left(\tilde{A}_{iT}^{\eta} \mid z_M\right), Par_i, \sigma, M-T\right]\phi(z_i)\,dz_i\right)\phi(z_M)\,dz_M \tag{A18}
$$

Expressed as a proportion of the portfolio's initial market value, an MTM portfolio capital allocation consistent with a target solvency margin of α is,

$$
\frac{\sum_{i=1}^{n} B_{i0} - {}_{MTM}\,B_{F0}^{P}}{\sum_{i=1}^{n} B_{i0}} =
$$

$$
1 - \frac{1}{B_{i0}}\left[\begin{array}{l} e^{-r_f M} Par_i - \hat{\alpha}\, e^{-r_f T}\left(\int_{-\infty}^{\infty} Put\left[\left(\tilde{A}_{iT} \mid z_M = \Phi^{-1}(1-\alpha)\right), Par_i, \sigma, M-T\right]\phi(z_i)\,dz_i\right) \\ - e^{r_f T}\int_{-\infty}^{\hat{z}_M}\left(\int_{-\infty}^{\infty} Put\left[\left(\tilde{A}_{iT}^{\eta} \mid z_M\right), Par_i, \sigma, M-T\right]\phi(z_i)\,dz_i\right)\phi(z_M)\,d \end{array}\right] \tag{A19}
$$

References

Basel Committee on Banking Supervision (2004). *Basel II: international Convergence of Capital Measurement and Capital Standards: A Revised Framework.* Bank for International Settlements, June. Available at www.bis.org.

Basel Committee on Banking Supervision (2002). *Overview Paper for the Impact Study,* Bank for International Settlements, October. Available at www.bis.org

Basel Committee on Banking Supervision (2001). *The Internal Ratings-Based Approach Consultative Document.* Bank for International Settlements, May. Available at www.bis.org.

Black, F. and J.C. Cox (1976). "Valuing Corporate Securities: Some Effects of Bond Indenture Provisions," *Journal of Finance,* Vol. 31, pp. 351-367.

Black, F. and M. Scholes (1973). "The pricing of options and corporate liabilities," *The Journal of Political Economy,* Vol. 81, pp. 637-54.

Eom, Young, Jean Helwege and Jing-zhi Huang, (2004). "Structural Models of Corporate Bond Pricing: An Empirical Analysis," *Review of Financial Studies*, Vol. 17, pp. 499-544.

Gordy, Michael (2003). "A risk-factor model foundation for ratings-based bank capital rules," *Journal of Financial Intermediation*, Vol. 12, pp. 199-232.

Finger, Chris (1999). "Conditional approaches for CreditMetrics portfolio distributions," CreditMetrics Monitor, pp. 14-33.

Jones, E.P., S. Mason, and E. Rosenfeld (1984). "Contingent Claims Analysis of Corporate Capital Structures: An Empirical Investigation," Journal of Finance, Vol .39, pp. 611-625.

History of the Eighties—Lessons for the Future. Volume I: An Examination of the Banking Crisis of the 1980s and Early 1990s. Washington D.C.: Federal Deposit Insurance Corporation, 1997.

Kupiec, Paul (2004). "Estimating economic capital allocations for market and credit risks," *Journal of Risk,* Vol. 6, No. 4.

Kupiec, Paul (1999). "Risk Capital and VAR," *Journal of Derivatives,* Vol. 7, No. 2, pp. 41-52.

Merton, Robert (1974). "On the pricing of corporate debt: The risk structure of interest rates," *The Journal of Finance,* Vol. 29, pp. 449-70.

Ogden, J. (1987). "Determinants of the Ratings and Yields on Corporate Bonds: Tests of the Contingent Claims Model," *The Journal of Financial Research,* Vol. 10, pp. 329-339.

Schönbucher, P. (2000). “Factor Models for Portfolio Credit Risk,” memo, Department of Statistics, Bonn University.

Vasicek, O.A., (1991). “Limiting loan loss probability distribution,” KMV Corporation, working paper.